Chapter 1 • Introduction

1.1 A gas at 20°C may be *rarefied* if it contains less than 10^{12} molecules per mm^3. If Avogadro's number is 6.023E23 molecules per mole, what air pressure does this represent?

Solution: The mass of one molecule of air may be computed as

$$\text{m} = \frac{\text{Molecular weight}}{\text{Avogadro's number}} = \frac{28.97\ \text{mol}^{-1}}{6.023\text{E}23\ \text{molecules/g}\cdot\text{mol}} = 4.81\text{E–}23\ \text{g}$$

Then the density of air containing 10^{12} molecules per mm^3 is, in SI units,

$$\rho = \left(10^{12}\ \frac{\text{molecules}}{\text{mm}^3}\right)\left(4.81\text{E–}23\ \frac{\text{g}}{\text{molecule}}\right)$$

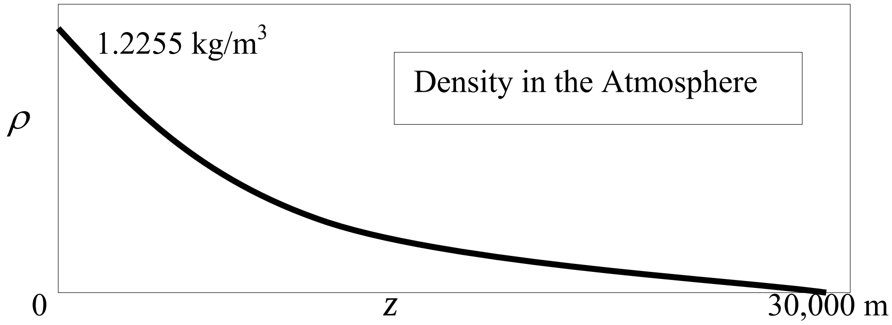
$$= 4.81\text{E–}11\ \frac{\text{g}}{\text{mm}^3} = 4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}$$

Finally, from the perfect gas law, Eq. (1.13), at 20°C = 293 K, we obtain the pressure:

$$\text{p} = \rho\text{RT} = \left(4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}\right)\left(287\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}\right)(293\ \text{K}) = \mathbf{4.0\ Pa} \quad \textit{Ans.}$$

P1.2 Table A.6 lists the density of the standard atmosphere as a function of altitude. Use these values to estimate, crudely, say, within a factor of 2, the number of molecules of air in the entire atmosphere of the earth.

Solution: Make a plot of density ρ versus altitude z in the atmosphere, from Table A.6:



This writer's approximation: The curve is approximately an exponential, $\rho \approx \rho_o \exp(-b\,z)$, with b approximately equal to 0.00011 per meter. Integrate this over the entire atmosphere, with the radius of the earth equal to 6377 km:

$$m_{atmosphere} = \int \rho\, d(vol) \approx \int_0^{\infty} [\rho_o\, e^{-b\,z}](4\pi R_{earth}^2\, dz) =$$

$$= \frac{\rho_o\, 4\pi R_{earth}^2}{b} = \frac{(1.2255\, kg/m^3)4\pi(6.377E6\, m)^2}{0.00011/m} \approx 5.7E18\ kg$$

Dividing by the mass of one molecule ≈ 4.8E−23 g (see Prob. 1.1 above), we obtain the total number of molecules in the earth's atmosphere:

$$N_{\text{molecules}} = \frac{\text{m(atmosphere)}}{\text{m(one molecule)}} = \frac{\text{5.7E21 grams}}{\text{4.8E−23 gm/molecule}} \approx \mathbf{1.2E44}\ \textit{molecules} \quad \textit{Ans.}$$

This estimate, though crude, is within 10 per cent of the exact mass of the atmosphere.

1.3 For the triangular element in Fig. P1.3, show that a tilted free liquid surface, in contact with an atmosphere at pressure pa, must undergo shear stress and hence begin to flow.

Solution: Assume zero shear. Due to element weight, the pressure along the lower and right sides must vary linearly as shown, to a higher value at point C. Vertical forces are presumably in balance with element weight included. But horizontal forces are out of balance, with the unbalanced force being to the left, due to the shaded excess-pressure triangle on the right side BC. Thus hydrostatic pressures cannot keep the element in balance, and shear and flow result.

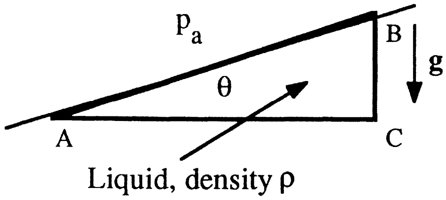
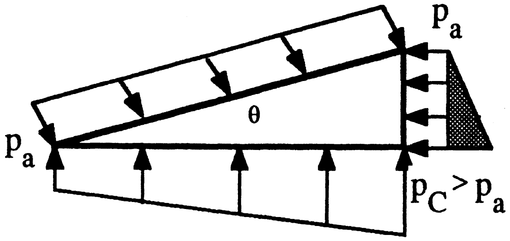


Fig. P1.3



P1.4 Sand, and other granular materials, definitely *flow*, that is, you can pour them from a container or a hopper. There are whole textbooks on the “transport” of granular materials [54]. Therefore, is sand a *fluid*? Explain.

Solution: Granular materials do indeed *flow*, at a rate that can be measured by “flowmeters”. But they are *not* true fluids, because they can support a small shear stress without flowing. They may rest at a finite angle without flowing, which is not possible for liquids (see Prob. P1.3). The maximum such angle, above which sand begins to flow, is called the *angle of repose*. A familiar example is sugar, which pours easily but forms a significant angle of repose on a heaping spoonful. The physics of granular materials are complicated by effects such as particle cohesion, clumping, vibration, and size segregation. See Ref. 54 to learn more.

1.5 A formula for estimating the mean free path of a perfect gas is:

$$\ell = 1.26\frac{\mu}{\rho\sqrt{RT}} = 1.26\frac{\mu}{\mathrm{p}}\sqrt{RT} \qquad (1)$$

where the latter form follows from the ideal-gas law, $\rho = p/RT$. What are the dimensions of the constant "1.26"? Estimate the mean free path of air at 20°C and 7 kPa. Is air *rarefied* at this condition?

Solution: We know the dimensions of every term except "1.26":

$$\{\ell\} = \{L\} \quad \{\mu\} = \left\{\frac{M}{LT}\right\} \quad \{\rho\} = \left\{\frac{M}{L^3}\right\} \quad \{R\} = \left\{\frac{L^2}{T^2\Theta}\right\} \quad \{T\} = \{\Theta\}$$

Therefore the above formula (first form) may be written dimensionally as

$$\{L\} = \{1.26?\}\frac{\{M/L \cdot T\}}{\{M/L^3\}\sqrt{[\{L^2/T^2 \cdot \Theta\}\{\Theta\}]}} = \{1.26?\}\{L\}$$

Since we have {L} on both sides, {1.26} = {unity}, that is, the constant is dimensionless. The formula is therefore dimensionally homogeneous and should hold for any unit system.

For air at 20°C = 293 K and 7000 Pa, the density is $\rho = p/RT = (7000)/[(287)(293)] = 0.0832\ \text{kg/m}^3$. From Table A-2, its viscosity is $1.80\text{E}{-5}\ \text{N}\cdot\text{s/m}^2$. Then the formula predicts a mean free path of

$$\ell = 1.26\frac{1.80\text{E}{-5}}{(0.0832)[(287)(293)]^{1/2}} \approx \mathbf{9.4E{-}7\ m} \quad \textit{Ans.}$$

This is quite small. We would judge this gas to approximate a continuum if the physical scales in the flow are greater than about 100 ℓ, that is, greater than about 94 μm.

P1.6 The Saybolt Universal Viscometer, now obsolete but still sold in scientific catalogs, measures the kinematic viscosity of lubricants [Ref. 49, p. 40]. A container, held at constant temperature, is filled with 60 ml of fluid. Measure the time *t* for the fluid to drain from a small hole or short tube in the bottom. This time unit, called *Saybolt universal seconds*, or SUS, is correlated with kinematic viscosity ν, in centistokes (1 stoke = 1 cm^2/s), by the following curve-fit formula:

$$\nu = 0.215t - \frac{145}{t} \qquad \text{for} \quad 40 < t < 100\ \text{SUS}$$

(*a*) Comment on the dimensionality of this equation. (*b*) Is the formula physically correct? (*c*) Since ν varies strongly with temperature, how does temperature enter into the formula? (*d*) Can we easily convert ν from centistokes to mm^2/s?

Solution: (*a*) The formula is dimensionally *inconsistent*. The right-hand side does not have obvious kinematic viscosity units. The constants 0.215 and 145 must conceal (dimensional) information on temperature, gravity, fluid density, and container shape. (*b*) The formula correctly predicts that the time to drain increases with fluid viscosity. (*c*) The time *t* will reflect changes in ν, and the constants 0.215 and 145 vary slightly (±1%) with temperature [Ref. 49, p. 43]. (*d*) Yes, no conversion necessary; the units of centistoke and mm^2/s are exactly the same..

P1.7 Convert the following inappropriate quantities into SI units: (*a*) a velocity of 3,937 yards per hour; (*b*) a volume flow rate of 4,903 acre-feet of water per week; and (*c*) a mass flow rate of 25,616 gallons per day of SAE 30W oil at 20ºC.

Solution: Most of what we need is on the inside front cover of the text.

(*a*) One yard equals 3 ft. Thus 3937 yards = 11811 ft x (0.3048 m/ft) = 3600 m. One hour = 3600 s. Thus, finally, the velocity in SI units is (3600 m)/(3600 s) = **1.00 m/s** Ans.(*a*)

(*b*) One acre = 4046.9 m^2, and 1 ft = 0.3048 m, thus 4903 acre-ft = (4903)(4046.9)(0.3048) = 6,048,000 m^3. One week = (7 days)(24 h/day)(3600 s/h) = 604,800 s. Finally,

4903 acre-ft per week = (6,048,000 m^3)/(604,800 s) = **10.0 m^3/s** Ans.(*b*)

(*c*) From Table A.3, the density of SAE 30W oil at 20ºC is 891 kg/m^3. Meanwhile, 25616 gallons x (0.0037854 m^3/gal) = 96.97 m^3. One day = (24 h/day) x (3600 s/h) = 86400 s.

Finally, 25616 gal/day = (891 kg/m^3)(96.97 m^3)/(86400 s) = **1.00 kg/s** Ans.(*c*)

1.8 Suppose that bending stress σ in a beam depends upon bending moment M and beam area moment of inertia I and is proportional to the beam half-thickness y. Suppose also that, for the particular case M = 2900 in·lbf, y = 1.5 in, and I = 0.4 in^4, the predicted stress is 75 MPa. Find the only possible dimensionally homogeneous formula for σ.

Solution: We are given that σ = y fcn(M,I) and we are *not* to study up on strength of materials but only to use dimensional reasoning. For homogeneity, the right hand side must have dimensions of stress, that is,

$$\{\sigma\} = \{y\}\{\text{fcn}(M,I)\}, \quad \text{or:} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\{\text{fcn}(M,I)\}$$

$$\text{or:} \quad \text{the function must have dimensions } \{\text{fcn}(M,I)\} = \left\{\frac{M}{L^2T^2}\right\}$$

Therefore, to achieve dimensional homogeneity, we somehow must combine bending moment, whose dimensions are $\{ML^2T^{-2}\}$, with area moment of inertia, $\{I\} = \{L^4\}$, and end up with $\{ML^{-2}T^{-2}\}$. Well, it is clear that {I} contains neither mass {M} nor time {T} dimensions, but the bending moment contains both mass and time and in exactly the combination we need, $\{MT^{-2}\}$. Thus it must be that σ is proportional to M also. Now we have reduced the problem to:

$$\sigma = \text{yM fcn(I)}, \quad \text{or} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\left\{\frac{ML^2}{T^2}\right\}\{\text{fcn(I)}\}, \quad \text{or:} \quad \{\text{fcn(I)}\} = \mathbf{\{L^{-4}\}}$$

We need just enough *I*'s to give dimensions of $\{L^{-4}\}$: we need the formula to be exactly *inverse* in *I*. The correct dimensionally homogeneous beam bending formula is thus:

$$\boldsymbol{\sigma} = \mathbf{C}\frac{\mathbf{My}}{\mathbf{I}}, \quad \text{where } \{C\} = \{\text{unity}\} \quad \textit{Ans.}$$

The formula admits to an arbitrary dimensionless constant C whose value can only be obtained from known data. Convert stress into English units: σ = (75 MPa)/(6894.8) = 10880 lbf/in^2. Substitute the given data into the proposed formula:

$$\sigma = 10880\ \frac{\text{lbf}}{\text{in}^2} = C\frac{My}{I} = C\frac{(2900\ \text{lbf}\cdot\text{in})(1.5\ \text{in})}{0.4\ \text{in}^4}, \quad \text{or:} \quad \mathbf{C \approx 1.00} \quad \textit{Ans.}$$

The data show that C = 1, or $\boldsymbol{\sigma} = \mathbf{My/I}$, our old friend from strength of materials.

P1.9 An inverted conical container, 26 inches in diameter and 44 inches high, is filled with a liquid at 20°C and weighed. The liquid weight is found to be 5030 ounces. (*a*) What is the density of the fluid, in kg/m^3? (*b*) What fluid might this be? Assume standard gravity, $g = 9.807$ m/s^2.

Solution: First find the volume of the liquid in m^3, from our high school cone-volume formula:

$$\text{Liquid Vol.} = \frac{\pi}{3}R^2 h = \frac{\pi}{3}(13\,in)^2\,(44\,in) = 7787\,in^3 = 4.506\ ft^3 = 0.1276\,m^3$$

Then find the mass of liquid in kilograms:

$$Liquid\ \ mass = 5030\,oz \div 16 = 314.4\,lbm \times 0.45359 = 142.6\ kg$$

$$\text{Then liquid density} = \frac{mass}{volume} = \frac{142.6\,kg}{0.1276\,m^3} = \mathbf{1117}\ \frac{kg}{m^3} \quad Ans.(a)$$

(*b*) From Appendix Table A.3, this could very well be **ethylene glycol**. *Ans.*(*b*)

1.10 The Stokes-Oseen formula [10] for drag on a sphere at low velocity V is:

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

where D = sphere diameter, μ = viscosity, and ρ = density. Is the formula homogeneous?

Solution: Write this formula in dimensional form, using Table 1-2:

$$\{F\} = \{3\pi\}\{\mu\}\{D\}\{V\} + \left\{\frac{9\pi}{16}\right\}\{\rho\}\{V\}^2\{D\}^2 \;?$$

$$\text{or:}\quad \left\{\frac{ML}{T^2}\right\} = \{1\}\left\{\frac{M}{LT}\right\}\{L\}\left\{\frac{L}{T}\right\} + \{1\}\left\{\frac{M}{L^3}\right\}\left\{\frac{L^2}{T^2}\right\}\{L^2\} \;?$$

where, hoping for homogeneity, we have assumed that all constants ($3,\pi,9,16$) are *pure*, i.e., {unity}. Well, yes indeed, all terms have dimensions $\{ML/T^2\}$! Therefore the Stokes-Oseen formula (derived in fact from a theory) is ***dimensionally homogeneous.***

P1.11 In English Engineering units, the specific heat c_p of air at room temperature is approximately 0.24 Btu/(lbm-°F). When working with kinetic energy relations, it is more appropriate to express c_p as a velocity-squared per absolute degree. Give the numerical value, in this form, of c_p for air in (*a*) SI units, and (*b*) BG units.

Solution: From Appendix C, *Conversion Factors*, 1 Btu = 1055.056 J (or N-m) = 778.17 ft-lbf, and 1 lbm = 0.4536 kg = (1/32.174) slug. Thus the conversions are:

$$\text{SI units: } 0.24\frac{Btu}{lbm\,^\circ F} = 0.24\frac{1055.056\,N\cdot m}{(0.4536\,kg)(1K/1.8)} = 1005\frac{N\cdot m}{kg\cdot K} = \mathbf{1005}\frac{m^2}{s^2\,K} \quad Ans.(a)$$

$$\text{BG units: } 0.24\frac{Btu}{lbm\,^\circ F} = 0.24\frac{778.17\;ft\cdot lbf}{[(1/32.174)slug](1\,^\circ R)} = 6009\frac{ft\cdot lbf}{slug\cdot^\circ R} = \mathbf{6009}\frac{ft^2}{s^2\,^\circ R} \quad Ans.(b)$$

1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad \textit{Ans.}$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if $\Delta p = 35$ psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that $Q\Delta p$ has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s}; \quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2}; \quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*

1.14 The volume flow Q over a dam is proportional to dam width B and also varies with gravity g and excess water height H upstream, as shown in Fig. P1.14. What is the only possible dimensionally homogeneous relation for this flow rate?

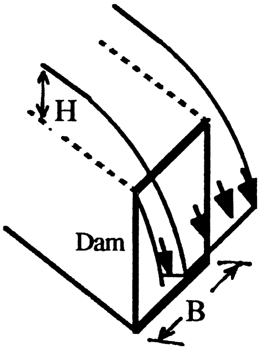


Fig. P1.14

Solution: So far we know that Q = B fcn(H,g). Write this in dimensional form:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} = \{B\}\{f(H,g)\} = \{L\}\{f(H,g)\},$$

$$\text{or:} \quad \{f(H,g)\} = \left\{\frac{L^2}{T}\right\}$$

So the function fcn(H,g) must provide dimensions of $\{L^2/T\}$, but only *g* contains *time*. Therefore g must enter in the form $g^{1/2}$ to accomplish this. The relation is now

$Q = Bg^{1/2}\text{fcn}(H)$, or: $\{L^3/T\} = \{L\}\{L^{1/2}/T\}\{\text{fcn}(H)\}$, or: $\{\text{fcn}(H)\} = \{\mathbf{L^{3/2}}\}$In order for fcn(H) to provide dimensions of $\{L^{3/2}\}$, the function must be a 3/2 power. Thus the final desired homogeneous relation for dam flow is:

$$\mathbf{Q = C\,B\,g^{1/2}\,H^{3/2}}, \quad \text{where C is a dimensionless constant} \quad \textit{Ans.}$$

P1.15 Mott [49] recommends the following formula for the friction head loss h_f, in ft, for flow through a pipe of length L_o and diameter D (both in ft):

$$h_f = L_o\left(\frac{Q}{0.551AC_hD^{0.63}}\right)^{1.852}$$

where Q is the volume flow rate in ft^3/s, A is the pipe cross-section area in ft^2, and C_h is a dimensionless coefficient whose value is approximately 100. Determine the dimensions of the constant 0.551.

Solution: Write out the dimensions of each of the terms in the formula:

$$\{h_f\}=\{L\}\ ;\ \{L_o\}=\{L\}\ ;\ \{Q\}=\{L^3/T\}\ ;\ \{A\}=\{L^2\}\ ;\ \{C_h\}=\{1\}\ ;\ \{D\}=\{L\}$$

Use these dimensions in the equation to determine {0.551}. Since h_f and L_o have the same dimensions {*L*}, it follows that the quantity in parentheses must be dimensionless:

$$\{(\frac{Q}{0.551AC_hD^{0.63}})\} = \{1\} = \{\frac{L^3/T}{\{0.551\}\{L^2\}\{1\}\{L\}^{0.63}}\} = \{\frac{L^{0.37}}{\{0.551)T}\} = \{1\}$$

It follows that $\{0.551\} = \{\mathbf{L^{0.37}/T}\}$ *Ans.*

The constant has dimensions; therefore *beware*. The formula is valid only for *water* flow at high (*turbulent*) velocities. The density and viscosity of water are hidden in the constant 0.551, and the wall roughness is hidden (approximately) in the numerical value of C_h.

1.16 Test the dimensional homogeneity of the boundary-layer *x*-momentum equation:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\rho g_x+\frac{\partial \tau}{\partial y}$$

Solution: This equation, like **all** theoretical partial differential equations in mechanics, is dimensionally homogeneous. Test each term in sequence:

$$\left\{\rho u\frac{\partial u}{\partial x}\right\}=\left\{\rho v\frac{\partial u}{\partial y}\right\}=\frac{M}{L^3}\frac{L}{T}\frac{L/T}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial p}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

$$\{\rho g_x\}=\frac{M}{L^3}\frac{L}{T^2}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial \tau}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

All terms have dimension $\{ML^{-2}T^{-2}\}$. This equation may use *any* consistent units.

1.17 Investigate the consistency of the Hazen-Williams formula from hydraulics:

$$Q=61.9D^{2.63}\left(\frac{\Delta p}{L}\right)^{0.54}$$

What are the dimensions of the constant "61.9"? Can this equation be used with confidence for a variety of liquids and gases?

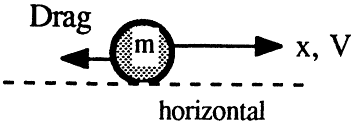
Solution: Write out the dimensions of each side of the equation:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} \overset{?}{=} \{61.9\}\{D^{2.63}\}\left\{\frac{\Delta p}{L}\right\}^{0.54} = \{61.9\}\{L^{2.63}\}\left\{\frac{M/LT^2}{L}\right\}^{0.54}$$

The constant 61.9 has *fractional* dimensions: $\{61.9\} = \mathbf{\{L^{1.45}T^{0.08}M^{-0.54}\}}$ *Ans.*

Clearly, the formula is extremely inconsistent and cannot be used with confidence for any given fluid or condition or units. Actually, the Hazen-Williams formula, still in common use in the watersupply industry, is valid only for **water** flow in smooth pipes larger than 2-in. diameter and turbulent velocities less than 10 ft/s and (certain) English units. This formula should be held at arm's length and given a vote of "No Confidence."

***1.18** ("*" means "difficult"—not just a plug-and-chug, that is) For small particles at low velocities, the first (linear) term in Stokes' drag law, Prob. 1.10, is dominant, hence F = KV, where K is a constant. Suppose a particle of mass m is constrained to move horizontally from the initial position x = 0 with initial velocity V = Vo. Show (a) that its velocity will decrease exponentially with time; and (b) that it will stop after travelling a distance x = mV$_o$/K.



Solution: Set up and solve the differential equation for forces in the *x*-direction:

$$\sum F_x = -\text{Drag} = ma_x, \quad \text{or:} \quad -KV = m\frac{dV}{dt}, \quad \text{integrate} \int_{V_o}^{V} \frac{dV}{V} = -\int_0^t \frac{m}{K}dt$$

$$\text{Solve} \quad \mathbf{V = V_o e^{-mt/K}} \quad \text{and} \quad \mathbf{x = \int_0^t V\,dt = \frac{mV_o}{K}\left(1 - e^{-mt/K}\right)} \quad \textit{Ans.}\text{(a,b)}$$

Thus, as asked, V drops off exponentially with time, and, as $t \to \infty, x = K\dfrac{V_o}{m}$

P1.19 In his study of the circular hydraulic jump formed by a faucet flowing into a sink, Watson [53] proposes a parameter combining volume flow rate *Q*, density ρ and viscosity μ of the fluid, and depth *h* of the water in the sink. He claims that the grouping is dimensionless, with *Q* in the numerator. Can you verify this?

Solution: Check the dimensions of these four variables, from Table 1.2:

$$\{Q\} = \{L^3/T\} \;;\; \{\rho\} = \{M/L^3\} \;;\; \{\mu\} = \{M/LT\} \;;\; \{h\} = \{L\}$$

Can we make this dimensionless? First eliminate mass {*M*} by dividing density by viscosity, that is, ρ/μ has units $\{T/L^2\}$. (I am pretending that kinematic viscosity is unfamiliar to the students in this introductory chapter.) Then combine ρ/μ and *Q* to eliminate time: $(\rho/\mu)Q$ has units {L}. Finally, divide that by a single depth *h* to form a dimensionless group:

$$\{\frac{\rho Q}{\mu h}\} = \frac{\{M/L^3\}\{L^3/T\}}{\{M/LT\}\{L\}} = \{1\} = \text{dimensionless} \quad \textit{Ans.} \quad \text{Watson is correct.}$$

P1.20 Books on porous media and atomization claim that the viscosity μ and surface tension Υ of a fluid can be combined with a characteristic velocity *U* to form an important dimensionless parameter. (*a*) Verify that this is so. (*b*) Evaluate this parameter for water at 20°C and a velocity of 3.5 cm/s. NOTE: Extra credit if you know the name of this parameter.

Solution: We know from Table 1.2 that $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{U\} = \{LT^{-1}\}$, and $\{\Upsilon\} = \{FL^{-1}\} = \{MT^{-2}\}$. To eliminate mass {M}, we must divide μ by Υ, giving $\{\mu/\Upsilon\} = \{TL^{-1}\}$. Multiplying by the velocity will thus cancel all dimensions:

$$\frac{\mu U}{\Upsilon} \text{ is dimensionless, as is its inverse, } \frac{\Upsilon}{\mu U} \qquad \textit{Ans.}(a)$$

The grouping is called the *Capillary Number*. (*b*) For water at 20°C and a velocity of 3.5 cm/s, use Table A.3 to find μ = 0.001 kg/m-s and Υ = 0.0728 N/m. Evaluate

$$\frac{\mu U}{\Upsilon} = \frac{(0.001\,kg/m-s)(0.035\,m/s)}{(0.0728\,kg/s^2)} = \mathbf{0.00048}\,,\; \frac{\Upsilon}{\mu U} = \mathbf{2080} \quad \textit{Ans.}(b)$$

P1.21 In 1908, Prandtl's student Heinrich Blasius proposed the following formula for the wall shear stress τ_w at a position *x* in viscous flow at velocity *V* past a flat surface:

$$\tau_w = 0.332\,\rho^{1/2}\,\mu^{1/2}\,V^{3/2}\,x^{-1/2}$$

Determine the dimensions of the constant 0.332.

Solution: From Table 1.2 we find the dimensions of each term in the equation:

$$\{\tau_w\}=\{ML^{-1}T^{-2}\}\ ;\ \{\rho\}=\{ML^{-3}\}\ ;\ \{\mu\}=\{ML^{-1}T^{-1}\}\ ;\ \{V\}=\{LT^{-1}\}\ ;\ \{x\}=\{L\}$$

Use these dimensions in the equation to determine {0.332}:

$$\{\frac{M}{LT^2}\} = \{0.332\}\,\{\frac{M}{L^3}\}^{1/2}\,\{\frac{M}{LT}\}^{1/2}\,\{\frac{L}{T}\}^{3/2}\,\{L\}^{-1/2}$$

$$\text{Clean up:}\quad \{\frac{M}{LT^2}\} = \{0.332\}\,\{\frac{M}{LT^2}\}\ ,\quad \text{or:}\quad \mathbf{\{0.332\}} = \mathbf{\{1\}}\qquad \textit{Ans.}$$

The constant 0.332 is *dimensionless*. Blasius was one of the first workers to deduce dimensionally consistent viscous-flow formulas without empirical constants.

P1.22 The *Ekman number*, Ek, arises in geophysical fluid dynamics. It is a dimensionless parameter combining seawater density ρ, a characteristic length *L*, seawater viscosity μ, and the Coriolis frequency $\Omega \sin\phi$, where Ω is the rotation rate of the earth and ϕ is the latitude angle. Determine the correct form of Ek if the viscosity is in the numerator.

Solution : First list the dimensions of the various quantities:

$$\{\rho\} = \{ML^{-3}\}\ ;\ \{L\} = \{L\}\ ;\ \{\mu\} = \{ML^{-1}T^{-1}\}\ ;\ \{\Omega\sin\phi\} = \{T^{-1}\}$$

Note that $\sin\phi$ is itself dimensionless, so the Coriolis frequency has the dimensions of Ω. Only ρ and μ contain mass {M}, so if μ is in the numerator, ρ must be in the denominator. That combination μ/ρ we know to be the kinematic viscosity, with units $\{L^2T^{-1}\}$. Of the two remaining variables, only $\Omega \sin\phi$ contains time $\{T^{-1}\}$, so it must be in the denominator. So far, we have the grouping $\mu/(\rho\,\Omega \sin\phi)$, which has the dimensions $\{L^2\}$. So we put the length-squared into the denominator and we are finished:

$$\text{Dimensionless Ekman number:}\quad \text{Ek} = \frac{\mu}{\rho L^2\,\Omega\sin\phi}\qquad \textit{Ans.}$$

P1.23 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the energy released by an atomic bomb explosion. He assumed that the energy released, E, was a function of blast wave radius R, air density ρ, and time t. Arrange these variables into a single dimensionless group, which we may term the *blast wave number*.

Solution: These variables have the dimensions $\{E\} = \{ML^2/T^2\}$, $\{R\} = \{L\}$, $\{\rho\} = \{M/L^3\}$, and $\{t\} = \{T\}$. Multiplying E by t^2 eliminates time, then dividing by ρ eliminates mass, leaving $\{L^5\}$ in the numerator. It becomes dimensionless when we divide by R^5. Thus

$$\text{Blast wave number} \quad = \quad \frac{E\,t^2}{\rho\,R^5}$$

P1.24 Air, assumed to be an ideal gas with $k = 1.40$, flows isentropically through a nozzle. At section 1, conditions are sea level standard (see Table A.6). At section 2, the temperature is –50°C. Estimate (a) the pressure, and (b) the density of the air at section 2.

Solution: From Table A.6, $p_1 = 101350$ Pa, $T_1 = 288.16$ K, and $\rho_1 = 1.2255$ kg/m^3. Convert to absolute temperature, $T_2 = -50°\text{C} = 223.26$ K. Then, for a perfect gas with constant k,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1.4/(1.4-1)} = (0.7744)^{3.5} = 0.4087$$

$$\text{Thus} \quad p_2 = (0.4087)(101350\,Pa) = \mathbf{41,400}\ Pa \qquad Ans.(a)$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1/(1.4-1)} = (0.7744)^{2.5} = 0.5278$$

$$\text{Thus} \quad \rho_2 = (0.5278)(1.2255\,kg/m^3) = \mathbf{0.647}\ kg/m^3 \qquad Ans.(b)$$

Alternately, once p_2 was known, we could have simply computed ρ_2 from the ideal-gas law.

$$\rho_2 = p_2/RT_2 = (41400)/[287(223.16)] = 0.647\ \text{kg/m}^3$$

1.25 A tank contains 0.9 m^3 of helium at 200 kPa and 20°C. Estimate the total mass of this gas, in kg, (a) on earth; and (b) on the moon. Also, (c) how much heat transfer, in MJ, is required to expand this gas at constant temperature to a new volume of 1.5 m^3?

Solution: First find the density of helium for this condition, given R = 2077 $m^2/(s^2\cdot K)$ from Table A-4. Change 20°C to 293 K:

$$\rho_{He} = \frac{p}{R_{He}T} = \frac{200000\ N/m^2}{(2077\ J/kg\cdot K)(293\ K)} \approx 0.3286\ kg/m^3$$

Now mass is *mass*, no matter where you are. Therefore, on the moon or wherever,

$$m_{He} = \rho_{He}\upsilon = (0.3286\ kg/m^3)(0.9\ m^3) \approx \mathbf{0.296\ kg} \quad \textit{Ans.}\ (a,b)$$

For part (c), we expand a constant mass isothermally from 0.9 to 1.5 m^3. The first law of thermodynamics gives

$$dQ_{added} - dW_{by\ gas} = dE = mc_v\Delta T = 0 \quad \text{since } T_2 = T_1 \text{ (isothermal)}$$

Then the heat added equals the work of expansion. Estimate the work done:

$$W_{1\text{-}2} = \int_1^2 p\, d\upsilon = \int_1^2 \frac{m}{\upsilon} RT\, d\upsilon = mRT \int_1^2 \frac{d\upsilon}{\upsilon} = mRT \ln(\upsilon_2/\upsilon_1),$$

$$\text{or:} \quad W_{1\text{-}2} = (0.296\ kg)(2077\ J/kg\cdot K)(293\ K)\ln(1.5/0.9) = Q_{1\text{-}2} \approx \mathbf{92000\ J} \quad \textit{Ans.}\ (c)$$

1.26 A tire has a volume of 3.0 ft^3 and a 'gage' pressure (above atmospheric pressure) of 32 psi at 75°F. If the ambient pressure is sea-level standard, what is the weight of air in the tire?

Solution: Convert the temperature from 75°F to 535°R. Convert the pressure to psf:

$$p = (32\ lbf/in^2)(144\ in^2/ft^2) + 2116\ lbf/ft^2 = 4608 + 2116 \approx 6724\ lbf/ft^2$$

From this compute the density of the air in the tire:

$$\rho_{air} = \frac{p}{RT} = \frac{6724\ lbf/ft^2}{(1717\ ft\cdot lbf/slug\cdot{}^\circ R)(535^\circ R)} = 0.00732\ slug/ft^3$$

Then the total weight of air in the tire is

$$W_{air} = \rho g\upsilon = (0.00732\ slug/ft^3)(32.2\ ft/s^2)(3.0\ ft^3) \approx \mathbf{0.707\ lbf} \quad \textit{Ans.}$$

1.27 Given temperature and specific volume data for steam at 40 psia [Ref. 23]:

T, °F:	400	500	600	700	800
v, ft^3/lbm:	12.624	14.165	15.685	17.195	18.699

Is the ideal gas law reasonable for this data? If so, find a least-squares value for the gas constant R in m^2/(s^2·K) and compare with Table A-4.

Solution: The units are awkward but we can compute R from the data. At 400°F,

$$\text{“R”}_{400°\text{F}} = \frac{pV}{T} = \frac{(40\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)(12.624\ \text{ft}^3/\text{lbm})(32.2\ \text{lbm/slug})}{(400+459.6)°\text{R}} \approx 2721\ \frac{\text{ft·lbf}}{\text{slug°R}}$$

The metric conversion factor, from the inside cover of the text, is “5.9798”: R_{metric} = 2721/5.9798 = 455.1 m^2/(s^2·K). Not bad! This is only 1.3% less than the ideal-gas approximation for steam in Table A-4: **461** m^2/(s^2·K). Let’s try all the five data points:

T, °F:	400	500	600	700	800
R, m^2/(s^2·K):	455	457	459	460	460

The total variation in the data is only ±0.6%. Therefore steam *is* nearly an ideal gas in this (high) temperature range and for this (low) pressure. We can take an average value:

$$p = 40\ \text{psia},\ 400°\text{F} \le T \le 800°\text{F}:\quad \mathbf{R_{steam}} \approx \frac{1}{5}\sum_{i=1}^{5} R_i \approx \mathbf{458\ \frac{J}{kg\cdot K} \pm 0.6\%} \quad \textit{Ans.}$$

With such a small uncertainty, we don’t really *need* to perform a least-squares analysis, but if we wanted to, it would go like this: We wish to minimize, for all data, the sum of the squares of the deviations from the perfect-gas law:

$$\text{Minimize}\ E = \sum_{i=1}^{5}\left(R - \frac{pV_i}{T_i}\right)^2 \quad \text{by differentiating}\ \frac{\partial E}{\partial R} = 0 = \sum_{i=1}^{5} 2\left(R - \frac{pV_i}{T_i}\right)$$

$$\text{Thus}\ R_{\text{least-squares}} = \frac{p}{5}\sum_{i=1}^{5}\frac{V_i}{T_i} = \frac{40(144)}{5}\left[\frac{12.624}{860°\text{R}} + \cdots + \frac{18.699}{1260°\text{R}}\right](32.2)$$

For this example, then, least-squares amounts to summing the (V/T) values and converting the units. The English result shown above gives $R_{\text{least-squares}}$ ≈ 2739 ft·lbf/slug·°R. Convert this to metric units for our (highly accurate) least-squares estimate:

$$R_{\text{steam}} \approx 2739/5.9798 \approx \mathbf{458 \pm 0.6\%\ J/kg{\cdot}K} \quad \textit{Ans.}$$

1.28 Wet air, at 100% relative humidity, is at 40°C and 1 atm. Using Dalton's law of partial pressures, compute the density of this wet air and compare with dry air.

Solution: Change T from 40°C to 313 K. Dalton's law of partial pressures is

$$p_{tot} = 1\text{ atm} = p_{air} + p_{water} = \frac{m_a}{\upsilon}R_aT + \frac{m_w}{\upsilon}R_wT$$

$$\text{or:}\quad m_{tot} = m_a + m_w = \frac{p_a\upsilon}{R_aT} + \frac{p_w\upsilon}{R_wT}\quad \text{for an ideal gas}$$

where, from Table A-4, $R_{air} = 287$ and $R_{water} = 461\ m^2/(s^2\cdot K)$. Meanwhile, from Table A-5, at 40°C, the vapor pressure of saturated (100% humid) water is 7375 Pa, whence the partial pressure of the air is $p_a = 1\text{ atm} - p_w = 101350 - 7375 = 93975$ Pa.

Solving for the mixture density, we obtain

$$\rho = \frac{m_a + m_w}{\upsilon} = \frac{p_a}{R_aT} + \frac{p_w}{R_wT} = \frac{93975}{287(313)} + \frac{7375}{461(313)} = 1.046 + 0.051 \approx \mathbf{1.10\ \frac{kg}{m^3}}\quad \textit{Ans.}$$

By comparison, the density of dry air for the same conditions is

$$\rho_{\text{dry air}} = \frac{p}{RT} = \frac{101350}{287(313)} = 1.13\ \frac{kg}{m^3}$$

Thus, at 40°C, wet, 100% humidity, air is *lighter* than dry air, by about **2.7%**.

1.29 A tank holds 5 ft^3 of air at 20°C and 120 psi (gage). Estimate the energy in ft-lbf required to compress this air isothermally from one atmosphere (14.7 psia = 2116 psfa).

Solution: Integrate the work of compression, assuming an ideal gas:

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = -\int_1^2 \frac{mRT}{\upsilon}d\upsilon = -mRT\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = p_2\upsilon_2\ln\left(\frac{p_2}{p_1}\right)$$

where the latter form follows from the ideal gas law for isothermal changes. For the given numerical data, we obtain the quantitative work done:

$$W_{1\text{-}2} = p_2\upsilon_2\ \ln\left(\frac{p_2}{p_1}\right) = \left(134.7\times 144\ \frac{lbf}{ft^2}\right)(5\ ft^3)\ \ln\left(\frac{134.7}{14.7}\right) \approx \mathbf{215{,}000\ ft\cdot lbf}\quad \textit{Ans.}$$

1.30 Repeat Prob. 1.29 if the tank is filled with compressed *water* rather than air. Why is the result thousands of times less than the result of 215,000 ft·lbf in Prob. 1.29?

Solution: First evaluate the density change of water. At 1 atm, $\rho_o \approx 1.94$ slug/ft^3. At 120 psi(gage) = 134.7 psia, the density would rise slightly according to Eq. (1.22):

$$\frac{p}{p_o} = \frac{134.7}{14.7} \approx 3001\left(\frac{\rho}{1.94}\right)^7 - 3000, \quad \text{solve } \rho \approx 1.940753 \text{ slug/ft}^3,$$

$$\text{Hence } m_{water} = \rho\upsilon = (1.940753)(5 \text{ ft}^3) \approx 9.704 \text{ slug}$$

The density change is extremely small. Now the work done, as in Prob. 1.29 above, is

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = \int_1^2 p\,d\left(\frac{m}{\rho}\right) = \int_1^2 p\,\frac{m\,d\rho}{\rho^2} \approx p_{avg} m \frac{\Delta\rho}{\rho_{avg}^2} \quad \text{for a linear pressure rise}$$

$$\text{Hence } W_{1\text{-}2} \approx \left(\frac{14.7+134.7}{2} \times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)(9.704 \text{ slug})\left(\frac{0.000753}{1.9404^2}\ \frac{\text{ft}^3}{\text{slug}}\right) \approx \mathbf{21\ ft\cdot lbf} \quad \textit{Ans.}$$

[Exact integration of Eq. (1.22) would give the same numerical result.] Compressing water (extremely small $\Delta\rho$) takes *ten thousand times less energy* than compressing air, which is why it is safe to test high-pressure systems with water but dangerous with air.

P1.31 One cubic foot of argon gas at 10°C and 1 atm is compressed isentropically to a new pressure of 600 kPa. (*a*) What will be its new density and temperature? (*b*) If allowed to cool, at this new volume, back to 10°C, what will be the final pressure? Assume constant specific heats.

Solution: This is an exercise in having students recall their thermodynamics. From Table A.4, for argon gas, $R = 208$ m^2/(s^2-K) and $k = 1.67$. Note $T_1 = 283K$. First compute the initial density:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101350\,N/m^2}{(208\,m^2/s^2\cdot K)(283K)} = 1.72\ kg/m^3$$

For an isentropic process at constant *k*,

$$\frac{p_2}{p_1} = \frac{600{,}000\,Pa}{101{,}350\,Pa} = 5.92 = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{\rho_2}{1.72}\right)^{1.67}, \quad \text{Solve } \rho_2 = \mathbf{4.99\ kg/m^3} \quad \textit{Ans.(a)}$$

$$\frac{p_2}{p_1} = 5.92 = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_2}{283K}\right)^{1.67/0.67}, \quad \text{Solve } T_2 = \mathbf{578\,K} = 305^\circ C \quad \textit{Ans.(a)}$$

(*b*) Cooling at constant volume means ρ stays the same and the new temperature is 283K. Thus

$$p_3 = \rho_3 RT_3 = (5.00\frac{kg}{m^3})(208\frac{m^2}{s^2K})(283K) = 294{,}000\,Pa = \mathbf{294\ kPa} \quad Ans.(b)$$

1.32 A blimp is approximated by a prolate spheroid 90 m long and 30 m in diameter. Estimate the weight of 20°C gas within the blimp for (a) helium at 1.1 atm; and (b) air at 1.0 atm. What might the difference between these two values represent (Chap. 2)?

Solution: Find a handbook. The volume of a prolate spheroid is, for our data,

$$\upsilon = \frac{2}{3}\pi LR^2 = \frac{2}{3}\pi(90\text{ m})(15\text{ m})^2 \approx 42412\text{ m}^3$$

Estimate, from the ideal-gas law, the respective densities of helium and air:

$$\text{(a)}\quad \rho_{\text{helium}} = \frac{p_{He}}{R_{He}T} = \frac{1.1(101350)}{2077(293)} \approx 0.1832\ \frac{kg}{m^3};$$

$$\text{(b)}\quad \rho_{\text{air}} = \frac{p_{air}}{R_{air}T} = \frac{101350}{287(293)} \approx 1.205\ \frac{kg}{m^3}.$$

Then the respective gas weights are

$$W_{He} = \rho_{He} g\upsilon = \left(0.1832\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(42412\text{ m}^3) \approx \mathbf{76000\ N} \quad \textit{Ans.}\text{ (a)}$$

$$W_{air} = \rho_{air} g\upsilon = (1.205)(9.81)(42412) \approx \mathbf{501000\ N} \quad \textit{Ans.}\text{ (b)}$$

The difference between these two, **425000 N**, is the *buoyancy*, or lifting ability, of the blimp. [See Section 2.8 for the principles of buoyancy.]

P1.33 Experimental data [55] for the density of n-pentane liquid for high pressures, at 50°C, are listed as follows:

Pressure, MPa	0.01	10.23	20.70	34.31
Density, kg/m^3	586.3	604.1	617.8	632.8

Interestingly, this data does not fit the author's suggested liquid state relation, Eq. (1.19), very well. Therefore (*a*) fit the data, as best you can, to a second order polynomial. Use your curve-fit to estimate (*b*) the isentropic bulk modulus of n-pentane at 1 atm, and (*c*) the speed of sound of n-pentane at a pressure of 25 MPa.

Solution: The writer used his Excel spreadsheet, which has a least-squares polynomial feature for tabulated data. The writer hit "Add Polynomial Trend Line" and received this 2nd-order curve-fit.

$$p \approx 1.702E9 - 6.278E6\rho + 5755\rho^2 \qquad Ans.(a)$$

with p in Pa and ρ in kg/m^3. The error, for the four given data points, is less than 0.5%.

(*b*) The isentropic bulk modulus can be estimated by differentiating the p-ρ relation at the atmospheric density of 586.3 kg/m^3.

$$B = \rho\left(\frac{\partial p}{\partial \rho}\right)_s = \rho(11510\rho - 6.278\text{E}6)\big|_{\rho=586.3} \approx \mathbf{2.76E8}\,Pa \qquad Ans.(b)$$

(*c*) First examine *Ans.*(*a*) to see what density corresponds to p = 25 MPa. We find, approximately, by iteration or EES, that $\rho \approx 624$ kg/m^3. Then the speed of sound at this pressure is

$$a|_{25MPa} = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{11510(624) - 6.278E6} = \sqrt{904240} \approx \mathbf{950}\ m/s \qquad Ans.(c)$$

We are assuming, reasonably, that the given p-ρ relation is approximately isentropic.

1.34 Consider steam at the following state near the saturation line: (p_1, T_1) = (1.31 MPa, 290°C). Calculate and compare, for an ideal gas (Table A.4) and the Steam Tables (or the EES software), (a) the density ρ_1; and (b) the density ρ_2 if the steam expands isentropically to a new pressure of 414 kPa. Discuss your results.

Solution: From Table A.4, for steam, $k \approx 1.33$, and $R \approx 461\ \mathrm{m^2/(s^2 \cdot K)}$. Convert T_1 = 563 K. Then,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1{,}310{,}000\ Pa}{(461\ m^2/s^2K)(563\ K)} = \mathbf{5.05}\ \frac{kg}{m^3} \quad \textit{Ans.}\ \text{(a)}$$

$$\frac{\rho_2}{\rho_1} = \frac{\rho_2}{5.05} = \left(\frac{p_2}{p_1}\right)^{1/k} = \left(\frac{414\ kPa}{1310\ kPa}\right)^{1/1.33} = 0.421, \quad \textit{or}: \quad \rho_2 = \mathbf{2.12}\ \frac{kg}{m^3} \quad \textit{Ans.}\ \text{(b)}$$

For EES or the Steam Tables, just program the properties for steam or look it up:

EES real steam: $\rho_1 = \mathbf{5.23}\ \mathrm{kg/m^3}$ *Ans.* (a), $\rho_2 = \mathbf{2.16}\ \mathrm{kg/m^3}$ *Ans.* (b)

The ideal-gas error is only about 3%, even though the expansion approached the saturation line.

1.35 In Table A-4, most common gases (air, nitrogen, oxygen, hydrogen, CO, NO) have a specific heat ratio k = 1.40. Why do argon and helium have such high values? Why does NH_3 have such a low value? What is the lowest k for any gas that you know?

Solution: In elementary kinetic theory of gases [21], k is related to the number of "degrees of freedom" of the gas: $k \approx 1 + 2/N$, where N is the number of different modes of translation, rotation, and vibration possible for the gas molecule.

Example: Monotomic gas, N = 3 (translation only), thus $k \approx 5/3$

This explains why helium and argon, which are monatomic gases, have $k \approx 1.67$.

Example: Diatomic gas, N = 5 (translation plus 2 rotations), thus $k \approx 7/5$

This explains why air, nitrogen, oxygen, NO, CO and hydrogen have $k \approx 1.40$.

But NH_3 has *four* atoms and therefore more than 5 degrees of freedom, hence k will be less than 1.40. Most tables list k = 1.32 for NH_3 at about 20°C, implying N ≈ 6.

The lowest k known to this writer is for *uranium hexafluoride*, $^{238}UF_6$, which is a very complex, heavy molecule with many degrees of freedom. The estimated value of k for this heavy gas is $k \approx \mathbf{1.06}$.

P1.36 The isentropic bulk modulus *B* of a fluid is defined in Eq. (1.39). (*a*) What are its dimensions? Using theoretical p-ρ relations for a gas or liquid, estimate the bulk modulus, in Pa, of (*b*) chlorine at 100°C and 10 atm; and (*c*) water, at 20°C and 1000 atm.

Solution: (*a*) Given $B = \rho(\partial p/\partial\rho)_s$, note that the density dimensions cancel. Therefore the dimensions of bulk modulus *B* are the same as pressure: $\{ML^{-1}T^{-2}\}$. *Ans.*(*a*)

(*b*) For an ideal gas, we may differentiate to find a formula for bulk modulus:

$$B_{gas} = \rho\left(\frac{\partial p}{\partial \rho}\right)_s = \rho(kRT) = \rho\left(k\frac{p}{\rho}\right) = k\,p$$

$$Chlorine:\ k = 1.34\ ,\ \ p = 10\ atm\ ,\ \ B = (1.34)(10) = 13.4\ atm = \mathbf{1.36E6}\ Pa \quad Ans.(b)$$

(*c*) For water, differentiate the suggested curve-fit, Eq. (1.19):

$$\frac{p}{p_o} = (\hat{B}+1)\left(\frac{\rho}{\rho_o}\right)^n - \hat{B}\ ;\ \ Water: \hat{B} \approx 3000\ ,\ \ n \approx 7$$

$$\text{Bulk modulus} = \rho\frac{dp}{d\rho} = \rho\left[\frac{n(\hat{B}+1)\,p_o}{\rho_o}\right]\left(\frac{\rho}{\rho_o}\right)^n;\ \text{Now find } \rho \text{ at } 1000\,atm:$$

$$1000 = (3001)\left(\frac{\rho}{\rho_o}\right)^7 - 3000\ ,\ \ solve\ \left(\frac{\rho}{\rho_o}\right) = 1.0419\ ;\ \text{then, finally,}$$

$$B_{water} = (1.0419)(7)(3001)(1\,atm)(1.0419)^7 \approx 29200\,atm \approx \mathbf{2.96E9}\ Pa \qquad Ans.(c)$$

This is 35% more than the low-pressure bulk modulus listed for water in Table A.3. We apologize for using the same symbol *B* for bulk modulus and also for the curve-fit constant in Eq. (1.19). It won't happen again.

1.37 A near-ideal gas has M = 44 and c_V = 610 J/(kg·K). At 100°C, what are (a) its specific heat ratio, and (b) its speed of sound?

Solution: The gas constant is $R = \Lambda/M = 8314/44 \approx 189$ J/(kg·K). Then

$$c_v = R/(k-1),\quad \text{or:}\quad k = 1 + R/c_v = 1 + 189/610 \approx \mathbf{1.31} \quad \textit{Ans.}\text{ (a)} \quad [\text{It is probably } N_2O]$$

With k and R known, the speed of sound at 100ºC = 373 K is estimated by

$$a = \sqrt{kRT} = \sqrt{1.31[189\ m^2/(s^2\cdot K)](373\ K)} \approx \mathbf{304\ m/s} \quad \textit{Ans.}\text{ (b)}$$

1.38 In Fig. P1.38, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

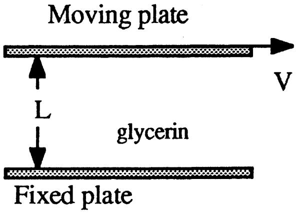


Fig. P1.38

Solution: (a) For glycerin at 20°C, from Table 1.4, $\mu \approx 1.5\ \mathrm{N\cdot s/m^2}$. The shear stress is found from Eq. (1) of Ex. 1.8:

$$\tau = \frac{\mu V}{h} = \frac{(1.5\ \mathrm{Pa{\cdot}s})(5.5\ \mathrm{m/s})}{(0.006\ \mathrm{m})} \approx \mathbf{1380\ Pa} \quad \textit{Ans.}\ \text{(a)}$$

The density of glycerin at 20°C is 1264 $\mathrm{kg/m^3}$. Then the Reynolds number is defined by Eq. (1.24), with L = h, and is found to be decidedly laminar, Re < 1500:

$$\mathrm{Re_L} = \frac{\rho VL}{\mu} = \frac{(1264\ \mathrm{kg/m^3})(5.5\ \mathrm{m/s})(0.006\ \mathrm{m})}{1.5\ \mathrm{kg/m\cdot s}} \approx \mathbf{28} \quad \textit{Ans.}\ \text{(b)}$$

1.39 Knowing $\mu \approx 1.80\mathrm{E}{-5}\ \mathrm{Pa\cdot s}$ for air at 20°C from Table 1-4, estimate its viscosity at 500°C by (a) the power-law, (b) the Sutherland law, and (c) the Law of Corresponding States, Fig. 1.5. Compare with the accepted value $\mu(500°\mathrm{C}) \approx 3.58\mathrm{E}{-5}\ \mathrm{Pa\cdot s}$.

Solution: First change T from 500°C to 773 K. (a) For the power-law for air, n ≈ 0.7, and from Eq. (1.30a),

$$\mu = \mu_o (T/T_o)^n \approx (1.80\mathrm{E}{-5})\left(\frac{773}{293}\right)^{0.7} \approx \mathbf{3.55E{-5}\ \frac{kg}{m{\cdot}s}} \quad \textit{Ans.}\ \text{(a)}$$

This is less than 1% low. (b) For the Sutherland law, for air, S ≈ 110 K, and from Eq. (1.30b),

$$\mu = \mu_o \left[\frac{(T/T_o)^{1.5}(T_o + S)}{(T + S)}\right] \approx (1.80\mathrm{E}{-5})\left[\frac{(773/293)^{1.5}(293+110)}{(773+110)}\right]$$

$$= \mathbf{3.52E{-5}\ \frac{kg}{m{\cdot}s}} \quad \textit{Ans.}\ \text{(b)}$$

This is only 1.7% low. (c) Finally use Fig. 1.5. Critical values for air from Ref. 3 are:

$$\text{Air:}\quad \mu_c \approx 1.93\text{E}-5\ \text{Pa·s} \quad T_c \approx 132\ \text{K} \quad (\text{“mixture” estimates})$$

At 773 K, the temperature ratio is $T/T_c = 773/132 \approx 5.9$. From Fig. 1.5, read $\mu/\mu_c \approx 1.8$. Then our critical-point-correlation estimate of air viscosity is only 3% low:

$$\mu \approx 1.8\mu_c = (1.8)(1.93\text{E–}5) \approx \mathbf{3.5E–5}\ \frac{\mathbf{kg}}{\mathbf{m·s}} \quad \textit{Ans. } (c)$$

1.40 Curve-fit the viscosity data for water in Table A-1 in the form of Andrade's equation,

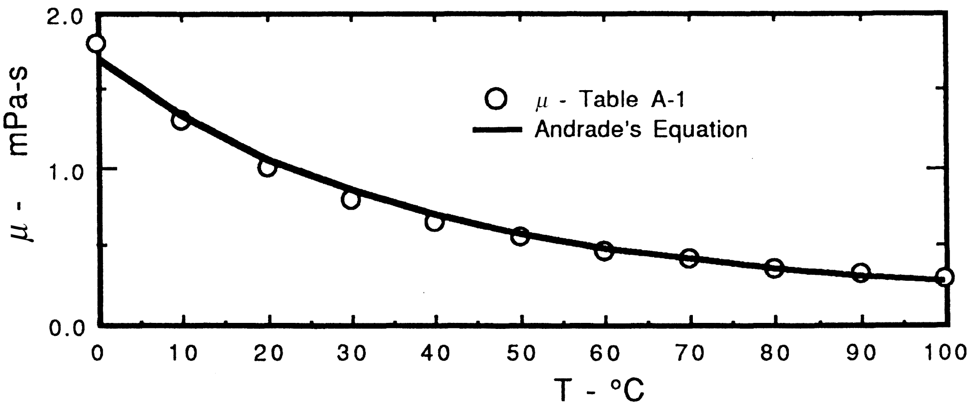
$$\mu \approx A\exp\left(\frac{B}{T}\right) \quad \text{where T is in °K and A and B are curve-fit constants.}$$

Solution: This is an alternative formula to the log-quadratic law of Eq. (1.31). We have eleven data points for water from Table A-1 and can perform a least-squares fit to Andrade's equation:

$$\text{Minimize}\quad E = \sum_{i=1}^{11}[\mu_i - A\exp(B/T_i)]^2, \quad \text{then set}\quad \frac{\partial E}{\partial A} = 0 \quad \text{and} \quad \frac{\partial E}{\partial B} = 0$$

The result of this minimization is: **A ≈ 0.0016 kg/m·s, B ≈ 1903°K.** *Ans.*

The data and the Andrade's curve-fit are plotted. The error is ±7%, so Andrade's equation is not as accurate as the log-quadratic correlation of Eq. (1.31).



P1.41 An aluminum cylinder weighing 30 N, 6 cm in diameter and 40 cm long, is falling concentrically through a long vertical sleeve of diameter 6.04 cm. The clearance is filled with SAE 50 oil at 20°C. Estimate the *terminal* (zero acceleration) fall velocity. Neglect air drag and assume a linear velocity distribution in the oil. [HINT: You are given diameters, not radii.]

Solution: From Table A.3 for SAE 50 oil, $\mu = 0.86$ kg/m-s. The clearance is the difference in *radii*: 3.02 – 3.0 cm = 0.02 cm = 0.0002 m. At terminal velocity, the cylinder weight must balance the viscous drag on the cylinder surface:

$$W = \tau_{wall} A_{wall} = (\mu \frac{V}{C})(\pi D L), \quad \text{where } C = \text{clearance} = r_{sleeve} - r_{cylinder}$$

$$\text{or:} \quad 30\,N = [0.86 \frac{kg}{m-s})(\frac{V}{0.0002\,m})]\,\pi (0.06\,m)(0.40\,m)$$

$$\text{Solve for } V = \mathbf{0.0925}\,m/s \qquad \textit{Ans.}$$

1.42 Some experimental values of μ of helium at 1 atm are as follows:

T, °K:	200	400	600	800	1000	1200
μ, kg/m·s:	1.50E–5	2.43E–5	3.20E–5	3.88E–5	4.50E–5	5.08E–5

Fit these values to either (a) a power-law, or (b) a Sutherland law, Eq. (1.30a,b).

Solution: (a) The power-law is straightforward: put the values of μ and T into, say, an Excel graph, take logarithms, plot them, and make a linear curve-fit. The result is:

$$\text{Power-law curve-fit:} \quad \mu_{He} \approx \mathbf{1.505E-5} \left(\frac{\mathbf{T\ °K}}{\mathbf{200\ K}}\right)^{\mathbf{0.68}} \qquad \textit{Ans.}\ (a)$$

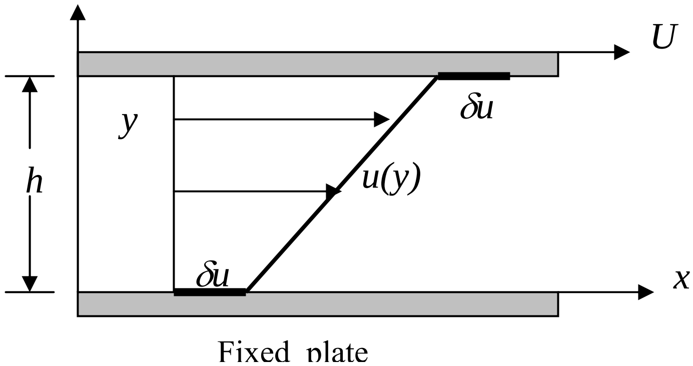
The accuracy is less than ±1%. (b) For the Sutherland fit, we can emulate Prob. 1.41 and perform the least-squares summation, $E = \Sigma[\mu i - \mu o(Ti/200)^{1.5}(200 + S)/(Ti + S)]^2$ and minimize by setting $\partial E/\partial S = 0$. We can try $\mu_o = 1.50E{-}5$ kg/m·s and $T_o = 200°K$ for starters, and it works OK. The best-fit value of $S \approx 95.1°K$. Thus the result is:

$$\text{Sutherland law:}\quad \frac{\mu_{\text{Helium}}}{\mathbf{1.50E{-}5\ kg/m{\cdot}s}} \approx \frac{\mathbf{(T/200)^{1.5}(200+95.1°K)}}{\mathbf{(T+95.1°K)}} \pm \mathbf{4\%} \quad \textit{Ans.}\ \text{(b)}$$

For the complete range 200–1200°K, the power-law is a better fit. The Sutherland law improves to ±1% if we drop the data point at 200°K.

P1.43 For the flow between two parallel plates of Fig. 1.8, reanalyze for the case of *slip flow* at both walls. Use the simple slip condition, $u_{\text{wall}} = \ell (du/dy)_{\text{wall}}$, where ℓ is the mean free path of the fluid. (*a*) Sketch the expected velocity profile and (*b*) find an expression for the shear stress at each wall.

Solution: As in Fig. 1.8, the shear stress remains constant between the two plates. The analysis is correct up to the relation $u = a + b\,y$. There would be equal slip velocities, δu, at both walls, as shown in the following sketch



Ans. (*a*)

Because of slip at the walls, the boundary conditions are different for $u = a + b\,y$:

$$\text{At } y = 0:\quad u = \delta u = a = \ell\left(\frac{du}{dy}\right)_{y=0} = \ell\, b$$

$$At\ y = h:\quad u = U - \delta u = a + bh = U - \ell\, b = \ell\, b + bh,\ or:\ b = \frac{U}{h+2\ell}$$

One way to write the final solution is

$$u = \delta u + \left(\frac{U}{h+2\ell}\right)y \quad , \quad where \quad \delta u = \left(\frac{\ell}{h+2\ell}\right)U \quad ,$$

$$and \quad \tau_{wall} = \mu\left(\frac{du}{dy}\right)_{wall} = \frac{\mu U}{h+2\ell} \qquad Ans.(b)$$

P1.44 SAE 50 oil at 20°C fills the concentric annular space between an inner cylinder, $r_i = 5$ cm, and an outer cylinder, $r_o = 6$ cm. The length of the cylinders is 120 cm. If the outer cylinder is fixed and the inner cylinder rotates at 900 rev/min, use the linear profile approximation to estimate the power, in watts, required to maintain the rotation. Neglect any temperature change of the oil.

Solution: Convert $\Omega_i = 900$ rpm x $(2\pi/60) = 94.25$ rad/s. For SAE 50 oil, from Table A.3, $\mu = 0.86$ kg/m-s. Then the rotational velocity of the inner cylinder, and its related shear stress, are:

$$V_i = \Omega_i r_i = (94.25\, rad/s)(0.05\, m) = 4.71\ m/s$$

$$\tau_i = \mu\frac{\Delta V}{\Delta r} = \left(0.86\frac{kg}{m-s}\right)\left(\frac{4.71-0\, m/s}{0.06-0.05\, m}\right) = 405\frac{kg}{m-s^2} = 405\, Pa$$

The total moment of this stress about the centerline is

$$M = \int r_i\, dF = \int_0^{2\pi} r_i\,(\tau_i\, r_i\, d\theta\, L) = \tau_i\, 2\pi r_i^2 L = (405)(2\pi)(0.05)^2(1.2) = 7.64\ N-m$$

Then the power required is $P = \Omega_i M$ = (94.25 rad/s)(7.64 N-m) = **720** watts *Ans.*

1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

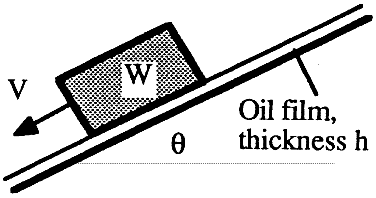


Fig. P1.45

Solution: Let "x" be down the incline, in the direction of V. By "terminal" velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W\sin\theta - \tau A = W\sin\theta - \left(\mu\frac{V}{h}\right)A = ma_x = 0,$$

$$\text{or:}\quad V_{\text{terminal}} = \frac{\mathbf{hW\sin\theta}}{\boldsymbol{\mu}\mathbf{A}} \quad \textit{Ans.}$$

P1.46 A simple and popular model for two non-newtonian fluids in Fig. 1.9*a* is the *power-law*:

$$\tau \approx C\,(\frac{du}{dy})^n$$

where C and n are constants fit to the fluid [15]. From Fig. 1.9*a*, deduce the values of the exponent n for which the fluid is (*a*) newtonian; (*b*) dilatant; and (*c*) pseudoplastic. (*d*) Consider the specific model constant $C = 0.4$ N-s^n/m^2, with the fluid being sheared between two parallel plates as in Fig. 1.8. If the shear stress in the fluid is 1200 Pa, find the velocity V of the upper plate for the cases (*d*) $n = 1.0$; (*e*) $n = 1.2$; and (*f*) $n = 0.8$.

Solution: By comparing the behavior of the model law with Fig. 1.9*a*, we see that

(*a*) Newtonian: $n = 1$; (*b*) Dilatant: $n > 1$; (*c*) Pseudoplastic: $n < 1$ *Ans.*(*a,b,c*)

From the discussion of Fig. 1.8, it is clear that the shear stress is *constant* in a fluid sheared between two plates. The velocity profile remains a straight line (if the flow is laminar), and the strain rate $du/dy = V/h$. Thus, for this flow, the model becomes $\tau = C(V/h)^n$. For the three given numerical cases, we calculate:

$$(d)\ n=1:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^1}{m^2})(\frac{V}{0.001m})^1\ ,\quad \text{solve}\quad V = \mathbf{3.0}\frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}(d)$$

$$(e)\ n=1.2:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{1.2}}{m^2})(\frac{V}{0.001m})^{1.2},\quad \text{solve}\quad V = \mathbf{0.79}\frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}(e)$$

$$(f)\ n=0.8:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{0.8}}{m^2})(\frac{V}{0.001m})^{0.8}\ ,\quad \text{solve}\quad V = \mathbf{22}\frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}(f)$$

A small change in the exponent n can sharply change the numerical values.

P1.47 Data for the apparent viscosity of average human blood, at normal body temperature of 37°C, varies with shear strain rate, as shown in the following table.

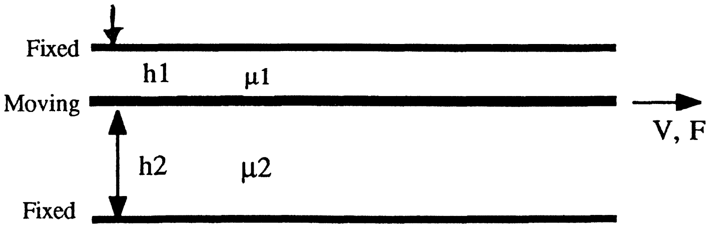
Shear strain rate, s^{-1}	1	10	100	1000
Apparent viscosity, kg/(m•s)	0.011	0.009	0.006	0.004

(*a*) Is blood a nonnewtonian fluid? (*b*) If so, what type of fluid is it? (*c*) How do these viscosities compare with plain water at 37°C?

Solution: (*a*) By definition, since viscosity varies with strain rate, blood is a *nonnewtonian fluid*.

(*b*) Since the apparent viscosity decreases with strain rate, it must be a *pseudoplastic* fluid, as in Fig. 1.9(*a*). The decrease is too slight to call this a "plastic" fluid. (*c*) These viscosity values are from six to fifteen times the viscosity of pure water at 37°C, which is about 0.00070 kg/m-s. The viscosity of the liquid part of blood, called *plasma*, is about 1.8 times that of water. Then there is a sharp increase of blood viscosity due to *hematocrit*, which is the percentage, by volume, of red cells and platelets in the blood. For normal human beings, the hematocrit varies from 40% to 60%, which makes this blood about six times the viscosity of plasma.

1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2}\right) A \quad \textit{Ans. (a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is *no necessary relation* between the two viscosities.

1.49 An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length *L*. Let the rotational rate be Ω (rad/s) and the applied torque be *M*. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu\frac{\Delta V}{\Delta r} \approx \mu\frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau(r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi\mu\Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\boldsymbol{\mu \approx M(r_o - r_i)\Big/\left\{2\pi\Omega r_i^3 L\right\}}$ *Ans.*

1.50 A simple viscometer measures the time *t* for a solid sphere to fall a distance *L* through a test fluid of density ρ. The fluid viscosity μ is then given by

$$\mu \approx \frac{W_{net}t}{3\pi DL} \quad if \quad t \ge \frac{2\rho DL}{\mu}$$

where *D* is the sphere diameter and W_{net} is the sphere net weight in the fluid.
(a) Show that both of these formulas are dimensionally homogeneous. (b) Suppose that a 2.5 mm diameter aluminum sphere (density 2700 kg/m^3) falls in an oil of density 875 kg/m^3. If the time to fall 50 cm is 32 s, estimate the oil viscosity and verify that the inequality is valid.

Solution: (a) Test the dimensions of each term in the two equations:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} \quad \text{and} \quad \left\{\frac{W_{\text{net}}\, t}{(3\pi)DL}\right\} = \left\{\frac{(ML/T^2)(T)}{(1)(L)(L)}\right\} = \left\{\frac{M}{LT}\right\} \quad \textbf{Yes}, \text{ dimensions OK.}$$

$$\{t\} = \{T\} \quad \text{and} \quad \left\{\frac{2\rho DL}{\mu}\right\} = \left\{\frac{(1)(M/L^3)(L)(L)}{M/LT}\right\} = \{T\} \quad \textbf{Yes}, \text{ dimensions OK.} \quad \textit{Ans.}\text{ (a)}$$

(b) Evaluate the two equations for the data. We need the net weight of the sphere in the fluid:

$$W_{\text{net}} = (\rho_{\text{sphere}} - \rho_{\text{fluid}})g(Vol)_{\text{fluid}} = (2700 - 875 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(\pi/6)(0.0025 \text{ m})^3$$
$$= 0.000146 \text{ N}$$

$$\text{Then } \mu = \frac{W_{\text{net}}t}{3\pi DL} = \frac{(0.000146\ N)(32\ s)}{3\pi(0.0025\ m)(0.5\ m)} = \mathbf{0.40}\ \frac{kg}{m\cdot s} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Check } t = 32\ s \text{ compared to } \frac{2\rho \text{DL}}{\mu} = \frac{2(875\ kg/m^3)(0.0025\ m)(0.5\ m)}{0.40\ kg/m\cdot s}$$
$$= 5.5\ s \text{ OK, } t \text{ is greater}$$

P1.51 An approximation for the boundary-layer shape in Figs. 1.6*b* and P1.51 is the formula

$$u(y) \approx U\sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in Fig. P.151. If the fluid is helium at 20°C and 1 atm, and if U = 10.8 m/s and δ = 3 mm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa; and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

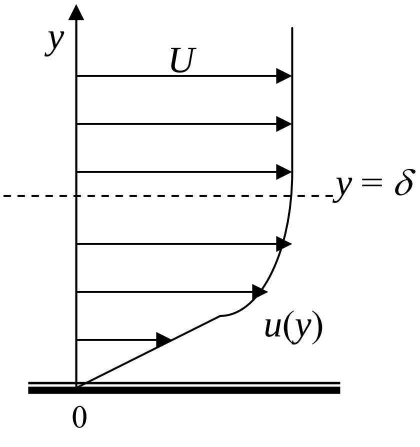


Fig. P1.51

Solution: From Table A.4, for helium, take R = 2077 m^2/(s^2-K) and μ = 1.97E-5 kg/m-s. (*a*) Then the wall shear stress is calculated as

$$\tau_w = \mu\frac{\partial u}{\partial y}|_{y=0} = \mu(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta})_{y=0} = \frac{\pi\,\mu U}{2\delta}$$

$$\text{Numerical values: } \tau_w = \frac{\pi(1.97E-5\,kg/m-s)(10.8\,m/s)}{2(0.003\,m)} = \mathbf{0.11\ Pa} \qquad \textit{Ans.(a)}$$

A very small shear stress, but it has a profound effect on the flow pattern.

(*b*) The variation of shear stress across the boundary layer is a cosine wave, $\tau = \mu\,(du/dy)$:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos(\frac{\pi y}{2\delta}) = \tau_w\cos(\frac{\pi y}{2\delta}) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or: } \mathbf{y} = \frac{\mathbf{2\delta}}{\mathbf{3}} \qquad \textit{Ans.(b)}$$

1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-drive power P required as a function of (h, L, V, B, μ). Neglect air drag. What power P in watts is required if the belt moves at 2.5 m/s over SAE 30W oil at 20°C, with L = 2 m, b = 60 cm, and h = 3 cm?

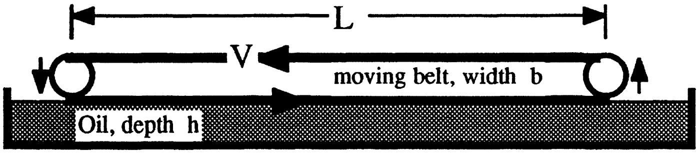


Fig. P1.52

Solution: The power is the viscous resisting force times the belt velocity:

$$P = \tau_{oil}A_{belt}V_{belt} \approx \left(\mu\frac{V}{h}\right)(bL)V = \boldsymbol{\mu V^2 b\frac{L}{h}} \qquad \textit{Ans.}$$

(b) For SAE 30W oil, $\mu \approx 0.29$ kg/m·s. Then, for the given belt parameters,

$$P = \mu V^2 bL/h = \left(0.29\ \frac{kg}{m\cdot s}\right)\left(2.5\ \frac{m}{s}\right)^2(0.6\ m)\frac{2.0\ m}{0.03\ m} \approx 73\ \frac{kg\cdot m^2}{s^3} = \mathbf{73\ W} \qquad \textit{Ans.}\text{ (b)}$$

1.53* A solid cone of base r₀ and initial angular velocity ω_0 is rotating inside a conical seat. Neglect air drag and derive a formula for the cone's angular velocity ω(t) if there is no applied torque.

Solution: At any radial position r < r₀ on the cone surface and instantaneous rate ω,

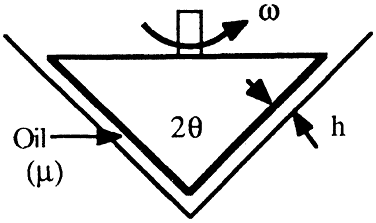


Fig. P1.53

$$d(\text{Torque}) = r\tau\, dA_w = r\left(\mu\frac{r\omega}{h}\right)\left(2\pi r\frac{dr}{\sin\theta}\right),$$

$$\text{or:}\quad \text{Torque } M = \int_0^{r_o}\frac{\mu\omega}{h\sin\theta}2\pi r^3\, dr = \frac{\pi\mu\omega r_o^4}{2h\sin\theta}$$

We may compute the cone's slowing down from the angular momentum relation:

$$M = -I_o\frac{d\omega}{dt},\quad \text{where } I_o(\text{cone}) = \frac{3}{10}mr_o^2,\quad m = \text{cone mass}$$

Separating the variables, we may integrate:

$$\int_{\omega_0}^{w}\frac{d\omega}{\omega} = -\frac{\pi\mu r_o^4}{2hI_o\sin\theta}\int_0^t dt,\quad \text{or:}\quad \boldsymbol{\omega = \omega_o\, \exp\left[-\frac{5\pi\mu r_o^2 t}{3mh\sin\theta}\right]}\quad \textit{Ans.}$$

1.54* A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

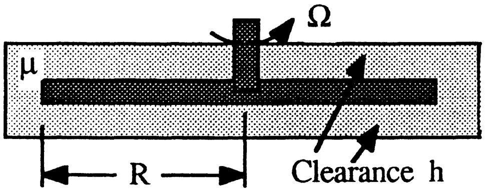


Fig. P1.54

Solution: At any r ≤ R, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\tau\, dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\, dr,$$

$$\text{or:}\quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\, dr = \boldsymbol{\frac{\pi\mu\Omega R^4}{h}}\quad \textit{Ans.}$$

P1.55 A block of weight *W* is being pulled over a table by another weight W_o, as shown in Fig. P1.55. Find an algebraic formula for the steady velocity *U* of the block if it slides on an oil film of thickness *h* and viscosity *μ*. The block bottom area *A* is in contact with the oil. Neglect the cord weight and the pulley friction.

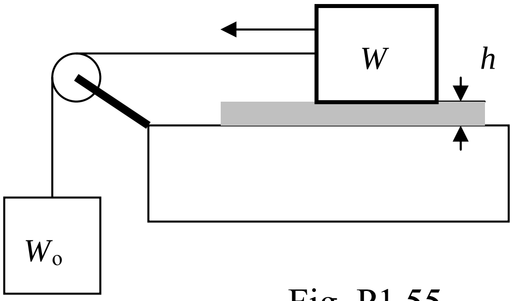


Fig. P1.55

Solution: This problem is a lot easier to *solve* than to set up and sketch. For steady motion,

$$\sum F_{x,block} = 0 = \tau A - W_o = (\mu \frac{U}{h})A - W_o$$

$$\text{Solve for} \quad U = \frac{W_o h}{\mu A} \qquad \textit{Ans.}$$

there is no acceleration, and the falling weight balances the viscous resistance of the oil film: The block weight *W* has no effect on steady horizontal motion except to smush the oil film.

1.56* For the cone-plate viscometer in Fig. P1.56, the angle is very small, and the gap is filled with test liquid *μ*. Assuming a linear velocity profile, derive a formula for the viscosity *μ* in terms of the torque M and cone parameters.

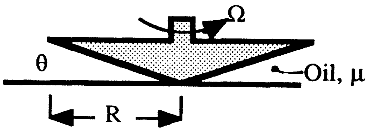


Fig. P1.56

Solution: For any radius r ≤ R, the liquid gap is h = r tan*θ*. Then

$$\text{d(Torque)} = \text{dM} = \tau \text{dA}_w r = \left(\mu \frac{\Omega r}{r \tan\theta}\right)\left(2\pi r \frac{dr}{\cos\theta}\right) r, \quad \text{or}$$

$$M = \frac{2\pi\Omega\mu}{\sin\theta}\int_0^R r^2\,dr = \frac{2\pi\Omega\mu R^3}{3\sin\theta}, \quad \text{or:} \quad \mu = \frac{\mathbf{3M\sin\theta}}{\mathbf{2\pi\Omega R^3}} \qquad \textit{Ans.}$$

P1.57 Extend the steady flow between a fixed lower plate and a moving upper plate, from Fig. 1.8, to the case of two immiscible liquids between the plates, as in Fig. P1.57.

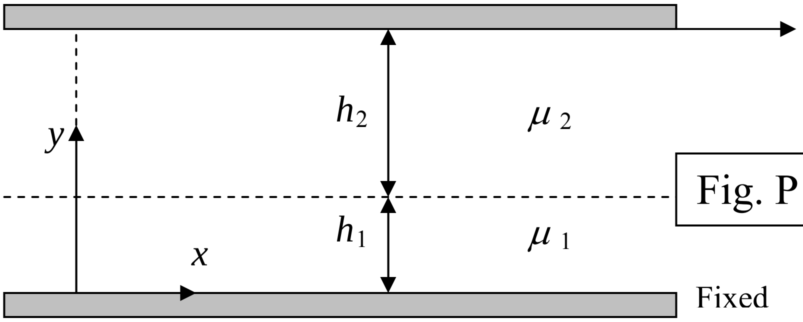
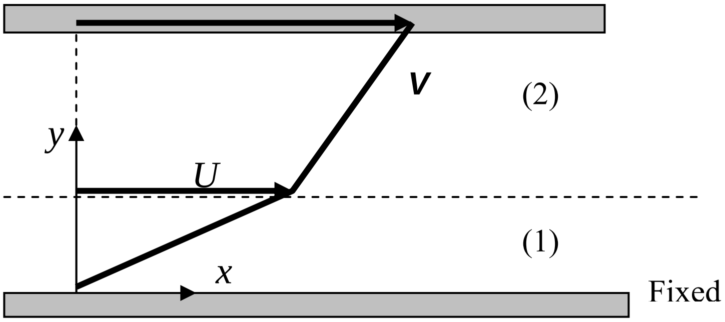


Fig. P1.57

(*a*) Sketch the expected no-slip velocity distribution *u*(*y*) between the plates. (*b*) Find an analytic expression for the velocity *U* at the interface between the two liquid layers. (*c*) What is the result if the viscosities and layer thicknesses are equal?

Solution: We begin with the hint, from Fig. 1.8, that the shear stress is constant between the two plates. The velocity profile would be a straight line in each layer, with different slopes:



Ans. (*a*)

Here we have drawn the case where $\mu_2 > \mu_1$, hence the upper profile slope is less. (*b*) Set the two shear stresses equal, assuming no-slip at each wall:

$$\tau_1 = \mu_1 \frac{(U-0)}{h_1} = \tau_2 = \mu_2 \frac{(V-U)}{h_2}$$

$$\text{Solve for} \quad U = V\left[\frac{1}{1+\dfrac{\mu_1 h_2}{\mu_2 h_1}}\right] \qquad \textit{Ans.}(b)$$

(*c*) For equal viscosities and layer thicknesses, we get the simple result $U = V/2$. *Ans.*

1.58 The laminar-pipe-flow example of Prob. 1.14 leads to a *capillary viscometer* [29], using the formula $\mu = \pi r_o^4 \Delta p/(8LQ)$. Given $r_o = 2$ mm and L = 25 cm. The data are

Q, m^3/hr:	0.36	0.72	1.08	1.44	1.80
Δp, kPa:	159	318	477	1274	1851

Estimate the fluid viscosity. What is wrong with the last two data points?

Solution: Apply our formula, with consistent units, to the first data point:

$$\Delta p = 159\,\text{kPa}: \quad \mu \approx \frac{\pi r_o^4 \Delta p}{8LQ} = \frac{\pi(0.002\text{ m})^4(159000\text{ N/m}^2)}{8(0.25\text{ m})(0.36/3600\text{ m}^3/\text{s})} \approx 0.040\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

Do the same thing for all five data points:

Δp, kPa:	159	318	477	1274	1851
μ, N·s/m^2:	**0.040**	**0.040**	**0.040**	*0.080*(?)	*0.093*(?) *Ans.*

The last two estimates, though measured properly, are *incorrect*. The Reynolds number of the capillary has risen above 2000 and the flow is *turbulent*, which requires a different formula.

1.59 A solid cylinder of diameter D, length L, density ρ_s falls due to gravity inside a tube of diameter D_o. The clearance, $(D_o - D) \ll D$, is filled with a film of viscous fluid (ρ, μ). Derive a formula for terminal fall velocity and apply to SAE 30 oil at 20°C for a steel cylinder with D = 2 cm, D_o = 2.04 cm, and L = 15 cm. Neglect the effect of any air in the tube.

Solution: The geometry is similar to Prob. 1.47, only vertical instead of horizontal. At terminal velocity, the cylinder weight should equal the viscous drag:

$$a_z = 0: \quad \Sigma F_z = -W + \text{Drag} = -\rho_s g \frac{\pi}{4} D^2 L + \left[\mu \frac{V}{(D_o - D)/2}\right] \pi D L,$$

$$\text{or:} \quad V = \frac{\boldsymbol{\rho_s g D (D_o - D)}}{\boldsymbol{8\mu}} \quad \textit{Ans.}$$

For the particular numerical case given, $\rho_{steel} \approx 7850\ \text{kg/m}^3$. For SAE 30 oil at 20°C, $\mu \approx 0.29$ kg/m·s from Table 1.4. Then the formula predicts

$$V_{terminal} = \frac{\rho_s g D(D_o - D)}{8\mu} = \frac{(7850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.02\ \text{m})(0.0204 - 0.02\ \text{m})}{8(0.29\ \text{kg/m}\cdot\text{s})}$$

$$\approx \mathbf{0.265\ m/s} \quad \textit{Ans.}$$

P1.60 Pipelines are cleaned by pushing through them a close-fitting cylinder called a *pig*. The name comes from the squealing noise it makes sliding along. Ref. 50 describes a new non-toxic pig, driven by compressed air, for cleaning cosmetic and beverage pipes. Suppose the pig diameter is 5-15/16 in and its length 26 in. It cleans a 6-in-diameter pipe at a speed of 1.2 m/s. If the clearance is filled with glycerin at 20°C, what pressure difference, in pascals, is needed to drive the pig? Assume a linear velocity profile in the oil and neglect air drag.

Solution: Since the problem calls for *pascals*, convert everything to SI units:
Find the shear stress in the oil, multiply that by the cylinder wall area to get the required force, and divide the force by the area of the cylinder face to find the required pressure difference.

Comment: The Reynolds number of the clearance flow, Re = $\rho VC/\mu$, is approximately 0.8.

$$D_{cyl} = (5\tfrac{15}{16}\,in)(0.0254\frac{m}{in}) = 0.1508\,m \quad ; \quad L = (26\,in)(0.0254\frac{m}{in}) = 0.6604\,m$$

$$\text{Clearance} = C = (D_{pipe} - D_{cyl})/2 = (6 - 5\tfrac{15}{16}\,in)(0.0254\frac{m}{in})/2 = 0.000794\,m$$

$$\text{Table A.3, glycerin:} \quad \mu = 1.49\ kg/m-s \ , \quad \rho = 1260\ kg/m^3 \quad (\rho\ \text{not needed})$$

$$\text{Shear stress } \tau = \mu V/C = (1.49\ kg/m-s)(1.2\,m/s)/0.000794\,m = 2252\,N/m^2$$

$$\text{Shear force } F = \tau A_{wall} = \tau(\pi D_{cyl} L) =$$

$$(2252\,N/m^2)[\pi(0.1508\,m)(0.6604\,m)] = 705\,N$$

$$\text{Finally,} \quad \Delta p = \frac{F}{A_{cyl\ face}} = \frac{705\,N}{(\pi/4)(0.1508\,m)^2} = \mathbf{39500\ Pa} \quad \textit{Ans.}$$

1.61 An air-hockey puck has m = 50 g and D = 9 cm. When placed on a 20°C air table, the blower forms a 0.12-mm-thick air film under the puck. The puck is struck with an initial velocity of 10 m/s. How long will it take the puck to (a) slow down to 1 m/s; (b) stop completely? Also (c) how far will the puck have travelled for case (a)?

Solution: For air at 20°C take $\mu \approx 1.8\text{E}{-}5$ kg/m·s. Let *A* be the bottom area of the puck, $A = \pi D^2/4$. Let *x* be in the direction of travel. Then the only force acting in the *x* direction is the air drag resisting the motion, assuming a linear velocity distribution in the air:

$$\sum F_x = -\tau A = -\mu \frac{V}{h} A = m\frac{dV}{dt}, \quad \text{where h = air film thickness}$$

Separate the variables and integrate to find the velocity of the decelerating puck:

$$\int_{V_o}^{V} \frac{dV}{V} = -K\int_0^t dt, \quad \text{or} \quad V = V_o e^{-Kt}, \quad \text{where } K = \frac{\mu A}{mh}$$

Integrate again to find the displacement of the puck:

$$x = \int_0^t V\,dt = \frac{V_o}{K}[1 - e^{-Kt}]$$

Apply to the particular case given: air, $\mu \approx 1.8\text{E}{-}5$ kg/m·s, m = 50 g, D = 9 cm, h = 0.12 mm, V_o = 10 m/s. First evaluate the time-constant K:

$$K = \frac{\mu A}{mh} = \frac{(1.8\text{E–5 kg/m}\cdot\text{s})[(\pi/4)(0.09\text{ m})^2]}{(0.050\text{ kg})(0.00012\text{ m})} \approx 0.0191\text{ s}^{-1}$$

(a) When the puck slows down to 1 m/s, we obtain the time:

$$V = 1\text{ m/s} = V_o e^{-Kt} = (10\text{ m/s})e^{-(0.0191\text{ s}^{-1})t}, \quad \text{or} \quad t \approx \mathbf{121\ s} \quad \textit{Ans.}\text{ (a)}$$

(b) The puck will stop completely only when $\mathbf{e^{-Kt} = 0}$**, or: $t = \infty$** *Ans.* (b)
(c) For part (a), the puck will have travelled, in 121 seconds,

$$x = \frac{V_o}{K}(1 - e^{-Kt}) = \frac{10\text{ m/s}}{0.0191\text{ s}^{-1}}[1 - e^{-(0.0191)(121)}] \approx \mathbf{472\ m} \quad \textit{Ans.}\text{(c)}$$

This may perhaps be a little unrealistic. But the air-hockey puck *does* decelerate slowly!

1.62 The hydrogen bubbles in Fig. 1.13 have D ≈ 0.01 mm. Assume an "air-water" interface at 30°C. What is the excess pressure within the bubble?

Solution: At 30°C the surface tension from Table A-1 is 0.0712 N/m. For a droplet or bubble with one spherical surface, from Eq. (1.32),

$$\Delta p = \frac{2Y}{R} = \frac{2(0.0712\ \text{N/m})}{(5\text{E–}6\ \text{m})} \approx \mathbf{28500\ Pa} \quad \textit{Ans.}$$

1.63 Derive Eq. (1.37) by making a force balance on the fluid interface in Fig. 1.9c.

Solution: The surface tension forces YdL1 and YdL2 have a slight vertical component. Thus summation of forces in the vertical gives the result

$$\sum F_z = 0 = 2YdL_2 \sin(d\theta_1/2) \\ + 2YdL_1 \sin(d\theta_2/2) - \Delta p\, dA$$

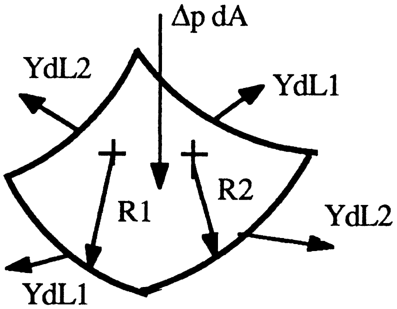


Fig. 1.9c

But dA = dL1dL2 and sin(dθ/2) ≈ dθ/2, so we may solve for the pressure difference:

$$\Delta p = Y\frac{dL_2 d\theta_1 + dL_1 d\theta_2}{dL_1 dL_2} = Y\left(\frac{d\theta_1}{dL_1} + \frac{d\theta_2}{dL_2}\right) = \mathbf{Y}\left(\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}}\right) \quad \textit{Ans.}$$

P1.64 Determine the maximum diameter of a solid aluminum ball, density ρ_o = 2700 kg/m^3, which will “float” on a clean water-air surface at 20°C.

Solution: At maximum-weight condition, the ball will float with the surface tension supporting force nearly vertical. As shown in the figure, the weight of the ball will be balanced by the upward surface tension force.

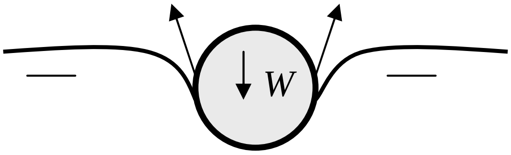


Fig. P1.64

$$\Upsilon \pi D = \rho_o g \frac{\pi}{6} D^3$$

$$or \quad D_{\max} = \sqrt{\frac{6\Upsilon}{\rho_o g}}$$

From Table A.3, surface tension Y for a clean water-air surface is 0.0728 N/m. Thus compute

$$D_{\max} = \sqrt{\frac{6(0.0728 N/m)}{(2700 kg/m^3)(9.81 m/s^2)}} = 0.00406 m = \mathbf{4.06} mm \quad Ans.$$

1.65 The system in Fig. P1.65 is used to estimate the pressure p_1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C. Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water; and (b) mercury.

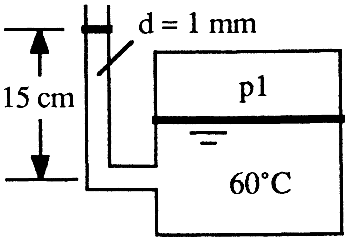


Fig. P1.65

Solution: This is a somewhat more realistic variation of Ex. 1.9. Use values from that example for contact angle θ:
(a) Water at 60°C: $\gamma \approx 9640$ N/m^3, $\theta \approx 0°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.0662 \text{ N/m})\cos(0°)}{(9640 \text{ N/m}^3)(0.001 \text{ m})} = 0.0275 \text{ m},$$

$$\text{or:} \quad \Delta h_{true} = 15.0 - 2.75 \text{ cm} \approx \mathbf{12.25 \text{ cm} (+22\% \text{ error})} \quad Ans. \text{ (a)}$$

(b) Mercury at 60°C: $\gamma \approx 132200$ N/m^3, $\theta \approx 130°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.47 \text{ N/m})\cos 130°}{(132200 \text{ N/m}^3)(0.001 \text{ m})} = -0.0091 \text{ m},$$

or: $\Delta h_{true} = 15.0 + 0.91 \approx \mathbf{15.91\,cm\,(-6\%\,error)}$ *Ans.* (b)

1.66 A thin wire ring, 3 cm in diameter, is lifted from a water surface at 20°C. What is the lift force required? Is this a good method? Suggest a ring material.

Solution: In the literature this ring-pull device is called a DuNouy Tensiometer. The forces are very small and may be measured by a calibrated soft-spring balance. Platinum-iridium is recommended for the ring, being noncorrosive and highly wetting to most liquids. There are two surfaces, inside and outside the ring, so the total force measured is

$$F = 2(Y\pi D) = 2Y\pi D$$

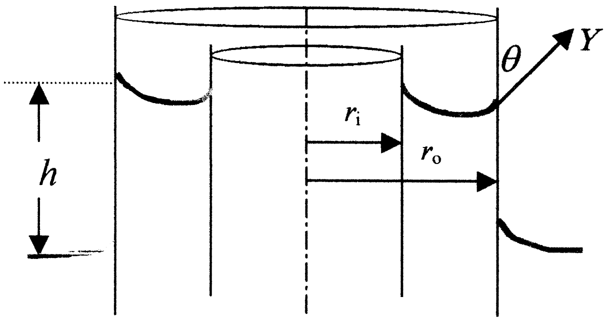
This is crude—commercial devices recommend multiplying this relation by a correction factor $f = O(1)$ which accounts for wire diameter and the distorted surface shape.

For the given data, $Y \approx 0.0728$ N/m (20°C water/air) and the estimated pull force is

$$F = 2\pi(0.0728\text{ N/m})(0.03\text{ m}) \approx \mathbf{0.0137\ N} \quad \textit{Ans.}$$

For further details, see, e.g., F. Daniels et al., *Experimental Physical Chemistry*, 7th ed., McGraw-Hill Book Co., New York, 1970.

1.67 A vertical concentric annulus, with outer radius r_o and inner radius r_i, is lowered into fluid of surface tension Y and contact angle $\theta < 90°$. Derive an expression for the capillary rise h in the annular gap, if the gap is very narrow.



Solution: For the figure above, the force balance on the annular fluid is

$$Y\cos\theta(2\pi r_o + 2\pi r_i) = \rho g\pi\left(r_o^2 - r_i^2\right)h$$

Cancel where possible and the result is

$$\boldsymbol{h = 2Y\cos\theta/\{\rho g(r_o - r_i)\}} \quad \textit{Ans.}$$

1.68* Analyze the shape $\eta(x)$ of the water-air interface near a wall, as shown. Assume small slope, $R^{-1} \approx d^2\eta/dx^2$. The pressure difference across the interface is $\Delta p \approx \rho g\eta$, with a contact angle θ at x = 0 and a horizontal surface at x = ∞. Find an expression for the maximum height *h*.

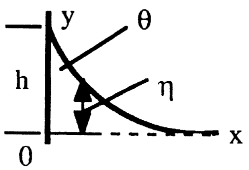


Fig. P1.68

Solution: This is a two-dimensional surface-tension problem, with single curvature. The surface tension rise is balanced by the weight of the film. Therefore the differential equation is

$$\Delta p = \rho g\eta = \frac{Y}{R} \approx Y\frac{d^2\eta}{dx^2} \quad \left(\frac{d\eta}{dx} << 1\right)$$

This is a second-order differential equation with the well-known solution,

$$\eta = C_1\exp[Kx] + C_2\exp[-Kx], \quad K = \sqrt{(\rho g/Y)}$$

To keep η from going infinite as x = ∞, it must be that **C1 = 0**. The constant C2 is found from the maximum height at the wall:

$$\eta|_{x=0} = h = C_2\exp(0), \quad \text{hence } C_2 = h$$

Meanwhile, the contact angle shown above must be such that,

$$\frac{d\eta}{dx}\bigg|_{x=0} = -\cot(\theta) = -hK, \quad \text{thus } h = \frac{\cot\theta}{K}$$

The complete (small-slope) solution to this problem is:

$$\boldsymbol{\eta = h\exp[-(\rho g/Y)^{1/2}x]}, \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if $x = \infty$. It requires "small slope" and therefore the contact angle should be in the range $70° < \theta < 110°$.

1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maximum diameter needle able to float on the surface.

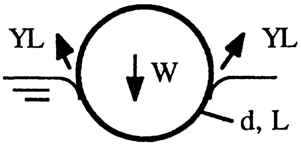


Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g\frac{\pi}{4}d^2L, \quad \text{or:} \quad \mathbf{d_{max}} \approx \sqrt{\frac{\mathbf{8Y}}{\boldsymbol{\pi\rho g}}} \quad \textit{Ans.} \text{ (a)}$$

(b) Calculate d_{max} for a steel needle (SG ≈ 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi\rho g}} = \sqrt{\frac{8(0.073 \text{ N/m})}{\pi(7.84 \times 998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}} \approx 0.00156 \text{ m} \approx \mathbf{1.6\ mm} \quad \textit{Ans.} \text{ (b)}$$

1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

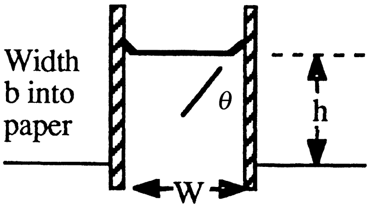


Fig. P1.70

Solution: With *b* the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho gWhb = 2(Yb\cos\theta), \quad \text{or:} \quad h \approx \frac{\mathbf{2Y\cos\theta}}{\boldsymbol{\rho g W}} \quad \textit{Ans.}$$

For water at 20°C, $Y \approx 0.0728$ N/m, $\rho g \approx 9790$ N/m^3, and $\theta \approx 0°$. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728\text{ N/m})\cos 0°}{(9790\text{ N/m}^3)(0.0005\text{ m})} \approx 0.030\text{ m} \approx \mathbf{30\ mm} \quad \textit{Ans.}$$

1.71* A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1\upsilon_1 + \rho_2\upsilon_2 = m_3 = \rho_3\upsilon_3,$$

$$\text{or:}\quad \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_aD_3^3 + 8YD_3^2 = \left(p_aD_2^3 + 8YD_2^2\right) + \left(p_aD_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

Solution: From Table A-5 at 84°C, vapor pressure $p_v \approx 55.4$ kPa. We may use this value to interpolate in the standard altitude, Table A-6, to estimate

$$z \approx \mathbf{4800\ m} \quad \textit{Ans.}$$

1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read $p_v = 2.337$ kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\text{ kg/m}^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\text{ (a)}$$

If we decrease water temperature to 5°C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

$$\mathrm{Ca} = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans*. (b)

1.74 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally-spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

Solution: The absolute maximum length L occurs when the pump inlet pressure is slightly greater than 20 kPa. The pump increases this by 1.3 MPa and friction drops the pressure over a distance L until it again reaches 20 kPa. In other words, quite simply,

$$1.3\text{ MPa} = 1{,}300{,}000\text{ Pa} = (150\text{ Pa/m})L, \quad \text{or} \quad L_{max} \approx \mathbf{8660\ m} \quad \textit{Ans.}$$

It makes more sense to have the pump inlet at 1 atm, not 20 kPa, dropping L to about 8 km.

1.75 An airplane flies at 555 mi/h. At what altitude in the standard atmosphere will the airplane's Mach number be exactly 0.8?

Solution: First convert V = 555 mi/h x 0.44704 = 248.1 m/s. Then the speed of sound is

$$a = \frac{V}{\mathrm{Ma}} = \frac{248.1 m/s}{0.8} = 310 m/s$$

Reading in Table A.6, we estimate the altitude to be approximately **7500 m**. *Ans*.

1.76 Estimate the speed of sound of steam at 200°C and 400 kPa, (a) by an ideal-gas approximation (Table A.4); and (b) using EES (or the Steam Tables) and making small isentropic changes in pressure and density and approximating Eq. (1.38).
Solution: (a) For steam, $k \approx 1.33$ and $R = 461$ m^2/s^2·K. The ideal gas formula predicts:

$$a \approx \sqrt{(kRT)} = \sqrt{\{1.33(461\text{ m}^2/\text{s}^2\cdot\text{K})(200+273\text{ K})\}} \approx \mathbf{539\ m/s} \quad \textit{Ans.}\text{(a)}$$

(b) *The old way to do this*: We use the formula $a = \sqrt{(\partial p/\partial \rho)_s} \approx \sqrt{\{\Delta p|_s/\Delta\rho|_s\}}$ for small isentropic changes in p and ρ. From EES, at 200°C and 400 kPa, the entropy is $s = 1.872$ kJ/kg·K. Raise and lower the pressure 1 kPa at the same entropy. At $p = 401$ kPa, $\rho = 1.87565$ kg/m^3. At $p = 399$ kPa, $\rho = 1.86849$ kg/m^3. Thus $\Delta\rho = 0.00716$ kg/m^3, $\Delta p = 2000$ Pa, and the formula for sound speed predicts:

$$a \approx \sqrt{\{\Delta p|_s/\Delta\rho|_s\}} = \sqrt{\{(2000\ \text{N/m}^2)/(0.00716\ \text{kg/m}^3)\}} = 529\ m/s \quad \textit{Ans.}\text{(b)}$$

Again, as in Prob. 1.34, the ideal gas approximation is within 2% of a Steam-Table solution.

(b1) *The new way to do this*: EES now has a speed-of-sound thermophysical function:

a = SOUNDSPEED(steam, p = 400, T = 200) = **528.8 m/s** *Ans.*(b1)

NOTE: You have to specify a unit system in EES. Here we used kilopascals and degrees C.

1.77 The density of gasoline varies with pressure approximately as follows:

p, atm:	1	500	1000	1500
ρ, lbm/ft^3:	42.45	44.85	46.60	47.98

Estimate (a) its speed of sound, and (b) its bulk modulus at 1 atm.

Solution: For a crude estimate, we could just take differences of the first two points:

$$a \approx \sqrt{(\Delta p/\Delta\rho)} \approx \sqrt{\left\{\frac{(500-1)(2116)\ \text{lbf/ft}^2}{(44.85-42.45)/32.2\ \text{slug/ft}^3}\right\}} \approx 3760\ \frac{\text{ft}}{\text{s}} \approx \mathbf{1150\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$B \approx \rho a^2 = [42.45/32.2\ \text{slug/ft}^3](3760\ \text{ft/s})^2 \approx 1.87\text{E}7\ \frac{\text{lbf}}{\text{ft}^2} \approx \mathbf{895\ MPa} \quad \textit{Ans.}\ \text{(b)}$$

For more accuracy, we could fit the data to the nonlinear equation of state for liquids, Eq. (1.22). The best-fit result for gasoline (data above) is $n \approx 8.0$ and $B \approx 900$.

Equation (1.22) is too simplified to show temperature or entropy effects, so we assume that it approximates "isentropic" conditions and thus differentiate:

$$\frac{p}{p_a} \approx (B+1)(\rho/\rho_a)^n - B, \quad \text{or:} \quad a^2 = \frac{dp}{d\rho} \approx \frac{n(B+1)p_a}{\rho_a}(\rho/\rho_a)^{n-1}$$

$$\text{or, at 1 atm,} \quad a_{\text{liquid}} \approx \sqrt{n(B+1)p_a/\rho_a}$$

The bulk modulus of gasoline is thus approximately:

$$\text{“B”} = \rho \frac{dp}{d\rho}\Big|_{1\text{ atm}} = n(B+1)p_a = (8.0)(901)(101350\text{ Pa}) \approx \mathbf{731\ MPa} \quad \textit{Ans.}\text{ (b)}$$

And the speed of sound in gasoline is approximately,

$$a_{1\text{ atm}} = [(8.0)(901)(101350\text{ Pa})/(680\text{ kg/m}^3)]^{1/2} \approx \mathbf{1040\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

1.78 Sir Isaac Newton measured sound speed by timing the difference between seeing a cannon's puff of smoke and hearing its boom. If the cannon is on a mountain 5.2 miles away, estimate the air temperature in °C if the time difference is (a) 24.2 s; (b) 25.1 s.

Solution: Cannon booms are finite (shock) waves and travel slightly faster than sound waves, but what the heck, assume it's close enough to sound speed:

$$\text{(a)}\quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{24.2} = 345.8\ \frac{m}{s} = \sqrt{1.4(287)T}, \quad T \approx 298\text{ K} \approx \mathbf{25°C} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{25.1} = 333.4\ \frac{m}{s} = \sqrt{1.4(287)T}, \quad T \approx 277\text{ K} \approx \mathbf{4°C} \quad \textit{Ans.}\text{ (b)}$$

P1.79 From Table A.3, the density of glycerin at standard conditions is about 1260 kg/m^3. At a very high pressure of 8000 lb/in^2, its density increases to approximately 1275 kg/m^3. Use this data to estimate the speed of sound of glycerin, in ft/s.

Solution: For a liquid, we simplify Eq. (1.38) to a pressure-density ratio, without knowing if the process is isentropic or not. This should give satisfactory accuracy:

$$a^2\,|_{glycerin} \approx \frac{\Delta p}{\Delta \rho} = \frac{(8000-15\,lb/in^2)(6895\,Pa/psi)}{(1275-1260)\,kg/m^3} = 3.67E6\ \frac{m^2}{s^2}$$

$$\textit{Hence}\quad a \approx \sqrt{3.67E6} \approx 1920\,m/s \approx \mathbf{6300}\ ft/s \qquad \textit{Ans.}$$

The accepted value, in Table 9.1, is 6100 ft/s. This accuracy (3%) is very good, considering the small change in density (1.2%).

P1.80 In Problem P1.24, for the given data, the air velocity at section 2 is 1180 ft/s. What is the Mach number at that section?

Solution: First convert 1180 ft/s to 360 m/s. With T_2 given as 223 K, evaluate the speed of sound at section 2, assuming, as stated, an ideal gas:

$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287\,m^2/s^2 - K)(223\,K)} = 299\ m/s$$

$$\text{Then}\quad Ma_2 = \frac{V_2}{a_2} = \frac{360\,m/s}{299\,m/s} \approx \mathbf{1.20} \quad \textit{Ans.}$$

This kind of calculation will be a large part of the material in Chap. 9, Compressible Flow.

1.81 Repeat Ex. 1.13 by letting the velocity components increase linearly with time:

$$\mathbf{V} = \mathrm{Kxt}\mathbf{i} - \mathrm{Kyt}\mathbf{j} + 0\mathbf{k}$$

Solution: The flow is unsteady and two-dimensional, and Eq. (1.44) still holds:

$$\text{Streamline:}\quad \frac{dx}{u} = \frac{dy}{v}, \quad \text{or:}\quad \frac{dx}{Kxt} = \frac{dy}{-Kyt}$$

The terms K and t both vanish and leave us with the same result as in Ex. 1.13, that is,

$$\int dx/x = -\int dy/y, \quad \text{or:}\quad \mathbf{xy = C} \quad \textit{Ans.}$$

The streamlines have exactly the same “stagnation flow” shape as in Fig. 1.13. However, the flow *is* accelerating, and the mass flow between streamlines is constantly increasing.

1.82 A velocity field is given by u = V cosθ, v = V sinθ, and w = 0, where V and θ are constants. Find an expression for the streamlines of this flow.

Solution: Equation (1.44) may be used to find the streamlines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{V\cos\theta} = \frac{dy}{V\sin\theta}, \quad \text{or:} \quad \frac{dy}{dx} = \tan\theta$$

$$\text{Solution:} \quad y = (\tan\theta)\,x + constant \quad Ans.$$

The streamlines are straight parallel lines which make an angle θ with the *x* axis. In other words, this velocity field represents a *uniform stream V moving upward at angle θ.*

1.83* A two-dimensional *unsteady* velocity field is given by u = x(1 + 2t), v = y. Find the time-varying streamlines which pass through some reference point (x_o,y_o). Sketch some.

Solution: Equation (1.44) applies with time as a parameter:

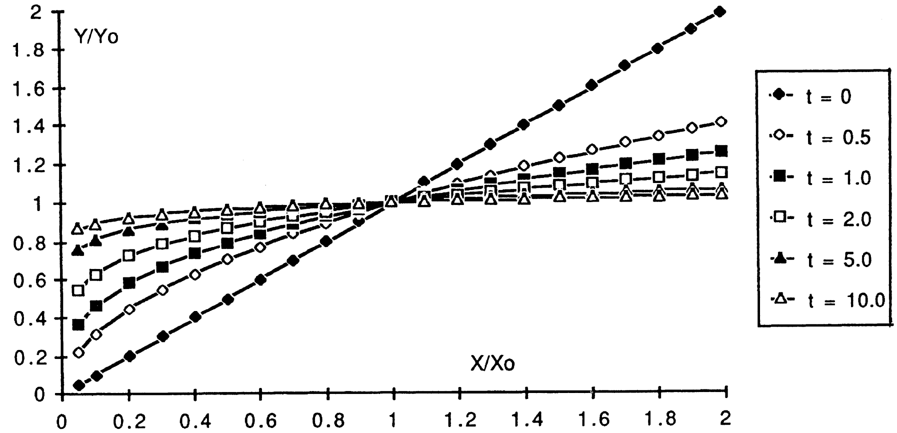
$$\frac{dx}{u} = \frac{dx}{x(1+2t)} = \frac{dy}{v} = \frac{dy}{y}, \quad \text{or:} \quad \ln(y) = \frac{1}{1+2t}\ln(x) + \text{constant}$$

$$\text{or:} \quad y = Cx^{1/(1+2t)}, \quad \text{where C is a constant}$$

In order for all streamlines to pass through y = y_o at x = x_o, the constant must be such that:

$$\boldsymbol{y = y_o(x/x_o)^{1/(1+2t)}} \quad Ans.$$

Some streamlines are plotted on the next page and are seen to be strongly time-varying.



P1.84 In the early 1900's, the British chemist Sir Cyril Hinshelwood quipped that fluid dynamics was divided into "workers who observed things they could not explain and workers who explained things they could not observe". To what historic situation was he referring?

Solution: He was referring to the split between hydraulics engineers, who performed experiments but had no theory for what they were observing, and theoretical hydrodynamicists, who found numerous mathematical solutions, for inviscid flow, that were not verified by experiment.

1.85-a Report to the class on the achievements of *Evangelista Torricelli.*

Solution: Torricelli's biography is taken from a goldmine of information which I did not put in the references, preferring to let the students find it themselves: C. C. Gillespie (ed.), *Dictionary of Scientific Biography*, 15 vols., Charles Scribner's Sons, New York, 1976.

Torricelli (1608–1647) was born in Faenza, Italy, to poor parents who recognized his genius and arranged through Jesuit priests to have him study mathematics, philosophy, and (later) hydraulic engineering under Benedetto Castelli. His work on dynamics of projectiles attracted the attention of Galileo himself, who took on Torricelli as an assistant in 1641. Galileo died one year later, and Torricelli was appointed in his place as "mathematician and philosopher" by Duke Ferdinando II of Tuscany. He then took up residence in Florence, where he spent his five happiest years, until his death in 1647. In 1644 he published his only known printed work, *Opera Geometrica*, which made him famous as a mathematician and geometer.

In addition to many contributions to geometry and calculus, Torricelli was the first to show that a zero-drag projectile formed a *parabolic* trajectory. His tables of trajectories for various angles and initial velocities were used by Italian artillerymen. He was an excellent machinist and constructed—and sold—the very finest telescope lenses in Italy.

Torricelli's hydraulic studies were brief but stunning, leading Ernst Mach to proclaim him the 'founder of hydrodynamics.' He deduced his theorem that the velocity of efflux from a hole in a tank was equal to $\sqrt{(2gh)}$, where h is the height of the free surface above the hole. He also showed that the efflux jet was parabolic and even commented on water-droplet breakup and the effect of air resistance. By experimenting with various liquids in closed tubes—including mercury (from mines in Tuscany)—he thereby invented the *barometer*. From barometric pressure (about 30 feet of water) he was able to explain why siphons did not work if the elevation change was too large. He also was the first to explain that winds were produced by temperature and *density differences* in the atmosphere and not by "evaporation."

1.85-b Report to the class on the achievements of *Henri de Pitot*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Pitot (1695–1771) was born in Aramon, France, to patrician parents. He hated to study and entered the military instead, but only for a short time. Chance reading of a textbook obtained in Grenoble led him back to academic studies of mathematics, astronomy, and engineering. In 1723 he became assistant to Réamur at the French Academy of Sciences and in 1740 became a civil engineer upon his appointment as a director of public works in Languedoc Province. He retired in 1756 and returned to Aramon until his death in 1771.

Pitot's research was apparently mediocre, described as "competent solutions to minor problems without lasting significance"—not a good recommendation for tenure nowadays! His *lasting* contribution was the invention, in 1735, of the instrument which bears his name: a glass tube bent at right angles and inserted into a moving stream with the opening facing upstream. The water level in the tube rises a distance h above the surface, and Pitot correctly deduced that the stream velocity $\approx \sqrt{(2gh)}$. This is still a basic instrument in fluid mechanics.

1.85-c Report to the class on the achievements of *Antoine Chézy*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Chézy (1718–1798) was born in Châlons-sur-Marne, France, studied engineering at the Ecole des Ponts et Chaussées and then spent his entire career working for this school, finally being appointed Director one year before his death. His chief contribution was to study the flow in open channels and rivers, resulting in a famous formula, used even today, for the average velocity:

$$V \approx \text{const}\sqrt{AS/P}$$

where A is the cross-section area, S the bottom slope, and P the wetted perimeter, i.e., the length of the bottom and sides of the cross-section. The "constant" depends primarily on the roughness of the channel bottom and sides. [See Chap. 10 for further details.]

1.85-d Report to the class on the achievements of *Gotthilf Heinrich Ludwig Hagen*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Hagen (1884) was born in Königsberg, East Prussia, and studied there, having among his teachers the famous mathematician Bessel. He became an engineer, teacher, and writer and published a handbook on hydraulic engineering in 1841. He is best known for his study in 1839 of pipe-flow resistance, for water flow at heads of 0.7 to 40 cm, diameters of 2.5 to 6 mm, and lengths of 47 to 110 cm. The measurements indicated that the pressure drop was proportional to Q at low heads and proportional (approximately) to Q^2 at higher heads, where "strong movements" occurred—turbulence. He also showed that Δp was approximately proportional to D^{-4}.

Later, in an 1854 paper, Hagen noted that the difference between laminar and turbulent flow was clearly visible in the efflux jet, which was either "smooth or fluctuating," and in glass tubes, where sawdust particles either "moved axially" or, at higher Q, "came into whirling motion." Thus Hagen was a true pioneer in fluid mechanics experimentation. Unfortunately, his achievements were somewhat overshadowed by the more widely publicized 1840 tube-flow studies of J. L. M. Poiseuille, the French physician.

1.85-e Report to the class on the achievements of *Julius Weisbach*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a) and also from Rouse and Ince [Ref. 12].

Weisbach (1806–1871) was born near Annaberg, Germany, the 8th of nine children of working-class parents. He studied mathematics, physics, and mechanics at Göttingen and Vienna and in 1931 became instructor of mathematics at Freiberg Gymnasium. In 1835 he was promoted to full professor at the Bergakademie in Freiberg. He published 15 books and 59 papers, primarily on hydraulics. He was a skilled laboratory worker and summarized his results in *Experimental-Hydraulik* (Freiberg, 1855) and in the *Lehrbuch* *der Ingenieur- und Maschinen-Mechanik* (Brunswick, 1845), which was still in print 60 years later. There were 13 chapters on hydraulics in this latter treatise. Weisbach modernized the subject of fluid mechanics, and his discussions and drawings of flow patterns would be welcome in any 20th century textbook—see Rouse and Ince [23] for examples.

Weisbach was the first to write the pipe-resistance head-loss formula in modern form: $h_{f(pipe)} = f(L/D)(V^2/2g)$, where f was the dimensionless 'friction factor,' which Weisbach noted was not a constant but related to the pipe flow parameters [see Sect. 6.4]. He was also the first to derive the "weir equation" for volume flow rate Q over a dam of crest length L:

$$Q \approx \frac{2}{3}C_w(2g)^{1/2}\left[\left(H+\frac{V^2}{2g}\right)^{3/2}-\left(\frac{V^2}{2g}\right)^{3/2}\right] \approx \frac{2}{3}C_w(2g)^{1/2}H^{3/2}$$

where H is the upstream water head level above the dam crest and C_W is a dimensionless weir coefficient ≈ O(unity). [see Sect. 10.7] In 1860 Weisbach received the first Honorary Membership awarded by the German engineering society, the *Verein Deutscher Ingenieure.*

1.85-f Report to the class on the achievements of *George Gabriel Stokes*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Stokes (1819–1903) was born in Skreen, County Sligo, Ireland, to a clergical family associated for generations with the Church of Ireland. He attended Bristol College and Cambridge University and, upon graduation in 1841, was elected Fellow of Pembroke College, Cambridge. In 1849, he became Lucasian Professor at Cambridge, a post once held by Isaac Newton. His 60-year career was spent primarily at Cambridge and resulted in many honors: President of the Cambridge Philosophical Society (1859), secretary (1854) and president (1885) of the Royal Society of London, member of Parliament (1887–1891), knighthood (1889), the Copley Medal (1893), and Master of Pembroke College (1902). A true 'natural philosopher,' Stokes systematically explored hydrodynamics, elasticity, wave mechanics, diffraction, gravity, acoustics, heat, meteorology, and chemistry. His primary research output was from 1840–1860, for he later became tied down with administrative duties.

In hydrodynamics, Stokes has several formulas and fields named after him:

(1) The equations of motion of a linear viscous fluid: the *Navier-Stokes equations*.
(2) The motion of nonlinear deep-water surface waves: *Stokes waves*.
(3) The drag on a sphere at low Reynolds number: *Stokes' formula,* $F = 3\pi\mu VD$.
(4) Flow over immersed bodies for $Re \ll 1$: *Stokes flow*.
(5) A metric (CGS) unit of kinematic viscosity, ν: $1\ cm^2/s = 1$ *stoke.*
(6) A relation between the 1st and 2nd coefficients of viscosity: *Stokes' hypothesis.*
(7) A stream function for axisymmetric flow: *Stokes' stream function* [see Chap. 8].

Although Navier, Poisson, and Saint-Venant had made derivations of the equations of motion of a viscous fluid in the 1820's and 1830's, Stokes was quite unfamiliar with the French literature. He published a completely independent derivation in 1845 of the *Navier-Stokes equations* [see Sect. 4.3], using a 'continuum-calculus' rather than a 'molecular' viewpoint, and showed that these equations were directly analogous to the motion of elastic solids. Although not really new, Stokes' equations were notable for being the first to replace the mysterious French 'molecular coefficient' ε by the coefficient of absolute viscosity, μ.

1.85-g Report to the class on the achievements of *Moritz Weber*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Weber (1871–1951) was professor of naval mechanics at the Polytechnic Institute of Berlin. He clarified the principles of similitude (dimensional analysis) in the form used today. It was he who named the Froude number and the Reynolds number in honor of those workers. In a 1919 paper, he developed a dimensionless surface-tension (capillarity) parameter [see Sect. 5.4] which was later named the *Weber number* in his honor.

1.85-h Report to the class on the achievements of *Theodor von Kármán.*

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Another good reference is his ghost-written (by Lee Edson) autobiography, *The Wind and Beyond*, Little-Brown, Boston, 1967.

Kármán (1881–1963) was born in Budapest, Hungary, to distinguished and well-educated parents. He attended the Technical University of Budapest and in 1906 received a fellowship to Göttingen, where he worked for six years with Ludwig Prandtl, who had just developed boundary layer theory. He received a doctorate in 1912 from Göttingen and was then appointed director of aeronautics at the Polytechnic Institute of Aachen. He remained at Aachen until 1929, when he was named director of the newly formed Guggenheim Aeronautical Laboratory at the California Institute of Technology. Kármán developed CalTech into a premier research center for aeronautics. His leadership spurred the growth of the aerospace industry in southern California. He helped found the Jet Propulsion Laboratory and the Aerojet General Corporation. After World War II, Kármán founded a research arm for NATO, the Advisory Group for Aeronautical Research and Development, whose renowned educational institute in Brussels is now called the Von Kármán Center.

Kármán was uniquely skilled in integrating physics, mathematics, and fluid mechanics into a variety of phenomena. His most famous paper was written in 1912 to explain the puzzling alternating vortices shed behind cylinders in a steady-flow experiment conducted by K. Hiemenz, one of Kármán's students—these are now called *Kármán vortex streets* [see Fig. 5.2a]. Shed vortices are thought to have caused the destruction by winds of the Tacoma Narrows Bridge in 1940 in Washington State.

Kármán wrote 171 articles and 5 books and his methods had a profound influence on fluid mechanics education in the 20th century.

1.85-i Report to the class on the achievements of *Paul Richard Heinrich Blasius.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Blasius (1883–1970) was Ludwig Prandtl's first graduate student at Göttingen. His 1908 dissertation gave the analytic solution for the laminar boundary layer on a flat plate [see Sect. 7.4]. Then, in two papers in 1911 and 1913, he gave the first demonstration that pipe-flow resistance could be nondimensionalized as a plot of friction factor versus Reynolds number—the first "Moody-type" chart. His correlation, $f \approx 0.316\ Re_d^{-1/4}$, is still is use today. He later worked on analytical solutions of boundary layers with variable pressure gradients.

1.85-j Report to the class on the achievements of *Ludwig Prandtl.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Ludwig Prandtl (1875–1953) is described by Rouse and Ince [23] as the father of modern fluid mechanics. Born in Munich, the son of a professor, Prandtl studied engineering and received a doctorate in elasticity. But his first job as an engineer made him aware of the lack of correlation between theory and experiment in fluid mechanics. He conducted research from 1901–1904 at the Polytechnic Institute of Hanover and presented a seminal paper in 1904, outlining the new concept of "boundary layer theory." He was promptly hired as professor and director of applied mechanics at the University of Gottingen, where he remained throughout his career. He, and his dozens of famous students, started a new "engineering science" of fluid mechanics, emphasizing (1) mathematical analysis based upon by physical reasoning; (2) new experimental techniques; and (3) new and inspired flow-visualization schemes which greatly increased our understanding of flow phenomena.

In addition to boundary-layer theory, Prandtl made important contributions to (1) wing theory; (2) turbulence modeling; (3) supersonic flow; (4) dimensional analysis; and (5) instability and transition of laminar flow. He was a legendary engineering professor.

1.85-k Report to the class on the achievements of *Osborne Reynolds*.

Solution: The following notes are from Rouse and Ince [Ref. 12].

Osborne Reynolds (1842–1912) was born in Belfast, Ireland, to a clerical family and studied mathematics at Cambridge University. In 1868 he was appointed chair of engineering at a college which is now known as the University of Manchester Institute of Science and Technology (UMIST). He wrote on wide-ranging topics—mechanics, electricity, navigation—and developed a new hydraulics laboratory at UMIST. He was the first person to demonstrate cavitation, that is, formation of vapor bubbles due to high velocity and low pressure. His most famous experiment, still performed in the undergraduate laboratory at UMIST (see Fig. 6.5 in the text) demonstrated transition of laminar pipe flow into turbulence. He also showed in this experiment that the viscosity was very important and led him to the dimensionless stability parameter $\rho VD/\mu$ now called the *Reynolds number* in his honor. Perhaps his most important paper, in 1894, extended the Navier-Stokes equations (see Eqs. 4.38 of the text) to time-averaged randomly fluctuating turbulent flow, with a result now called the *Reynolds equations* of turbulence. Reynolds also contributed to the concept of the *control volume* which forms the basis of integral analysis of flow (Chap. 3).

1.85-l Report to the class on the achievements of *John William Strutt, Lord Rayleigh.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

John William Strutt (1842–1919) was born in Essex, England, and inherited the title Lord Rayleigh. He studied at Cambridge University and was a traditional hydrodynamicist in the spirit of Euler and Stokes. He taught at Cambridge most of his life and also served as president of the Royal Society. He is most famous for his work (and his textbook) on the theory of sound. In 1904 he won the Nobel Prize for the discovery of argon gas. He made at least five important contributions to hydrodynamics: (1) the equations of bubble dynamics in liquids, now known as *Rayleigh-Plesset theory*; (2) the theory of nonlinear surface waves; (3) the capillary (surface tension) instability of jets; (4) the “heat-transfer analogy” to laminar flow; and (5) dimensional similarity, especially related to viscosity data for argon gas and later generalized into group theory which previewed Buckingham’s Pi Theorem. He ended his career as president, in 1909, of the first British committee on aeronautics.

1.85-m Report to the class on the achievements of *Daniel Bernoulli.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Daniel Bernoulli (1700–1782) was born in Groningen, Holland, his father, Johann, being a Dutch professor. He studied at the University of Basel, Switzerland, and taught mathematics for a few years at St. Petersburg, Russia. There he wrote, and published in 1738, his famous treatise *Hydrodynamica*, for which he is best known. This text contained numerous ingenious drawings illustrating various flow phenomena. Bernoulli used energy concepts to establish proportional relations between kinetic and potential energy, with pressure work added only in the abstract. Thus he never actually derived the famous equation now bearing his name (Eq. 3.77 of the text), later derived in 1755 by his friend Leonhard Euler. Daniel Bernoulli never married and thus never contributed additional members to his famous family of mathematicians.

1.85-n Report to the class on the achievements of *Leonhard Euler.*

Solution: The following notes are from Rouse and Ince [Ref. 12].

Leonhard Euler (1707–1783) was born in Basel, Switzerland, and studied mathematics under Johann Bernoulli, Daniel’s father. He succeeded Daniel Bernoulli as professor of mathematics at the St. Petersburg Academy, leaving there in 1741 to join the faculty of Berlin University. He lost his sight in 1766 but continued to work, aided by a prodigious memory, and produced a vast output of scientific papers, dealing with mathematics, optics, mechanics, hydrodynamics, and celestial mechanics (for which he is most famous today). His famous paper of 1755 on fluid flow derived the full inviscid equations of fluid motion (Eqs. 4.36 of the text) now called *Euler’s equations*. He used a fixed coordinate system, now called the *Eulerian frame of reference*. The paper also presented, for the first time, the correct form of Bernoulli’s equation (Eq. 3.77 of the text). Separately, in 1754 he produced a seminal paper on the theory of reaction turbines, leading to *Euler’s turbine equation* (Eq. 11.11 of the text).

P1.86 A right circular cylinder volume υ is to be calculated from the measured base radius R and height H. If the uncertainty in R is 2% and the uncertainty in H is 3%, estimate the overall uncertainty in the calculated volume.

Solution: The formula for volume is, of course, $\upsilon = \pi R^2 H$. There are two terms to be calculated on the right-hand side of Eq. (1.43):

$$\frac{\partial \upsilon}{\partial R}\delta R \;=\; 2\pi R H\,\delta R \qquad ; \qquad \frac{\partial \upsilon}{\partial H}\delta H \;=\; \pi R^2\;\delta H$$

Expressed as a ratio to the volume, these become

$$\frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,R} \;=\; \frac{2\pi R H\,\delta R}{\pi R^2 H} \;=\; 2\frac{\delta R}{R} \qquad ; \qquad \frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,H} = \frac{\pi R^2\,\delta H}{\pi R^2 H} \;=\; \frac{\delta H}{H}$$

Note $\delta R/R = 2\%$ and $\delta H/H = 3\%$. The overall uncertainty in the volume is a root-mean-square:

$$\frac{\delta \upsilon}{\upsilon} = [(2\frac{\delta R}{R})^2 \;+\; (\frac{\delta H}{H})^2]^{1/2} \;=\; [\{2(2\%)\}^2 + (3\%)^2]^{1/2} = \;(16+9)^{1/2} = \;(25)^{1/2} \;=\; \mathbf{5\%} \qquad \textit{Ans.}$$

Because it is doubled, the error in radius contributes more to the overall uncertainty.

P1.87 A dimensionless parameter, important in natural convection heat transfer of fluids, is the *Grashof number*:

$$\mathrm{Gr} \;=\; \frac{g\;\beta\;\rho^2\;L^3\;\Delta T}{\mu^2}$$

where g is the acceleration of gravity, β is the thermal expansion coefficient, ρ the density, L a characteristic length, ΔT a temperature difference, and μ the viscosity. If the uncertainty of each of these variables is ±2 per cent, determine the overall uncertainty of the Grashof number.

Solution: This grouping of variables fits the conditions of Eq. (1.44):

$$\begin{aligned}\frac{\delta \mathrm{Gr}}{\mathrm{Gr}} &= [(\frac{\delta g}{g})^2 + (\frac{\delta \beta}{\beta})^2 + (2\frac{\delta \rho}{\rho})^2 + (3\frac{\delta L}{L})^2 + (\frac{\delta\,\Delta T}{\Delta T}) + (2\frac{\delta \mu}{\mu})^2]^{1/2} \\ &= \;[(0\%)^2 + (2\%)^2 + \{2(2\%)\}^2 + \{3(2\%)\}^2 + (2\%)^2 + \{2(2\%)\}^2]^{1/2} \\ &= \;\sqrt{76} \;=\; \pm\mathbf{8.7\%} \qquad \textit{Ans.}\end{aligned}$$

The fact that the length error is cubed contributes about two-thirds of this uncertainty.

P1.88 The device in Fig. P1.54 is called a *rotating disk viscometer* [29]. Suppose that R = 5 cm and h = 1 mm. (*a*) If the torque required to rotate the disk at 900 r/min is 0.537 N·m, what is the viscosity of the fluid? (*b*) If the uncertainty in each parameter (M, R, h, Ω) is ±1%, what is the overall uncertainty in the viscosity?

Solution: From Prob. 1.54, the analytical result for the required torque M is

$$M = \frac{\pi \mu \Omega R^4}{h}, \qquad \text{rewrite this as} \quad \mu = \frac{M h}{\pi \Omega R^4}$$

(a) First convert Ω to radians: Ω = 900 r/min x 2π/60 = 94.2 rad/s. Then apply the formula:

$$\mu = \frac{M h}{\pi \Omega R^4} = \frac{(0.537\,N-m)(0.001\,m)}{\pi(94.2 rad/s)(0.05m)^4} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\ \frac{kg}{m-s}} \quad \textit{Ans.}(a)$$

This could be SAE 30W oil!

(*b)* in calculating uncertainty, the only complication is the term R^4, whose uncertainty, from Eq. (1.44), is 4 times the uncertainty in R:

$$\frac{\delta R^4}{R^4} \approx 4\frac{\delta R}{R}$$

That said, the overall uncertainty in viscosity is calculated as

$$\frac{\delta\mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta h}{h})^2 + (\frac{\delta\Omega}{\Omega})^2 + (\frac{\delta R^4}{R^4})^2]^{1/2}$$

$$= [(1\%)^2 + (1\%)^2 + (1\%)^2 + \{4(1\%)\}^2]^{1/2} = \sqrt{19} = \mathbf{4.4\%} \quad \textit{Ans.}(b)$$

Clearly the error in R is responsible for almost all of the uncertainty in the viscosity.

P1.89 For the cone-plate viscometer of Fig. P1.56, suppose R = 6 cm and θ = 3°. (*a*) If the torque M required to rotate the cone at 600 r/min is 0.157 N·m, what is the viscosity of the fluid? (*b*) If the uncertainty in each parameter (M, R, θ, Ω) is ±2%, what is the overall uncertainty in the viscosity?

Solution: First convert 600 r/min to (600)(2π/60) = 62.8 rad/s. (a) From Prob. 1.5, the analytical result for the measured viscosity μ is calculated as

$$\mu = \frac{3M\sin\theta}{2\pi\Omega R^3} = \frac{3(0.157N-m)\sin(3^\circ)}{2\pi(62.8\,rad/s)(0.06m)^3} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\frac{kg}{m-s}} \quad Ans.(a)$$

Once again, this could be SAE 30W oil! (*b*) The only complication in the uncertainty calculation is that the error in R^3 is three times the error in *R*:

$$\frac{\delta R^3}{R^3} = \frac{(\frac{\partial R^3}{\partial R})\delta R}{R^3} = \frac{3R^2\,\delta R}{R^3} = 3\frac{\delta R}{R}$$

Then the overall uncertainty is computed as

$$\frac{\delta\mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta\theta}{\theta})^2 + (\frac{\delta\Omega}{\Omega})^2 + (\frac{\delta R^3}{R^3})^2]^{1/2}$$
$$= [(2\%)^2 + (2\%)^2 + (2\%)^2 + \{3(2\%)\}^2]^{1/2} = \sqrt{48} = \mathbf{6.9\%} \quad Ans.(b)$$

The radius R, whose error is tripled, dominates the uncertainty.

P1.90 The dimensionless *drag coefficient* C_D of a sphere, to be studied in Chaps. 5 and 7, is

$$C_D = \frac{F}{(1/2)\rho V^2(\pi/4)D^2}$$

where *F* is the drag force, ρ the fluid density, *V* the fluid velocity, and *D* the sphere diameter. If the uncertainties of these variables are *F* (±3%), ρ (±1.5%), *V* (±2%), and *D* (±1%), what is the overall uncertainty in the measured drag coefficient?

Solution: Since *F* and ρ occur alone, i.e. to the 1st power, their uncertainties are as stated. However, both *V* and *D* are squared, so the relevant uncertainties are doubled:

$$\frac{\delta V^2}{V^2} = 2\frac{\delta V}{V} \quad ; \quad \frac{\delta D^2}{D^2} = 2\frac{\delta D}{D}$$

Putting together the four different uncertainties in the definition of C_D, we obtain

$$\frac{\delta C_D}{C_D} = [(\frac{\delta F}{F})^2 + (\frac{\delta\rho}{\rho})^2 + (\frac{\delta V^2}{V^2})^2 + (\frac{\delta D^2}{D^2})^2]^{1/2}$$
$$= [(3\%)^2 + (1.5\%)^2 + \{2(2\%)\}^2 + \{2(1\%)\}^2] = \sqrt{31.25} = \mathbf{5.6\%} \quad Ans.$$

The largest contribution comes from the uncertainty in velocity.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE-1.1 The absolute viscosity μ of a fluid is primarily a function of
(a) density **(b) temperature** (c) pressure (d) velocity (e) surface tension

FE-1.2 Carbon dioxide, at 20°C and 1 atm, is compressed isentropically to 4 atm. Assume CO_2 is an ideal gas. The final temperature would be
(*a*) **130°C** , (*b*) 162°C , (*c*) 171°C , (*d*) 237°C , (*e*) 313°C

FE-1.3 Helium has a molecular weight of 4.003. What is the weight of 2 cubic meters of helium at 1 atmosphere and 20°C?
(a) 3.3 N (b) 6.5 N (c) 11.8 N (d) 23.5 N (e) 94.2 N

FE-1.4 An oil has a kinematic viscosity of 1.25E–4 m^2/s and a specific gravity of 0.80. What is its dynamic (absolute) viscosity in kg/(m·s)?
(a) 0.08 **(b) 0.10** (c) 0.125 (d) 1.0 (e) 1.25

FE-1.5 Consider a soap bubble of diameter 3 mm. If the surface tension coefficient is 0.072 N/m and external pressure is 0 Pa gage, what is the bubble's internal gage pressure?
(a) –24 Pa (b) +48 Pa (c) +96 Pa **(d) +192 Pa** (e) –192 Pa

FE-1.6 The only possible dimensionless group which combines velocity V, body size L, fluid density ρ, and surface tension coefficient σ is:
(a) $L\rho\sigma/V$ (b) $\rho VL^2/\sigma$ (c) $\rho\sigma V^2/L$ (d) $\sigma LV^2/\rho$ **(e)** $\boldsymbol{\rho LV^2/\sigma}$

FE-1.7 Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5-mm-thick layer of oil of specific gravity 0.80 and kinematic viscosity 1.25E–4 m^2/s. What is the average shear stress in the oil?
(a) 80 Pa (b) 100 Pa (c) 125 Pa (d) 160 Pa (e) 200 Pa

FE-1.8 Carbon dioxide has a specific heat ratio of 1.30 and a gas constant of 189 J/(kg·°C). If its temperature rises from 20°C to 45°C, what is its internal energy rise?
(a) 12.6 kJ/kg **(b) 15.8 kJ/kg** (c) 17.6 kJ/kg (d) 20.5 kJ/kg (e) 25.1 kJ/kg

(b)

FE-1.9 A certain water flow at 20°C has a critical cavitation number, where bubbles form, $Ca \approx 0.25$, where $Ca = 2(p_a - p_{vap})/(\rho V^2)$. If p_a = 1 atm and the vapor pressure is 0.34 psia, for what water velocity will bubbles form?
(a) 12 mi/hr (b) 28 mi/hr (c) 36 mi/hr (d) 55 mi/hr **(e) 63 mi/hr**

FE-1.10 Example 1.10 gave an analysis which predicted that the viscous moment on a rotating disk was $M = \pi\mu\Omega R^4/(2h)$. If the uncertainty of each of the four variables (μ, Ω, R, h) is 1.0 %, what is the estimated overall uncertainty of the moment M?
(*a*) 4.0 % , (*b*) **4.4 %** , (*c*) 5.0 % , (*d*) 6.0 % , (*e*) 7.0 %

COMPREHENSIVE PROBLEMS

C1.1 Sometimes equations can be developed and practical problems solved by knowing nothing more than the dimensions of the key parameters. For example, consider the heat loss through a window in a building. Window efficiency is rated in terms of “R value,” which has units of ft^2·hr·°F/Btu. A certain manufacturer offers a double-pane window with R = 2.5 and also a triple-pane window with R = 3.4. Both windows are 3 ft by 5 ft. On a given winter day, the temperature difference between inside and outside is 45°F. (a) Develop and equation for window heat loss Q, in time period Δt, as a function of window area A, R value, and temperature difference ΔT. How much heat is lost through the above (a) double-pane window, or (b) triple-pane window, in 24 hours? (c) Suppose the building is heated with propane gas, at $3.25 per gallon, burning at 80% efficiency. Propane has 90,000 Btu of available energy per gallon. In a 24-hour period, how much money would a homeowner save, per window, by installing a triple-pane rather than a double-pane window? (d) Finally, suppose the homeowner buys 20 such triple-pane windows for the house. A typical winter equals about 120 heating days at ΔT = 45°F. Each triple-pane window costs $85 more than the double-pane window. Ignoring interest and inflation, how many years will it take the homeowner to make up the additional cost of the triple-pane windows from heating bill savings?

Solution: (a) The function Q = fcn(Δt, R, A, ΔT) must have units of Btu. The only combination of units which accomplishes this is:

$$Q = \frac{\Delta \mathbf{t} \Delta T A}{R} \quad Ans. \qquad Thus\ Q_{lost} = \frac{(24\ hr)(45°F)(3\ ft\cdot 5\ ft)}{2.5\ ft^2\cdot hr\cdot °F/Btu} = \mathbf{6480\ Btu} \quad Ans.\,\text{(a)}$$

(b) Triple-pane window: use R = 3.4 instead of 2.5 to obtain $Q_{3\text{-pane}}$ = **4760 Btu** *Ans.* (b)

(c) The savings, using propane, for one triple-pane window for one 24-hour period is:

$$\Delta Cost = \frac{\$3.25/gal}{90000\,Btu/gal}(6480-4760Btu)\frac{1}{0.80_{efficiency}} = \$0.078 = \mathbf{7.8\ cents} \quad Ans.(c)$$

(d) Extrapolate to 20 windows, 120 cold days per year, and $85 extra cost per window:

$$Pay-back\ time = \frac{\$85/window}{(0.078\$/window/day)(120\,days/year)} = \mathbf{9\ years} \qquad Ans.(d)$$

Not a very good investment. We are using '$' and 'windows' as "units" in our equations!

C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed Vo, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (Vo, *m*, L, h, μ, W). (c) Compute X for the case Vo = 4 m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

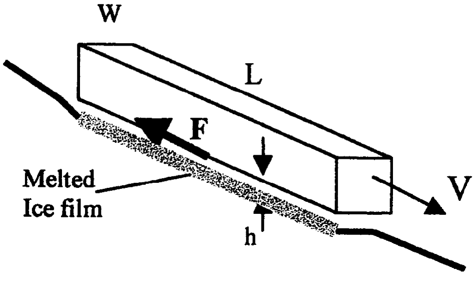
Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h}, \quad F = \tau A = \frac{\mu VLW}{h} \quad Ans.\ \text{(a)}$$

(b) Use **F** = m**a** to find the stopping distance:

$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)



Separate and integrate once to find the velocity, once again to find the distance traveled:

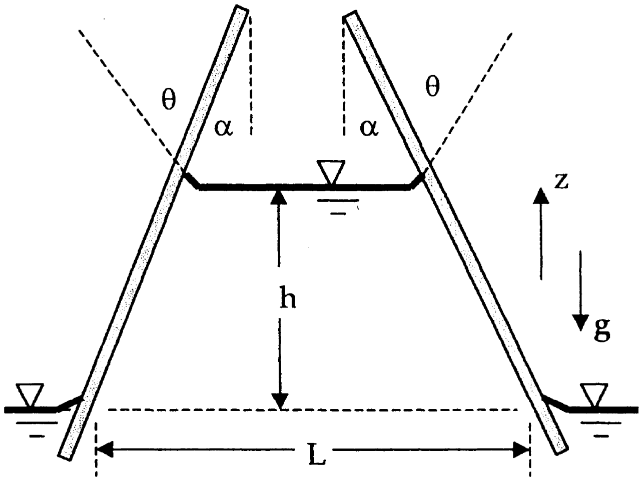
$$\int \frac{dV}{V} = -\int \frac{2\mu LW}{mh}dt, \quad or: \quad V = V_o e^{\frac{-2\mu LW}{mh}t}, \quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW} \quad Ans.\ \text{(b)}$$

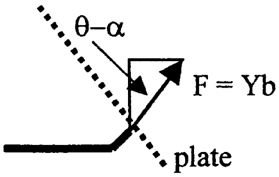
(c) Apply our specific numerical values to a 100-kg (!) person:

$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\,(!)} \quad \textit{Ans.}\ \text{(c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.

C1.3 Two thin flat plates are tilted at an angle α and placed in a tank of known surface tension Y and contact angle θ, as shown. At the free surface of the liquid in the tank, the two plates are a distance L apart, and of width b into the paper. (a) What is the total z-directed force, due to surface tension, acting on the liquid column between plates? (b) If the liquid density is ρ, find an expression for Y in terms of the other variables.





Solution: (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude F = Yb, as shown. Its vertical component is F $\cos(\theta-\alpha)$, as shown. There are two plates. Therefore, the total *z*-directed force on the liquid column is

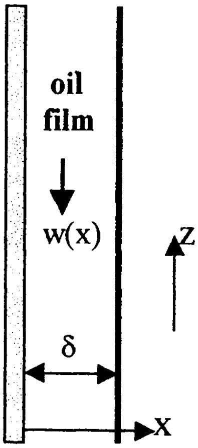
$$\mathbf{F_{vertical} = 2Yb\cos(\theta-\alpha)} \quad \textit{Ans.}\ \text{(a)}$$

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is W = $\rho g\{bh(L - h\tan\alpha)\}$. Set W equal to F and solve for

$$\mathbf{Y = [\rho gbh(L - h\tan\alpha)]/[2\cos(\theta-\alpha)]} \quad \textit{Ans.}\ \text{(b)}$$

C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown. The film is fully developed, that is, its thickness δ and velocity profile w(x) are independent of distance z down the plate. Assume that the atmosphere offers no shear resistance to the film surface.

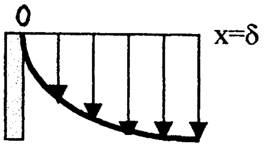
(a) Sketch the approximate shape of the velocity profile w(x), keeping in mind the boundary conditions.

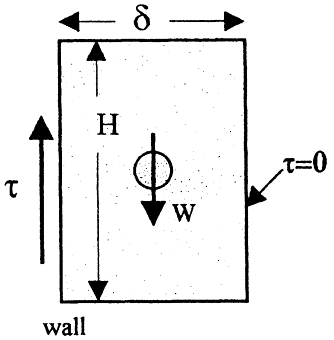


(b) Suppose film thickness d is measured, along with the slope of the velocity profile at the wall, (dw/dx)wall, with a laser-Doppler anemometer (Chap. 6). Find an expression for μ as a function of ρ, δ, (dw/dx)wall, and g. Note that both w and (dw/dx)wall will be negative as shown.

Solution: (a) The velocity profile must be such that there is no slip (w = 0) at the wall and no shear (dw/dx = 0) at the film surface. This is shown at right. *Ans.* (a)

(b) Consider a freebody of any vertical length H of film, as at right. Since there is no acceleration (fully developed film), the weight of the film must exactly balance the shear force on the wall:





$$W = \rho g(H\delta b) = \tau_{wall}(Hb), \quad \tau_{wall} = -\mu\frac{dw}{dx}\Big|_{wall}$$

Solve this equality for the fluid viscosity:

$$\boldsymbol{\mu = \frac{-\rho g\delta}{(dw/dx)_{wall}}} \quad \textit{Ans.}\text{ (b)}$$

C1.5 Viscosity can be measured by flow through a thin-bore or *capillary* tube if the flow rate is low. For length *L*, (small) diameter *D* << *L*, pressure drop Δ*p*, and (low) volume flow rate *Q*, the formula for viscosity is $\mu = D^4\Delta p/(CLQ)$, where *C* is a constant. (a) Verify that *C* is dimensionless. The following data are for water flowing through a 2-mm-diameter tube which is 1 meter long. The pressure drop is held constant at Δ*p* = 5 kPa.

T, °C:	10.0	40.0	70.0
Q, L/min:	0.091	0.179	0.292

(b) Using proper SI units, determine an average value of *C* by accounting for the variation with temperature of the viscosity of water.

Solution: (a) Check the dimensions of the formula and solve for {*C*}:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} = \left\{\frac{D^4\Delta p}{CLQ}\right\} = \left\{\frac{L^4(ML^{-1}T^{-2})}{\{C\}(L)(L^3/T)}\right\} = \left\{\frac{M}{LT\{C\}}\right\},$$

$$\textit{therefore}\ \ \boldsymbol{\{C\} = \{1\}}\ \ \textbf{\textit{Dimensionless}}\ \ \textit{Ans.}\text{ (a)}$$

(b) Use the given data, with values of μwater from Table A.1, to evaluate C, with $L = 1$ m, $D = 0.002$ m, and $\Delta p = 5000$ Pa. Convert the flow rate from L/min to m^3/s.

T, °C:	10.0	40.0	70.0
Q, m^3/s:	1.52E−6	2.98E−6	4.87E−6
μwater, kg/m-s:	1.307E−3	0.657E−3	0.405E−3
$\boldsymbol{C = D^4\Delta p/(\mu LQ)}$:	40.3	40.9	40.6

The estimated value of **$C = 40.6 \pm 0.3$**. The theoretical value (Chap. 4) is $C = 128/\pi = 40.74$.

C1.6 The *rotating-cylinder viscometer* in Fig. C1.6 shears the fluid in a narrow clearance, Δr, as shown. Assume a linear velocity distribution in the gaps. If the driving torque M is measured, find an expression for μ by (a) neglecting, and (b) including the bottom friction.

Solution: (a) The fluid in the annular region has the same shear stress analysis as Prob. 1.49:

$$M = \int R\,dF = \int (R)(\tau)\,dA \int_0^{2\pi} R\left(\mu\frac{\Omega R}{\Delta R}\right) RL\,d\theta = 2\pi\mu\frac{\Omega R^3 L}{\Delta R},$$

$$\text{or:}\quad \mu = \frac{\boldsymbol{M\Delta R}}{\boldsymbol{2\pi\Omega R^3 L}} \quad \textit{Ans.}\text{(a)}$$

(b) Now add in the moment of the (variable) shear stresses on the bottom of the cylinder:

$$M_{bottom} = \int r\tau\,dA = \int_0^R r\left(\mu\frac{\Omega r}{\Delta R}\right)2\pi r\,dr$$

$$= \frac{2\pi\Omega\mu}{\Delta R}\int_0^R r^3\,dr = \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$\textit{Thus}\ \ M_{total} = \frac{2\pi\Omega\mu R^3 L}{\Delta R} + \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$\textit{Solve for}\ \ \mu = \frac{\boldsymbol{M\Delta R}}{\boldsymbol{2\pi\Omega R^3(L + R/4)}} \quad \textit{Ans.}\ \text{(b)}$$

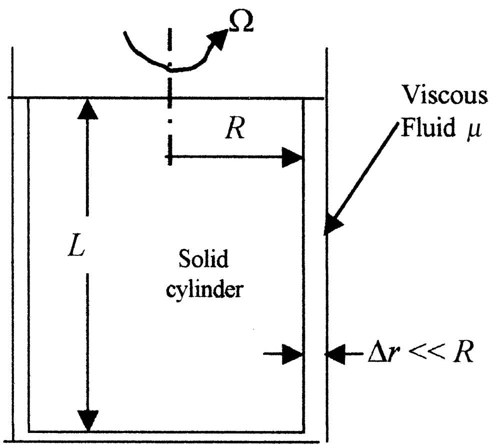


Fig. C1.6

C1.7 SAE 10W oil at 20°C flows past a flat surface, as in Fig. 1.4(b). The velocity profile $u(y)$ is measured, with the following results:

y, m:	0.0	0.003	0.006	0.009	0.012	0.015
u, m/s:	0.0	1.99	3.94	5.75	7.29	8.46

Using your best interpolating skills, estimate the shear stress in the oil (a) at the wall ($y = 0$); and (b) at $y = 15$ mm.

Solution: For SAE10W oil, from Table A.3, read $\mu = 0.104$ kg/m·s. We need to estimate the derivative (du/dy) at the two values of y, then compute $\tau = \mu(du/dy)$.

Method 1: Use a Newton-Raphson three-point derivative estimate.
At three equally-spaced points, $du/dy|_{y_o} \approx (-3u_o + 4u_1 - u_2)/(2\Delta y)$. Thus

(a) at $y = 0$: $du/dy|_{y=0} \approx [-3(0.00) + 4(1.99) - (3.94)]/(2\{0.003\}) = 670\ \text{s}^{-1}$

$$\text{Then}\quad \tau = \mu(du/dy) = (670\ \text{s}^{-1})(0.104\ \text{kg/m}\cdot\text{s}) \approx \mathbf{70\ Pa}\quad \textit{Ans.}\ \text{(a)}$$

(b) at $y = 0.015$ m: $du/dy|_{y=0} \approx [3(8.46) - 4(7.29) + (5.75)]/(2\{0.003\}) = 328\ \text{s}^{-1}$

$$\text{Then}\quad \tau = \mu(du/dy) = (328\ \text{s}^{-1})(0.104\ \text{kg/m}\cdot\text{s}) \approx \mathbf{34\ Pa}\quad \textit{Ans.}\ \text{(b)}$$

Method 2: Type the six data points into Excel and run a cubic "trendline" fit. The result is

$$u \approx 656.2y + 4339.8y^2 - 699163y^3$$

Differentiating this polynomial at $y = 0$ gives $du/dy \approx 656.2\ \text{s}^{-1}$, $\tau \approx \mathbf{68\ Pa}$ *Ans.* (a)

Differentiating this polynomial at $y = 0.015$ gives $du/dy \approx 314\ \text{s}^{-1}$, $\tau \approx \mathbf{33\ Pa}$ *Ans.* (b)

C1.8 A mechanical device, which uses the rotating cylinder of Fig. C1.6, is the *Stormer viscometer* [Ref. 29 of Chap. 1]. Instead of being driven at constant Ω, a cord is wrapped around the shaft and attached to a falling weight W. The time t to turn the shaft a given number of revolutions (usually 5) is measured and correlated with viscosity. The Stormer formula is

$$t = A\mu/(W - B)$$

where A and B are constants which are determined by calibrating the device with a known fluid. Here are calibration data for a Stormer viscometer tested in glycerol, using a weight of 50 N:

μ, kg/m·s:	0.23	0.34	0.57	0.84	1.15
t, sec:	15	23	38	56	77

(a) Find reasonable values of *A* and *B* to fit this calibration data. [*Hint*: The data are not very sensitive to the value of *B*.] (b) A more viscous fluid is tested with a 100-N weight and the measured time is 44 s. Estimate the viscosity of this fluid.

Solution: (a) The data fit well, with a standard deviation of about 0.17 s in the value of *t*, to the values

$$A \approx \mathbf{3000} \quad \text{and} \quad B \approx \mathbf{3.5} \quad \textit{Ans.}\text{ (a)}$$

(b) With a new fluid and a new weight, the values of A and B should nevertheless be the same:

$$t = 44\ s \approx \frac{A\mu}{W-B} = \frac{3000\mu}{100\ N - 3.5}, \quad \textit{solve for } \mu_{new\ fluid} \approx \mathbf{1.42}\ \frac{kg}{m\cdot s} \quad \textit{Ans.}\text{ (b)}$$

C1.9 The lever in Fig. P1.101 has a weight *W* at one end and is tied to a cylinder at the other end. The cylinder has negligible weight and buoyancy and slides upward through a film of heavy oil of viscosity μ. (*a*) If there is no acceleration (uniform lever rotation) derive a formula for the rate of fall V_2 of the weight. Neglect the lever weight. Assume a linear velocity profile in the oil film. (*b*) Estimate the fall velocity of the weight if $W = 20$ N, $L_1 = 75$ cm, $L_2 = 50$ cm, $D = 100$ cm, $L = 22$ cm, $\Delta R = 1$ mm, and the oil is glycerin at 20°C.

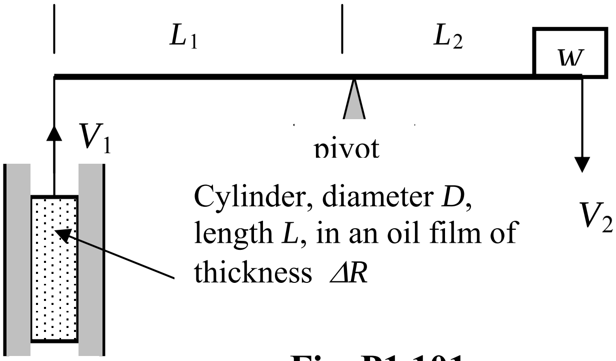

L_1
L_2
W
V_1
pivot
V_2
Cylinder, diameter *D*, length *L*, in an oil film of thickness ΔR

Fig. P1.101

Solution: (a) If the motion is uniform, no acceleration, then the moments balance about the pivot:

$$\sum M_{pivot} = 0 = WL_2 - F_1L_1, \quad \text{where} \quad F_1 = \tau_w A_w = (\mu\frac{V_1}{\Delta R})(\pi DL)$$

Since the lever is rigid, the endpoint velocities vary according to their lengths from the pivot:

$$V_1 = \Omega L_1 \ ; \ V_2 = \Omega L_2 \qquad \therefore \ V_1/L_1 = V_2/L_2$$

Combine these two relations to obtain the desired solution for a highly viscous fluid:

$$V_2 = V_1(L_2/L_1) = \frac{W\,\Delta R}{\mu\,\pi D L}(L_2/L_1)^2 \qquad \textit{Ans.}(a)$$

(*b*) For glycerin at 20°C, from Table A.3, $\mu = 1.49$ kg/m-s. The formula above yields

$$V_2 = \frac{W\,\Delta R}{\mu\,\pi D L}\left(\frac{L_2}{L_1}\right)^2 = \frac{(20\,N)(0.001\,m)}{(1.49\,N-s/m^2)\pi(0.1\,m)(0.22\,m)}\left(\frac{75\,cm}{50\,cm}\right)^2 \approx \mathbf{0.44}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}(b)$$

C1.10 A popular gravity-driven instrument is the Cannon-Ubbelohde viscometer, shown in Fig. C1.10. The test liquid is drawn up above the bulb on the right side and allowed to drain by gravity through the capillary tube below the bulb. The time *t* for the meniscus to pass from upper to lower timing marks is recorded. The kinematic viscosity is computed by the simple formula $\nu = Ct$, where C is a calibration constant. For ν in the range of 100-500 mm^2/s, the recommended constant is C = 0.50 mm^2/s^2, with an accuracy less than 0.5%.

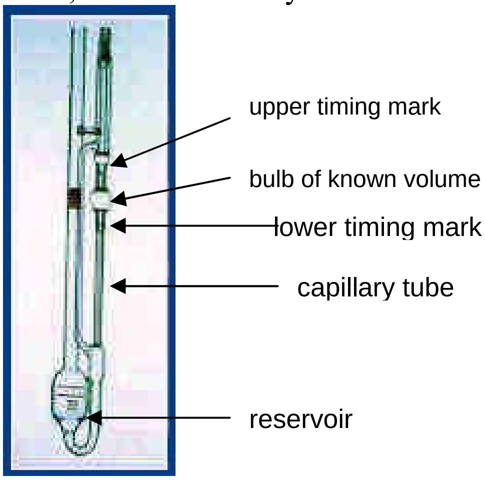


Fig. C1.10

The Cannon-Ubbelohde viscometer.

(*a*) What liquids from Table A.3 are in this viscosity range? (*b*) Is the calibration formula dimensionally consistent? (*c*) What system properties might the constant C depend upon? (*d*) What problem in this chapter hints at a formula for estimating the viscosity?

Solution: (*a*) Very hard to tell, because values of ν are not listed – sorry, I'll add these values if I remember. It turns out that only *three* of these 17 liquids are in the 100-500 mm^2/s range: SAE 10W, 10W30, and 30W oils, at 120, 194, and 326 mm^2/s respectively. (*b*) No, the formula is dimensionally inconsistent because C has units; thus C = 0.5 is appropriate *only* for a mm^2/s result. (*c*) Hidden in C are the length and diameter of the capillary tube and the acceleration of gravity - see Eq. (6.12) later. (*d*) Problem P1.58 gives a formula for flow through a capillary tube. If the flow is vertical and gravity driven, $\Delta p = \rho g L$ and $Q = (\pi/4)d^2V$, leaving a new formula as follows: $\nu \approx gd^2/(32V)$.

C1.11 Mott [Ref. 49, p. 38] discusses a simple falling-ball viscometer, which we can analyze later in Chapter 7. A small ball of diameter *D* and density ρ_b falls though a tube of test liquid. The fall velocity *V* is calculated by the time to fall a measured distance. The formula for calculating the viscosity of the fluid is

$$\mu = \frac{(\rho_b - \rho)\, g\, D^2}{18\, V}$$

This result is limited by the requirement that the Reynolds number ($\rho VD/\mu$) be less than 1.0. Suppose a steel ball (SG = 7.87) of diameter 2.2 mm falls in SAE 25W oil (SG = 0.88) at 20°C. The measured fall velocity is 8.4 cm/s. (*a*) What is the viscosity of the oil, in kg/m-s? (*b*) Is the Reynolds number small enough for a valid estimate?

Solution: Relating SG to water, Eq. (1.7), the steel density is 7.87(1000) = 7870 kg/m^3 and the oil density is 0.88(1000) = 880 kg/m^3. Using SI units, the formula predicts

$$\mu_{oil} = \frac{(7870 - 880\, kg/m^3)(9.81\, m/s^2)(0.0022\, m)^2}{18(0.084\, m/s)} \approx \mathbf{0.22\, \frac{kg}{m-s}} \quad \textit{Ans.}$$

$$\text{Check} \quad \text{Re} = \frac{\rho VD}{\mu} = \frac{(880)(0.084)(0.0022)}{0.22} = 0.74 < 1.0 \quad \textit{OK}$$

As mentioned, we shall analyze this falling sphere problem in Chapter 7.

C1.12 A solid aluminum disk (SG = 2.7) is 2 inches in diameter and 3/16 inch thick. It slides steadily down a 14° incline that is coated with a castor oil (SG = 0.96) film one hundredth of an inch thick. The steady slide velocity is 2 cm/s. Using Figure A.1 and a linear oil velocity profile assumption, estimate the *temperature* of the castor oil.

Solution: This problem reviews complicated units, volume and weight, shear stress, and viscosity. It fits the sketch in Fig. P1.45. The writer converts to SI units.

$$W = \rho_{alum}\, g\, A h = (2700\, kg/m^3)(9.81\, m/s^2)\{\pi[(1/12)(0.3048)]^2\}(3/16/12)(0.3048) = 0.256\, N$$

The weight component along the incline balances the shear stress, in the castor oil, times the bottom flat area of the disk.

$$W\sin\theta = \mu\frac{V}{h}A\ ,\ \text{or}:\ (0.256N)\sin(14^\circ) = \mu\left[\frac{0.02\,m/s}{(1/100/12)(0.3048)\,m}\right]\pi[(1/12)(0.3048)m]^2$$

$$\text{Solve for}\quad \mu_{oil}\ \approx\ 0.39\,kg/(m-s)$$

Looking on Fig. A.1 for castor oil, this viscosity corresponds approximately to **30°C** *Ans.* The specific gravity of the castor oil was a red herring and is not needed. Note also that, since *W* is proportional to disk bottom area *A*, that area cancels out and is not needed.

Chapter 2 • Pressure Distribution in a Fluid

2.1 For the two-dimensional stress field in Fig. P2.1, let

$$\sigma_{xx} = 3000 \text{ psf} \quad \sigma_{yy} = 2000 \text{ psf}$$
$$\sigma_{xy} = 500 \text{ psf}$$

Find the shear and normal stresses on plane AA cutting through at 30°.

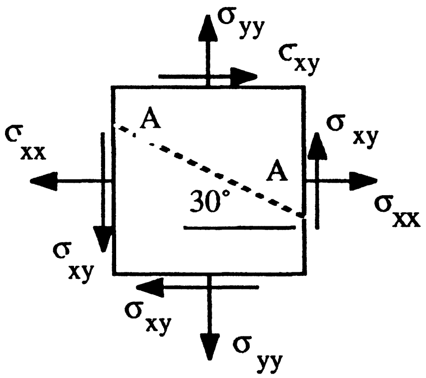
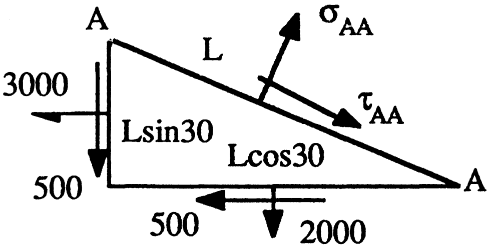


Fig. P2.1

Solution: Make cut "AA" so that it just hits the bottom right corner of the element. This gives the freebody shown at right. Now sum forces normal and tangential to side AA. Denote side length AA as "L."



$$\sum F_{n,AA} = 0 = \sigma_{AA}L - (3000\sin 30 + 500\cos 30)L\sin 30 - (2000\cos 30 + 500\sin 30)L\cos 30$$

Solve for $\sigma_{AA} \approx$ **2683 lbf/ft^2** *Ans.* (a)

$$\sum F_{t,AA} = 0 = \tau_{AA}L - (3000\cos 30 - 500\sin 30)L\sin 30 - (500\cos 30 - 2000\sin 30)L\cos 30$$

Solve for $\tau_{AA} \approx$ **683 lbf/ft^2** *Ans.* (b)

2.2 For the stress field of Fig. P2.1, change the known data to σ_{xx} = 2000 psf, σ_{yy} = 3000 psf, and $\sigma_n(AA)$ = 2500 psf. Compute σ_{xy} and the shear stress on plane AA.

Solution: Sum forces normal to and tangential to AA in the element freebody above, with $\sigma_n(AA)$ known and σ_{xy} unknown:

$$\sum F_{n,AA} = 2500L - (\sigma_{xy}\cos 30° + 2000\sin 30°)L\sin 30° - (\sigma_{xy}\sin 30° + 3000\cos 30°)L\cos 30° = 0$$

$$\text{Solve for}\quad \sigma_{xy} = (2500 - 500 - 2250)/0.866 \approx \mathbf{-289\ lbf/ft^2}\quad \textit{Ans.}\text{ (a)}$$

In like manner, solve for the shear stress on plane AA, using our result for σ_{xy}:

$$\sum F_{t,AA} = \tau_{AA}L - (2000\cos 30° + 289\sin 30°)L\sin 30° + (289\cos 30° + 3000\sin 30°)L\cos 30° = 0$$

$$\text{Solve for}\quad \tau_{AA} = 938 - 1515 \approx \mathbf{-577\ lbf/ft^2}\quad \textit{Ans.}\text{ (b)}$$

This problem and Prob. 2.1 can also be solved using Mohr's circle.

2.3 A vertical clean glass piezometer tube has an inside diameter of 1 mm. When a pressure is applied, water at 20°C rises into the tube to a height of 25 cm. After correcting for surface tension, estimate the applied pressure in Pa.

Solution: For water, let $Y = 0.073$ N/m, contact angle $\theta = 0°$, and $\gamma = 9790$ N/m^3. The capillary rise in the tube, from Example 1.9 of the text, is

$$h_{cap} = \frac{2Y\cos\theta}{\gamma R} = \frac{2(0.073\ N/m)\cos(0°)}{(9790\ N/m^3)(0.0005\ m)} = 0.030\ \text{m}$$

Then the rise due to applied pressure is less by that amount: $h_{press} = 0.25\text{ m} - 0.03\text{ m} = 0.22$ m. The applied pressure is estimated to be $p = \gamma h_{press} = (9790\text{ N/m}^3)(0.22\text{ m}) \approx$ **2160 Pa** *Ans.*

P2.4 Pressure gages, such as the Bourdon gage in Fig. P2.4, are calibrated with a deadweight piston. If the Bourdon gage is designed to rotate the pointer 10 degrees for every 2 psig of internal pressure, how many degrees does the pointer rotate if the piston and weight together total 44 newtons?

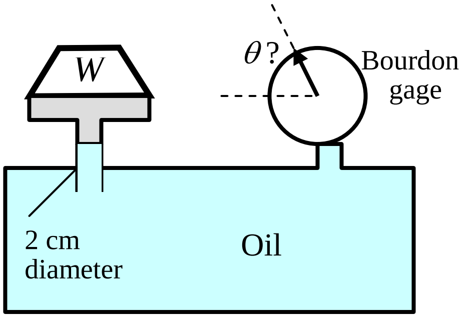


Fig. P2.4

Solution: The deadweight, divided by the piston area, should equal the pressure applied to the Bourdon gage. Stay in SI units for the moment:

$$p_{Bourdon} = \frac{F}{A_{piston}} = \frac{44\,N}{(\pi/4)(0.02m)^2} = 140{,}060\ Pa \div 6894.8 = 20.3\ \frac{lbf}{in^2}$$

At 10 degrees for every 2 psig, the pointer should move approximately **100 degrees**. *Ans.*

2.5 Denver, Colorado, has an average altitude of 5300 ft. On a U.S. standard day, pressure gage A reads 83 kPa and gage B reads 105 kPa. Express these readings in gage or vacuum pressure, whichever is appropriate.

Solution: We can find atmospheric pressure by either interpolating in Appendix Table A.6 or, more accurately, evaluate Eq. (2.27) at 5300 ft ≈ 1615 m:

$$\mathrm{p_a = p_o}\left(1-\frac{\mathrm{Bz}}{\mathrm{T_o}}\right)^{\mathrm{g/RB}} = (101.35\ \mathrm{kPa})\left[1-\frac{(0.0065\ \mathrm{K/m})(1615\ \mathrm{m})}{288.16\ \mathrm{K}}\right]^{5.26} \approx 83.4\ \mathrm{kPa}$$

Therefore:

$$\text{Gage A} = 83\ \text{kPa} - 83.4\ \text{kPa} = -0.4\ \text{kPa (gage)} = +\mathbf{0.4}\ \text{kPa (vacuum)}$$

$$\text{Gage B} = 105\ \text{kPa} - 83.4\ \text{kPa} = \mathbf{21.6}\ \text{kPa (gage)} \quad \textit{Ans.}$$

2.6 Express standard atmospheric pressure as a head, $h = p/\rho g$, in (a) feet of glycerin; (b) inches of mercury; (c) meters of water; and (d) mm of ethanol.

Solution: Take the specific weights, $\gamma = \rho g$, from Table A.3, divide p_{atm} by γ:

(a) Glycerin: $h = (2116\ \text{lbf/ft}^2)/(78.7\ \text{lbf/ft}^3) \approx$ **26.9 ft** *Ans.* (a)

(b) Mercury: $h = (2116\ \text{lbf/ft}^2)/(846\ \text{lbf/ft}^3) = 2.50\ \text{ft} \approx$ **30.0 inches** *Ans.* (b)

(c) Water: $h = (101350\ \text{N/m}^2)/(9790\ \text{N/m}^3) \approx$ **10.35 m** *Ans.* (c)

(d) Ethanol: $h = (101350\ \text{N/m}^2)/(7740\ \text{N/m}^3) = 13.1\ \text{m} \approx$ **13100 mm** *Ans.* (d)

P2.7 La Paz, Bolivia is at an altitude of approximately 12,000 ft. Assume a standard atmosphere. How high would the liquid rise in a *methanol* barometer, assumed at 20°C? [HINT: Don't forget the vapor pressure.]

Solution: Convert 12,000 ft to 3658 meters, and Table A.6, or Eq. (2.20), give

$$p_{LaPaz} = p_o\left(1-\frac{Bz}{T_o}\right)^{g/(RB)} = 101350\left[1-\frac{(0.0065)(3658)}{288.16}\right]^{5.26} \approx 64,400\ Pa$$

From Table A.3, methanol has $\rho = 791$ kg/m^3 and a large vapor pressure of 13,400 Pa. Then the manometer rise h is given by

$$p_{LaPaz} - p_{vap} = 64400 - 13400 = \rho_{methanol}\, g\, h = (791)(9.81)\, h$$

$$Solve\ \ for \quad h_{methanol} = \mathbf{6.57}\ m \qquad Ans.$$

2.8 A diamond mine is 2 miles below sea level. (a) Estimate the air pressure at this depth. (b) If a barometer, accurate to 1 mm of mercury, is carried into this mine, how accurately can it estimate the depth of the mine?

Solution: (a) Convert 2 miles = 3219 m and use a linear-pressure-variation estimate:

$$\text{Then} \quad p \approx p_a + \gamma h = 101{,}350\ \text{Pa} + (12\ \text{N/m}^3)(3219\ \text{m}) = 140{,}000\ \text{Pa} \approx \mathbf{140\ kPa} \quad Ans.\ \text{(a)}$$

Alternately, the troposphere formula, Eq. (2.27), predicts a slightly higher pressure:

$$p \approx p_a(1 - Bz/T_o)^{5.26} = (101.3\ \text{kPa})[1 - (0.0065\ \text{K/m})(-3219\ \text{m})/288.16\ \text{K}]^{5.26}$$

$$= \mathbf{147\ kPa} \quad Ans.\ \text{(a)}$$

(b) The gage pressure at this depth is approximately 40,000/133,100 ≈ 0.3 m Hg or 300 mm Hg ±1 mm Hg or ±0.3% error. Thus the error in the actual depth is 0.3% of 3220 m or about **±10 m** if all other parameters are accurate. *Ans*. (b)

P2.9 A storage tank, 26 ft in diameter and 36 ft high, is filled with SAE 30W oil at 20°C. (*a*) What is the gage pressure, in lbf/in^2, at the bottom of the tank? (*b*) How does your result in (*a*) change if the tank diameter is reduced to 15 ft? (*c*) Repeat (*a*) if leakage has caused a layer of 5 ft of water to rest at the bottom of the (full) tank.

Solution: This is a straightforward problem in hydrostatic pressure. From Table A.3, the density of SAE 30W oil is 891 kg/m^3 ÷ 515.38 = 1.73 slug/ft^3. (*a*) Thus the bottom pressure is

$$p_{bottom} = \rho_{oil}\, g\, h = (1.73\frac{slug}{ft^3})(32.2\frac{ft}{s^2})(36\ ft) = 2005\frac{lbf}{ft^2} = \mathbf{13.9}\frac{lbf}{in^2}\ gage \quad Ans.(a)$$

(*b*) The tank diameter has nothing to do with it, just the depth: $p_{\text{bottom}} = \mathbf{13.9}$ psig. *Ans*.(*b*) (*c*) If we have 31 ft of oil and 5 ft of water ($\rho = 1.94$ slug/ft^3), the bottom pressure is

$$p_b = \rho_{oil} g h_{oil} + \rho_{water} g h_{water} = (1.73)(32.2)(31) + (1.94)(32.2)(5) =$$

$$= \quad 1727 + 312 \;=\; 2039\,\frac{lbf}{ft^2} = \mathbf{14.2}\,\frac{lbf}{in^2} \quad \textit{Ans.(c)}$$

2.10 A closed tank contains 1.5 m of SAE 30 oil, 1 m of water, 20 cm of mercury, and an air space on top, all at 20°C. If p_{bottom} = 60 kPa, what is the pressure in the air space?

Solution: Apply the hydrostatic formula down through the three layers of fluid:

$$p_{bottom} = p_{air} + \gamma_{oil} h_{oil} + \gamma_{water} h_{water} + \gamma_{mercury} h_{mercury}$$

$$\text{or:} \quad 60000\text{ Pa} = p_{air} + (8720\text{ N/m}^3)(1.5\text{ m}) + (9790)(1.0\text{ m}) + (133100)(0.2\text{ m})$$

Solve for the pressure in the air space: $p_{air} \approx$ **10500 Pa** *Ans.*

2.11 In Fig. P2.11, sensor A reads 1.5 kPa (gage). All fluids are at 20°C. Determine the elevations Z in meters of the liquid levels in the open piezometer tubes B and C.

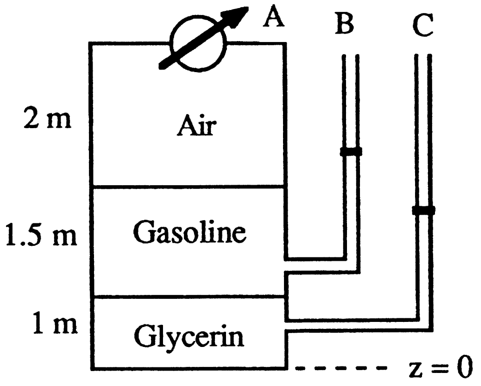


Fig. P2.11

Solution: (B) Let piezometer tube B be an arbitrary distance H above the gasoline-glycerin interface. The specific weights are $\gamma_{air} \approx 12.0$ N/m^3, $\gamma_{gasoline} = 6670$ N/m^3, and $\gamma_{glycerin} = 12360$ N/m^3. Then apply the hydrostatic formula from point A to point B:

$$1500\text{ N/m}^2 + (12.0\text{ N/m}^3)(2.0\text{ m}) + 6670(1.5 - H) - 6670(Z_B - H - 1.0) = p_B = 0\text{ (gage)}$$

Solve for Z_B = **2.73 m** (23 cm above the gasoline-air interface) *Ans.* (b)

Solution (C): Let piezometer tube C be an arbitrary distance Y above the bottom. Then

$$1500 + 12.0(2.0) + 6670(1.5) + 12360(1.0 - Y) - 12360(Z_C - Y) = p_C = 0\text{ (gage)}$$

Solve for Z_C = **1.93 m** (93 cm above the gasoline-glycerin interface) *Ans.* (c)

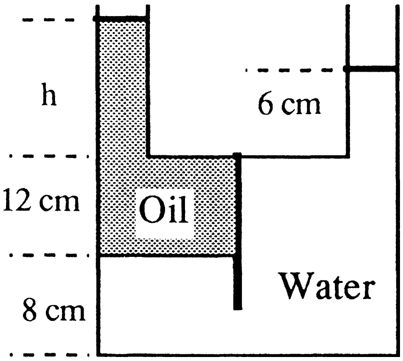


Fig. P2.12

2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m^3?

Solution: For water take the density = 998 kg/m^3. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08\text{ m} \approx \mathbf{8.0\ cm}$ *Ans.*

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

Solution: Take water = 9790 N/m^3 and gasoline = 6670 N/m^3. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

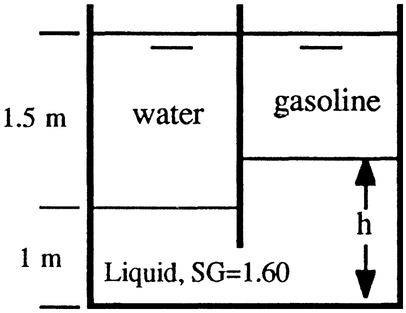


Fig. P2.13

$$p_{bottom} = (9790\text{ N/m}^3)(1.5\text{ m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

Solve for $h = \mathbf{1.52\ m}$ *Ans.*

P2.14 For the three-liquid system shown, compute h_1 and h_2. Neglect the air density.

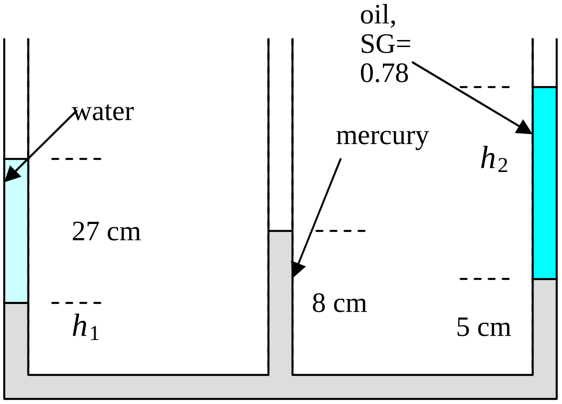


Fig. P2.14

Solution: The pressures at the three top surfaces must all be atmospheric, or zero gage pressure. Compute $\gamma_{oil} = (0.78)(9790) = 7636\ \text{N/m}^3$. Also, from Table 2.1, $\gamma_{water} = 9790\ \text{N/m}^3$ and $\gamma_{mercury} = 133100\ \text{N/m}^3$. The surface pressure equality is

$$(9790\frac{N}{m^3})(0.27\,m) + (133100\frac{N}{m^3})h_1 = (133100\frac{N}{m^3})(0.08m) = (7636\frac{N}{m^3})h_2 + (133100\frac{N}{m^3})(0.05m)$$

$$\text{or}: \quad 2643 + 133100\,h_1 = 10648\,Pa = 7836\,h_2 + 6655$$

$$\text{Solve for} \quad h_1 = 0.060m = \mathbf{6.0\ cm}, \quad h_2 = 0.523m = \mathbf{52.3\ cm} \quad \textit{Ans.}$$

2.15 In Fig. P2.15 all fluids are at 20°C. Gage A reads 15 lbf/in^2 absolute and gage B reads 1.25 lbf/in^2 less than gage C. Com-pute (a) the specific weight of the oil; and (b) the actual reading of gage C in lbf/in^2 absolute.

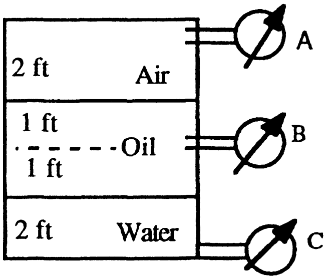


Fig. P2.15

Solution: First evaluate $\gamma_{air} = (p_A/RT)g = [15 \times 144/(1717 \times 528)](32.2) \approx 0.0767$ lbf/ft^3. Take $\gamma_{water} = 62.4$ lbf/ft^3. Then apply the hydrostatic formula from point B to point C:

$$p_B + \gamma_{oil}(1.0\text{ ft}) + (62.4)(2.0\text{ ft}) = p_C = p_B + (1.25)(144)\text{ psf}$$

$$\text{Solve for}\quad \gamma_{oil} \approx \mathbf{55.2\ lbf/ft^3} \quad \textit{Ans.}\text{ (a)}$$

With the oil weight known, we can now apply hydrostatics from point A to point C:

$$p_C = p_A + \sum \rho g h = (15)(144) + (0.0767)(2.0) + (55.2)(2.0) + (62.4)(2.0)$$

$$\text{or:}\quad p_C = 2395\text{ lbf/ft}^2 = \mathbf{16.6\ psi} \quad \textit{Ans.}\text{ (b)}$$

P2.16 If the absolute pressure at the interface between water and mercury in Fig. P2.16 is 93 kPa, what, in lbf/ft^2, is (*a*) the pressure at the surface, and (*b*) the pressure at the bottom of the container?

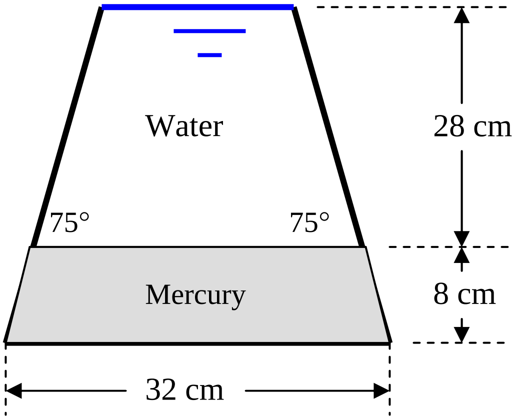


Fig. P2.16

Solution: Do the whole problem in SI units and then convert to BG at the end. The bottom width and the slanted 75-degree walls are irrelevant red herrings. Just go up and down:

$$p_{surface} = p_{interface} - \gamma_{water}\,\Delta h = 93000\,Pa - (9790\,N/m^3)(0.28m) =$$

$$= 90260\,Pa \div 47.88 = \mathbf{1885}\ lbf/ft^2 \quad Ans.(a)$$

$$p_{bottom} = p_{interface} + \gamma_{mercury}\,\Delta h = 93000\,Pa + (133100\,N/m^3)(0.08m) =$$

$$= 103650\,Pa \div 47.88 = \mathbf{2165}\ lbf/ft^2 \quad Ans.(b)$$

2.17 All fluids in Fig. P2.17 are at 20°C. If p = 1900 psf at point A, determine the pressures at B, C, and D in psf.

Solution: Using a specific weight of 62.4 lbf/ft^3 for water, we first compute p_B and p_D:

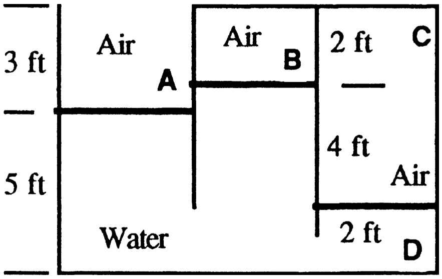


Fig. P2.17

$$p_B = p_A - \gamma_{water}(z_B - z_A) = 1900 - 62.4(1.0\text{ ft}) = \mathbf{1838\ lbf/ft^2} \quad \textit{Ans.}\text{ (pt. B)}$$

$$p_D = p_A + \gamma_{water}(z_A - z_D) = 1900 + 62.4(5.0\text{ ft}) = \mathbf{2212\ lbf/ft^2} \quad \textit{Ans.}\text{ (pt. D)}$$

Finally, moving up from D to C, we can neglect the air specific weight to good accuracy:

$$p_C = p_D - \gamma_{water}(z_C - z_D) = 2212 - 62.4(2.0\text{ ft}) = \mathbf{2087\ lbf/ft^2} \quad \textit{Ans.}\text{ (pt. C)}$$

The air near C has $\gamma \approx 0.074$ lbf/ft^3 times 6 ft yields less than 0.5 psf correction at C.

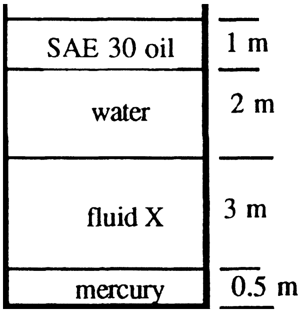


Fig. P2.18

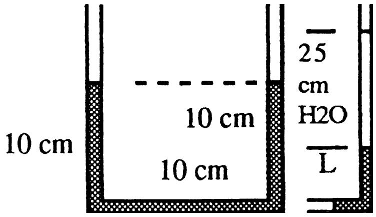
2.18 All fluids in Fig. P2.18 are at 20°C. If atmospheric pressure = 101.33 kPa and the bottom pressure is 242 kPa absolute, what is the specific gravity of fluid X?

Solution: Simply apply the hydrostatic formula from top to bottom:

$$p_{bottom} = p_{top} + \sum \gamma h,$$

$$\text{or:} \quad 242000 = 101330 + (8720)(1.0) + (9790)(2.0) + \gamma_X(3.0) + (133100)(0.5)$$

$$\text{Solve for} \quad \gamma_X = 15273\text{ N/m}^3, \quad \text{or:} \quad SG_X = 15273/9790 = 1.56 \quad \textit{Ans.}$$



2.19 The U-tube at right has a 1-cm ID and contains mercury as shown. If 20 cm^3 of water is poured into the right-hand leg, what will be the free surface height in each leg after the sloshing has died down?

Solution: First figure the height of water added:

$$20\ cm^3 = \frac{\pi}{4}(1\ cm)^2 h, \quad \text{or} \quad h = 25.46\ cm$$

Then, at equilibrium, the new system must have 25.46 cm of water on the right, and a 30-cm length of mercury is somewhat displaced so that "L" is on the right, 0.1 m on the bottom, and "0.2 – L" on the left side, as shown at right. The bottom pressure is constant:

$$p_{atm} + 133100(0.2 - L) = p_{atm} + 9790(0.2546) + 133100(L), \quad \text{or:} \quad L \approx 0.0906\ m$$

Thus right-leg-height = 9.06 + 25.46 = **34.52 cm** *Ans.*
left-leg-height = 20.0 – 9.06 = **10.94 cm** *Ans.*

2.20 The hydraulic jack in Fig. P2.20 is filled with oil at 56 lbf/ft^3. Neglecting piston weights, what force F on the handle is required to support the 2000-lbf weight shown?

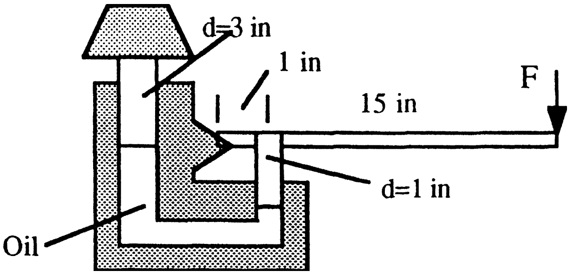


Fig. P2.20

Solution: First sum moments clockwise about the hinge A of the handle:

$$\sum M_A = 0 = F(15 + 1) - P(1),$$

or: F = P/16, where P is the force in the small (1 in) piston.

Meanwhile figure the pressure in the oil from the weight on the large piston:

$$p_{oil} = \frac{W}{A_{3\text{-in}}} = \frac{2000\ lbf}{(\pi/4)(3/12\ ft)^2} = 40744\ psf,$$

$$\text{Hence}\quad P = p_{oil}A_{small} = (40744)\frac{\pi}{4}\left(\frac{1}{12}\right)^2 = 222\ \text{lbf}$$

Therefore the handle force required is $F = P/16 = 222/16 \approx \mathbf{14\ lbf}$ *Ans.*

2.21 In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

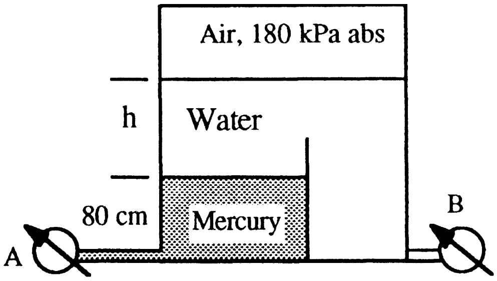


Fig. P2.21

Solution: Apply the hydrostatic formula from the air to gage A:

$$p_A = p_{air} + \sum \gamma h$$

$$= 180000 + (9790)h + 133100(0.8) = 350000\ \text{Pa},$$

Solve for $h \approx \mathbf{6.49\ m}$ *Ans.* (a)

Then, with *h* known, we can evaluate the pressure at gage B:

$$p_B = 180000 + 9790(6.49 + 0.80) = 251000\ \text{Pa} \approx \mathbf{251\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

2.22 The fuel gage for an auto gas tank reads proportional to the bottom gage pressure as in Fig. P2.22. If the tank accidentally contains 2 cm of water plus gasoline, how many centimeters "h" of air remain when the gage reads "full" in error?

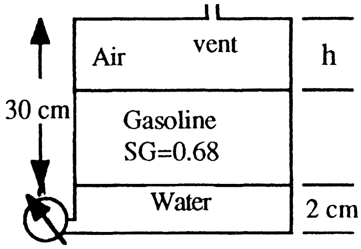


Fig. P2.22

Solution: Given $\gamma_{\text{gasoline}} = 0.68(9790) = 6657\ \text{N/m}^3$, compute the gage pressure when "full":

$$p_{\text{full}} = \gamma_{\text{gasoline}}(\text{full height}) = (6657\ \text{N/m}^3)(0.30\ \text{m}) = 1997\ \text{Pa}$$

Set this pressure equal to 2 cm of water plus "Y" centimeters of gasoline:

$$p_{\text{full}} = 1997 = 9790(0.02\ \text{m}) + 6657Y, \quad \text{or} \quad Y \approx 0.2706\ \text{m} = 27.06\ \text{cm}$$

Therefore the air gap $h = 30\ \text{cm} - 2\ \text{cm(water)} - 27.06\ \text{cm(gasoline)} \approx$ **0.94 cm** *Ans.*

2.23 In Fig. P2.23 both fluids are at 20°C. If surface tension effects are negligible, what is the density of the oil, in kg/m³?

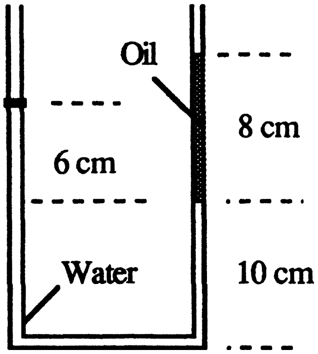


Fig. P2.23

Solution: Move around the U-tube from left atmosphere to right atmosphere:

$$p_a + (9790\ \text{N/m}^3)(0.06\ \text{m}) - \gamma_{\text{oil}}(0.08\ \text{m}) = p_a,$$

$$\text{solve for} \quad \gamma_{\text{oil}} \approx 7343\ \text{N/m}^3,$$

$$\text{or:} \quad \rho_{\text{oil}} = 7343/9.81 \approx \mathbf{748\ kg/m^3} \quad \textit{Ans.}$$

2.24 In Prob. 1.2 we made a crude integration of atmospheric density from Table A.6 and found that the atmospheric mass is approximately $m \approx 6\text{E}18$ kg. Can this result be used to estimate sea-level pressure? Can sea-level pressure be used to estimate m?

Solution: Yes, atmospheric pressure is essentially a result of the weight of the air above. Therefore the air weight divided by the surface area of the earth equals sea-level pressure:

$$p_{\text{sea-level}} = \frac{W_{\text{air}}}{A_{\text{earth}}} = \frac{m_{\text{air}} g}{4\pi R_{\text{earth}}^2} \approx \frac{(6.0\text{E}18\ \text{kg})(9.81\ \text{m/s}^2)}{4\pi(6.377\text{E}6\ \text{m})^2} \approx 115000\ Pa \quad \textit{Ans.}$$

This is a little off, thus our mass estimate must have been a little off. If global average sea-level pressure is actually 101350 Pa, then the mass of atmospheric air must be more nearly

$$m_{\text{air}} = \frac{A_{\text{earth}} p_{\text{sea-level}}}{g} \approx \frac{4\pi(6.377\text{E}6\ \text{m})^2(101350\ \text{Pa})}{9.81\ \text{m/s}^2} \approx \mathbf{5.28E18\ kg} \quad \textit{Ans.}$$

***P2.25** As measured by NASA's Viking landers, the atmosphere of Mars, where g = 3.71 m/s^2, is almost entirely carbon dioxide, and the surface pressure averages 700 Pa. The temperature is cold and drops off exponentially: $T \approx T_o\, e^{-Cz}$, where $C \approx$ 1.3E-5 m^{-1} and $T_o \approx$ 250 K. For example, at 20,000 m altitude, $T \approx$ 193 K. (*a*) Find an analytic formula for the variation of pressure with altitude. (*b*) Find the altitude where pressure on Mars has dropped to 1 pascal.

Solution: (*a*) The analytic formula is found by integrating Eq. (2.17) of the text:

$$\ln\left(\frac{p}{p_o}\right) = -\frac{g}{R}\int_0^z \frac{dz}{T} = -\frac{g}{R}\int_0^z \frac{dz}{T_o e^{-Cz}} = -\frac{g}{RT_oC}(e^{Cz}-1)$$

$$\text{or, finally,} \qquad p = p_o \exp\left[-\frac{g}{RT_oC}(e^{Cz}-1)\right] \qquad \textit{Ans.}(a)$$

(*b*) From Table A.4 for CO$_2$, R = 189 m^2/(s^2-K). Substitute p = 1 Pa to find the altitude:

$$p = 1Pa = p_o \exp\left[-\frac{g}{RT_oC}(e^{Cz}-1)\right] = (700\,Pa)\exp\left[-\frac{3.71 m/s^2}{(189)(250)(1.3E-5)}\{e^{(1.3\text{E}-5)z}-1\}\right]$$

$$or:\ \ln\left(\frac{1}{700}\right) = -6.55 = -6.04\{e^{(1.3\text{E}-5)z}-1\},\quad \text{Solve for} \quad z \approx \mathbf{56{,}500}\ m \qquad \textit{Ans.}(b)$$

P2.26 For gases over large changes in height, the linear approximation, Eq. (2.14), is inaccurate. Expand the troposphere power-law, Eq. (2.20), into a power series and show that the linear approximation $p \approx p_a - \rho_a g z$ is adequate when

$$\delta z << \frac{2T_o}{(n-1)B}, \qquad \text{where} \quad n = \frac{g}{RB}$$

Solution: The power-law term in Eq. (2.20) can be expanded into a series:

$$\left(1-\frac{Bz}{T_o}\right)^n = 1 - n\frac{Bz}{T_o} + \frac{n(n-1)}{2!}\left(\frac{Bz}{T_o}\right)^2 - \ldots\ldots \qquad \text{where} \quad n = \frac{g}{RB}$$

Multiply by p_a, as in Eq. (2.20), and note that $p_a nB/T_o = (p_a/RT_o)gz = \rho_a gz$. Then the series may be rewritten as follows:

$$p = p_a - \rho_a gz\left(1 - \frac{n-1}{2}\frac{Bz}{T_o} + \,.....\,\right)$$

For the linear law to be accurate, the 2nd term in parentheses must be much less than unity. If the starting point is not at $z = 0$, then replace z by δz:

$$\frac{n-1}{2}\frac{B\delta z}{T_o} << 1 \quad , \qquad \text{or:} \qquad \delta z << \frac{2T_o}{(n-1)B} \qquad \textit{Ans.}$$

2.27 This is an *experimental* problem: Put a card or thick sheet over a glass of water, hold it tight, and turn it over without leaking (a glossy postcard works best). Let go of the card. Will the card stay attached when the glass is upside down? **Yes**: This is essentially a *water barometer* and, in principle, could hold a column of water up to 10 ft high!

P2.28 A correlation of numerical results indicates that, all other things being equal, the horizontal distance traveled by a well-hit baseball varies inversely as the cube root of the air density. If a home-run ball hit in New York City travels 400 ft, estimate the distance it would travel in (*a*) Denver, Colorado; and (*b*) La Paz, Bolivia.

Solution: New York City is approximately at sea level, so use the Standard Atmosphere, Table A.6, and take $\rho_{air} = 1.2255\ kg/m^3$. Modify Eq. (2.20) for density instead of pressure:

$$\frac{\rho}{\rho_a} = \left(1 - \frac{Bz}{T_o}\right)^{(g/RB)-1} = \left(1 - \frac{0.0065z}{288.16}\right)^{4.26}$$

Using nominal altitudes from almanacs, apply this formula to Denver and La Paz:

$$(a)\quad \text{Denver, Colorado:}\ z \approx 5280\ ft = 1609\,m\ ;\quad \rho \approx 1.047\ kg/m^3$$

$$(b)\quad \text{La Paz, Bolivia:}\ z \approx 12000\ ft = 3660\,m\ ;\quad \rho \approx 0.849\ kg/m^3$$

Finally apply this to the 400-ft home-run ball:

$$(a)\quad \text{Denver: Distance traveled} = (400\,ft)\left(\frac{1.2255}{1.047}\right)^{1/3} \approx \mathbf{421\,ft} \qquad Ans.(a)$$

$$(b)\quad \text{La Paz: Distance traveled} = (400\,ft)\left(\frac{1.2255}{0.849}\right)^{1/3} \approx \mathbf{452\,ft} \qquad Ans.(b)$$

In Denver, balls go 5% further, as attested to by many teams visiting Coors Field.

P2.29 An airplane flies at a Mach number of 0.82 at a standard altitude of 24,000 ft. (*a*) What is the plane's velocity, in mi/h? (*b*) What is the standard density at that altitude?

Solution: (a) Convert 24,000 ft to 7315 m. Find the standard temperature from Eq. (2.19):

$$T = T_o - B\,z = 288.16K - (0.0065m)(7315m) = 240.6\,K$$

From the (absolute) temperature, we compute the speed of sound and hence the velocity:

$$a = \sqrt{kRT} = \sqrt{1.4(287\,m^2/s^2 - K)(240.6K)} = 311\,m/s$$
$$V = (Ma)\,a = (0.82)(311m/s) = 255\,m/s \div 0.44704 = \mathbf{570}\ mi/h \qquad Ans.(a)$$

(*b*) Given ρ_o = 1.2255 kg/m^3, the power-law density formula is evaluated at T = 240.6 K:

$$\rho = \rho_o\left(\frac{T}{T_o}\right)^{\frac{g}{RB}-1} = (1.2255)\left(\frac{240.6}{288.16}\right)^{5.26-1} = \mathbf{0.568}\ kg/m^3 \qquad Ans.(b)$$

P2.30 For the traditional equal-level manometer measurement in Fig. E2.3, water at 20°C flows through the plug device from *a* to *b*. The manometer fluid is mercury. If L = 12 cm and h = 24 cm, (*a*) what is the pressure drop through the device? (*b*) If the water flows through the pipe at a velocity V = 18 ft/s, what is the *dimensionless loss coefficient* of the device, defined by $K = \Delta p/(\rho V^2)$? We will study loss coefficients in Chap. 6.

Solution: Gather density data: $\rho_{\text{mercury}} = 13550\ \text{kg/m}^3$, $\rho_{\text{water}} = 998\ \text{kg/m}^3$. Example 2.3, by going down from (*a*) to the mercury level, jumping across, and going up to (*b*), found the very important formula for this type of equal-leg manometer:

$$\Delta p = p_a - p_b = (\rho_{merc} - \rho_{water})\,g\,h = (13550 - 998\,kg/m^3)(9.81\,m/s^2)(0.24\,m)$$

$$or: \quad \Delta p = \mathbf{29,600}\ Pa \qquad Ans.(a)$$

(*b*) The loss coefficient calculation is straightforward, but we check the units to make sure. Convert the velocity from 18 ft/s to 5.49 m/s. Then

$$K = \frac{\Delta p}{\rho V^2} = \frac{29600\ N/m^2}{(998\,kg/m^2)(5.49\,m/s)^2} = \frac{29600\ N/m^2}{30080\,N/m^2} = \mathbf{0.98} \qquad Ans.(b)$$

2.31 In Fig. P2.31 determine Δp between points A and B. All fluids are at 20°C.

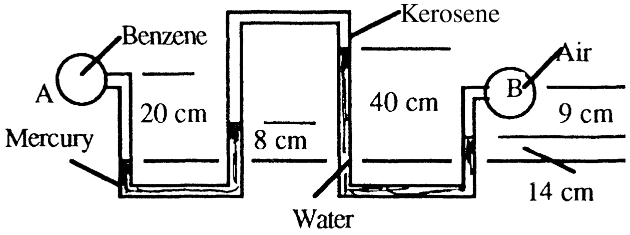


Fig. P2.31

Solution: Take the specific weights to be

Benzene:	$8640\ \text{N/m}^3$	Mercury:	$133100\ \text{N/m}^3$
Kerosene:	$7885\ \text{N/m}^3$	Water:	$9790\ \text{N/m}^3$

and γ_{air} will be small, probably around $12\ \text{N/m}^3$. Work your way around from A to B:

$$p_A + (8640)(0.20\text{ m}) - (133100)(0.08) - (7885)(0.32) + (9790)(0.26) - (12)(0.09)$$

$$= p_B, \quad \text{or, after cleaning up,} \quad p_A - p_B \approx \mathbf{8900\ Pa} \quad Ans.$$

2.32 For the manometer of Fig. P2.32, all fluids are at 20°C. If $p_B - p_A = 97$ kPa, determine the height H in centimeters.

Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\ \mathrm{N/m^3})(\mathrm{H\ meters}) - 8096(0.18) +$$

$$+133100(0.18 + \mathrm{H} + 0.35) = p_B = p_A + 97000.$$

$$\text{Solve for}\quad \mathrm{H} \approx 0.226\ \mathrm{m} = \mathbf{22.6\ cm}\quad \textit{Ans.}$$

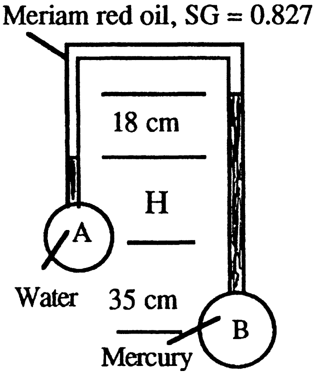


Fig. P2.32

2.33 In Fig. P2.33 the pressure at point A is 25 psi. All fluids are at 20°C. What is the air pressure in the closed chamber B?

Solution: Take $\gamma = 9790$ N/m^3 for water, 8720 N/m^3 for SAE 30 oil, and (1.45)(9790) = 14196 N/m^3 for the third fluid. Convert the pressure at A from 25 lbf/in^2 to 172400 Pa. Compute hydrostatically from point A to point B:

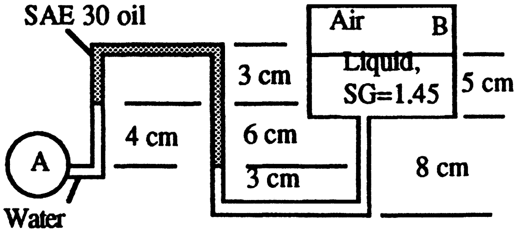


Fig. P2.33

$$p_A + \sum \gamma h = 172400 - (9790\ \text{N/m}^3)(0.04\ \text{m}) + (8720)(0.06) - (14196)(0.10)$$

$$= p_B = 171100\ \text{Pa} \div 47.88 \div 144 = \mathbf{24.8\ psi} \quad \textit{Ans.}$$

2.34 To show the effect of manometer dimensions, consider Fig. P2.34. The containers (a) and (b) are cylindrical and are such that $p_a = p_b$ as shown. Suppose the oil-water interface on the right moves up a distance $\Delta h < h$. Derive a formula for the difference $p_a - p_b$ when (a) $d \ll D$; and (b) $d = 0.15D$. What is the % difference?

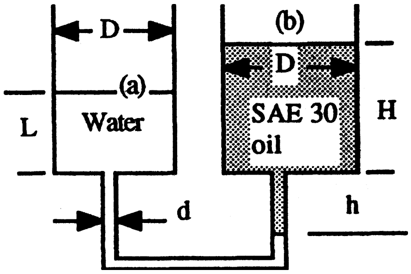


Fig. P2.34

Solution: Take $\gamma = 9790\ \text{N/m}^3$ for water and $8720\ \text{N/m}^3$ for SAE 30 oil. Let "H" be the height of the oil in reservoir (b). For the condition shown, $p_a = p_b$, therefore

$$\gamma_{water}(L + h) = \gamma_{oil}(H + h), \quad \text{or:} \quad H = (\gamma_{water}/\gamma_{oil})(L + h) - h \qquad (1)$$

Case (a), $d \ll D$: When the meniscus rises Δh, there will be no significant change in reservoir levels. Therefore we can write a simple hydrostatic relation from (a) to (b):

$$p_a + \gamma_{water}(L + h - \Delta h) - \gamma_{oil}(H + h - \Delta h) = p_b,$$

$$\text{or:} \quad \mathbf{p_a - p_b = \Delta h(\gamma_{water} - \gamma_{oil})} \quad \textit{Ans.}\text{ (a)}$$

where we have used Eq. (1) above to eliminate H and L. Putting in numbers to compare later with part (b), we have $\Delta p = \Delta h(9790 - 8720) = \mathit{1070}\ \Delta h$, with Δh in meters.

Case (b), $d = 0.15D$. Here we must account for reservoir volume changes. For a rise $\Delta h < h$, a volume $(\pi/4)d^2\Delta h$ of *water* leaves reservoir (a), decreasing "L" by $\Delta h(d/D)^2$, and an identical volume of oil enters reservoir (b), increasing "H" by the same amount $\Delta h(d/D)^2$. The hydrostatic relation between (a) and (b) becomes, for this case,

$$p_a + \gamma_{water}[L - \Delta h(d/D)^2 + h - \Delta h] - \gamma_{oil}[H + \Delta h(d/D)^2 + h - \Delta h] = p_b,$$

$$\text{or:} \quad p_a - p_b = \mathbf{\Delta h[\gamma_{water}(1 + d^2/D^2) - \gamma_{oil}(1 - d^2/D^2)]} \quad \textit{Ans.}\text{ (b)}$$

where again we have used Eq. (1) to eliminate H and L. If d is not small, this is a *considerable* difference, with surprisingly large error. For the case d = 0.15 D, with water and oil, we obtain $\Delta p = \Delta h[1.0225(9790) - 0.9775(8720)] \approx 1486\ \Delta h$ or **39% more** than (a).

2.35 Water flows upward in a pipe slanted at 30°, as in Fig. P2.35. The mercury manometer reads h = 12 cm. What is the pressure difference between points (1) and (2) in the pipe?

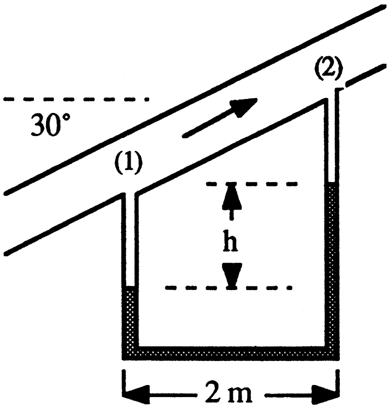


Fig. P2.35

Solution: The vertical distance between points 1 and 2 equals (2.0 m)tan 30° or 1.155 m. Go around the U-tube hydrostatically from point 1 to point 2:

$$p_1 + 9790h - 133100h - 9790(1.155\text{ m}) = p_2,$$

$$\text{or:}\quad p_1 - p_2 = (133100 - 9790)(0.12) + 11300 = \mathbf{26100\ Pa} \quad \textit{Ans.}$$

2.36 In Fig. P2.36 both the tank and the slanted tube are open to the atmosphere. If L = 2.13 m, what is the angle of tilt ϕ of the tube?

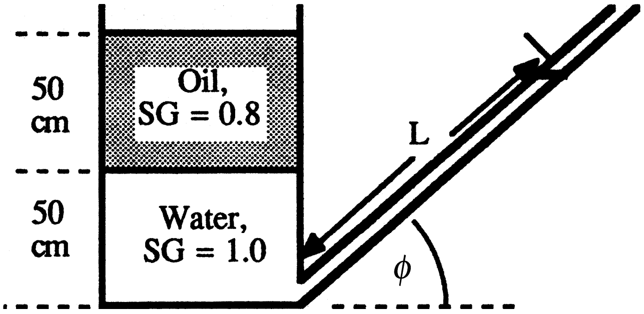


Fig. P2.36

Solution: Proceed hydrostatically from the oil surface to the slanted tube surface:

$$p_a + 0.8(9790)(0.5) + 9790(0.5) - 9790(2.13\sin\phi) = p_a,$$

$$\text{or:}\quad \sin\phi = \frac{8811}{20853} = 0.4225, \quad \text{solve } \phi \approx \mathbf{25°} \quad \textit{Ans.}$$

2.37 The inclined manometer in Fig. P2.37 contains Meriam red oil, SG = 0.827. Assume the reservoir is very large. If the inclined arm has graduations 1 inch apart, what should θ be if each graduation represents 1 psf of the pressure p_A?

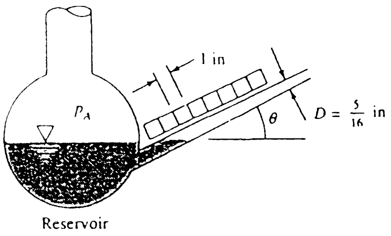


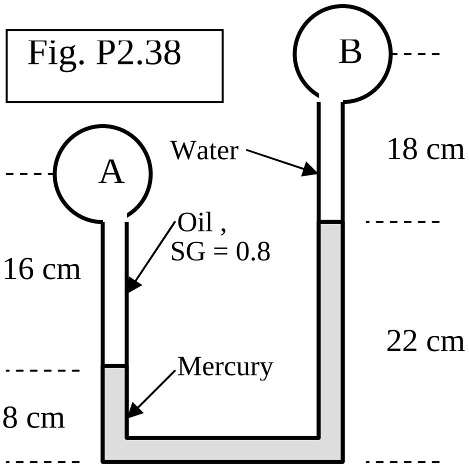
Fig. P2.37

Solution: The specific weight of the oil is (0.827)(62.4) = 51.6 lbf/ft^3. If the reservoir level does not change and ΔL = 1 inch is the scale marking, then

$$p_A(\text{gage}) = 1\,\frac{\text{lbf}}{\text{ft}^2} = \gamma_{oil}\Delta z = \gamma_{oil}\Delta L\ \sin\theta = \left(51.6\,\frac{\text{lbf}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ ft}\right)\sin\theta,$$

$$\text{or: } \sin\theta = 0.2325 \quad \text{or: } \theta = \mathbf{13.45°} \quad \textit{Ans.}$$

P2.38 If the pressure in container A is 150 kPa, compute the pressure in container B.



Solution: The specific weights are

γ_{oil} = (0.8)(9790) = 7832 N/m^3,

$\gamma_{mercury}$ = 133,100 N/m^3, and

γ_{water} = 9790 N/m^3.

Go down 16 cm from A to the mercury interface, jump across and go up 14 cm (22 cm - 8 cm) to the right-side mercury interface, and then up 18 cm of water to point B. Of course, you could also go straight down to the bottom of the tube and then across and up. Calculate

$$p_A + (7832\,N/m^3)(0.16m) - (133100)(0.14) - (9790)(0.18) = p_B$$

$$\text{Given } p_A = 150{,}000\,Pa, \quad \text{solve for } p_B = 130{,}900\,Pa \approx \mathbf{131}\ kPa \quad \textit{Ans.}$$

2.39 In Fig. P2.39 the right leg of the manometer is open to the atmosphere. Find the gage pressure, in Pa, in the air gap in the tank. Neglect surface tension.

Solution: The two 8-cm legs of air are negligible (only 2 Pa). Begin at the right mercury interface and go to the air gap:

$$\begin{aligned}&0\ \text{Pa-gage} + (133100\ \text{N/m}^3)(0.12 + 0.09\ \text{m})\\ &\quad - (0.8 \times 9790\ \text{N/m}^3)(0.09 - 0.12 - 0.08\ \text{m})\\ &\qquad = p_{\text{airgap}}\end{aligned}$$

or: $p_{\text{airgap}} = 27951\ \text{Pa} - 2271\ \text{Pa} \approx \mathbf{25700\ Pa\text{-}gage}$ *Ans.*

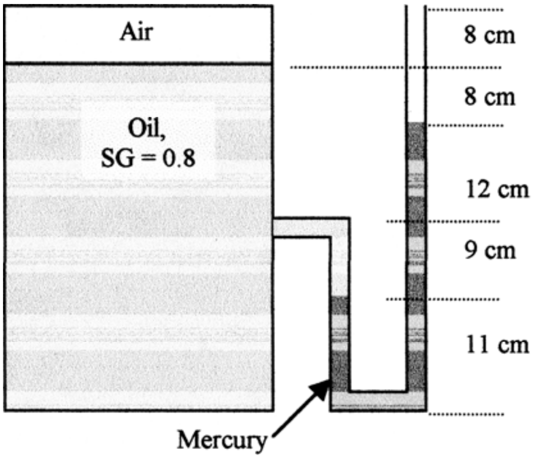


Fig. P2.39

2.40 In Fig. P2.40 the pressures at A and B are the same, 100 kPa. If water is introduced at A to increase p_A to 130 kPa, find and sketch the new positions of the mercury menisci. The connecting tube is a uniform 1-cm in diameter. Assume no change in the liquid densities.

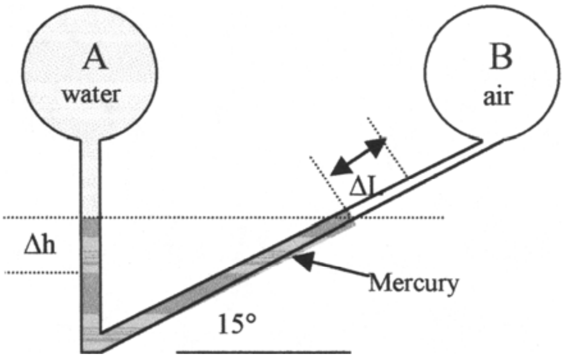


Fig. P2.40

Solution: Since the tube diameter is constant, the volume of mercury will displace a distance Δh down the left side, equal to the volume increase on the right side; Δh = ΔL. Apply the hydrostatic relation to the pressure change, beginning at the right (air/mercury) interface:

$$p_B + \gamma_{Hg}(\Delta L \sin\theta + \Delta h) - \gamma_W(\Delta h + \Delta L \sin\theta) = p_A \quad \text{with} \quad \Delta h = \Delta L$$

$$\text{or:} \quad 100{,}000 + 133100(\Delta h)(1+\sin 15^\circ) - 9790(\Delta h)(1+\sin 15^\circ) = p_A = 130{,}000\ \text{Pa}$$

$$\text{Solve for} \quad \Delta h = (30{,}000\ \text{Pa})/[(133100 - 9790\ \text{N/m}^2)(1+\sin 15^\circ)] = \mathbf{0.193\ m} \quad \textit{Ans.}$$

The mercury in the left (vertical) leg will drop 19.3 cm, the mercury in the right (slanted) leg will rise 19.3 cm along the slant and 5 cm in vertical elevation.

2.41 The system in Fig. P2.41 is at 20°C. Determine the pressure at point A in pounds per square foot.

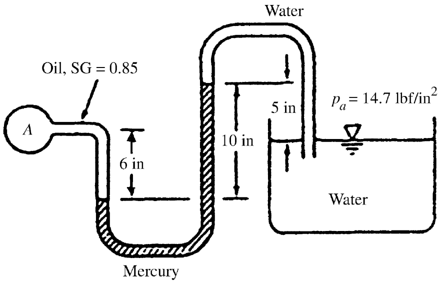


Fig. P2.41

Solution: Take the specific weights of water and mercury from Table 2.1. Write the hydrostatic formula from point A to the water surface:

$$p_A + (0.85)(62.4\ \text{lbf/ft}^3)\left(\frac{6}{12}\ \text{ft}\right) - (846)\left(\frac{10}{12}\right) + (62.4)\left(\frac{5}{12}\right) = p_{atm} = (14.7)(144)\ \frac{\text{lbf}}{\text{ft}^2}$$

$$\text{Solve for} \quad p_A = \mathbf{2770\ lbf/ft^2} \quad \textit{Ans.}$$

2.42 Small pressure differences can be measured by the two-fluid manometer in Fig. P2.42, where ρ_2 is only slightly larger than ρ_1. Derive a formula for $p_A - p_B$ if the reservoirs are very large.

Solution: Apply the hydrostatic formula from A to B:

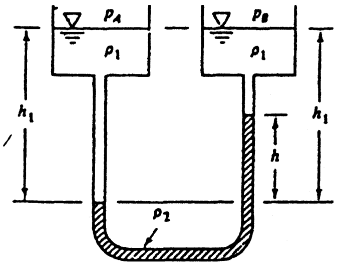


Fig. P2.42

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$$

$$\text{Solve for} \quad \mathbf{p_A - p_B = (\rho_2 - \rho_1)gh} \quad \textit{Ans.}$$

If $(\rho_2 - \rho_1)$ is very small, *h* will be very large for a given Δp (a sensitive manometer).

2.43 The traditional method of measuring blood pressure uses a *sphygmomanometer*, first recording the highest (*systolic*) and then the lowest (*diastolic*) pressure from which flowing "Korotkoff" sounds can be heard. Patients with dangerous hypertension can exhibit systolic pressures as high as 5 $\mathrm{lbf/in^2}$. Normal levels, however, are 2.7 and 1.7 $\mathrm{lbf/in^2}$, respectively, for systolic and diastolic pressures. The manometer uses mercury and air as fluids. (a) How high should the manometer tube be? (b) Express normal systolic and diastolic blood pressure in millimeters of mercury.

Solution: (a) The manometer height must be at least large enough to accommodate the largest systolic pressure expected. Thus apply the hydrostatic relation using 5 $\mathrm{lbf/in^2}$ as the pressure,

$$h = p_B/\rho g = (5\ \mathrm{lbf/in^2})(6895\ \mathrm{Pa/lbf/in^2})/(133100\ \mathrm{N/m^3}) = 0.26\ \mathrm{m}$$

So make the height about **30 cm**. *Ans.* (a)

(b) Convert the systolic and diastolic pressures by dividing them by mercury's specific weight.

$$h_{\text{systolic}} = (2.7\ \mathrm{lbf/in^2})(144\ \mathrm{in^2/ft^2})/(846\ \mathrm{lbf/ft^3}) = 0.46\ \mathrm{ft\ Hg} = 140\ \mathrm{mm\ Hg}$$

$$h_{\text{diastolic}} = (1.7\ \mathrm{lbf/in^2})(144\ \mathrm{in^2/ft^2})/(846\ \mathrm{lbf/ft^3}) = 0.289\ \mathrm{ft\ Hg} = 88\ \mathrm{mm\ Hg}$$

The systolic/diastolic pressures are thus **140/88 mm Hg**. *Ans.* (b)

2.44 Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The mercury manometer reads a 6-in height. The pressure drop $p_2 - p_1$ is partly due to friction and partly due to gravity. Determine the total pressure drop and also the part due to friction only. Which part does the manometer read? Why?

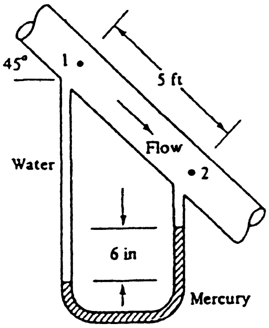


Fig. P2.44

Solution: Let "h" be the distance down from point 2 to the mercury-water interface in the right leg. Write the hydrostatic formula from 1 to 2:

$$p_1 + 62.4\left(5\sin 45° + h + \frac{6}{12}\right) - 846\left(\frac{6}{12}\right) - 62.4h = p_2,$$

$$p_1 - p_2 = \underbrace{(846 - 62.4)(6/12)}_{\text{....friction loss...}} - \underbrace{62.4(5\sin 45°)}_{\text{..gravity head..}} = 392 - 221$$

$$= \mathbf{171\,\frac{lbf}{ft^2}} \quad \textit{Ans.}$$

The manometer reads only the friction loss of 392 lbf/ft^2, not the gravity head of 221 psf.

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than $P_{\text{atmosphere}}$?

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$\begin{aligned} p_{atm} &+ (0.85)(9790)(0.4\text{ m}) \\ &- (133100)(0.15\text{ m}) - (12)(0.30\text{ m}) \\ &+ (9790)(0.45\text{ m}) = p_A, \end{aligned}$$

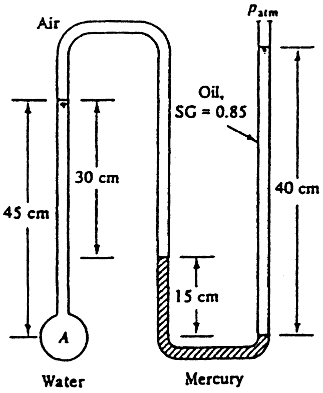


Fig. P2.45

or: $p_A = p_{atm} - 12200\text{ Pa} = \mathbf{12200\ Pa\ (vacuum)}$ *Ans.*

2.46 In Fig. P2.46 both ends of the manometer are open to the atmosphere. Estimate the specific gravity of fluid X.

Solution: The pressure at the bottom of the manometer must be the same regardless of which leg we approach through, left or right:

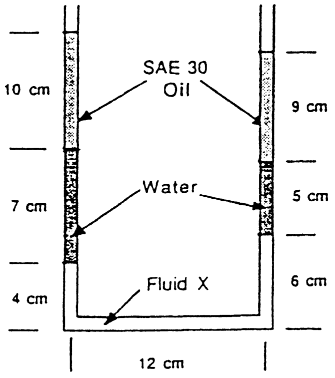


Fig. P2.46

$$p_{atm} + (8720)(0.1) + (9790)(0.07) + \gamma_X(0.04) \quad \text{(left leg)}$$

$$= p_{atm} + (8720)(0.09) + (9790)(0.05) + \gamma_X(0.06) \quad \text{(right leg)}$$

$$\text{or:} \quad \gamma_X = 14150\ \text{N/m}^3, \quad SG_X = \frac{14150}{9790} \approx \mathbf{1.45} \quad \textit{Ans.}$$

2.47 The cylindrical tank in Fig. P2.47 is being filled with 20°C water by a pump developing an exit pressure of 175 kPa. At the instant shown, the air pressure is 110 kPa and H = 35 cm. The pump stops when it can no longer raise the water pressure. Estimate "H" at that time.

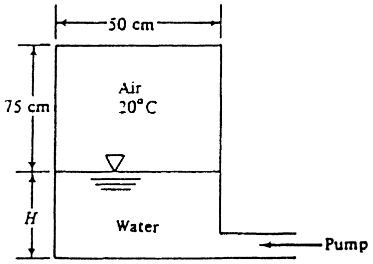


Fig. P2.47

Solution: At the end of pumping, the bottom water pressure must be 175 kPa:

$$p_{air} + 9790H = 175000$$

Meanwhile, assuming isothermal air compression, the final air pressure is such that

$$\frac{p_{air}}{110000} = \frac{Vol_{old}}{Vol_{new}} = \frac{\pi R^2(0.75\ \text{m})}{\pi R^2(1.1\ \text{m} - H)} = \frac{0.75}{1.1 - H}$$

where R is the tank radius. Combining these two gives a quadratic equation for H:

$$\frac{0.75(110000)}{1.1 - H} + 9790H = 175000, \quad \text{or} \quad H^2 - 18.98H + 11.24 = 0$$

The two roots are H = 18.37 m (ridiculous) or, properly, H = **0.612 m** *Ans.*

P2.48 The system in Fig. P2.49 is open to 1 atm on the right side.

(*a*) If L = 120 cm, what is the air pressure in container A?

(*b*) Conversely, if p_A = 135 kPa, what is the length L?

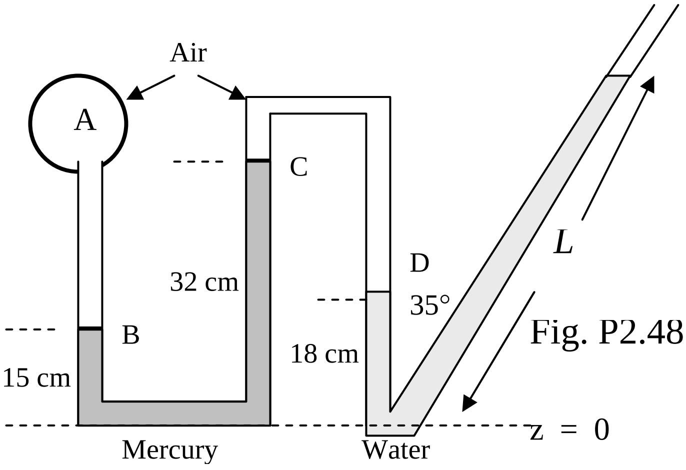


Fig. P2.48

Solution: (*a*) The vertical elevation of the water surface in the slanted tube is (1.2m)(sin55°) = 0.983 m. Then the pressure at the 18-cm level of the water, point D, is

$$p_D = p_{atm} + \gamma_{water}\,\Delta z = 101350\,Pa + (9790\frac{N}{m^3})(0.983 - 0.18m) = 109200\,Pa$$

Going up from D to C in air is negligible, less than 2 Pa. Thus $p_C \approx p_D$ = 109200 Pa. Going down from point C to the level of point B increases the pressure in mercury:

$$p_B = p_C + \gamma_{mercury}\,\Delta z_{C-B} = 109200 + (133100\frac{N}{m^3})(0.32 - 0.15m) = \mathbf{131800\,Pa} \quad Ans.(a)$$

This is the answer, since again it is negligible to go up to point A in low-density air.
(*b*) Given p_A = 135 kPa, go down from point A to point B with negligible air-pressure change, then jump across the mercury U-tube and go up to point C with a decrease:

$$p_C = p_B - \gamma_{mercury}\Delta z_{B-C} = 135000 - (133100)(0.32 - 0.15) = 112400\,Pa$$

Once again, $p_C \approx p_D \approx$ 112400 Pa, jump across the water and then go up to the surface:

$$p_{atm} = p_D - \gamma_{water}\,\Delta z = 112400 - 9790(z_{surface} - 0.18m) = 101350 Pa$$

$$\text{Solve for} \quad z_{surface} \approx 1.306\,m$$

$$\text{Then the slanted distance} \quad L = 1.306m/\sin 55^\circ = \mathbf{1.594\,m} \qquad Ans.(b)$$

2.49 Conduct an experiment: Place a thin wooden ruler on a table with a 40% overhang, as shown. Cover it with 2 full-size sheets of newspaper. (a) Estimate the total force on top of the newspaper due to air pressure. (b) With everyone out of the way, perform a karate chop on the outer end of the ruler. (c) Explain the results in b.

Results: (a) Newsprint is about 27 in (0.686 m) by 22.5 in (0.572 m). Thus the force is:

$$F = pA = (101325\ Pa)(0.686\ m)(0.572\ m)$$
$$= \mathbf{39700\ N!} \quad \textit{Ans.}$$

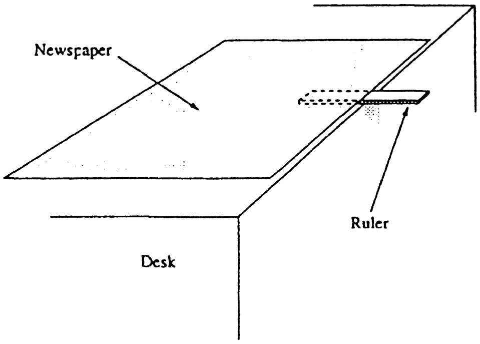


Fig. P2.48

(b) The newspaper will hold the ruler, which will probably *break* due to the chop. *Ans.*
(c) Chop is fast, air does not have time to rush in, partial vacuum under newspaper. *Ans.*

P2.50 A small submarine, with a hatch door 30 inches in diameter, is submerged in seawater. (*a*) If the water hydrostatic force on the hatch is 69,000 lbf, how deep is the sub? (*b*) If the sub is 350 ft deep, what is the hydrostatic force on the hatch?

Solution: In either case, the force is $p_{CG}A_{hatch}$. Stay with BG units. Convert 30 inches = 2.5 ft. For seawater, ρ = 1025 kg/m^3 ÷ 515.38 = 1.99 slug/ft^3, hence γ = (1.99)(32.2) = 64.0 lbf/ft^3.

$$(a)\ F = p_{cg}\,A = (\gamma\,h)\,A = 69{,}000\,lbf = (64\frac{lbf}{ft^3})\,h\,\frac{\pi}{4}(2.5\ ft)^2\ ;\quad h = \mathbf{220}\ ft \qquad Ans.(a)$$

$$(b)\ F = p_{cg}\,A = (\gamma\,h)\,A\ = (64\frac{lbf}{ft^3})(350\ ft)\,\frac{\pi}{4}(2.5\ ft)^2\ =\ \mathbf{110{,}000}\ lbf \qquad Ans.(b)$$

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

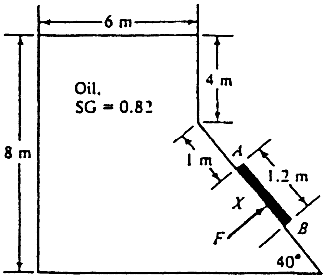


Fig. P2.51

Solution: The centroidal depth of the gate is

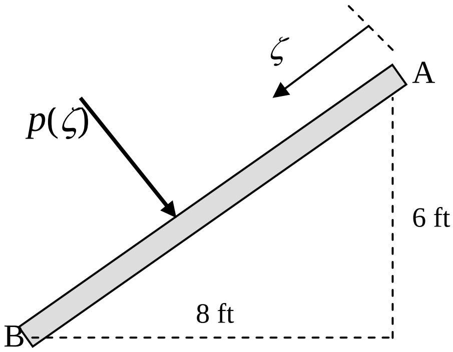
$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028\text{ m},$$

$$\text{hence}\quad F_{AB} = \gamma_{oil}h_{CG}A_{gate} = (0.82\times 9790)(5.028)(1.2\times 0.8) = \mathbf{38750\ N} \quad \textit{Ans.}$$

The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3\sin 40°}{(5.028)(1.2\times 0.8)} = -0.0153\text{ m}$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx$ **0.615 m** *Ans.*



P2.52 Example 2.5 calculated the force on plate AB and its line of action, using the moment-of-inertia approach. Some teachers say it is more instructive to calculate these by *direct integration* of the pressure forces. Using Figs. P2.52 and E2.5*a*, (*a*) find an expression for the pressure variation $p(\zeta)$ along the plate;

Fig. P2.52

(*b*) integrate this pressure to find the total force *F*;
(*c*) integrate the moments about point A to find the position of the center of pressure.

Solution: (*a*) Point A is 9 ft deep, and point B is 15 ft deep, and γ = 64 lbf/ft^3. Thus p_A = (64lbf/ft^3)(9ft) = 576 lbf/ft^2 and p_B = (64lbf/ft^3)(15ft) = 960 lbf/ft^2. Along the 10-ft length, pressure increases by (960-576)/10ft = 38.4 lbf/ft^2/ft. Thus the pressure is

$$p(\zeta) \;=\; 576 + 38.4\zeta \quad (lbf/ft^2) \qquad\qquad Ans.(a)$$

(b) Given that the plate width b = 5 ft. Integrate for the total force on the plate:

$$F \;=\; \int_{plate} p\,dA = \int p\,b\,d\zeta = \int_0^{10} (576 + 38.4\zeta)(5\,ft)d\zeta \;=$$

$$= (5)(576\zeta + 38.4\zeta^2/2)\Big|_0^{10} \;=\; 28800 + 9600 \;=\; \mathbf{38{,}400\ lbf} \qquad Ans.(b)$$

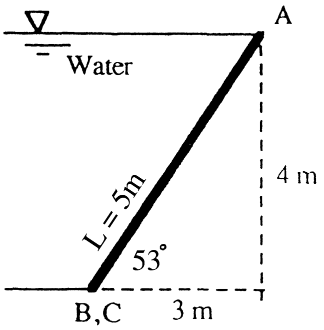
(c) Find the moment of the pressure forces about point A and divide by the force:

The center of pressure is 5.417 ft down the plate from Point A.

$$M_A \;=\; \int_{plate} p\zeta\,b\,dA \;=\; \int_0^{10} \zeta(576 + 38.4\zeta)(5\,ft)d\zeta \;=$$

$$= (5)(576\zeta^2/2 + 38.4\zeta^3/3)\Big|_0^{10} = 144000 + 64000 \;=\; 208{,}000\,ft - lbf$$

$$Then \qquad \zeta_{CP} \;=\; \frac{M_A}{F} = \frac{208000\ ft - lbf}{38400\,lbf} \;=\; \mathbf{5.42\ ft} \qquad Ans.(c)$$



2.53 Panel ABC in the slanted side of a water tank (shown at right) is an isoceles triangle with vertex at A and base BC = 2 m. Find the water force on the panel and its line of action.

Solution: (a) The centroid of ABC is 2/3 of the depth down, or 8/3 m from the surface. The panel area is (1/2)(2 m)(5 m) = 5 m^2. The water force is

$$F_{ABC} = \gamma h_{CG} A_{panel} = (9790)(2.67\text{ m})(5\text{ m}^2) = \mathbf{131{,}000\ N} \quad \textit{Ans.}\text{ (a)}$$

(b) The moment of inertia of ABC is (1/36)(2 m)(5 m)^3 = 6.94 m^4. From Eq. (2.44),

$$y_{CP} = -I_{xx}\sin\theta/(h_{CG}A_{panel}) = -6.94\sin(53°)/[2.67(5)] = \mathbf{-0.417\ m} \quad \textit{Ans.}\text{ (b)}$$

The centroid is 5(2/3) = 3.33 m down from A along the panel. The center of pressure is thus (3.33+0.417) = **3.75 m down from A**, or 1.25 m up from BC.

2.54 In Fig. P2.54, the hydrostatic force F is the same on the bottom of all three containers, even though the weights of liquid above are quite different. The three bottom shapes and the fluids are the same. This is called the *hydrostatic paradox*. Explain why it is true and sketch a freebody of each of the liquid columns.

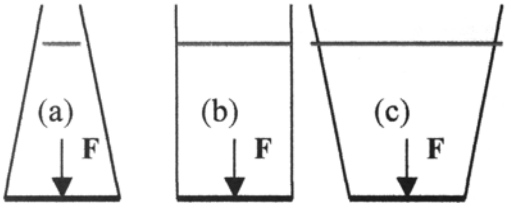
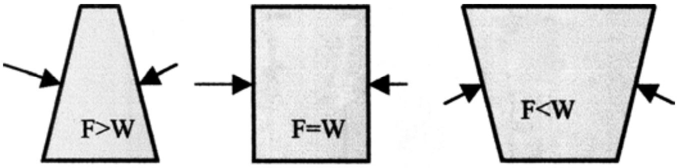


Fig. P2.54

Solution: The three freebodies are shown below. Pressure on the side-walls balances the forces. In (a), downward side-pressure components help add to a light W. In (b) side pressures are horizontal. In (c) upward side pressure helps reduce a heavy W.



(a) (b) (c)

2.55 Gate AB in Fig. P2.55 is 5 ft wide into the paper, hinged at A, and restrained by a stop at B. Compute (a) the force on stop B; and (b) the reactions at A if $h = 9.5$ ft.

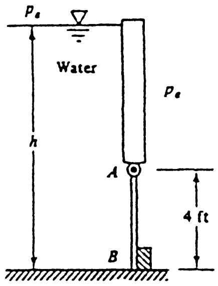


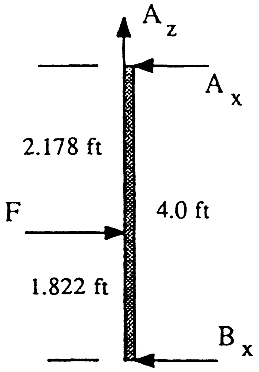
Fig. P2.55

Solution: The centroid of AB is 2.0 ft below A, hence the centroidal depth is $h + 2 - 4 = 7.5$ ft. Then the total hydrostatic force on the gate is

$$F = \gamma h_{CG} A_{gate} = (62.4\ \text{lbf/ft}^3)(7.5\ \text{ft})(20\ \text{ft}^2) = 9360\ \text{lbf}$$

The C.P. is below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(1/12)(5)(4)^3\sin 90°}{(7.5)(20)}$$
$$= -0.178\ \text{ft}$$



This is shown on the freebody of the gate at right. We find force Bx with moments about A:

$$\sum M_A = B_x(4.0) - (9360)(2.178) = 0,$$
$$\text{or:}\quad B_x = \mathbf{5100\ lbf} \quad \text{(to left)} \quad \textit{Ans.}\ \text{(a)}$$

The reaction forces at A then follow from equilibrium of forces (with *zero* gate weight):

$$\sum F_x = 0 = 9360 - 5100 - A_x, \quad \text{or:} \quad A_x = \mathbf{4260\ lbf} \quad \text{(to left)}$$
$$\sum F_z = 0 = A_z + W_{gate} \approx A_z, \quad \text{or:} \quad A_z = \mathbf{0\ lbf} \quad \textit{Ans.}\ \text{(b)}$$

2.56 For the gate of Prob. 2.55 above, stop "B" breaks if the force on it equals 9200 lbf. For what water depth *h* is this condition reached?

Solution: The formulas must be written in terms of the unknown centroidal depth h_{CG}:

$$h_{CG} = h - 2 \quad F = \gamma h_{CG} A = (62.4)h_{CG}(20) = 1248h_{CG}$$

$$y_{CP} = -\frac{I_{XX}\sin\theta}{h_{CG}A} = -\frac{(1/12)(5)(4)^3 \sin 90°}{h_{CG}(20)} = -\frac{1.333}{h_{CG}}$$

Then moments about A for the freebody in Prob. 2.155 above will yield the answer:

$$\sum M_A = 0 = 9200(4) - (1248h_{CG})\left(2 + \frac{1.333}{h_{CG}}\right), \quad \text{or} \quad h_{CG} = 14.08 \text{ ft}, \; h = \mathbf{16.08\ ft} \quad \textit{Ans.}$$

2.57 The tank in Fig. P2.57 is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on panel BC, (a) from a single formula; (b) by computing horizontal and vertical forces separately, in the spirit of curved surfaces.

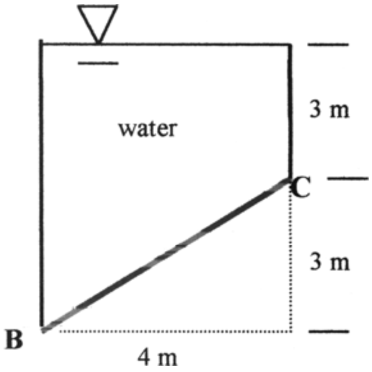


Fig. P2.57

Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790 \text{ N/m}^3)(3 + 1.5 \text{ m})(5 \text{ m} \times 2 \text{ m}) = 440{,}550 \text{ N} = \mathbf{441\ kN} \quad \textit{Ans.} \text{ (a)}$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

$$F_H = (9790)(4.5)(3 \times 2) = 264{,}330 \text{ N} = \mathbf{264\ kN} \quad \textit{Ans.} \text{ (b)}$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790)[(3)(4) + (1/2)(4)(3)](2) = 352{,}440 = \mathbf{352\ kN} \quad \textit{Ans.} \text{ (b)}$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2}$ **= 441 kN.**

2.58 In Fig. P2.58, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?

Solution: The centroidal depth is exactly

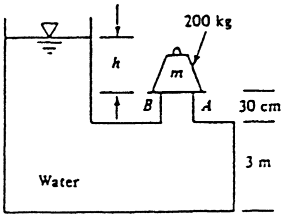


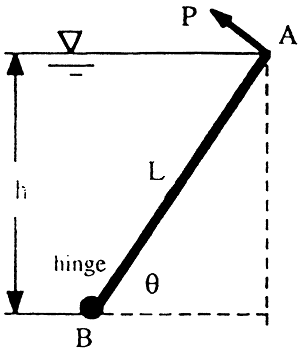
Fig. P2.58

equal to *h* and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG} A_{gate} = (9790\ \text{N/m}^3)\ h\frac{\pi}{4}(0.8\ \text{m})^2 = W = (200)(9.81)\ \text{N}$$

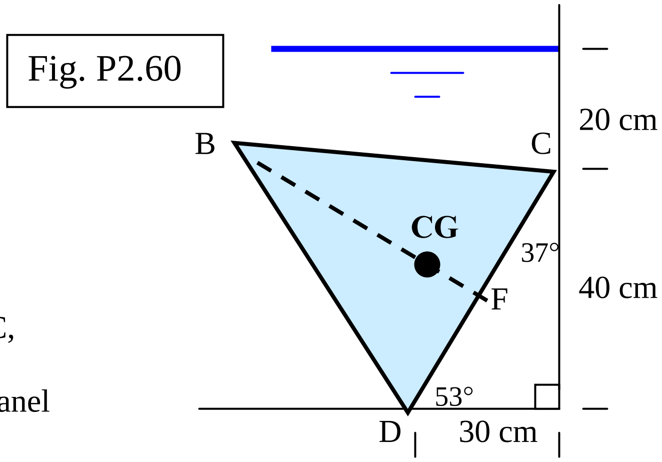
Solve for $h = \mathbf{0.40\ m}$ *Ans.*

2.59 Gate AB has length L, width b into the paper, is hinged at B, and has negligible weight. The liquid level h remains at the top of the gate for any angle θ. Find an analytic expression for the force P, perpendicular to AB, required to keep the gate in equilibrium.



Solution: The centroid of the gate remains at distance L/2 from A and depth h/2 below the surface. For any θ, then, the hydrostatic force is $F = \gamma(h/2)Lb$. The moment of inertia of the gate is $(1/12)bL^3$, hence $y_{CP} = -(1/12)bL^3\sin\theta/[(h/2)Lb]$, and the center of pressure is $(L/2 - y_{CP})$ from point B. Summing moments about hinge B yields

$$PL = F(L/2 - y_{CP}), \quad \text{or:} \quad \mathbf{P = (\gamma hb/4)[L - L^2\sin\theta/(3h)]} \quad \textit{Ans.}$$



P2.60 Determine the water hydrostatic force on one side of the vertical equilateral triangle panel BCD in Fig.. P2.60. Neglect atmospheric pressure.

Solution: Pythagoras says that the lengths of DC, BC, and BD are all 50 cm. The centroid of the panel would lie along the bisector BF, whose length is 50 sin60° = 43.3 cm.

The CG would be 1/3 of the way from F to B, or 14.4 cm away from F. The CG would be 14.4sin37° = 8.66 cm vertically above F, and F is 20 cm above the bottom. Thus the CG is 60 cm – 20 cm – 8.66 cm = 31.34 cm below the surface of the water. The hydrostatic force is

$$A_{triangle} = (1/2)(50\,cm)(43.3\,cm) = 1082\,cm^2 = 0.1082\,m^2$$

$$p_{CG} = \gamma_{water}\,h_{CG} = (9790\ N/m^3)(0.3134\,m) = 3068\,Pa$$

Finally, $F_{panel} = p_{CG}\,A = (3068\,Pa)(0.1082\,m^2) = \mathbf{332\ N}$ *Ans.*

2.61 Gate AB in Fig. P2.61 is a homo-geneous mass of 180 kg, 1.2 m wide into the paper, resting on smooth bottom B. All fluids are at 20°C. For what water depth *h* will the force at point B be zero?

Solution: Let $\gamma = 12360\ \text{N/m}^3$ for glycerin and $9790\ \text{N/m}^3$ for water. The centroid of AB is 0.433 m vertically below A, so $h_{CG} = 2.0 - 0.433 = 1.567$ m, and we may compute the glycerin force and its line of action:

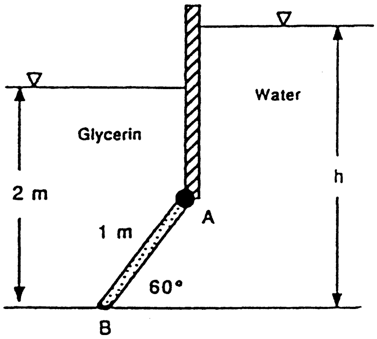
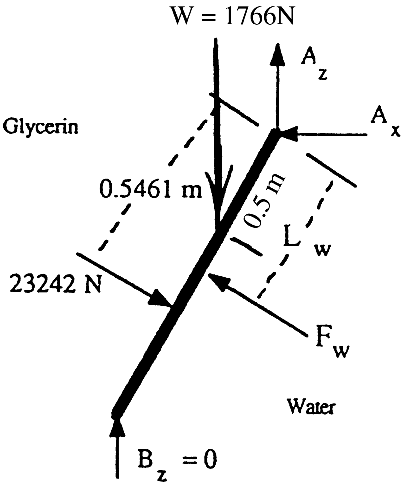


Fig. P2.61

$$F_g = \gamma \bar{h} A = (12360)(1.567)(1.2) = 23242\ \text{N}$$

$$y_{CP,g} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{(1.567)(1.2)} = -0.0461\ \text{m}$$

These are shown on the freebody at right. The water force and its line of action are shown without numbers, because they depend upon the centroidal depth on the water side:



$$F_w = (9790) h_{CG} (1.2)$$

$$y_{CP} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{h_{CG}(1.2)} = -\frac{0.0722}{h_{CG}}$$

The weight of the gate, W = 180(9.81) = 1766 N, acts at the centroid, as shown above. Since the force at B equals zero, we may sum moments counterclockwise about A to find the water depth:

$$\sum M_A = 0 = (23242)(0.5461) + (1766)(0.5 \cos 60°) - (9790) h_{CG} (1.2)(0.5 + 0.0722/h_{CG})$$

Solve for $h_{CG,water} = 2.09$ m, or: $h = h_{CG} + 0.433 = \mathbf{2.52\ m}$ *Ans.*

2.62 Gate AB in Fig. P2.62 is 15 ft long and 8 ft wide into the paper, hinged at B with a stop at A. The gate is 1-in-thick steel, SG = 7.85. Compute the 20°C water level *h* for which the gate will start to fall.

Solution: Only the length (*h* csc 60°) of the gate lies below the water. Only this part contributes to the hydrostatic force shown in the freebody at right:

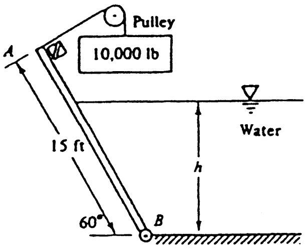


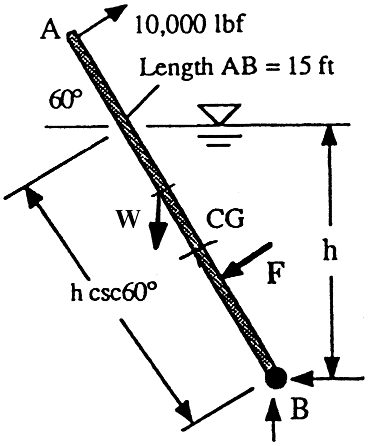
Fig. P2.62

$$F = \gamma h_{CG} A = (62.4)\left(\frac{h}{2}\right)(8h\csc 60°)$$

$$= 288.2h^2 \text{ (lbf)}$$

$$y_{CP} = -\frac{(1/12)(8)(h\csc 60°)^3\sin 60°}{(h/2)(8h\csc 60°)}$$

$$= -\frac{h}{6}\csc 60°$$



The weight of the gate is (7.85)(62.4 lbf/ft^3)(15 ft)(1/12 ft)(8 ft) = 4898 lbf. This weight acts downward at the CG of the *full gate* as shown (not the CG of the submerged portion). Thus, W is 7.5 ft above point B and has moment arm (7.5 cos 60° ft) about B.

We are now in a position to find *h* by summing moments about the hinge line B:

$$\sum M_B = (10000)(15) - (288.2h^2)[(h/2)\csc 60° - (h/6)\csc 60°] - 4898(7.5\cos 60°) = 0,$$

$$\text{or:}\quad 110.9h^3 = 150000 - 18369, \quad h = (131631/110.9)^{1/3} = \mathbf{10.6\ ft} \quad \textit{Ans.}$$

2.63 The tank in Fig. P2.63 has a 4-cm-diameter plug which will pop out if the hydrostatic force on it reaches 25 N. For 20°C fluids, what will be the reading *h* on the manometer when this happens?

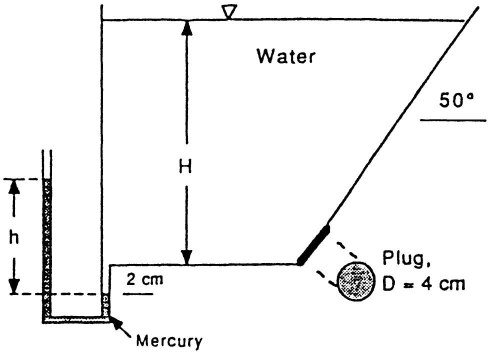


Fig. P2.63

Solution: The water depth when the plug pops out is

$$F = 25\ N = \gamma h_{CG} A = (9790)h_{CG}\frac{\pi(0.04)^2}{4}$$

$$\text{or}\quad h_{CG} = 2.032\ m$$

It makes little numerical difference, but the mercury-water interface is a little deeper than this, by the amount (0.02 sin 50°) of plug-depth, plus 2 cm of tube length. Thus

$$p_{atm} + (9790)(2.032 + 0.02\sin 50° + 0.02) - (133100)h = p_{atm},$$

$$\text{or:}\quad h \approx \mathbf{0.152\ m} \quad \textit{Ans.}$$

2.64 Gate ABC in Fig. P2.64 has a fixed hinge at B and is 2 m wide into the paper. If the water level is high enough, the gate will open. Compute the depth *h* for which this happens.

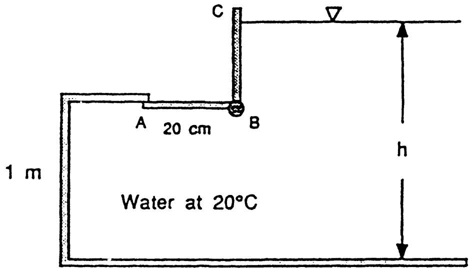
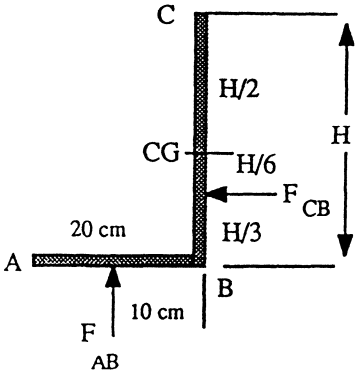


Fig. P2.64

Solution: Let H = (h − 1 meter) be the depth down to the level AB. The forces on AB and BC are shown in the freebody at right. The moments of these forces about B are equal when the gate opens:



$$\sum M_B = 0 = \gamma H(0.2)b(0.1)$$

$$= \gamma\left(\frac{H}{2}\right)(Hb)\left(\frac{H}{3}\right)$$

or: $H = 0.346$ m,

$h = H + 1 = \mathbf{1.346\ m}$ *Ans.*

This solution is independent of both the water density and the gate width *b* into the paper.

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

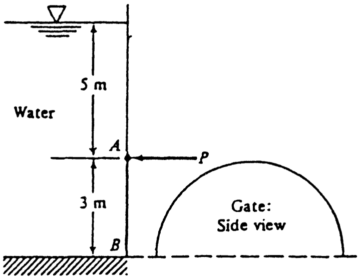
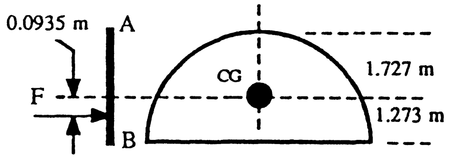


Fig. P2.65

Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is



$$F = \gamma h_{CG}A = (9790)(5.0 + 1.727)\frac{\pi}{2}(3)^2$$

$$= 931000\ \text{N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(0.10976)(3)^4\sin 90^\circ}{(5 + 1.727)(\pi/2)(3)^2} = -0.0935\ \text{m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \quad \text{or:} \quad P = \mathbf{366000\ N} \quad \textit{Ans.}$$

2.66 Dam ABC in Fig. P2.66 is 30 m wide into the paper and is concrete (SG ≈ 2.40). Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over? Would fluid seepage under the dam change your argument?

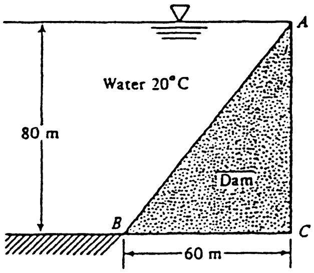
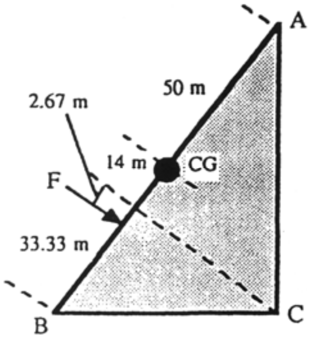


Fig. P2.66

Solution: The centroid of surface AB is 40 m deep, and the total force on AB is

$$F = \gamma h_{CG} A = (9790)(40)(100 \times 30)$$
$$= 1.175E9\ \text{N}$$



The line of action of this force is two-thirds of the way down along AB, or 66.67 m from A. This is seen either by inspection (A is at the surface) or by the usual formula:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(30)(100)^3\sin(53.13°)}{(40)(30 \times 100)} = -16.67\ \text{m}$$

to be added to the 50-m distance from A to the centroid, or 50 + 16.67 = 66.67 m. As shown in the figure, the line of action of F is 2.67 m to the left of a line up from C normal to AB. The moment of F about C is thus

$$M_C = FL = (1.175E9)(66.67 - 64.0) \approx \mathbf{3.13E9\ N \cdot m} \quad \textit{Ans.}$$

This moment is counterclockwise, hence it cannot tip over the dam. If there were *seepage* under the dam, the main support force at the bottom of the dam would shift to the left of point C and might indeed cause the dam to tip over.

2.67 Generalize Prob. 2.66 with length AB as "H", length BC as "L", and angle ABC as "θ", with width "b" into the paper. If the dam material has specific gravity "SG", with no seepage, find the critical angle θ_C for which the dam will just tip over to the right. Evaluate this expression for SG = 2.40.

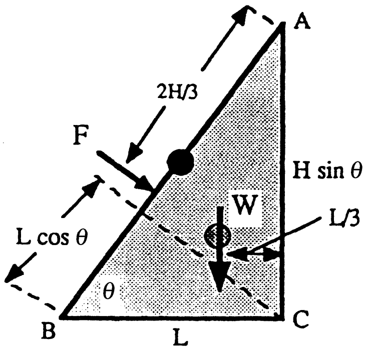


Fig. P2.67

Solution: By geometry, L = H$\cos\theta$ and the vertical height of the dam is H$\sin\theta$. The force F on surface AB is γ(H/2)(sinθ)Hb, and its position is at 2H/3 down from point A, as shown in the figure. Its moment arm about C is thus (H/3 − Lcosθ). Meanwhile the weight of the dam is W = (SG)γ(L/2)H(sinθ)b, with a moment arm L/3 as shown. Then summation of clockwise moments about C gives, for critical "tip-over" conditions,

$$\sum M_C = 0 = \left(\gamma \frac{H}{2}\sin\theta\, Hb\right)\left[\frac{H}{3} - L\cos\theta\right] - \left[SG(\gamma)\frac{L}{2}H\sin\theta\, b\right]\left(\frac{L}{3}\right) \quad \text{with } L = H\cos\theta.$$

$$\text{Solve for} \quad \cos^2\theta_c = \frac{1}{\mathbf{3+SG}} \quad \textit{Ans.}$$

Any angle greater than θ_c will cause tip-over to the right. For the particular case of concrete, SG ≈ 2.40, cosθ_c ≈ 0.430, or θ_c ≈ **64.5°**, which is greater than the given angle θ = 53.13° in Prob. 2.66, hence there was no tipping in that problem.

2.68 Isosceles triangle gate AB in Fig. P2.68 is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?

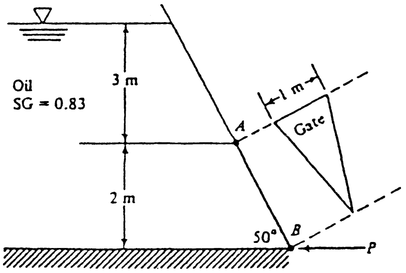


Fig. P2.68

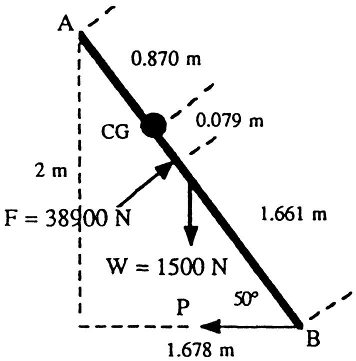
Solution: The gate is 2.0/sin 50° = 2.611 m long from A to B and its area is 1.3054 m^2. Its centroid is 1/3 of the way down from A, so the centroidal depth is 3.0 + 0.667 m. The force on the gate is

$$F = \gamma h_{CG}A = (0.83)(9790)(3.667)(1.3054)$$
$$= 38894 \text{ N}$$

The position of this force is below the centroid:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$= -\frac{(1/36)(1.0)(2.611)^3\sin 50°}{(3.667)(1.3054)} = -0.0791 \text{ m}$$



The force and its position are shown in the freebody at upper right. The gate weight of 1500 N is assumed at the centroid of the plate, with moment arm 0.559 meters about point A. Summing moments about point A gives the required force P:

$$\sum M_A = 0 = P(2.0) + 1500(0.559) - 38894(0.870 + 0.0791),$$

$$\text{Solve for} \quad P = \mathbf{18040\ N} \quad \textit{Ans.}$$

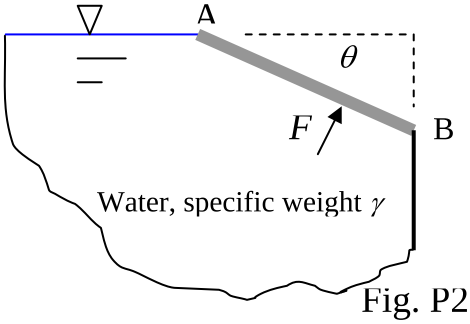


Fig. P2.69

P2.69 Consider the slanted plate AB of length L in Fig. P2.69. (*a*) Is the hydrostatic force F on the plate equal to the weight of the *missing water* above the plate? If not, correct this hypothesis. Neglect the atmosphere. (*b*) Can a "missing water" approach be generalized to *curved* plates of this type?

Solution: (*a*) The actual force F equals the pressure at the centroid times the plate area: But the weight of the "missing water" is

$$F = p_{CG}\,A_{plate} = \gamma\,h_{CG}\,L\,b = \gamma\frac{L\sin\theta}{2}L\,b = \frac{\gamma}{2}L^2\,b\sin\theta$$

$$W_{missing} = \gamma\,\upsilon_{missing} = \gamma\,[\frac{1}{2}(L\sin\theta)(L\cos\theta)b] = \frac{\gamma}{2}L^2\,b\sin\theta\cos\theta$$

Why the discrepancy? Because the actual plate force ***is not vertical***. Its vertical component is $F\cos\theta = W_{\text{missing}}$. The missing-water weight equals the ***vertical*** component of the force. *Ans*.(*a*) This same approach applies to ***curved*** plates with missing water. *Ans*.(*b*)

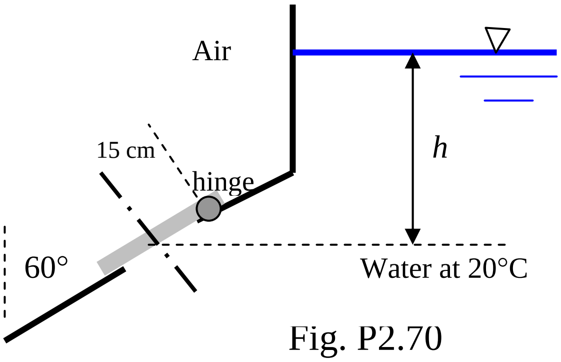


Fig. P2.70

P2.70 The swing-check valve in Fig. P2.70 covers a 22.86-cm diameter opening in the slanted wall. The hinge is 15 cm from the centerline, as shown. The valve will open when the hinge moment is 50 N-m. Find the value of *h* for the water to cause this condition.

Solution: For water, take γ = 9790 N/m^3. The hydrostatic force on the valve is

$$F = p_{CG}\,A = \gamma\, h(\pi) R^2 = (9790\frac{N}{m^3})\, h(\pi)(0.1143m)^2 = 401.8\ h$$

The center of pressure is slightly below the centerline by an amount

$$y_{CP} = -\frac{\gamma \sin\theta\, I_{xx}}{F} = \frac{(9790)\sin(30^\circ)(\pi/4)(0.1143)^4}{100.45h} = \frac{0.00653}{h}$$

The 60° angle in the figure is a red herring – we need the **30° angle** with the horizontal. Then the moment about the hinge is

$$M_{hinge} = F\,l = (401.8h)(0.15+\frac{0.00653}{h}) = 50\,N-m$$

$$\text{Solve for} \qquad h = \mathbf{0.79}\ m \qquad\qquad Ans.$$

Since y_{CP} is so small (2 mm), you don’t really need EES. Just iterate once or twice.

2.71 In Fig. P2.71 gate AB is 3 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.40). What sphere diameter is just right to close the gate?

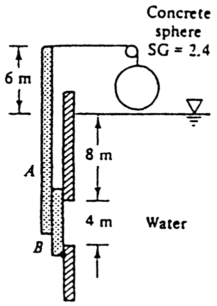


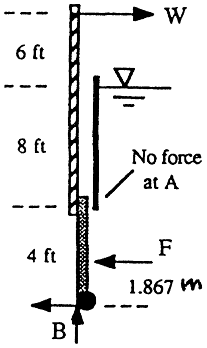
Fig. P2.71

Solution: The centroid of AB is 10 m down from the surface, hence the hydrostatic force is

$$F = \gamma h_{CG} A = (9790)(10)(4 \times 3)$$
$$= 1.175E6 \text{ N}$$

The line of action is slightly below the centroid:

$$y_{CP} = -\frac{(1/12)(3)(4)^3 \sin 90^\circ}{(10)(12)} = -0.133 \text{ m}$$



Sum moments about B in the freebody at right to find the pulley force or weight W:

$$\sum M_B = 0 = W(6+8+4 \text{ m}) - (1.175E6)(2.0 - 0.133 \text{ m}), \quad \text{or} \quad W = 121800 \text{ N}$$

Set this value equal to the weight of a solid concrete sphere:

$$W = 121800 \text{ N} = \gamma_{\text{concrete}} \frac{\pi}{6} D^3 = (2.4)(9790)\frac{\pi}{6} D^3, \quad \text{or:} \quad D_{\text{sphere}} = \mathbf{2.15 \text{ m}} \quad \textit{Ans.}$$

2.72 Gate B is 30 cm high and 60 cm wide into the paper and hinged at the top. What is the water depth *h* which will first cause the gate to open?

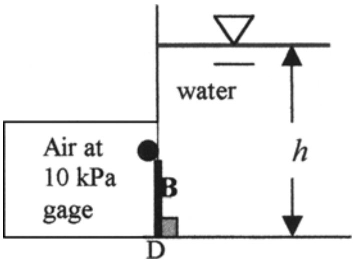


Fig. P2.72

Solution: The minimum height needed to open the gate can be assessed by calculating the hydrostatic force on each side of the gate and equating moments about the hinge. The air pressure causes a force, F_{air}, which acts on the gate at 0.15 m above point D.

$$F_{air} = (10{,}000 \text{ Pa})(0.3 \text{ m})(0.6 \text{ m}) = 1800 \text{ N}$$

Since the air pressure is uniform, F_{air} acts at the centroid of the gate, or 15 cm below the hinge. The force imparted by the water is simply the hydrostatic force,

$$F_w = (\gamma h_{CG} A)_w = (9790\ \mathrm{N/m^3})(h - 0.15\ \mathrm{m})(0.3\ \mathrm{m})(0.6\ \mathrm{m}) = 1762.2h - 264.3$$

This force has a center of pressure at,

$$y_{CP} = \frac{(1/12)(0.6)(0.3)^3(\sin 90^\circ)}{(h-0.15)(0.3)(0.6)} = \frac{0.0075}{h-0.15} \quad \text{with } h \text{ in meters}$$

Sum moments about the hinge and set equal to zero to find the minimum height:

$$\sum M_{hinge} = 0 = (1762.2h - 264.3)[0.15 + (0.0075/(h-0.15))] - (1800)(0.15)$$

This is quadratic in *h*, but let's simply solve by iteration: $h = \mathbf{1.12\ m}$ *Ans.*

2.73 Weightless gate AB is 5 ft wide into the paper and opens to let fresh water out when the ocean tide is falling. The hinge at A is 2 ft above the freshwater level. Find *h* when the gate opens.

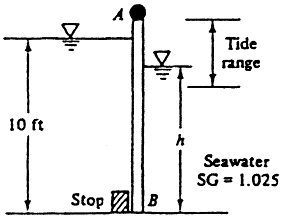
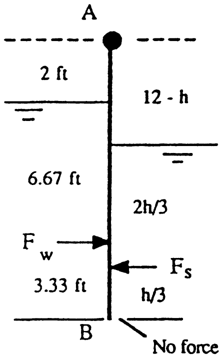


Fig. P2.73



Solution: There are two different hydrostatic forces and two different lines of action. On the water side,

$$F_w = \gamma h_{CG} A = (62.4)(5)(10 \times 5) = 15600\ \text{lbf}$$

positioned at 3.33 ft above point B. In the seawater,

$$F_s = (1.025 \times 62.4)\left(\frac{h}{2}\right)(5h) = 159.9h^2 \quad \text{(lbf)}$$

positioned at h/3 above point B. Summing moments about hinge point A gives the desired seawater depth h:

$$\sum M_A = 0 = (159.9h^2)(12 - h/3) - (15600)(12 - 3.33),$$

$$\text{or} \quad 53.3h^3 - 1918.8h^2 + 135200 = 0, \quad \text{solve for} \quad h = \mathbf{9.85\ ft} \quad \textit{Ans.}$$

2.74 Find the height H in Fig. P2.74 for which the hydrostatic force on the rectangular panel is the same as the force on the semicircular panel below.

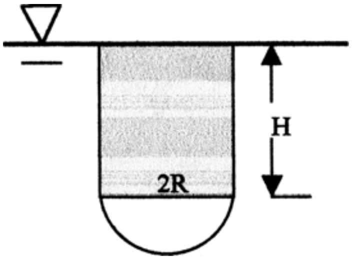


Fig. P2.74

Solution: Find the force on each panel and set them equal:

$$F_{rect} = \gamma h_{CG} A_{rect} = \gamma(H/2)[(2R)(H)] = \gamma R H^2$$

$$F_{semi} = \gamma h_{CG} A_{semi} = \gamma(H + 4R/3\pi)[(\pi/2)R^2]$$

Set them equal, cancel γ: $RH^2 = (\pi/2)R^2H + 2R^3/3$, or: $H^2 - (\pi/2)RH - 2R^2/3 = 0$

Finally, $H = \mathbf{R[\pi/4 + \{(\pi/4)^2 + 2/3\}^{1/2}] \approx 1.92R}$ *Ans.*

P2.75 The cap at point B on the 5-cm-diameter tube in Fig. P2.75 will be dislodged when the hydrostatic force on its base reaches 22 lbf. For what water depth h does this occur?

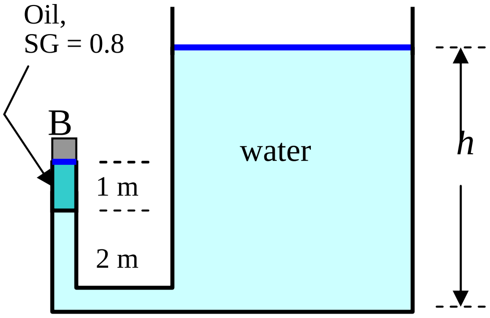


Fig. P2.75

Solution: Convert the cap force to SI units: 22 lbf x 4.4482 = 97.9 N. Then the “dislodging: pressure just under cap B will be

$$p_B = \frac{F}{A_{tube}} = \frac{97.9\,N}{(\pi/4)(0.05\,m)^2} = 49{,}800\ Pa\ \ (gage)$$

Begin at point B, go down and around the two fluids to the surface of the tank:

$$49800\,Pa + (0.8)(9790\frac{N}{m^3})(1m) + (9790\frac{N}{m^3})(2\,m) - (9790\frac{N}{m^3})(h) = p_{surface} = 0\ (gage)$$

$$\text{Solve for}\quad h = \frac{77250\,Pa}{9790\,N/m^3} = \mathbf{7.89}\ m \qquad \textit{Ans.}$$

2.76 Panel BC in Fig. P2.76 is circular. Compute (a) the hydrostatic force of the water on the panel; (b) its center of pressure; and (c) the moment of this force about point B.

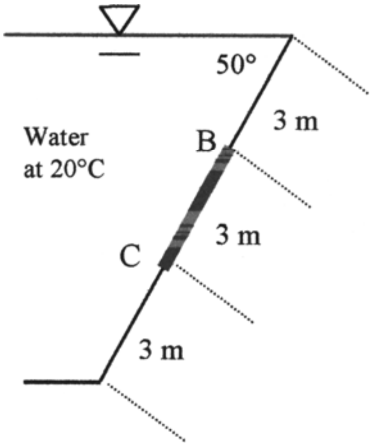


Fig. P2.76

Solution: (a) The hydrostatic force on the gate is:

$$\begin{aligned}F &= \gamma h_{CG} A \\ &= (9790\ \text{N/m}^3)(4.5\ \text{m})\sin 50°(\pi)(1.5\ \text{m})^2 \\ &= \mathbf{239\ kN} \quad \textit{Ans.}\ \text{(a)}\end{aligned}$$

(b) The center of pressure of the force is:

$$\begin{aligned}y_{CP} &= \frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{\frac{\pi}{4}r^4\sin\theta}{h_{CG}A} \\ &= \frac{\frac{\pi}{4}(1.5)^4\sin 50°}{(4.5\sin 50°)(\pi)(1.5^2)} = \mathbf{0.125}\ \boldsymbol{m} \quad \textit{Ans.}\ \text{(b)}\end{aligned}$$

Thus y is **1.625 m** down along the panel from B (or 0.125 m down from the center of the circle).

(c) The moment about B due to the hydrostatic force is,

$$M_B = (238550\ \text{N})(1.625\ \text{m}) = 387{,}600\ \text{N}\cdot\text{m} = \mathbf{388\ kN\cdot m} \quad \textit{Ans.}\ \text{(c)}$$

2.77 Circular gate ABC is hinged at B. Compute the force just sufficient to keep the gate from opening when h = 8 m. Neglect atmospheric pressure.

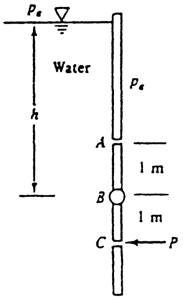


Fig. P2.77

Solution: The hydrostatic force on the gate is

$$\begin{aligned}F &= \gamma h_{CG} A = (9790)(8\ \text{m})(\pi\ \text{m}^2) \\ &= 246050\ \text{N}\end{aligned}$$

This force acts below point B by the distance

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)(1)^4\sin 90^\circ}{(8)(\pi)} = -0.03125\text{ m}$$

Summing moments about B gives P(1 m) = (246050)(0.03125 m), or P ≈ **7690 N** *Ans.*

P2.78 Panels AB and CD are each 120 cm wide into the paper. (*a*) Can you deduce, by inspection, which panel has the larger water force? (*b*) Even if your deduction is brilliant, calculate the panel forces anyway.

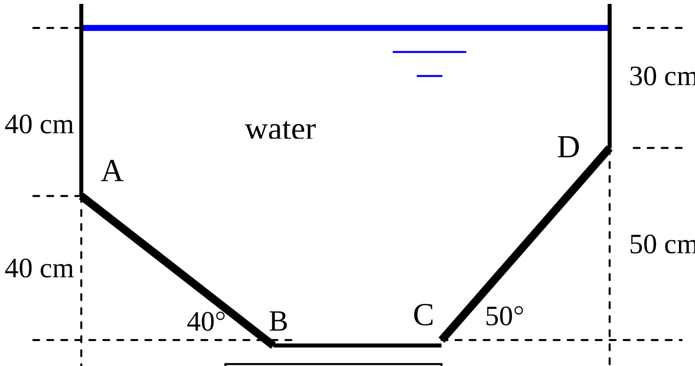


Fig. P2.78

Solution: (*a*) The writer is unable to deduce by inspection which panel force is larger. CD is longer than AB, but its centroid is not as deep. If you have a great insight, let me know.

(*b*) The length of AB is (40cm)/sin40° = 62.23 cm. The centroid of AB is 40+20 = 60 cm below the surface. The length of CD is (50cm)/sin50° = 65.27 cm. The centroid of AB is 30+25 = 55 cm below the surface. Calculate the two forces:

$$F_{AB} = \gamma h_{AB} A_{AB} = (9790\frac{N}{m^3})(0.6m)(0.6223m)(1.2m) = \mathbf{4390}\ N$$

$$F_{CD} = \gamma h_{CD} A_{CD} = (9790\frac{N}{m^3})(0.55m)(0.6527m)(1.2m) = \mathbf{4220}\ N \qquad Ans.(b)$$

It turns out that panel AB has the larger force, but it is only 4 percent larger.

2.79 Gate ABC in Fig. P2.79 is 1-m-square and hinged at B. It opens automatically when the water level is high enough. Neglecting atmospheric pressure, determine the lowest level *h* for which the gate will open. Is your result independent of the liquid density?

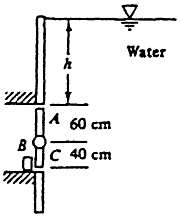
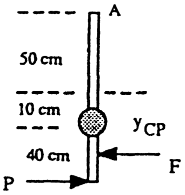


Fig. P2.79

Solution: The gate will open when the hydrostatic force F on the gate is *above B*, that is, when



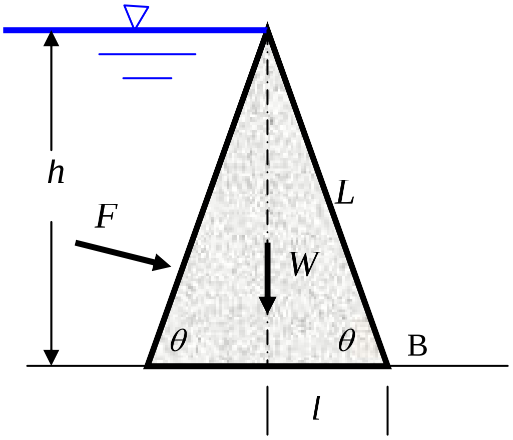
$$|y_{CP}| = \frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$= \frac{(1/12)(1\text{ m})(1\text{ m})^3\sin 90^\circ}{(h+0.5\text{ m})(1\text{ m}^2)} < 0.1\text{ m},$$

$$\text{or: } h+0.5 > 0.833\text{ m}, \quad \text{or: } \mathbf{h > 0.333\ m} \quad \textit{Ans.}$$

Indeed, this result is independent of the liquid density.

***P2.80** A concrete dam (SG = 2.5) is made in the shape of an isosceles triangle, as in Fig. P2.80. Analyze this geometry to find the range of angles θ for which the hydrostatic force will tend to tip the dam over at point B. The width into the paper is *b*.



Solution: The critical angle is when the hydrostatic force *F* causes a clockwise moment equal to the counterclockwise moment of the dam weight *W*. The length *L* of the slanted side of the dam is $L = h/\sin\theta$. The force *F* is two-thirds of the way down this face. The moment arm of the weight about point B is $l = h/\tan\theta$. The moment arm of *F* about point B is quite difficult, and you should check this:

$$\text{Moment arm of F about B is } \frac{L}{3} - 2l\cos\theta = \frac{1}{3}\frac{h}{\sin\theta} - \frac{2h}{\tan\theta}\cos\theta$$

Evaluate the two forces and then their moments:

$$F = \gamma\frac{h}{2}\frac{h}{\sin\theta}b \quad ; \quad W = SG\,\gamma\,\upsilon_{dam} = SG\,\gamma\,h\frac{h}{\tan\theta}b$$

$$\Sigma M_B = \frac{\gamma h^2 b}{2\sin\theta}\left(\frac{h}{3\sin\theta} - \frac{2h\cos\theta}{\tan\theta}\right) - \frac{SG\,\gamma h^2 b}{\tan\theta}\left(\frac{h}{\tan\theta}\right) \qquad \text{clockwise}$$

When the moment is negative (small θ), the dam is *stable*, it will not tip over. The moment is zero, for SG = 2.5, at $\theta = 77.4°$. Thus tipping is possible in the range $\boldsymbol{\theta > 77.4°}$. *Ans.* NOTE: This answer is independent of the numerical values of *h*, *g*, or *b* but requires SG = 2.5.

P2.81 For the semicircular cylinder CDE in Ex. 2.9, find the vertical hydrostatic force by integrating the vertical component of pressure around the surface from $\theta = 0$ to $\theta = \pi$.

Solution: A sketch is repeated here. At any position θ, as in Fig. P2.81, the vertical component of pressure is $p\cos\theta$. The depth down to this point is $h+R(1-\cos\theta)$, and the local pressure is γ times this depth. Thus

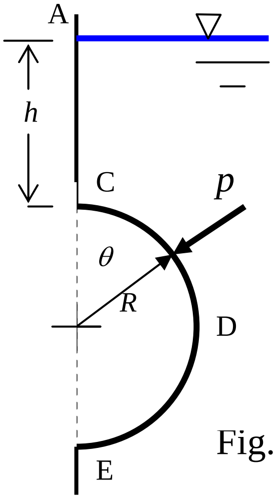


Fig. P2.81

$$F = \int p\cos\theta\, dA = \int_0^{\pi} \gamma[h+R(1-\cos\theta)](\cos\theta)[bR\,d\theta]$$

$$= \gamma bR(h+R)\int_0^{\pi}\cos\theta\, d\theta - \gamma bR^2\int_0^{\pi}\cos^2\theta\, d\theta = 0 - \gamma bR^2\frac{\pi}{2}$$

$$\text{Rewrite:}\quad F_{down} = -\gamma\frac{\pi}{2}R^2 b \qquad \textit{Ans.}$$

The negative sign occurs because the sign convention for *dF* was a *downward* force.

2.82 The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of hydrostatic force against the dam and the point CP where the resultant strikes the dam.

Solution: The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert}$$
$$= (9790\ \text{N/m}^3)(10\ \text{m})(20\times 50\ \text{m}^2)$$
$$\mathbf{= 97.9\ MN} \quad \textit{Ans.}$$

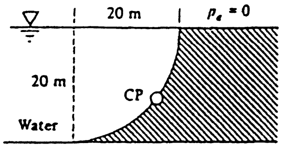
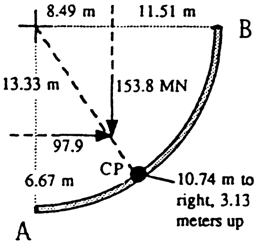


Fig. P2.82



This force acts 2/3 of the way down or 13.33 m from the surface, as in the figure. The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma(\text{Vol})_{\text{dam}} = (9790\ \text{N/m}^3)\frac{\pi}{4}(20\ \text{m})^2(50\ \text{m}) = \mathbf{153.8\ MN} \quad \textit{Ans.}$$

This vertical component acts through the centroid of the water above the dam, or $4R/3\pi = 4(20\ \text{m})/3\pi = 8.49$ m to the right of point A, as shown in the figure. The resultant hydrostatic force is $F = [(97.9\ \text{MN})^2 + (153.8\ \text{MN})^2]^{1/2} =$ **182.3 MN** acting down at an angle of **32.5°** from the vertical. The line of action of F strikes the circular-arc dam AB at the center of pressure CP, which is **10.74 m to the right and 3.13 m up from point A**, as shown in the figure. *Ans.*

2.83 Gate AB is a quarter-circle 10 ft wide and hinged at B. Find the force F just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf.

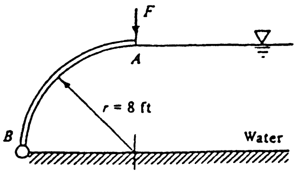


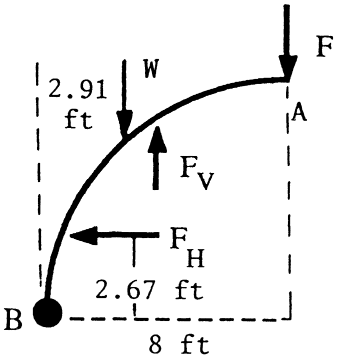
Fig. P2.83

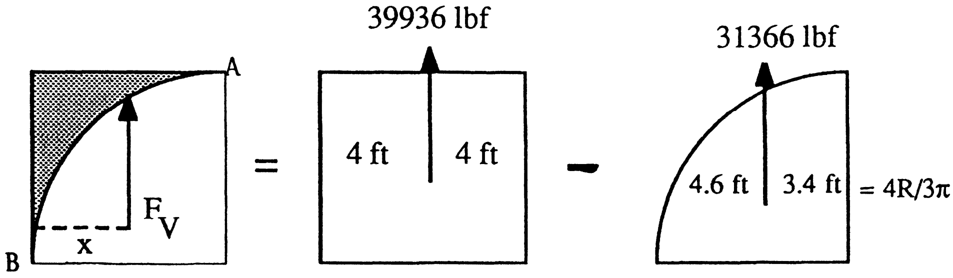
Solution: The horizontal force is computed as if AB were vertical:

$$F_H = \gamma h_{CG} A_{vert} = (62.4)(4\text{ ft})(8 \times 10\text{ ft}^2)$$
$$= 19968\text{ lbf} \quad \text{acting 5.33 ft below A}$$

The vertical force equals the weight of the missing piece of water above the gate, as shown below.

$$F_V = (62.4)(8)(8 \times 10) - (62.4)(\pi/4)(8)^2(10)$$
$$= 39936 - 31366 = 8570\text{ lbf}$$





The line of action *x* for this 8570-lbf force is found by summing moments from above:

$$\sum M_B(\text{of } F_V) = 8570x = 39936(4.0) - 31366(4.605), \quad \text{or} \quad x = 1.787\text{ ft}$$

Finally, there is the 3000-lbf gate weight W, whose centroid is $2R/\pi = 5.093$ ft from force F, or 8.0 – 5.093 = 2.907 ft from point B. Then we may sum moments about hinge B to find the force F, using the freebody of the gate as sketched at the top-right of this page:

$$\sum M_B(\text{clockwise}) = 0 = F(8.0) + (3000)(2.907) - (8570)(1.787) - (19968)(2.667),$$

$$\text{or} \quad F = \frac{59840}{8.0} = \mathbf{7480\ lbf} \quad \textit{Ans.}$$

P2.84 Panel AB is a parabola with its maximum at point A. It is 150 cm wide into the paper. Neglect atmospheric pressure. Find (*a*) the vertical and (*b*) horizontal water forces on the panel.

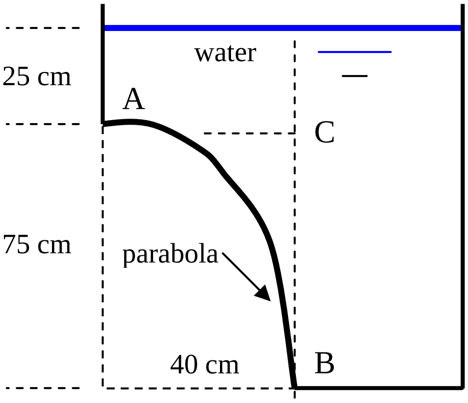


Fig. P2.84

Solution: (*b*) The horizontal force is calculated from the vertical projection of the panel (from point A down to the bottom). This is a rectangle, 75 cm by 150 cm, and its centroid is 37.5 cm below A, or (25 + 37.5) = 62.5 cm below the surface. Thus

$$F_H = p_{CG,H}\,A_{projected} = [9790\frac{N}{m^3}(0.625m)][0.75m(1.50m)] = \mathbf{6880}\ N \quad Ans.(b)$$

(*a*) The vertical force is the weight of water above the panel. This is in two parts (1) the weight of the rectangular portion above the line AC; and (2) the little curvy piece above the parabola and below line AC. Recall from Ex. 2.8 that the area under a parabola is two-thirds of the enclosed rectangle, so that little curvy piece is *one*-third of the rectangle. Thus, finally,

$$F_V = (9790)(0.25)(0.4)(1.5) + (9790)(\frac{1}{3})(0.75)(0.4)(1.5) =$$
$$= 1469\,N + 1469\,N \approx \mathbf{2940}\ N \qquad Ans.(a)$$

2.85 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

Solution: The horizontal component is

$$F_H = \gamma h_{CG} A_{vert} = (9790)(6)(2 \times 6)$$
$$= \mathbf{705000\ N} \quad \textit{Ans.}\ \text{(a)}$$

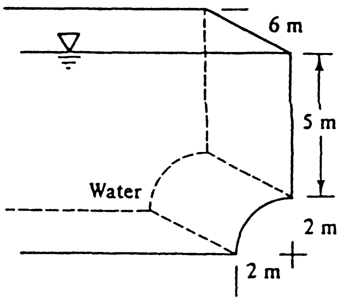


Fig. P2.85

The vertical component is the weight of the fluid above the quarter-circle panel:

$$F_V = W(2\text{ by }7\text{ rectangle}) - W(\text{quarter-circle})$$
$$= (9790)(2 \times 7 \times 6) - (9790)(\pi/4)(2)^2(6)$$
$$= 822360 - 184537 = \mathbf{638000\ N} \quad \textit{Ans.}\ \text{(b)}$$

2.86 The quarter circle gate BC in Fig. P2.86 is hinged at C. Find the horizontal force *P* required to hold the gate stationary. The width *b* into the paper is 3 m. Neglect the weight of the gate.

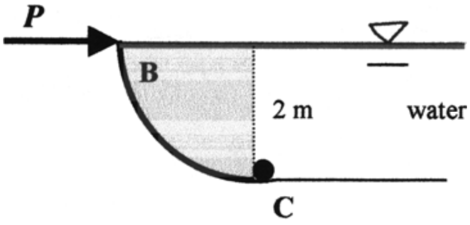


Fig. P2.86

Solution: The horizontal component of water force is

$$F_H = \gamma h_{CG} A = (9790\ \text{N/m}^3)(1\ \text{m})[(2\ \text{m})(3\ \text{m})] = 58{,}740\ \text{N}$$

This force acts 2/3 of the way down or 1.333 m down from the surface (0.667 m up from C). The vertical force is the weight of the quarter-circle of water above gate BC:

$$F_V = \gamma(\text{Vol})_{water} = (9790\ \text{N/m}^3)[(\pi/4)(2\ \text{m})^2(3\ \text{m})] = 92{,}270\ \text{N}$$

F_V acts down at $(4R/3\pi) = 0.849$ m to the left of C. Sum moments clockwise about point C:

$$\sum M_C = 0 = (2\text{ m})P - (58740\text{ N})(0.667\text{ m}) - (92270\text{ N})(0.849\text{ m}) = 2P - 117480$$

$$\text{Solve for} \quad P = 58{,}700\text{ N} = \mathbf{58.7\ kN} \quad \textit{Ans.}$$

2.87 The bottle of champagne (SG = 0.96) in Fig. P2.87 is under pressure as shown by the mercury manometer reading. Compute the net force on the 2-in-radius hemispherical end cap at the bottom of the bottle.

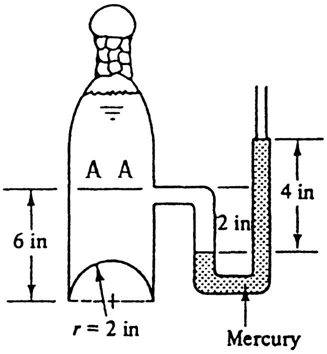


Fig. P2.87

Solution: First, from the manometer, compute the gage pressure at section AA in the champagne 6 inches above the bottom:

$$p_{AA} + (0.96\times 62.4)\left(\frac{2}{12}\text{ ft}\right) - (13.56\times 62.4)\left(\frac{4}{12}\text{ ft}\right) = p_{\text{atmosphere}} = 0\text{ (gage)},$$

$$\text{or:} \quad P_{AA} = 272\text{ lbf/ft}^2\text{ (gage)}$$

Then the force on the bottom end cap is vertical only (due to symmetry) and equals the force at section AA plus the weight of the champagne below AA:

$$F = F_V = p_{AA}(\text{Area})_{AA} + W_{\text{6-in cylinder}} - W_{\text{2-in hemisphere}}$$

$$= (272)\frac{\pi}{4}(4/12)^2 + (0.96\times 62.4)\pi(2/12)^2(6/12) - (0.96\times 62.4)(2\pi/3)(2/12)^3$$

$$= 23.74 + 2.61 - 0.58 \approx \mathbf{25.8\ lbf} \quad \textit{Ans.}$$

2.88 Circular-arc *Tainter* gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action. Does the force pass through point O?

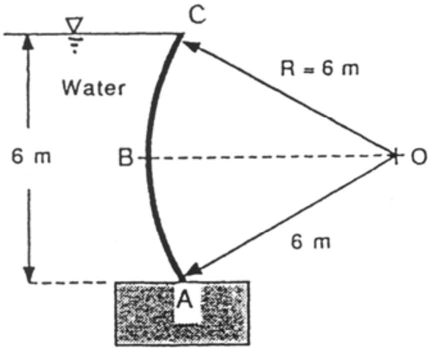
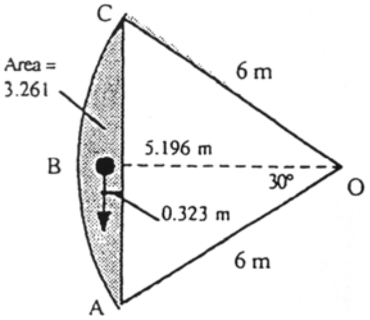


Fig. P2.88

Solution: The horizontal hydrostatic force is based on vertical projection:

$$F_H = \gamma h_{CG} A_{vert} = (9790)(3)(6\times 1) = 176220\ \text{N} \quad \text{at 4 m below C}$$

The vertical force is *upward* and equal to the weight of the missing water in the segment ABC shown shaded below. Reference to a good handbook will give you the geometric properties of a circular segment, and you may compute that the segment area is 3.261 m^2 and its centroid is 5.5196 m from point O, or 0.3235 m from vertical line AC, as shown in the figure. The vertical (upward) hydrostatic force on gate ABC is thus



$$F_V = \gamma A_{ABC}(\text{unit width}) = (9790)(3.2611)$$
$$= 31926\ \text{N} \quad \text{at 0.4804 m from B}$$

The net force is thus $F = [F_H^2 + F_V^2]^{1/2} = \mathbf{179100\ N}$ per meter of width, acting upward to the right at an angle of **10.27°** and passing through a point 1.0 m below and 0.4804 m to the right of point B. This force passes, as expected, *right through point O.*

2.89 The tank in the figure contains benzene and is pressurized to 200 kPa (gage) in the air gap. Determine the vertical hydrostatic force on circular-arc section AB and its line of action.

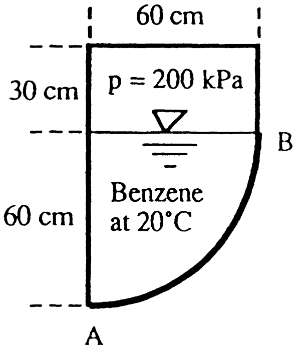


Fig. P2.89

Solution: Assume unit depth into the paper. The vertical force is the weight of benzene plus the force due to the air pressure:

$$F_V = \frac{\pi}{4}(0.6)^2(1.0)(881)(9.81) + (200{,}000)(0.6)(1.0) = \mathbf{122400\ \frac{N}{m}} \quad \textit{Ans.}$$

Most of this (120,000 N/m) is due to the air pressure, whose line of action is in the middle of the horizontal line through B. The vertical benzene force is 2400 N/m and has a line of action (see Fig. 2.13 of the text) at $4R/(3\pi) = 25.5$ cm to the right or A.

The moment of these two forces about A must equal to moment of the combined (122,400 N/m) force times a distance X to the right of A:

$$(120000)(30\text{ cm})+(2400)(25.5\text{ cm})=122400(X),\quad \textit{solve for}\ \ \mathbf{X=29.9\,cm}\quad \textit{Ans.}$$

The vertical force is **122400 N/m** (down), acting at **29.9 cm** to the right of A.

P2.90 The tank in Fig. P2.90 is 120 cm long into the paper. Determine the horizontal and vertical hydrostatic forces on the quarter-circle panel AB. The fluid is water at 20°C. Neglect atmospheric pressure.

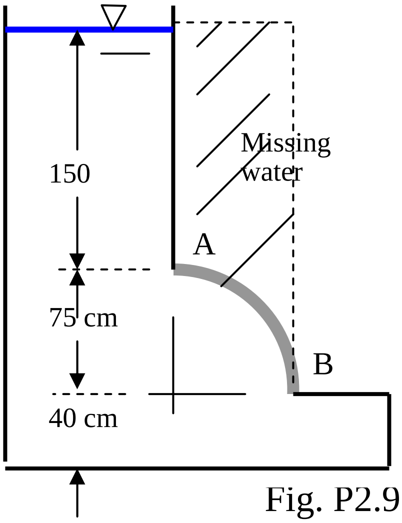


Fig. P2.90

Solution: For water at 20°C, take $\gamma = 9790\text{ N/m}^3$.

The vertical force on AB is the weight of the missing water above AB – see the dashed lines in Fig. P2.90. Calculate this as a rectangle plus a square-minus-a-quarter-circle:

$$\text{Missing water} = (1.5m)(0.75m)(1.2m)\ +\ (1-\pi/4)(0.75m)^2\ = 2.16+0.145\ = 2.305\,m^3$$

$$F_V\ =\ \gamma\upsilon\ =\ (9790\,N/m^3)(2.305\,m^3)\ \ =\ \ \mathbf{22{,}600\,N}\qquad (\textit{vertical force})$$

The horizontal force is calculated from the vertical projection of panel AB:

$$F_H = p_{CG}\,h\,A_{projection} = (9790\frac{N}{m^3})(1.5+\frac{0.75}{2}m)(0.75m)(1.2m)\ = \mathbf{16{,}500\,N}\ \ (\textit{horizontal force})$$

2.91 The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally-spaced bolts. What is the force in each bolt required to hold the dome down?

Solution: Assuming no leakage, the hydrostatic force required equals the *weight of missing water*, that is, the water in a 4-m-diameter cylinder, 6 m high, minus the hemisphere and the small pipe:

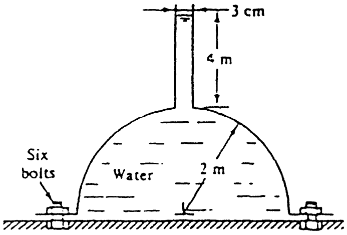


Fig. P2.91

$$F_{total} = W_{2\text{-m-cylinder}} - W_{2\text{-m-hemisphere}} - W_{3\text{-cm-pipe}}$$
$$= (9790)\pi(2)^2(6) - (9790)(2\pi/3)(2)^3 - (9790)(\pi/4)(0.03)^2(4)$$
$$= 738149 - 164033 - 28 = 574088 \text{ N}$$

The dome material helps with 30 kN of weight, thus the bolts must supply 574088–30000 or 544088 N. The force in each of 6 bolts is 544088/6 or $F_{bolt} \approx$ **90700 N** *Ans.*

2.92 A 4-m-diameter water tank consists of two half-cylinders, each weighing 4.5 kN/m, bolted together as in Fig. P2.92. If the end caps are neglected, compute the force in each bolt.

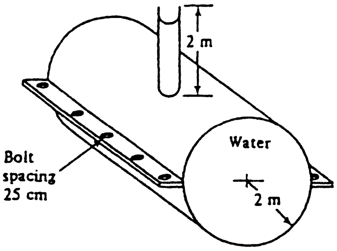
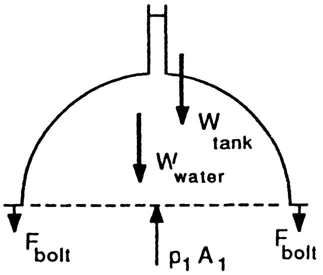


Fig. P2.92

Solution: Consider a 25-cm width of upper cylinder, as seen below. The water pressure in the bolt plane is

$$p_1 = \gamma h = (9790)(4) = 39160 \text{ Pa}$$



Then summation of vertical forces on this 25-cm-wide freebody gives

$$\sum F_z = 0 = p_1A_1 - W_{water} - W_{tank} - 2F_{bolt}$$

$$= (39160)(4\times 0.25) - (9790)(\pi/2)(2)^2(0.25) - (4500)/4 - 2F_{bolt},$$

Solve for $F_{one\ bolt} = \mathbf{11300\ N}$ *Ans.*

2.93 In Fig. P2.93 a one-quadrant spherical shell of radius R is submerged in liquid of specific weight γ and depth h > R. Derive an analytic expression for the hydrodynamic force F on the shell and its line of action.

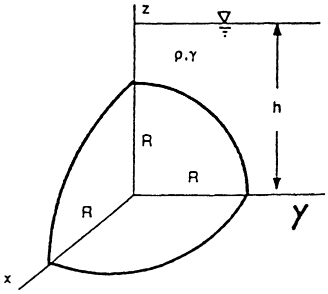


Fig. P2.93

Solution: The two horizontal components are identical in magnitude and equal to the force on the quarter-circle side panels, whose centroids are $(4R/3\pi)$ above the bottom:

$$\text{Horizontal components:}\quad F_x = F_y = \gamma h_{CG}A_{vert} = \gamma\left(h - \frac{4R}{3\pi}\right)\frac{\pi}{4}R^2$$

Similarly, the vertical component is the weight of the fluid above the spherical surface:

$$F_z = W_{cylinder} - W_{sphere} = \gamma\left(\frac{\pi}{4}R^2h\right) - \gamma\left(\frac{1}{8}\frac{4}{3}\pi R^3\right) = \gamma\frac{\pi}{4}R^2\left(h - \frac{2R}{3}\right)$$

There is no need to find the (complicated) centers of pressure for these three components, for we know that the resultant on a spherical surface *must pass through the center*. Thus

$$F = \left[F_x^2 + F_y^2 + F_z^2\right]^{1/2} = \boldsymbol{\gamma\frac{\pi}{4}R^2\left[(h - 2R/3)^2 + 2(h - 4R/3\pi)^2\right]^{1/2}} \quad \textit{Ans.}$$

P2.94 Find an analytic formula for the vertical and horizontal forces on each of the semi-circular panels AB in Fig. P2.94. The width into the paper is *b*. Which force is larger? Why?

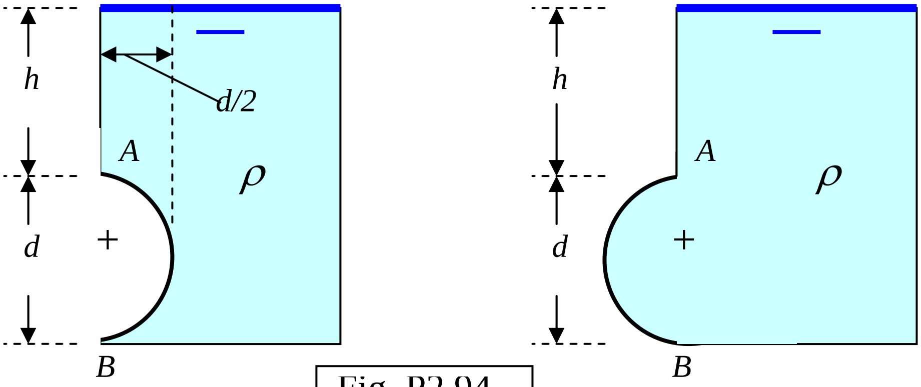


Fig. P2.94

Solution: It looks deceiving, since the bulging panel on the right has more water nearby, but these two forces are the same, except for their direction. The left-side figure is the same as Example 2.9, and its vertical force is *up*. The right-side figure has the same vertical force, but it is *down*. Both vertical forces equal the weight of water inside, or displaced by, the half-cylinder AB. Their horizontal forces equal the force on the projected plane AB.

$$F_H = p_{CG,AB}\,A_{projected} = \left[\rho g\left(h+\frac{d}{2}\right)\right](bd) \qquad Ans.$$

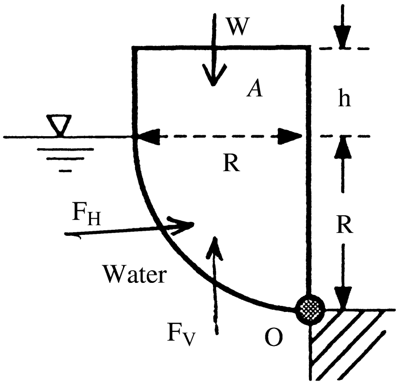
$$F_V = \rho g\,\upsilon_{half-cylinder} = \rho g\left[\frac{\pi}{2}\left(\frac{d}{2}\right)^2 b\right] \qquad Ans.$$

2.95 The uniform body A in the figure has width *b* into the paper and is in static equilibrium when pivoted about hinge O. What is the specific gravity of this body when (a) h = 0; and (b) h = R?

Solution: The water causes a horizontal and a vertical force on the body, as shown:

$$F_H = \gamma \frac{R}{2} Rb \quad at \ \frac{R}{3} \ above\ O,$$

$$F_V = \gamma \frac{\pi}{4} R^2 b \quad at \ \frac{4R}{3\pi} \ to\ the\ left\ of\ O$$



These must balance the moment of the body weight W about O:

$$\sum M_O = \frac{\gamma R^2 b}{2}\left(\frac{R}{3}\right) + \frac{\gamma \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \frac{\gamma_s \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \gamma_s R h b\left(\frac{R}{2}\right) = 0$$

$$Solve\ for: \quad SG_{body} = \frac{\gamma_s}{\gamma} = \left[\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{h}}{\mathbf{R}}\right]^{-1} \qquad Ans.$$

For h = 0, **SG = 3/2** *Ans.* (a). For h = R, **SG = 3/5** *Ans.* (b).

2.96 Curved panel BC is a 60° arc, perpendicular to the bottom at C. If the panel is 4 m wide into the paper, estimate the resultant hydrostatic force of the water on the panel.

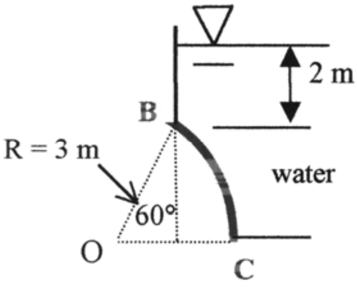
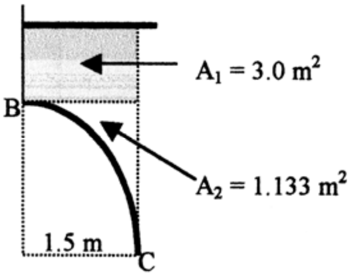


Fig. P2.96

Solution: The horizontal force is,

$$\begin{aligned} F_H &= \gamma h_{CG} A_h \\ &= (9790\ \text{N/m}^3)[2 + 0.5(3\sin 60°)\ \text{m}] \\ &\quad \times [(3\sin 60°)\text{m}(4\ \text{m})] \\ &= 335{,}650\ \text{N} \end{aligned}$$



The vertical component equals the weight of water above the gate, which is the sum of the rectangular piece above BC, and the curvy triangular piece of water just above arc BC—see figure at right. (The curvy-triangle calculation is messy and is not shown here.)

$$F_V = \gamma(\text{Vol})_{\text{above BC}} = (9790\ \text{N/m}^3)[(3.0 + 1.133\ \text{m}^2)(4\ \text{m})] = 161{,}860\ \text{N}$$

The resultant force is thus,

$$F_R = [(335{,}650)^2 + (161{,}860)^2]^{1/2} = 372{,}635\ \text{N} = \mathbf{373\ kN} \quad \textit{Ans.}$$

This resultant force acts along a line which passes through point O at

$$\theta = \tan^{-1}(161{,}860/335{,}650) = \mathbf{25.7°} \swarrow$$

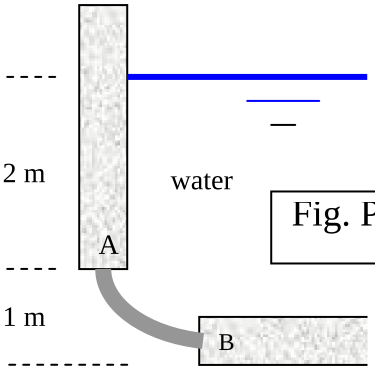


Fig. P2.97

P2.97 The contractor ran out of gunite mixture and finished the deep corner, of a 5-m-wide swimming pool, with a quarter-circle piece of PVC pipe, labeled AB in Fig. P2.97. Compute the (*a*) horizontal and (*b*) vertical water forces on the curved panel AB.

Solution: For water take $\gamma = 9790\ \text{N/m}^3$. (*a*) The horizontal force relates to the vertical projection of the curved panel AB:

$$F_{H,AB} = \gamma h_{CG} A_{projected} = (9790\frac{N}{m^3})(2.5\,m)[(1m)(5m)] = \mathbf{122,000}\,N \qquad Ans.(a)$$

(b) The vertical force is the weight of water above panel AB:

$$F_V = (9790\frac{N}{m^3})[(2m)(1m) + \frac{\pi}{4}(1m)^2](5\,m) = \mathbf{136,000}\,N \qquad Ans.(b)$$

2.98 Gate *ABC* in Fig. P2.98 is a quarter circle 8 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the gate and the line of action of the resultant force.

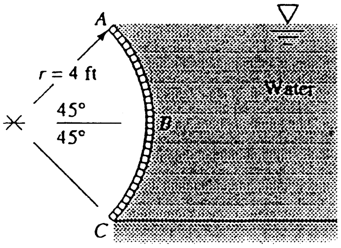
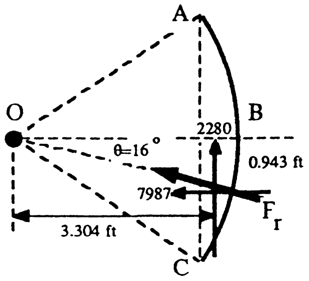


Fig. P2.98

Solution: The horizontal force is

$$F_h = \gamma h_{CG} A_h = (62.4)(2.828)(5.657 \times 8) = \mathbf{7987\ lbf} \leftarrow$$



located at

$$y_{cp} = -\frac{(1/12)(8)(5.657)^3}{(2.828)(5.657 \times 8)} = -0.943\ \text{ft}$$

$$\text{Area ABC} = (\pi/4)(4)^2 - (4\sin 45°)^2 = 4.566\ \text{ft}^2$$

$$\text{Thus}\quad F_v = \gamma \text{Vol}_{ABC} = (62.4)(8)(4.566) = \mathbf{2280\ lbf} \uparrow$$

The resultant is found to be

$$F_R = [(7987)^2 + (2280)^2]^{1/2} = \mathbf{8300\ lbf} \quad \text{acting at } \theta = 15.9° \text{ through the center O.} \quad \textit{Ans.}$$

P2.99 The mega-magnum cylinder in Fig. P2.99 has a hemispherical bottom and is pressurized with air to 75 kPa (gage). Determine (*a*) the horizontal and (*b*) the vertical hydrostatic forces on the hemisphere, in lbf.

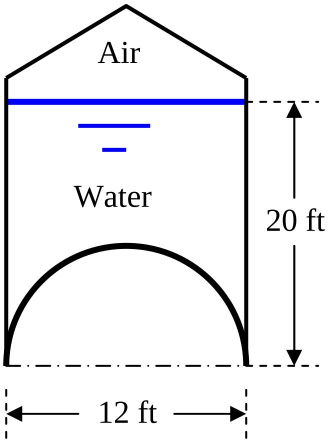


Fig. P2.99

Solution: Since the problem asks for BG units, convert the air pressure to BG: 75,000 Pa ÷ 47.88 = 1566 lbf/ft^2.

(*a*) By symmetry, the net horizontal force on the hemisphere is zero. *Ans.*(*a*)

(*b*) The vertical force is the sum of the air pressure term plus the weight of the water above:

$$F_V = p_{air} A_{surface} + \gamma_{water} \upsilon_{water} =$$

$$= (1566\frac{lbf}{ft^2})\pi(6\ ft)^2 + (62.4\frac{lbf}{ft^3})[\pi(6\ ft)^2(20\ ft) - \frac{1}{2}(\frac{4\pi}{3})(6\ ft)^3]$$

$$= 177{,}000\,lbf + 113{,}000\,lbf = \mathbf{290{,}000}\ lbf \qquad Ans.(b)$$

2.100 Pressurized water fills the tank in Fig. P2.100. Compute the hydrostatic force on the conical surface ABC.

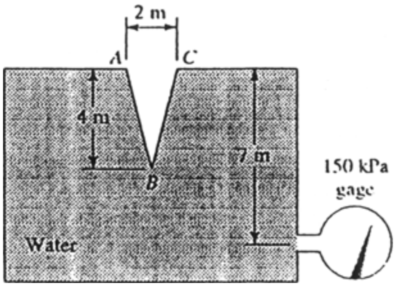


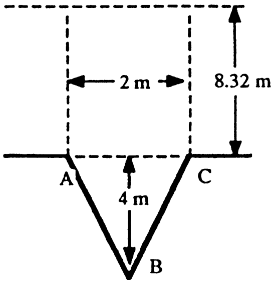
Fig. P2.100

Solution: The gage pressure is equivalent to a fictitious water level $h = p/\gamma = 150000/9790 = 15.32$ m above the gage or 8.32 m above AC. Then the vertical force on the cone equals the weight of fictitious water above ABC:

$$F_V = \gamma \text{Vol}_{above}$$

$$= (9790)\left[\frac{\pi}{4}(2)^2(8.32) + \frac{1}{3}\frac{\pi}{4}(2)^2(4)\right]$$

$$= \mathbf{297{,}000\ N} \quad Ans.$$



P2.101 The closed layered box in Fig. P2.101 has square horizontal cross-sections everywhere. All fluids are at 20°C. Estimate the gage pressure of the air if (*a*) the hydrostatic force on panel AB is 48 kN; or if (*b*) the hydrostatic force on the bottom panel BC is 97 kN.

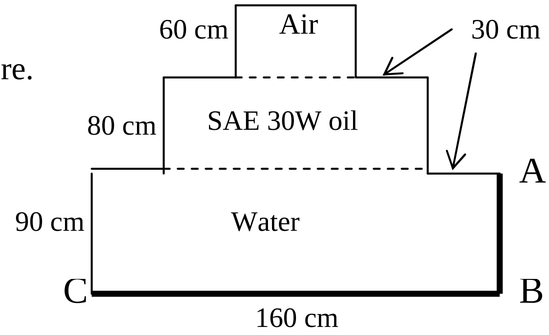


Fig. P2.101

Solution: At 20°C, take ρ_{oil} = 891 kg/m^3 and ρ_{water} = 998 kg/m^3. The wedding-cake shape of the box has nothing to do with the problem. (*a*) the force on panel AB equals the pressure at the panel centroid (45 cm down from A) times the panel area:

$$F_{AB} = p_{CG}A_{AB} = (p_{air} + \rho_{oil}gh_{oil} + \rho_{water}gh_{water-CG}), \quad \text{or:}$$
$$48000\,N = [p_{air} + (891)(9.81)(0.8m) + (998)(9.81)(0.45m)][(0.9m)(1.6m)]$$
$$= (p_{air} + 6993 + 4406\,Pa)(1.44\,m^2)\;; \quad \text{Solve} \quad p_{air} = \mathbf{22000\,Pa} \quad Ans.(a)$$

(*b*) The force on the bottom is handled similarly, except we go all the way to the bottom:

$$F_{BC} = p_{BC}A_{AB} = (p_{air} + \rho_{oil}gh_{oil} + \rho_{water}gh_{water}), \quad \text{or:}$$
$$97000\,N = [p_{air} + (891)(9.81)(0.8m) + (998)(9.81)(0.9m)][(1.6m)(1.6m)]$$
$$= (p_{air} + 6993 + 8812\,Pa)(2.56\,m^2)\;; \quad \text{Solve} \quad p_{air} = \mathbf{22000\,Pa} \quad Ans.(b)$$

2.102 A cubical tank is 3 × 3 × 3 m and is layered with 1 meter of fluid of specific gravity 1.0, 1 meter of fluid with SG = 0.9, and 1 meter of fluid with SG = 0.8. Neglect atmospheric pressure. Find (a) the hydrostatic force on the bottom; and (b) the force on a side panel.

Solution: (a) The force on the bottom is the bottom pressure times the bottom area:

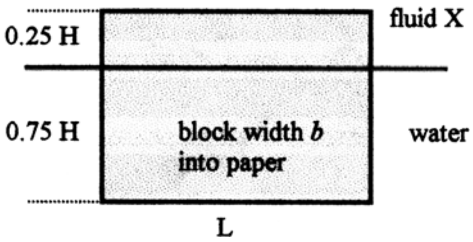
$$F_{\text{bot}} = p_{\text{bot}}A_{\text{bot}} = (9790\text{ N/m}^3)[(0.8\times 1\text{ m})+(0.9\times 1\text{ m})+(1.0\times 1\text{ m})](3\text{ m})^2$$
$$= \mathbf{238,000\ N} \quad \textit{Ans.}\text{ (a)}$$

(b) The hydrostatic force on the side panel is the sum of the forces due to each layer:

$$F_{\text{side}} = \sum\gamma h_{CG}A_{\text{side}} = (0.8\times 9790\text{ N/m}^3)(0.5\text{ m})(3\text{ m}^2)+(0.9\times 9790\text{ N/m}^3)(1.5\text{ m})(3\text{ m}^2)$$
$$+(9790\text{ N/m}^3)(2.5\text{ m})(3\text{ m}^2) = \mathbf{125,000\ kN} \quad \textit{Ans.}\text{ (b)}$$

2.103 A solid block, of specific gravity 0.9, floats such that 75% of its volume is in water and 25% of its volume is in fluid X, which is layered above the water. What is the specific gravity of fluid X?

Solution: The block is sketched below. A force balance is *W* = Σ*B*, or



$$0.9\gamma(\text{HbL}) = \gamma(0.75\text{HbL}) + \text{SG}_X\gamma(0.25\text{HbL})$$
$$0.9 - 0.75 = 0.25\text{SG}_X, \quad \boldsymbol{SG_X = 0.6} \quad \textit{Ans.}$$

2.104 The can in Fig. P2.104 floats in the position shown. What is its weight in newtons?

Solution: The can weight simply equals the weight of the displaced water (neglecting the air above):

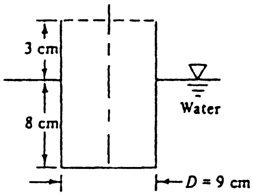


Fig. P2.104

$$W = \gamma \upsilon_{\text{displaced}} = (9790)\frac{\pi}{4}(0.09\text{ m})^2(0.08\text{ m}) = \mathbf{5.0\ N} \quad \textit{Ans.}$$

2.105 Archimedes, when asked by King Hiero if the new crown was pure gold (SG = 19.3), found the crown weight in air to be 11.8 N and in water to be 10.9 N. Was it gold?

Solution: The buoyancy is the difference between air weight and underwater weight:

$$B = W_{\text{air}} - W_{\text{water}} = 11.8 - 10.9 = 0.9\text{ N} = \gamma_{\text{water}}\upsilon_{\text{crown}}$$

$$\text{But also}\quad W_{\text{air}} = (SG)\gamma_{\text{water}}\upsilon_{\text{crown}},\quad \text{so}\ \ W_{\text{in water}} = B(SG-1)$$

$$\text{Solve for}\quad SG_{\text{crown}} = 1 + W_{\text{in water}}/B = 1 + 10.9/0.9 = \mathbf{13.1\ (not\ pure\ gold)} \quad \textit{Ans.}$$

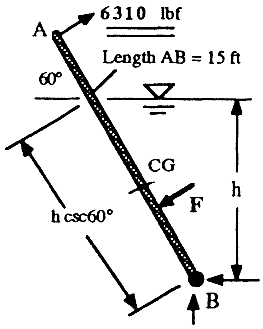
2.106 A spherical helium balloon is 2.5 m in diameter and has a total mass of 6.7 kg. When released into the U. S. Standard Atmosphere, at what altitude will it settle?

Solution: The altitude can be determined by calculating the air density to provide the proper buoyancy and then using Table A.3 to find the altitude associated with this density:

$$\rho_{air} = m_{balloon}/Vol_{sphere} = (6.7\ kg)/[\pi(2.5\ m^3)/6] = 0.819\ kg/m^3$$

From Table A.3, atmospheric air has $\rho = 0.819\ kg/m^3$ at an altitude of about **4000 m**. *Ans.*

2.107 Repeat Prob. 2.62 assuming that the 10,000 lbf weight is aluminum (SG = 2.71) and is hanging submerged in the water.



Solution: Refer back to Prob. 2.62 for details. The only difference is that the force applied to gate AB by the weight is less due to buoyancy:

$$F_{net} = \frac{(SG-1)}{SG}\gamma\upsilon_{body} = \frac{2.71-1}{2.71}(10000) = 6310\ lbf$$

This force replaces "10000" in the gate moment relation (see Prob. 2.62):

$$\sum M_B = 0 = 6310(15) - (288.2h^2)\left(\frac{h}{2}\csc 60° - \frac{h}{6}\csc 60°\right) - 4898(7.5\cos 60°)$$

or: $h^3 = 76280/110.9 = 688$, or: $h = \mathbf{8.83\ ft}$ *Ans.*

2.108 A 7-cm-diameter solid aluminum ball (SG = 2.7) and a solid brass ball (SG = 8.5) balance nicely when submerged in a liquid, as in Fig. P2.108. (*a*) If the fluid is water at 20°C, what is the diameter of the brass ball? (*b*) If the brass ball has a diameter of 3.8 cm, what is the density of the fluid?

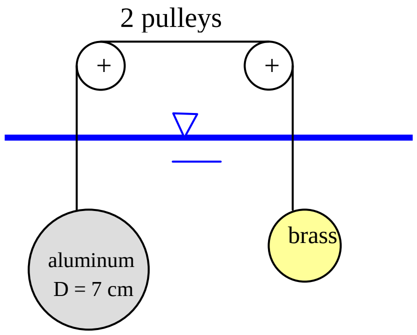


Fig. P2.108

Solution: For water, take $\gamma = 9790\ \text{N/m}^3$. If they balance, net weights are equal:

$$(SG_{alum} - SG_{fluid})\gamma_{water}\frac{\pi}{6}D_{alum}^3 = (SG_{brass} - SG_{fluid})\gamma_{water}\frac{\pi}{6}D_{brass}^3$$

We can cancel γ_{water} and $(\pi/6)$. (a) For water, $SG_{fluid} = 1$, and we obtain

$$(2.7-1)(0.07m)^3 = (8.5-1)D_{brass}^3 \quad ; \quad \text{Solve} \quad D_{brass} = \mathbf{0.0427\,m} \quad \textit{Ans.}(a)$$

(*b*) For this part, the fluid density (or specific gravity) is unknown:

$$(2.7 - SG_{fluid})(0.07\,m)^3 = (8.5 - SG_{fluid})(0.038m)^3 \;\; ; \;\; \text{Solve} \quad SG_{fluid} = \mathbf{1.595}$$

$$\text{Thus} \quad \rho_{fluid} = 1.595(998) = \mathbf{1592\,kg/m^3} \quad \textit{Ans.}(b)$$

According to Table A3, this fluid is probably *carbon tetrachloride*.

2.109 The float level h of a hydrometer is a measure of the specific gravity of the liquid. For stem diameter D and total weight W, if $h = 0$ represents SG = 1.0, derive a formula for h as a function of W, D, SG, and γ_o for water.

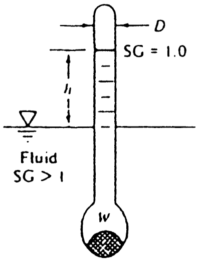


Fig. P2.109

Solution: Let submerged volume be υ_o when SG = 1. Let $A = \pi D^2/4$ be the area of the stem. Then

$$W = \gamma_o\upsilon_o = (SG)\gamma_o(\upsilon_o - Ah), \quad \text{or:} \quad \mathbf{h} = \frac{\mathbf{W(SG-1)}}{\mathbf{SG\gamma_o(\pi D^2/4)}} \quad \textit{Ans.}$$

P2.110 A solid sphere, of diameter 18 cm, floats in 20°C water with 1,527 cubic centimeters exposed above the surface. (*a*) What are the weight and specific gravity of this sphere? (*b*) Will it float in 20°C gasoline? If so, how many cubic centimeters will be exposed?

Solution: The total volume of the sphere is $(\pi/6)(18\text{ cm})^3 = 3054\text{ cm}^3$. Subtract the exposed portion to find the submerged volume = 3054 – 1527 = 1527 cm^3. Therefore the sphere is floating exactly half in and half out of the water. (*a*) Its weight and specific gravity are

$$W_{sphere} = \rho_{water}\, g\, \upsilon_{submerged} = (998\frac{kg}{m^3})(9.81\frac{m}{s^2})(1527E-6\,m^3) = \mathbf{14.95}\,N \quad Ans.(a)$$

$$\rho_{sphere} = \frac{W_{sphere}}{g\,\upsilon_{sphere}} = \frac{14.95}{(9.81)(3054E-6)} = 499\frac{kg}{m^3}\,, \quad SG_{sphere} = \frac{499}{1000} \approx \mathbf{0.50} \quad Ans.(a)$$

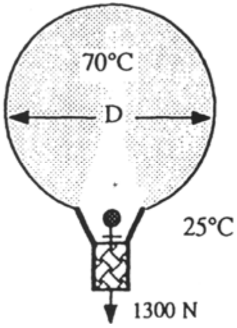
(*b*) From Table A.3, $\rho_{\text{gasoline}} = 680\text{ kg/m}^3 > \rho_{\text{sphere}}$. Therefore it ***floats*** in gasoline. *Ans.*(*b*)

(*c*) Neglecting air buoyancy on the exposed part, we compute the fraction of sphere volume that is exposed to be $(680 - 499\text{ kg/m}^3)/(680\text{ kg/m}^3) = 0.266$ or 26.6%. The volume exposed is

$$\upsilon_{exposed} = 0.266\,\upsilon_{sphere} = 0.266(3054\,cm^3) = \mathbf{813}\;cm^3 \quad Ans.(c)$$

Check buoyancy: the submerged volume, 2241 cm^3, times gasoline specific weight = 14.95 N .√

2.111 A hot-air balloon must support its own weight plus a person for a total weight of 1300 N. The balloon material has a mass of 60 g/m^2. Ambient air is at 25°C and 1 atm. The hot air inside the balloon is at 70°C and 1 atm. What diameter spherical balloon will just support the weight? Neglect the size of the hot-air inlet vent.



Solution: The buoyancy is due to the difference between hot and cold air density:

$$\rho_{\text{cold}} = \frac{p}{RT_{\text{cold}}} = \frac{101350}{(287)(273+25)} = 1.185\,\frac{\text{kg}}{\text{m}^3}; \quad \rho_{\text{hot}} = \frac{101350}{287(273+70)} = 1.030\,\frac{\text{kg}}{\text{m}^3}$$

The buoyant force must balance the known payload of 1300 N:

$$W = 1300\text{ N} = \Delta\rho g\text{ Vol} = (1.185-1.030)(9.81)\frac{\pi}{6}D^3,$$

$$\text{Solve for}\quad D^3 = 1628 \quad\text{or}\quad D_{\text{balloon}} \approx \mathbf{11.8\ m} \quad \textit{Ans.}$$

Check to make sure the balloon material is not excessively heavy:

$$W(\text{balloon}) = (0.06\text{ kg/m}^2)(9.81\text{ m/s}^2)(\pi)(11.8\text{ m})^2 \approx 256\text{ N} \quad \text{OK, only 20\% of } W_{\text{total}}.$$

2.112 The uniform 5-m-long wooden rod in the figure is tied to the bottom by a string. Determine (a) the string tension; and (b) the specific gravity of the wood. Is it also possible to determine the inclination angle θ?

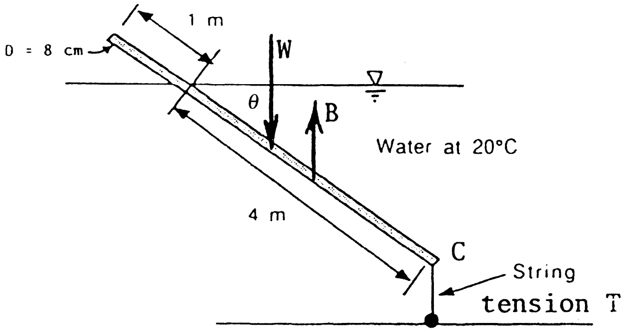


Fig. P2.112

Solution: The rod weight acts at the middle, 2.5 m from point C, while the buoyancy is 2 m from C. Summing moments about C gives

$$\sum M_C = 0 = W(2.5\sin\theta) - B(2.0\sin\theta), \quad \text{or} \quad W = 0.8B$$

$$\text{But} \quad B = (9790)(\pi/4)(0.08\text{ m})^2(4\text{ m}) = 196.8\text{ N}.$$

$$\text{Thus} \quad W = 0.8B = 157.5\text{ N} = SG(9790)(\pi/4)(0.08)^2(5\text{ m}), \quad \text{or:} \quad SG \approx \mathbf{0.64} \quad \textit{Ans.}\text{ (b)}$$

Summation of vertical forces yields

$$\text{String tension} \quad T = B - W = 196.8 - 157.5 \approx \mathbf{39\ N} \quad \textit{Ans.}\text{ (a)}$$

These results are independent of the angle θ, which cancels out of the moment balance.

2.113 A *spar buoy* is a rod weighted to float vertically, as in Fig. P2.113. Let the buoy be maple wood (SG = 0.6), 2 in by 2 in by 10 ft, floating in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added at the bottom so that $h = 18$ in?

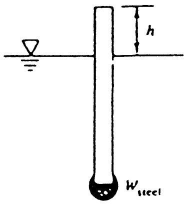


Fig. P2.113

Solution: The relevant volumes needed are

$$\text{Spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(10) = 0.278\ \text{ft}^3; \quad \text{Steel volume} = \frac{W_{steel}}{7.85(62.4)}$$

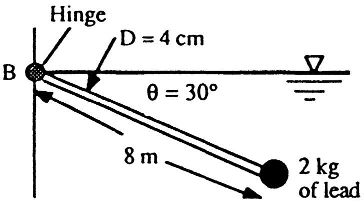
$$\text{Immersed spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(8.5) = 0.236\ \text{ft}^3$$

The vertical force balance is: buoyancy $B = W_{wood} + W_{steel}$,

$$\text{or:}\quad 1.025(62.4)\left[0.236 + \frac{W_{steel}}{7.85(62.4)}\right] = 0.6(62.4)(0.278) + W_{steel}$$

$$\text{or:}\quad 15.09 + 0.1306W_{steel} = 10.40 + W_{steel}, \quad \text{solve for}\quad W_{steel} \approx \mathbf{5.4\ lbf}\quad \textit{Ans.}$$

2.114 The uniform rod in the figure is hinged at B and in static equilibrium when 2 kg of lead (SG = 11.4) are attached at its end. What is the specific gravity of the rod material? What is peculiar about the rest angle $\theta = 30°$?



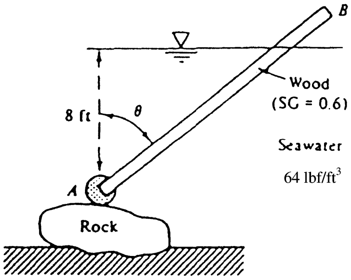
Solution: First compute buoyancies: $B_{rod} = 9790(\pi/4)(0.04)^2(8) = 98.42$ N, and $W_{lead} = 2(9.81) = 19.62$ N, $B_{lead} = 19.62/11.4 = 1.72$ N. Sum moments about B:

$$\sum M_B = 0 = (SG-1)(98.42)(4\cos 30°) + (19.62 - 1.72)(8\cos 30°) = 0$$

$$\text{Solve for}\quad \mathbf{SG_{rod} = 0.636}\quad \textit{Ans.}\ \text{(a)}$$

The angle θ drops out! The rod is neutrally stable for **any tilt angle!** *Ans.* (b)

2.115 The 2 inch by 2 inch by 12 ft spar buoy from Fig. P2.113 has 5 lbm of steel attached and has gone aground on a rock. If the rock exerts no moments on the spar, compute the angle of inclination θ.



Solution: Let ζ be the submerged length of spar. The relevant forces are:

$$W_{wood} = (0.6)(64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(12) = 12.8 \text{ lbf} \quad \textit{at distance } 6\sin\theta \textit{ to the right of } A\downarrow$$

$$Buoyancy = (64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)\zeta = 1.778\zeta \quad \textit{at distance } \frac{\zeta}{2}\sin\theta \textit{ to the right of } A\uparrow$$

The steel force acts right through A. Take moments about A:

$$\sum M_A = 0 = 12.8(6\sin\theta) - 1.778\zeta\left(\frac{\zeta}{2}\sin\theta\right)$$

$$\textit{Solve for} \quad \zeta^2 = 86.4, \quad \textit{or} \quad \zeta = 9.295\, ft \quad (\textit{submerged length})$$

Thus the angle of inclination $\theta = \cos^{-1}(8.0/9.295) = \mathbf{30.6°}$ *Ans.*

P2.116 The deep submersible vehicle ALVIN, in the chapter-opener photo, has a titanium (SG = 4.50) spherical passenger compartment with an inside diameter of 78.08 in and a wall thickness of 1.93 in. (*a*) Would the empty sphere float in seawater? (*b*) Would it float if it contained 1000 lbm of people and equipment inside? (*c*) What wall thickness would cause the empty sphere to be neutrally buoyant?

Solution: First convert the data to metric: D_i = 78.08 inches = 1.983 m, D_o = 78.08 + 2(1.93) = 81.94 inches = 2.081 m, $\rho_{titanium}$ = 4.50(1000) = 4500 kg/m^3, $\rho_{seawater}$ = 1025 kg/m^3. (*a*) Compute the weight of the sphere and compare to the buoyancy.

$$\text{Weight} = (4500\frac{kg}{m^3})(9.81\frac{m}{s^2})\frac{\pi}{6}[(2.081m)^3 - (1.983m)^3] = 28{,}084\ N$$

$$\text{Buoyancy} = (1025\frac{kg}{m^3})(9.81\frac{m}{s^2})\frac{\pi}{6}(2.081m)^3 = 47{,}466\,N$$

You could also add in the weight of the *air* inside the sphere, but that is only 49 N. We see that the buoyancy exceeds the weight by 19 kN, or more than 2 tons.

The sphere would float nicely. *Ans*.(*a*)

(*b*) If we add 1000 lbm of people and equipment, that would weigh 1000 lbf and be another (1000)(4.4482) = 4448 N of weight, making the total sphere weight 28084+49+4448 = 32581 N.

The excess buoyancy is 47466-32581 = 14885 N, **it will still float nicely**. *Ans*. (*b*)

NOTE: The entire ALVIN, not just the sphere, floats, which is a good safety feature.

(*c*) For neutral buoyancy, equate *W* (empty) and *B* and solve for the outside diameter:

$$W = (4500)(9.81)\frac{\pi}{6}[D_o^3 - (1.983)^3] + 49N = (1025)(9.81)\frac{\pi}{6}D_o^3$$

$$\text{Solve for} \quad D_o = 2.1615\,m,\ \text{thickness} = (2.1615 - 1.983)/2 = 0.0893\,m = \mathbf{3.51\,in} \quad Ans.(c)$$

Slightly less than doubling the wall thickness would create neutral buoyancy for the sphere.

2.117 The balloon in the figure is filled with helium and pressurized to 135 kPa and 20°C. The balloon material has a mass of 85 g/m^2. Estimate (a) the tension in the mooring line, and (b) the height in the standard atmosphere to which the balloon will rise if the mooring line is cut.

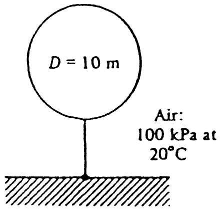


Fig. P2.117

Solution: (a) For helium, from Table A-4, R = 2077 m^2/s^2/K, hence its weight is

$$W_{helium} = \rho_{He} g \upsilon_{balloon} = \left[\frac{135000}{2077(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 1139\ N$$

Meanwhile, the total weight of the balloon material is

$$W_{balloon} = \left(0.085\ \frac{kg}{\mathrm{m}^2}\right)\left(9.81\ \frac{\mathrm{m}}{s^2}\right)[\pi(10\ \mathrm{m})^2] = 262\ N$$

Finally, the balloon buoyancy is the weight of displaced air:

$$B_{air} = \rho_{air} g \upsilon_{balloon} = \left[\frac{100000}{287(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 6108\ N$$

The difference between these is the tension in the mooring line:

$$T_{line} = B_{air} - W_{helium} - W_{balloon} = 6108 - 1139 - 262 \approx \mathbf{4700\ N} \quad \textit{Ans.}\ \text{(a)}$$

(b) If released, and the balloon remains at 135 kPa and 20°C, equilibrium occurs when the balloon air buoyancy exactly equals the total weight of 1139 + 262 = 1401 N:

$$B_{air} = 1401\ N = \rho_{air}(9.81)\frac{\pi}{6}(10)^3, \quad or: \quad \rho_{air} \approx 0.273\ \frac{kg}{\mathrm{m}^3}$$

From Table A-6, this standard density occurs at approximately

$$\mathbf{Z \approx 12,800\ m} \quad \textit{Ans.}\ \text{(b)}$$

P2.118 An intrepid treasure-salvage group has discovered a steel box, containing gold doubloons and other valuables, resting in 80 ft of seawater. They estimate the weight of the box and treasure (in air) at 7000 lbf. Their plan is to attach the box to a sturdy balloon, inflated with air to 3 atm pressure. The empty balloon weighs 250 lbf. The box is 2 ft wide, 5 ft long, and 18 in high. What is the proper diameter of the balloon to ensure an upward lift force on the box that is 20% more than required?

Solution: The specific weight of seawater is approximately 64 lbf/ft^3. The box volume is (2ft)(5ft)(1.5ft) = 12 ft^3, hence the buoyant force on the box is (64)(12) = 768 lbf. Thus the balloon must develop a net upward force of 1.2(7000-768lbf) = 7478 lbf. The air weight in the balloon is negligible, but we can compute it anyway. The air density is:

$$\text{At } p = 3atm,\ \rho_{air} = \frac{p}{RT} = \frac{3(2116 lbf/ft^2)}{(1716\, ft^2/s^2-{}^\circ R)(520^\circ R)} = 0.0071\frac{slug}{ft^3}$$

Hence the air specific weight is (0.0071)(32.2) = 0.23 lbf/ft^3, much less than the water. Accounting for balloon weight, the desired net buoyant force on the balloon is

$$F_{net} = (64 - 0.23 lbf/ft^3)(\pi/6)D_{balloon}^3 - 250\, lbf = 7478\, lbf$$

$$\text{Solve for } D^3 = 231.4\, lbf^3 \quad , \quad D_{balloon} \approx \mathbf{6.14\,ft} \quad \textit{Ans.}$$

2.119 With a 5-lbf-weight placed at one end, the uniform wooden beam in the figure floats at an angle θ with its upper right corner at the surface. Determine (a) θ; (b) γwood.

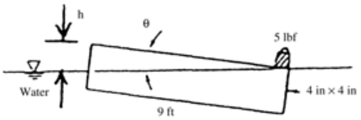


Fig. P2.119

Solution: The total wood volume is (4/12)^2(9) = 1 ft^3. The exposed distance $h = 9\tan\theta$. The vertical forces are

$$\sum F_z = 0 = (62.4)(1.0) - (62.4)(h/2)(9)(4/12) - (SG)(62.4)(1.0) - 5\text{ lbf}$$

The moments of these forces about point C at the right corner are:

$$\sum M_C = 0 = \gamma(1)(4.5) - \gamma(1.5h)(6\text{ ft}) - (SG)(\gamma)(1)(4.5\text{ ft}) + (5\text{ lbf})(0\text{ ft})$$

where $\gamma = 62.4$ lbf/ft^3 is the specific weight of water. Clean these two equations up:

$$1.5h = 1 - SG - 5/\gamma \quad \text{(forces)} \qquad 2.0h = 1 - SG \quad \text{(moments)}$$

Solve simultaneously for SG ≈ **0.68** *Ans.* (b); h = 0.16 ft; $\theta \approx$ **1.02°** *Ans.* (a)

2.120 A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?

Solution: The total beam volume is $3(.1)^2 = 0.03\ \text{m}^3$, and therefore its weight is $W = (0.65)(9790)(0.03) = 190.9$ N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is $B = (9790)(0.1)^2H = 97.9H$ newtons, acting at H/2 from the lower end. Sum moments about point A:

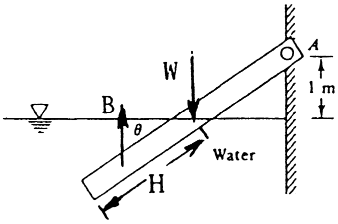


Fig. P2.120

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:}\quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225\ \text{m}$$

Geometry: 3 – H = 1.775 m is out of the water, or: $\sin\theta = 1.0/1.775$, or $\theta \approx$ **34.3°** *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b,h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

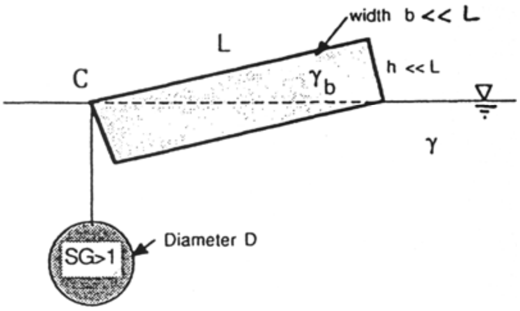


Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. } (a)$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{\text{sphere}} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that } D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG - 1)}}\right]^{\mathbf{1/3}} \quad \textit{Ans.}\ \text{(b)}$$

2.122 A uniform block of steel (SG = 7.85) will "float" at a mercury-water interface as in the figure. What is the ratio of the distances *a* and *b* for this condition?

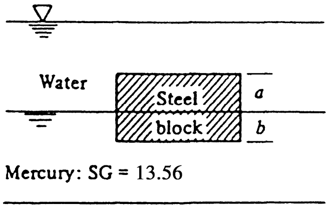


Fig. P2.122

Solution: Let *w* be the block width into the paper and let γ be the water specific weight. Then the vertical force balance on the block is

$$7.85\gamma(a + b)Lw = 1.0\gamma aLw + 13.56\gamma bLw,$$

$$\text{or:} \quad 7.85a + 7.85b = a + 13.56b, \quad \text{solve for } \frac{a}{b} = \frac{13.56 - 7.85}{7.85 - 1} = \mathbf{0.834} \quad \textit{Ans.}$$

P2.123 A barge has the trapezoidal shape shown in Fig. P2.123 and is 22 m long into the paper. If the total weight of barge and cargo is 350 tons, what is the draft *H* of the barge when floating in seawater?

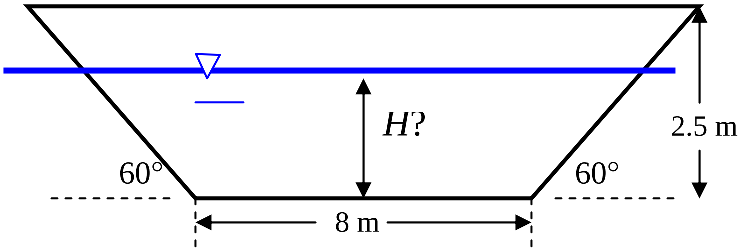


Fig. P2.123

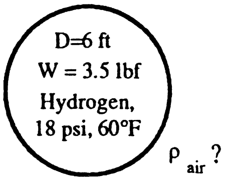
Solution: For seawater, let ρ = 1025 kg/m^3. The top of the barge has length [8m+2(2.5)tan60°] = 8 + 2.89 = 10.89 m. Thus the total volume of the barge is [(8+10.89m)/2](2.5m)(22m) = 519.4 m^3. In terms of seawater, this total volume would be equivalent to (519.4m^3)(1025kg/m^3)(9.81m/s^2) = 5.22E6N ÷ 4.4482lbf/N ÷ 2000lbf/ton =

587 tons. Thus a cargo of 350 tons = 700,000 lbf would fill the barge a bit more than halfway. Thus we solve the following equation for the draft to give W = 350 tons:

$$(22m)(H)(8+\frac{H}{\tan 60^\circ}m)(1025\frac{kg}{m^3})(9.81\frac{m}{s^2})(\frac{1}{4.4482 lbf/N}) = 700{,}000\,lbf$$

$$\text{Solve by iteration or EES:} \qquad H \approx \mathbf{1.58\,m} \qquad \textit{Ans.}$$

2.124 A balloon weighing 3.5 lbf is 6 ft in diameter. If filled with hydrogen at 18 psia and 60°F and released, at what U.S. standard altitude will it be neutral?



Solution: Assume that it remains at 18 psia and 60°F. For hydrogen, from Table A-4, $R \approx 24650\ \text{ft}^2/(\text{s}^2\cdot°\text{R})$. The density of the hydrogen in the balloon is thus

$$\rho_{H_2} = \frac{p}{RT} = \frac{18(144)}{(24650)(460+60)} \approx 0.000202\ \text{slug/ft}^3$$

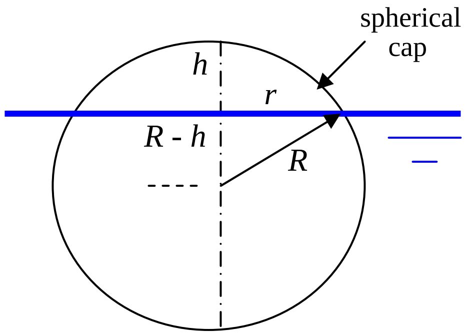
In the vertical force balance for neutral buoyancy, only the outside air density is unknown:

$$\sum F_z = B_{air} - W_{H_2} - W_{balloon} = \rho_{air}(32.2)\frac{\pi}{6}(6)^3 - (0.000202)(32.2)\frac{\pi}{6}(6)^3 - 3.5\ \text{lbf}$$

$$\text{Solve for} \quad \rho_{air} \approx 0.00116\ \text{slug/ft}^3 \approx 0.599\ \text{kg/m}^3$$

From Table A-6, this density occurs at a standard altitude of **6850 m ≈ 22500 ft**. *Ans.*

P2.125 A solid sphere, of diameter 20 cm, has a specific gravity of 0.7. (*a*) Will this sphere float in 20°C SAE 10W oil? If so, (*b*) how many cubic centimeters are exposed, and (*c*) how high will a spherical cap protrude above the surface? NOTE: If your knowledge of offbeat sphere formulas is lacking, you can "Ask Dr. Math" at Drexel University, http://mathforum.org/dr.math/. EES is recommended for the solution.



Solution: From Table A.3, the density of SAE 10W oil is 870 kg/m^3 > 700 kg/m^3. **The sphere floats in oil**. *Ans.*(*a*)

The volume of the sphere is (π/6)(20 cm)^3 = 4,189 cm^3. The fraction exposed is

(870-700)/870 = 0.195 or 19.5 %

The volume exposed is (0.195)(4189) = **818 cm^3**. *Ans.*(*b*)

From "Dr. Math", or a good math book, the sphere formulas relate to the distances (*r*, *h*, *R*) shown in the figure at right.

$$R = \frac{h^2+r^2}{2h} = 10\,cm \quad ; \quad \upsilon_{cap} = \frac{\pi}{6}(3r^2+h^2)h = 818\ cm^3$$

Knowing that *h* is small, of order 5 cm, you could guess your way to the answer. Or you could use EES and get the answer directly. In either case, the result is ***h* = 5.67 cm** *Ans.*(*c*)
The sphere would poke out of the water to a height of 5.67 centimeters.

2.126 A block of wood (SG = 0.6) floats in fluid X in Fig. P2.126 such that 75% of its volume is submerged in fluid X. Estimate the gage pressure of the air in the tank.

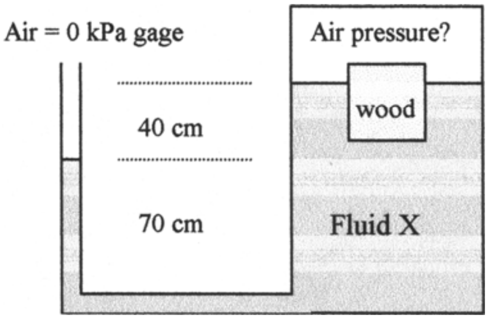


Fig. P2.126

Solution: In order to apply the hydrostatic relation for the air pressure calculation, the density of Fluid X must be found. The buoyancy principle is thus first applied. Let the block have volume V. Neglect the buoyancy of the air on the upper part of the block. Then

$$0.6\gamma_{water}V = \gamma_X(0.75V) + \cancel{\gamma_{air}(0.25V)}; \quad \gamma_X \approx 0.8\gamma_{water} = 7832\ N/m^3$$

The air gage pressure may then be calculated by jumping from the left interface into fluid X:

$$0\text{ Pa-gage} - (7832\text{ N/m}^3)(0.4\text{ m}) = p_{air} = -3130\text{ Pa-gage} = \mathbf{3130\ Pa\text{-}vacuum} \quad \textit{Ans.}$$

2.127* Consider a cylinder of specific gravity $S < 1$ floating vertically in water ($S = 1$), as in Fig. P2.127. Derive a formula for the stable values of D/L as a function of S and apply it to the case $D/L = 1.2$.

Solution: A vertical force balance provides a relation for h as a function of S and L,

$$\gamma\pi D^2h/4 = S\gamma\pi D^2L/4, \quad \text{thus } h = SL$$

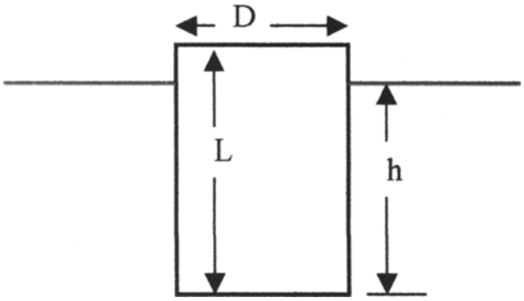


Fig. P2.127

To compute stability, we turn Eq. (2.52), centroid G, metacenter M, center of buoyancy B:

$$MB = I_o/v_{sub} = \frac{\frac{\pi}{4}(D/2)^4}{\frac{\pi}{4}D^2h} = MG + GB \quad \text{and substituting } h = SL, \quad \frac{D^2}{16SL} = MG + GB$$

where GB = L/2 − h/2 = L/2 − SL/2 = L(1 − S)/2. For neutral stability, MG = 0. Substituting,

$$\frac{D^2}{16SL} = 0 + \frac{L}{2}(1-S) \quad \textit{solving for } D/L, \quad \boldsymbol{\frac{D}{L} = \sqrt{8S(1-S)}} \quad \textit{Ans.}$$

If D/L = 1.2, $S^2 - S + 0.18 = 0$, or $\mathbf{0 \le S \le 0.235}$ and $\mathbf{0.765 \le S \le 1}$ for stability *Ans.*

2.128 The iceberg of Fig. 2.20 can be idealized as a cube of side length L as shown. If seawater is denoted as S = 1, the iceberg has S = 0.88. Is it stable?

Solution: The distance *h* is determined by

$$\gamma_w hL^2 = S\gamma_w L^3, \quad \text{or:} \quad h = SL$$

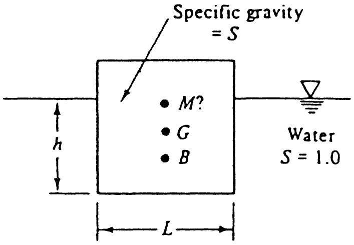


Fig. P2.128

The center of gravity is at L/2 above the bottom, and B is at h/2 above the bottom. The metacenter position is determined by Eq. (2.52):

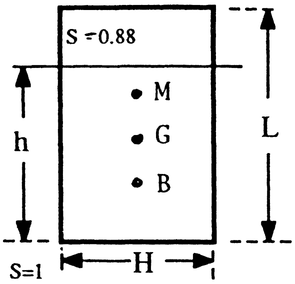
$$MB = I_o/\upsilon_{sub} = \frac{L^4/12}{L^2 h} = \frac{L^2}{12h} = \frac{L}{12S} = MG + GB$$

Noting that GB = L/2 – h/2 = L(1 – S)/2, we may solve for the metacentric height:

$$MG = \frac{L}{12S} - \frac{L}{2}(1-S) = 0 \quad \text{if} \quad S^2 - S + \frac{1}{6} = 0, \ \text{or:} \ S = 0.211 \quad \text{or} \quad 0.789$$

Instability: 0.211 < S < 0.789. Since the iceberg has S = 0.88 > 0.789, **it is stable**. *Ans.*

2.129 The iceberg of Prob. 2.128 may become unstable if its width decreases. Suppose that the height is L and the depth into the paper is L but the width decreases to H < L. Again with S = 0.88 for the iceberg, determine the ratio H/L for which the iceberg becomes unstable.



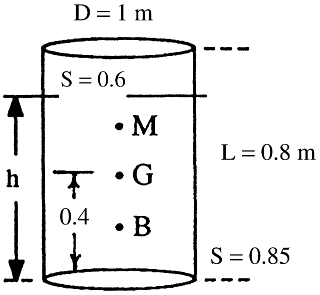
Solution: As in Prob. 2.128, the submerged distance h = SL = 0.88L, with G at L/2 above the bottom and B at h/2 above the bottom. From Eq. (2.52), the distance MB is

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{LH^3/12}{HL(SL)} = \frac{H^2}{12SL} = MG + GB = MG + \left(\frac{L}{2} - \frac{SL}{2}\right)$$

Then neutral stability occurs when MG = 0, or

$$\frac{H^2}{12SL} = \frac{L}{2}(1-S), \quad \text{or} \quad \frac{H}{L} = [6S(1-S)]^{1/2} = [6(0.88)(1-0.88)]^{1/2} = \mathbf{0.796} \quad \textit{Ans.}$$

2.130 Consider a wooden cylinder (SG = 0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (SG = 0.85)?



Solution: A vertical force balance gives

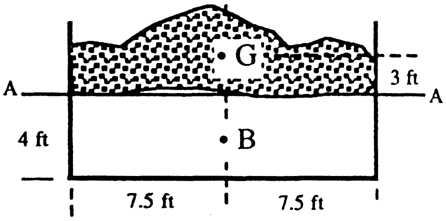
$$0.85\pi R^2 h = 0.6\pi R^2(0.8\text{ m}),$$
$$\text{or:}\quad h = 0.565\text{ m}$$

The point B is at h/2 = 0.282 m above the bottom. Use Eq. (2.52) to predict the metacenter location:

$$MB = I_o/\upsilon_{sub} = [\pi(0.5)^4/4]/[\pi(0.5)^2(0.565)] = 0.111\text{ m} = MG + GB$$

Now GB = 0.4 m – 0.282 m = 0.118 m, hence **MG** = 0.111 – 0.118 = **–0.007 m**. This float position is thus **slightly unstable**. The cylinder would turn over. *Ans.*

2.131 A barge is 15 ft wide and floats with a draft of 4 ft. It is piled so high with gravel that its center of gravity is 3 ft above the waterline, as shown. Is it stable?



Solution: Example 2.10 applies to this case, with L = 7.5 ft and H = 4 ft:

$$MA = \frac{L^2}{3H} - \frac{H}{2} = \frac{(7.5\text{ ft})^2}{3(4\text{ ft})} - \frac{4\text{ ft}}{2} = 2.69\text{ ft}, \quad \text{where "A" is the waterline}$$

Since G is **3** ft *above* the waterline, MG = 2.69 – 3.0 = **–0.31 ft, unstable**. *Ans.*

2.132 A solid right circular cone has SG = 0.99 and floats vertically as shown. Is this a stable position?

Solution: Let *r* be the radius at the surface and let *z* be the exposed height. Then

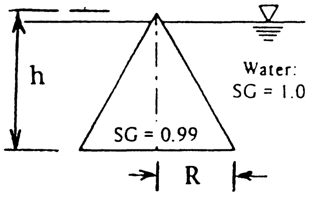


Fig. P2.132

$$\sum F_z = 0 = \gamma_w \frac{\pi}{3}(R^2h - r^2z) - 0.99\gamma_w \frac{\pi}{3}R^2h, \quad \text{with } \frac{z}{h} = \frac{r}{R}.$$

$$\text{Thus } \frac{z}{h} = (0.01)^{1/3} = 0.2154$$

The cone floats at a draft $\zeta = h - z = 0.7846h$. The centroid G is at 0.25h above the bottom. The center of buoyancy B is at the centroid of a frustrum of a (submerged) cone:

$$\zeta = \frac{0.7846h}{4}\left(\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right) = 0.2441h \quad \text{above the bottom}$$

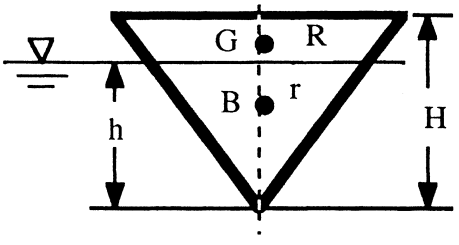
Then Eq. (2.52) predicts the position of the metacenter:

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{\pi(0.2154R)^4/4}{0.99\pi R^2h} = 0.000544\frac{R^2}{h} = MG + GB$$

$$= MG + (0.25h - 0.2441h) = MG + 0.0594h$$

Thus MG > 0 (**stability**) if $(R/h)^2 \geq 10.93$ or **R/h ≥ 3.31** *Ans.*

2.133 Consider a uniform right circular cone of specific gravity S < 1, floating with its vertex down in water, S = 1.0. The base radius is R and the cone height is H, as shown. Calculate and plot the stability parameter MG of this cone, in dimensionless form, versus H/R for a range of cone specific gravities S < 1.



Solution: The cone floats at height *h* and radius *r* such that B = W, or:

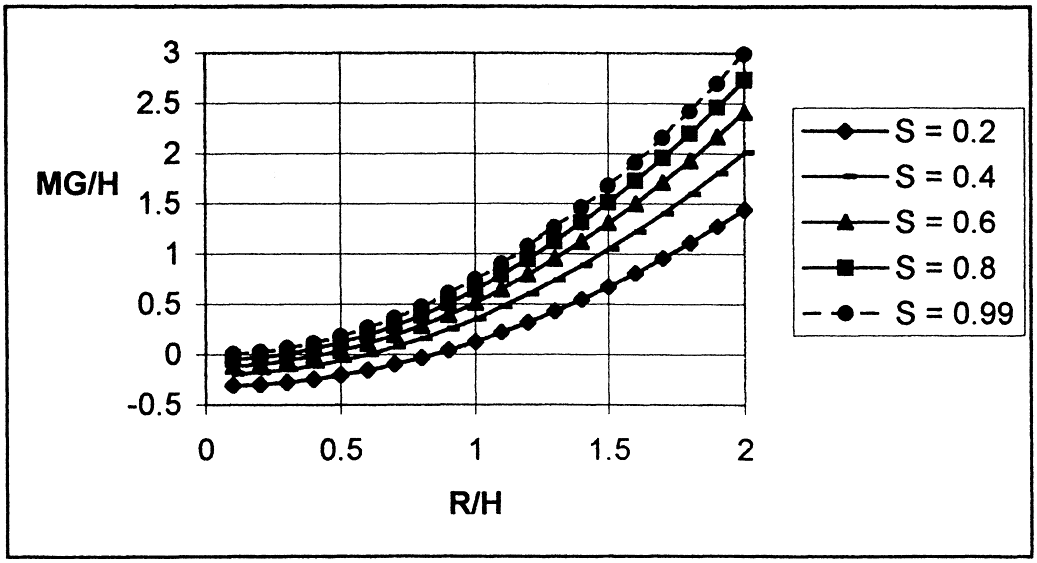
$$\frac{\pi}{3}r^2h(1.0)=\frac{\pi}{3}R^2H(S),\quad or:\quad \frac{h^3}{H^3}=\frac{r^3}{R^3}=S<1$$

Thus $r/R = h/H = S^{1/3} = \zeta$ for short. Now use the stability relation:

$$MG+GB=MG+\left(\frac{3H}{4}-\frac{3h}{4}\right)=\frac{I_o}{\upsilon_{sub}}=\frac{\pi r^4/4}{\pi r^2h/3}=\frac{3\zeta R^2}{4H}$$

Non-dimensionalize in the final form: $\dfrac{\mathbf{MG}}{\mathbf{H}}=\dfrac{\mathbf{3}}{\mathbf{4}}\left(\zeta\dfrac{\mathbf{R}^2}{\mathbf{H}^2}-\mathbf{1}+\zeta\right),\ \zeta=S^{1/3}$ *Ans.*

This is plotted below. Floating cones pointing *down* are stable unless slender, R << H.



2.134 When floating in water (SG = 1), an equilateral triangular body (SG = 0.9) might take *two* positions, as shown at right. Which position is more stable? Assume large body width into the paper.

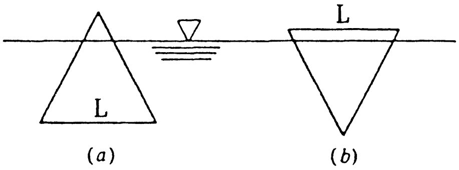


Fig. P2.134

Solution: The calculations are similar to the floating cone of Prob. 2.132. Let the triangle be L by L by L. List the basic results.

(a) Floating with point *up*: Centroid G is 0.289L above the bottom line, center of buoyancy B is 0.245L above the bottom, hence GB = (0.289 − 0.245)L ≈ 0.044L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.0068L = MG + GB = MG + 0.044L$$

$$\text{Hence}\quad MG = \mathbf{-0.037L}\quad \textbf{Unstable}\quad \textit{Ans.}\text{ (a)}$$

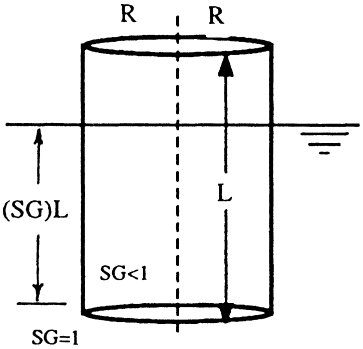
(b) Floating with point *down*: Centroid G is 0.577L above the bottom point, center of buoyancy B is 0.548L above the bottom point, hence GB = (0.577 − 0.548)L ≈ 0.0296L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.1826L = MG + GB = MG + 0.0296L$$

$$\text{Hence}\quad MG = \mathbf{+0.153L}\quad \textbf{Stable}\quad \textit{Ans.}\text{ (b)}$$

2.135 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG, floating in water (SG = 1) with its axis *vertical*. Show that the body is stable if

$$R/L > [2SG(1-SG)]^{1/2}$$

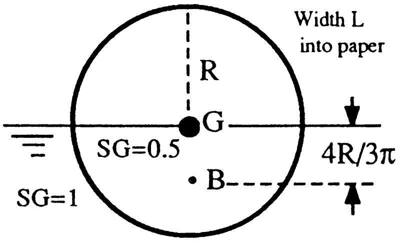


Solution: For a given SG, the body floats with a draft equal to (SG)L, as shown. Its center of gravity G is at L/2 above the bottom. Its center of buoyancy B is at (SG)L/2 above the bottom. Then Eq. (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{\pi R^4/4}{\pi R^2(SG)L} = \frac{R^2}{4(SG)L} = MG + GB = MG + \frac{L}{2} - SG\frac{L}{2}$$

$$\text{Thus}\quad MG > 0\ \text{(stability)}\quad \text{if}\ \ \mathbf{R^2/L^2 > 2SG(1-SG)}\quad \textit{Ans.}$$

For example, if SG = 0.8, stability requires that R/L > 0.566.

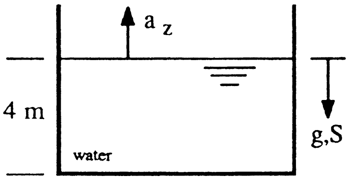


2.136 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG = 0.5, floating in water (SG = 1) with its axis *horizontal*. Show that the body is stable if L/R > 2.0.

Solution: For the given SG = 0.5, the body floats centrally with a draft equal to R, as shown. Its center of gravity G is exactly at the surface. Its center of buoyancy B is at the centroid of the immersed semicircle: 4R/(3π) below the surface. Equation (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{(1/12)(2R)L^3}{\pi(R^2/2)L} = \frac{L^2}{3\pi R} = MG + GB = MG + \frac{4R}{3\pi}$$

$$\text{or:}\quad MG = \frac{L^2}{3\pi R} - \frac{4R}{3\pi} > 0 \text{ (stability)} \quad \text{if } \mathbf{L/R > 2} \quad \textit{Ans.}$$



2.137 A tank of water 4 m deep receives a constant upward acceleration a_z. Determine (a) the gage pressure at the tank bottom if $a_z = 5\ m^2/s$; and (b) the value of a_z which causes the gage pressure at the tank bottom to be 1 atm.

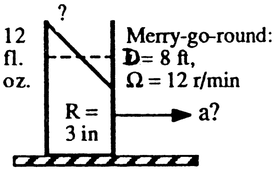
Solution: Equation (2.53) states that $\nabla p = \rho(\mathbf{g} - \mathbf{a}) = \rho(-\mathbf{k}g - \mathbf{k}a_z)$ for this case. Then, for part (a),

$$\Delta p = \rho(g + a_z)\Delta S = (998\ kg/m^3)(9.81 + 5\ m^2/s)(4\ m) = \mathbf{59100\ Pa\ (gage)} \quad \textit{Ans.}\ (a)$$

For part (b), we know $\Delta p = 1$ atm but we don't know the acceleration:

$$\Delta p = \rho(g + a_z)\Delta S = (998)(9.81 + a_z)(4.0) = 101350\ Pa \quad \text{if } \mathbf{a_z = 15.6\ \frac{m}{s^2}} \quad \textit{Ans.}\ (b)$$

2.138 A 12 fluid ounce glass, 3 inches in diameter, sits on the edge of a merry-go-round 8 ft in diameter, rotating at 12 r/min. How full can the glass be before it spills?



Solution: First, how high is the container? Well, 1 fluid oz. = 1.805 in^3, hence 12 fl. oz. = 21.66 in^3 = $\pi(1.5\text{ in})^2 h$, or *$h \approx 3.06$ in*—It is a fat, nearly square little glass. Second, determine the acceleration toward the center of the merry-go-round, noting that the angular velocity is Ω = (12 rev/min)(1 min/60 s)(2π rad/rev) = 1.26 rad/s. Then, for r = 4 ft,

$$a_x = \Omega^2 r = (1.26\text{ rad/s})^2(4\text{ ft}) = 6.32\text{ ft/s}^2$$

Then, for steady rotation, the water surface in the glass will slope at the angle

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{6.32}{32.2 + 0} = 0.196, \quad \text{or:} \quad \Delta h_{\text{left to center}} = (0.196)(1.5\text{ in}) = 0.294\text{ in}$$

Thus the glass should be filled to no more than 3.06 – 0.294 ≈ 2.77 inches
This amount of liquid is $\upsilon = \pi(1.5\text{ in})^2(2.77\text{ in}) = 19.6\text{ in}^3$ ≈ **10.8 fluid oz.** *Ans.*

2.139 The tank of liquid in the figure P2.139 accelerates to the right with the fluid in rigid-body motion. (a) Compute a_x in m/s^2. (b) Why doesn't the solution to part (a) depend upon fluid density? (c) Compute gage pressure at point A if the fluid is glycerin at 20°C.

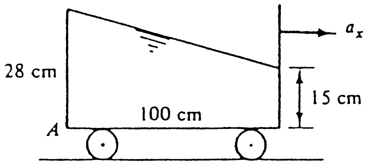


Fig. P2.139

Solution: (a) The slope of the liquid gives us the acceleration:

$$\mathbf{tan}\,\theta = \frac{a_x}{g} = \frac{28 - 15\text{ cm}}{100\text{ cm}} = 0.13, \quad \text{or:} \quad \theta = 7.4^\circ$$

$$\textit{thus} \quad a_x = 0.13g = 0.13(9.81) = \mathbf{1.28\ m/s^2} \quad \textit{Ans.}\ \text{(a)}$$

(b) Clearly, the solution to (a) is purely geometric and does not involve fluid density. *Ans.* (b)

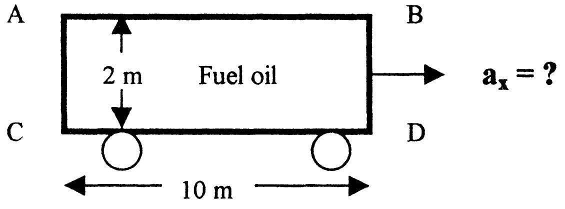
(c) From Table A-3 for glycerin, $\rho = 1260$ kg/m^3. There are many ways to compute p$_A$. For example, we can go straight down on the left side, using only gravity:

$$p_A = \rho g\Delta z = (1260\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.28\text{ m}) = \mathbf{3460\ Pa\ (gage)} \quad \textit{Ans.}\text{ (c)}$$

Or we can start on the right side, go down 15 cm with g and across 100 cm with a_x:

$$p_A = \rho g\Delta z + \rho a_x\Delta x = (1260)(9.81)(0.15) + (1260)(1.28)(1.00)$$
$$= 1854 + 1607 = \mathbf{3460\ Pa} \quad \textit{Ans.}\text{ (c)}$$

2.140 An elliptical-end fuel tank is 10 m long, with 3-m horizontal and 2-m vertical major axes, and filled completely with fuel oil ($\rho = 890$ kg/m^3). Let the tank be pulled along a horizontal road in rigid-body motion. Find the acceleration and direction for which (a) a constant-pressure surface extends from the top of the front end to the bottom of the back end; and (b) the top of the back end is at a pressure 0.5 atm lower than the top of the front end.



Solution: (a) We are given that the isobar or constant-pressure line reaches from point C to point B in the figure above, θ is *negative*, hence the tank is *decelerating*. The elliptical shape is immaterial, only the 2-m height. The isobar slope gives the acceleration:

$$\tan\theta_{C-B} = -\frac{2\text{ m}}{10\text{ m}} = -0.2 = \frac{a_x}{g}, \quad \textit{hence}\ a_x = -0.2(9.81) = \mathbf{-1.96\ m/s^2} \quad \textit{Ans.}\text{ (a)}$$

(b) We are now given that p$_A$ (back end top) is lower than p$_B$ (front end top)—see the figure above. Thus, again, the isobar must slope upward through B but not necessarily pass through point C. The pressure difference along line AB gives the correct *deceleration*:

$$\Delta p_{A-B} = -0.5(101325\ Pa) = \rho_{oil} a_x \Delta x_{A-B} = \left(890\ \frac{kg}{\text{m}^3}\right) a_x (10\text{ m})$$

$$\textit{solve for}\quad a_x = \mathbf{-5.69\ m/s^2} \quad \textit{Ans.}\text{ (b)}$$

This is more than part (a), so the isobar angle must be steeper:

$$\mathbf{tan}\,\theta = \frac{-5.69}{9.81} = -0.580, \quad \textit{hence}\ \ \theta_{isobar} = -30.1°$$

The isobar in part (a), line CB, has the angle θ(a) = $\tan^{-1}(-0.2) = -11.3°$.

2.141 The same tank from Prob. 2.139 is now accelerating while rolling *up* a 30° inclined plane, as shown. Assuming rigid-body motion, compute (a) the acceleration **a**, (b) whether the acceleration is up or down, and (c) the pressure at point A if the fluid is mercury at 20°C.

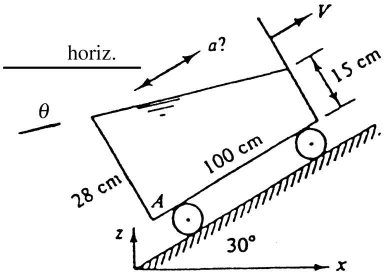


Fig. P2.141

Solution: The free surface is tilted at the angle $\theta = -30° + 7.41° = -22.59°$. This angle must satisfy Eq. (2.55):

$$\tan\theta = \tan(-22.59°) = -0.416 = a_x/(g + a_z)$$

But the 30° incline constrains the acceleration such that $a_x = 0.866a$, $a_z = 0.5a$. Thus

$$\tan\theta = -0.416 = \frac{0.866a}{9.81 + 0.5a}, \quad \text{solve for} \quad \mathbf{a \approx -3.80\ \frac{m}{s^2}\ (down)} \quad \textit{Ans.}\ \text{(a, b)}$$

The cartesian components are $a_x = -3.29$ m/s^2 and $a_z = -1.90$ m/s^2.
(c) The distance ΔS normal from the surface down to point A is (28 cosθ) cm. Thus

$$p_A = \rho[a_x^2 + (g + a_z)^2]^{1/2} = (13550)[(-3.29)^2 + (9.81 - 1.90)^2]^{1/2}(0.28\cos 7.41°)$$

$$\approx \mathbf{32200\ Pa\ (gage)} \quad \textit{Ans.}\ \text{(c)}$$

2.142 The tank of water in Fig. P2.142 is 12 cm wide into the paper. If the tank is accelerated to the right in rigid-body motion at 6 m/s^2, compute (a) the water depth at AB, and (b) the water force on panel AB.

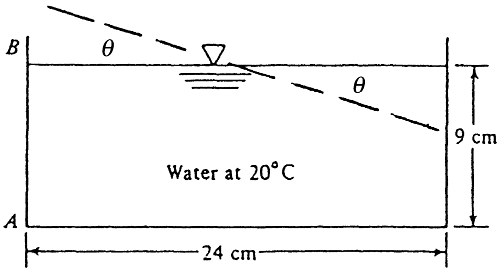


Fig. P2.142

Solution: From Eq. (2.55),

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta \approx 31.45°$$

Then surface point B on the left rises an additional $\Delta z = 12\tan\theta \approx 7.34$ cm,

$$\text{or:} \quad \text{water depth AB} = 9 + 7.34 \approx \mathbf{16.3\ cm} \quad \textit{Ans.}\ \text{(a)}$$

The water pressure on AB varies linearly due to gravity only, thus the water force is

$$F_{AB} = p_{CG}A_{AB} = (9790)\left(\frac{0.163}{2}\ \text{m}\right)(0.163\ \text{m})(0.12\ \text{m}) \approx \mathbf{15.7\ N} \quad \textit{Ans.}\ \text{(b)}$$

2.143 The tank of water in Fig. P2.143 is full and open to the atmosphere (p_{atm} = 15 psi = 2160 psf) at point A, as shown. For what acceleration a_x, in ft/s^2, will the pressure at point B in the figure be (a) atmospheric; and (b) zero absolute (neglecting cavitation)?

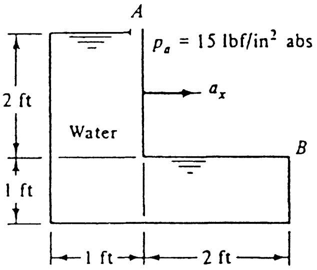


Fig. P2.143

Solution: (a) For $p_A = p_B$, the imaginary 'free surface isobar' should join points A and B:

$$\mathbf{tan}\,\theta_{AB} = \mathbf{tan}\,45° = 1.0 = a_x/g, \quad \text{hence} \quad a_x = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}\text{ (a)}$$

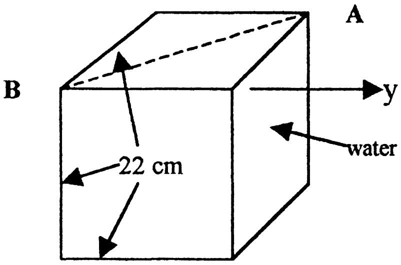
(b) For $p_B = 0$, the free-surface isobar must tilt even more than 45°, so that

$$p_B = 0 = p_A + \rho g \Delta z - \rho a_x \Delta x = 2160 + 1.94(32.2)(2) - 1.94 a_x(2),$$

$$\textit{solve} \quad a_x = \mathbf{589\ ft/s^2} \quad \textit{Ans.}\text{ (b)}$$

This is a very high acceleration (18 g's) and a very steep angle, $\theta = \tan^{-1}(589/32.2) = 87°$.

2.144 Consider a hollow cube of side length 22 cm, full of water at 20°C, and open to p_{atm} = 1 atm at top corner A. The top surface is horizontal. Determine the rigid-body accelerations for which the water at opposite top corner B will *cavitate*, for (a) horizontal, and (b) vertical motion.



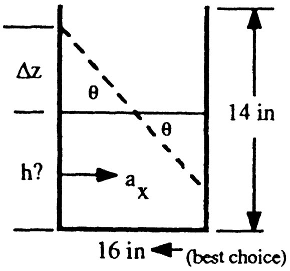
Solution: From Table A-5 the vapor pressure of the water is 2337 Pa. (a) Thus cavitation occurs first when accelerating horizontally along the diagonal AB:

$$p_A - p_B = 101325 - 2337 = \rho a_{x,AB} \Delta L_{AB} = (998) a_{x,AB} (0.22\sqrt{2}),$$

$$\textit{solve} \quad a_{x,AB} = \mathbf{319\ m/s^2} \quad \textit{Ans.}\text{ (a)}$$

If we moved along the *y* axis shown in the figure, we would need $a_y = 319\sqrt{2} = 451$ m/s^2. (b) For *vertical* acceleration, ***nothing would happen***, both points A and B would continue to be atmospheric, although the pressure at deeper points would change. *Ans.*

2.145 A fish tank 16-in by 27-in by 14-inch deep is carried in a car which may experience accelerations as high as 6 m/s^2. Assuming rigid-body motion, estimate the maximum water depth to avoid spilling. Which is the best way to align the tank?



Solution: The best way is to *align the 16-inch width with the car's direction of motion,* to minimize the vertical surface change Δz. From Eq. (2.55) the free surface angle will be

$$\tan\theta_{max} = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{thus} \quad \Delta z = \frac{16''}{2}\tan\theta = 4.9 \text{ inches } (\theta = 31.5°)$$

Thus the tank should contain no more than 14 – 4.9 ≈ **9.1 inches of water**. *Ans.*

2.146 The tank in Fig. P2.146 is filled with water and has a vent hole at point A. It is 1 m wide into the paper. Inside is a 10-cm balloon filled with helium at 130 kPa. If the tank accelerates to the right at 5 m/s/s, at what angle will the balloon lean? Will it lean to the left or to the right?

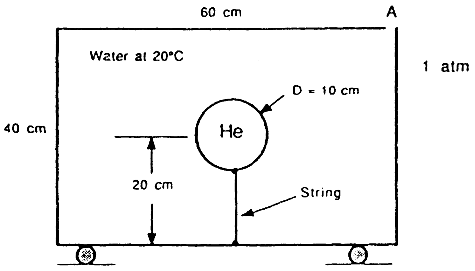
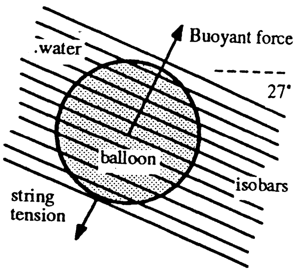


Fig. P2.146

Solution: The acceleration sets up pressure isobars which slant down and to the right, in both the water *and* in the helium. This means there will be a buoyancy force on the balloon up and to the right, as shown at right. It must be balanced by a string tension down and to the left. If we neglect balloon material weight, the balloon leans ***up and to the right*** at angle



$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{5.0}{9.81}\right) \approx \mathbf{27°} \quad \textit{Ans.}$$

measured from the vertical. This acceleration-buoyancy effect may seem counter-intuitive.

2.147 The tank of water in Fig. P2.147 accelerates uniformly by rolling without friction down the 30° inclined plane. What is the angle θ of the free surface? Can you explain this interesting result?

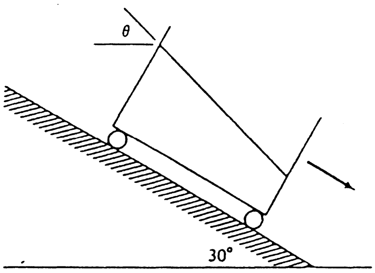


Fig. P2.147

Solution: If frictionless, $\Sigma F = W \sin\theta = ma$ along the incline and thus $a = g \sin 30° = 0.5g$.

$$\text{Thus } \tan\theta = \frac{a_x}{g + a_z} = \frac{0.5g\cos 30°}{g - 0.5g\sin 30°}; \quad \text{solve for } \theta = \mathbf{30°\,!} \quad \textit{Ans.}$$

The free surface aligns itself exactly parallel with the 30° incline.

P2.148 A child is holding a string onto which is attached a helium-filled balloon. (a) The child is standing still and suddenly accelerates forward. In a frame of reference moving with the child, which way will the balloon tilt, forward or backward? Explain. (b) The child is now sitting in a car that is stopped at a red light. The helium-filled balloon is not in contact with any part of the car (seats, ceiling, etc.) but is held in place by the string, which is held by the child. All the windows in the car are closed. When the traffic light turns green, the car accelerates forward. In a frame of reference moving with the car and child, which way will the balloon tilt, forward or backward? Explain. (c) Purchase or borrow a helium-filled balloon. Conduct a scientific experiment to see if your predictions in parts (a) and (b) are correct. If not, explain.

Solution: (a) Only the child and balloon accelerate, not the surrounding air. This is *not* rigid-body fluid motion. **The balloon will tilt backward** due to air drag. *Ans.*(a)

(b) Inside the car, the trapped air will accelerate with the car and the child, etc. This is rigid-body motion. **The balloon will tilt forward**, as in Prob. P2.146. *Ans.*(b)

(c) A student in the writer's class actually tried this experimentally. Our predictions were correct.

2.149 The waterwheel in Fig. P2.149 lifts water with 1-ft-diameter half-cylinder blades. The wheel rotates at 10 r/min. What is the water surface angle θ at pt. A?

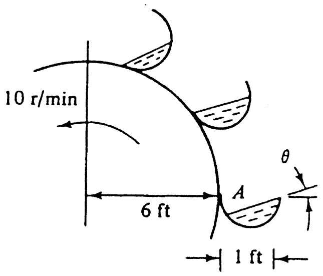


Fig. P2.149

Solution: Convert Ω = 10 r/min = 1.05 rad/s. Use an average radius R = 6.5 ft. Then

$$a_x = \Omega^2 R = (1.05)^2(6.5) \approx 7.13 \text{ ft/s}^2 \quad \text{toward the center}$$

$$\text{Thus} \quad \tan\theta = a_x/g = 7.13/32.2, \quad \text{or:} \quad \theta = \mathbf{12.5°} \quad \textit{Ans.}$$

2.150 A cheap accelerometer can be made from the U-tube at right. If L = 18 cm and D = 5 mm, what will *h* be if $a_x = 6 \text{ m/s}^2$?

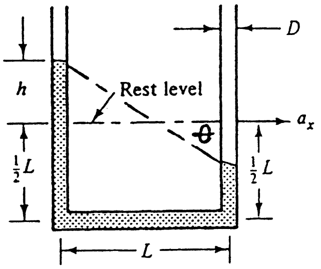


Fig. P2.150

Solution: We assume that the diameter is so small, $D \ll L$, that the free surface is a "point." Then Eq. (2.55) applies, and

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta = 31.5°$$

$$\text{Then} \quad h = (L/2)\tan\theta = (9 \text{ cm})(0.612) = \mathbf{5.5\ cm} \quad \textit{Ans.}$$

Since $h = (9 \text{ cm})a_x/g$, the scale readings are indeed linear in a_x, but I don't recommend it as an actual accelerometer, there are too many inaccuracies and disadvantages.

2.151 The U-tube in Fig. P2.151 is open at A and closed at D. What uniform acceleration a_x will cause the pressure at point C to be atmospheric? The fluid is water.

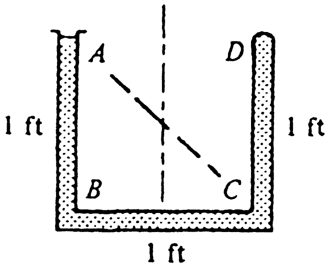
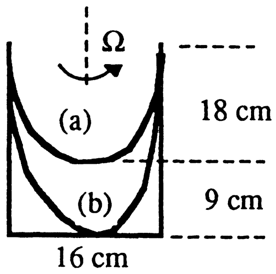


Fig. P2.151

Solution: If pressures at A and C are the same, the "free surface" must join these points:

$$\theta = 45°, \; a_x = g\tan\theta = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}$$

2.152 A 16-cm-diameter open cylinder 27 cm high is full of water. Find the central rigid-body rotation rate for which (a) one-third of the water will spill out; and (b) the bottom center of the can will be exposed.



Solution: (a) One-third will spill out if the resulting paraboloid surface is 18 cm deep:

$$h = 0.18\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for} \quad \Omega^2 = 552,$$

$$\Omega = 23.5\text{ rad/s} = \mathbf{224\ r/min} \quad \textit{Ans.}\text{ (a)}$$

(b) The bottom is barely exposed if the paraboloid surface is 27 cm deep:

$$h = 0.27\text{ m} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for } \Omega = 28.8\text{ rad/s} = \mathbf{275\ r/min} \quad \textit{Ans.}\text{ (b)}$$

P2.153 A cylindrical container, 14 inches in diameter, is used to make a mold for forming salad bowls. The bowls are to be 8 inches deep. The cylinder is half-filled with molten plastic, μ = 1.6 kg/(m-s), rotated steadily about the central axis, then cooled while rotating. What is the appropriate rotation rate, in r/min?

Solution: The molten plastic viscosity is a red herring, ignore. The appropriate final rotating surface shape is a paraboloid of radius 7 inches and depth 8 inches. Thus, from Fig. 2.23,

$$h = 8\ in = \frac{8}{12} ft = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (7/12\ ft)^2}{2(32.2\ ft/s^2)}$$

$$\text{Solve for} \quad \Omega = 11.2\frac{rad}{s} \times \frac{60}{2\pi} = \mathbf{107}\ \frac{r}{min} \qquad \textit{Ans.}$$

P2.154 A very tall 10-cm-diameter vase contains 1178 cm^3 of water. When spun steadily to achieve rigid-body rotation, a 4-cm-diameter dry spot appears at the bottom of the vase. What is the rotation rate, r/min, for this condition?

Solution: It is interesting that the answer has nothing to do with the water *density*. The value of 1178 cubic centimeters was chosen to make the rest depth a nice number:

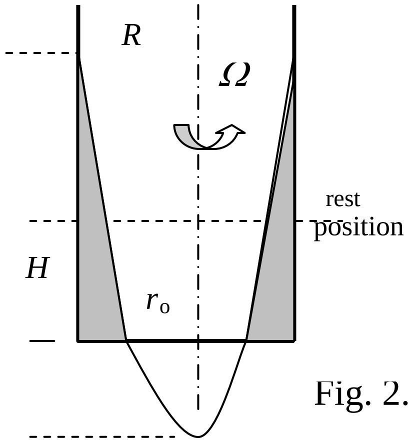


Fig. 2.154

$$\upsilon = 1178\,cm^3 = \pi(5cm)^2 H \text{ , solve } H = 15.0cm$$

One way would be to integrate and find the volume of the shaded liquid in Fig. P2.154 in terms of vase radius R and dry-spot radius r_o. That would yield the following formula:

$$d\upsilon = \pi(R^2 - r_o^2)\,dz \;,\quad \text{but } z = \Omega^2 r^2/2g \;,\quad \text{hence } dz = (\Omega^2 r/g)\,dr$$

$$\text{Thus}\quad \upsilon = \int_{r_o}^{R} \pi(R^2 - r_o^2)(\Omega^2 r/g)\,dr = \frac{\pi\Omega^2}{g}\int_{r_o}^{R}(R^2 r - r^3)\,dr = \frac{\pi\Omega^2}{g}\left(\frac{R^2 r^2}{2} - \frac{r^4}{4}\right)\Big|_{r_o}^{R}$$

$$\text{Finally:}\ \upsilon = \frac{\pi\Omega^2}{g}\left(\frac{R^4}{4} - \frac{R^2 r_o^2}{2} + \frac{r_o^4}{4}\right) = 0.001178\,m^3$$

$$\text{Solve for } R = 0.05m,\ r_o = 0.02m \ :\ \Omega^2 = 3336,\ \Omega = 57.8\frac{rad}{s} = \mathbf{552\,\frac{r}{min}} \quad \textit{Ans.}$$

The formulas in the text, concerning the paraboloids of "air", would, in the writer's opinion, be difficult to apply because of the free surface extending below the bottom of the vase.

2.155 For what uniform rotation rate in r/min about axis C will the U-tube fluid in Fig. P2.155 take the position shown? The fluid is mercury at 20°C.

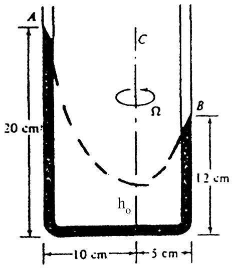


Fig. P2.155

Solution: Let ho be the height of the free surface at the centerline. Then, from Eq. (2.64),

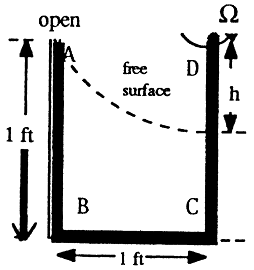
$$z_B = h_o + \frac{\Omega^2 R_B^2}{2g}; \quad z_A = h_o + \frac{\Omega^2 R_A^2}{2g}; \quad R_B = 0.05\text{ m} \quad \text{and} \quad R_A = 0.1\text{ m}$$

$$\text{Subtract:} \quad z_A - z_B = 0.08\text{ m} = \frac{\Omega^2}{2(9.81)}[(0.1)^2 - (0.05)^2],$$

$$\text{solve} \quad \Omega = 14.5\ \frac{\text{rad}}{\text{s}} = \mathbf{138}\ \frac{\mathbf{r}}{\mathbf{min}} \quad \textit{Ans.}$$

The fact that the fluid is mercury does not enter into this “kinematic” calculation.

2.156 Suppose the U-tube of Prob. 2.151 is rotated about axis DC. If the fluid is water at 122°F and atmospheric pressure is 2116 psfa, at what rotation rate will the fluid begin to vaporize? At what point in the tube will this happen?



Solution: At 122°F = 50°C, from Tables A-1 and A-5, for water, ρ = 988 kg/m^3 (or 1.917 slug/ft^3) and pv = 12.34 kPa (or 258 psf). When spinning around DC, the free surface comes down from point A to a position *below* point D, as shown. Therefore the fluid pressure is lowest at point D (*Ans.*). With *h* as shown in the figure,

$$p_D = p_{vap} = 258 = p_{atm} - \rho g h = 2116 - 1.917(32.2)h, \quad h = \Omega^2 R^2/(2g)$$

Solve for h ≈ 30.1 ft (!) Thus the drawing is wildly distorted and the dashed line falls **far below** point C! (The solution is correct, however.)

$$\text{Solve for} \quad \Omega^2 = 2(32.2)(30.1)/(1\text{ ft})^2 \quad \text{or:} \quad \Omega = 44\text{ rad/s} = \mathbf{420\ rev/min}. \quad \textit{Ans.}$$

2.157 The 45° V-tube in Fig. P2.157 contains water and is open at A and closed at C. (a) For what rigid-body rotation rate will the pressure be equal at points B and C? (b) For the condition of part (a), at what point in leg BC will the pressure be a minimum?

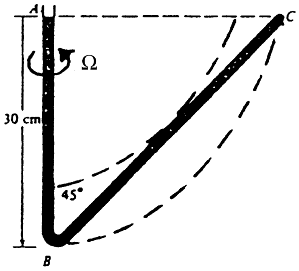


Fig. P2.157

Solution: (a) If pressures are equal at B and C, they must lie on a constant-pressure paraboloid surface as sketched in the figure. Taking $z_B = 0$, we may use Eq. (2.64):

$$z_C = 0.3\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2(0.3)^2}{2(9.81)}, \quad \text{solve for} \quad \Omega = 8.09\ \frac{\text{rad}}{\text{s}} = \mathbf{77}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans.}\ \text{(a)}$$

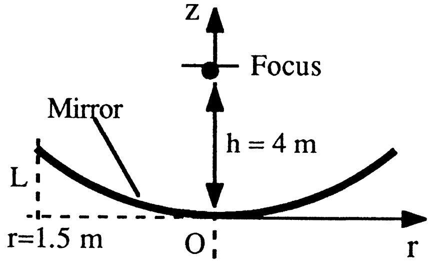
(b) The minimum pressure in leg BC occurs where the highest paraboloid pressure contour is tangent to leg BC, as sketched in the figure. This family of paraboloids has the formula

$$z = z_o + \frac{\Omega^2 r^2}{2g} = r\tan 45°, \quad \text{or:} \quad z_o + 3.333r^2 - r = 0 \quad \text{for a pressure contour}$$

$$\text{The minimum occurs when} \quad dz/dr = 0, \quad \text{or} \quad r \approx \mathbf{0.15\ m} \quad \textit{Ans.}\ \text{(b)}$$

The minimum pressure occurs *halfway between points B and C.*

2.158* It is desired to make a 3-m-diameter parabolic telescope mirror by rotating molten glass in rigid-body motion until the desired shape is achieved and then cooling the glass to a solid. The focus of the mirror is to be 4 m from the mirror, measured along the centerline. What is the proper mirror rotation rate, in rev/min?



Solution: We have to review our math book, or a handbook, to recall that the *focus* F of a parabola is the point for which all points on the parabola are equidistant from both the focus and a so-called "directrix" line (which is one focal length below the mirror).
For the focal length *h* and the *z-r* axes shown in the figure, the equation of the parabola is given by $\mathbf{r^2 = 4hz}$, with $h = 4$ m for our example.
Meanwhile the equation of the free-surface of the liquid is given by $\mathbf{z = r^2\Omega^2/(2g)}$.
Set these two equal to find the proper rotation rate:

$$z = \frac{r^2\Omega^2}{2g} = \frac{r^2}{4h}, \quad \text{or:} \quad \Omega^2 = \frac{g}{2h} = \frac{9.81}{2(4)} = 1.226$$

$$\textit{Thus} \quad \Omega = 1.107\ \frac{\text{rad}}{\text{s}}\left(\frac{60}{2\pi}\right) = \mathbf{10.6\ rev/min} \quad \textit{Ans.}$$

The focal point F is far above the mirror itself. If we put in $r = 1.5$ m and calculate the mirror depth "L" shown in the figure, we get L ≈ 14 centimeters.

2.159 The three-legged manometer in Fig. P2.159 is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

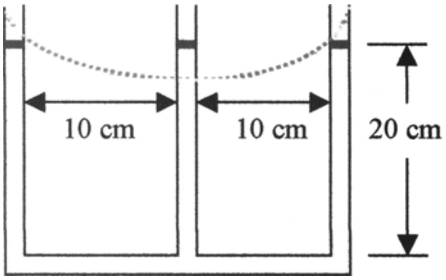


Fig. P2.159

Solution: (a) The free-surface during rotation is visualized as the dashed line in Fig. P2.159. The outer right and left legs experience an increase which is one-half that of the central leg, or $\Delta h_O = \Delta h_C/2$. The total displacement between outer and center menisci is, from Eq. (2.64) and Fig. 2.23, equal to $\Omega^2R^2/(2g)$. The center meniscus falls two-thirds of this amount and feeds the outer tubes, which each rise one-third of this amount above the rest position:

$$\boldsymbol{\Delta h_{outer} = \frac{1}{3}\Delta h_{total} = \frac{\Omega^2 R^2}{6g} \qquad \Delta h_{center} = -\frac{2}{3}\Delta h_{total} = -\frac{\Omega^2 R^2}{3g}} \quad \textit{Ans.}\text{ (a)}$$

For the particular case $R = 10$ cm and $\Omega = 120$ r/min $= (120)(2\pi/60) = 12.57$ rad/s, we obtain

$$\frac{\Omega^2 R^2}{2g} = \frac{(12.57\text{ rad/s})^2(0.1\text{ m})^2}{2(9.81\text{ m/s}^2)} = 0.0805\text{ m};$$

$$\Delta h_O \approx \mathbf{0.027\ m}\ (\textit{up}) \quad \Delta h_C \approx \mathbf{-0.054\ m}\ (\textit{down}) \quad \textit{Ans.}\text{ (b)}$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE-2.1 A gage attached to a pressurized nitrogen tank reads a gage pressure of 28 inches of mercury. If atmospheric pressure is 14.4 psia, what is the absolute pressure in the tank?

(a) 95 kPa (b) 99 kPa (c) 101 kPa **(d) 194 kPa** (e) 203 kPa

FE-2.2 On a sea-level standard day, a pressure gage, moored below the surface of the ocean (SG = 1.025), reads an absolute pressure of 1.4 MPa. How deep is the instrument?

(a) 4 m **(b) 129 m** (c) 133 m (d) 140 m (e) 2080 m

FE-2.3 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point A is 1 atmosphere, what is the absolute pressure at point B?

(a) 5.6 kPa (b) 10.9 kPa **(c) 106.9 kPa** (d) 112.2 kPa (e) 157.0 kPa

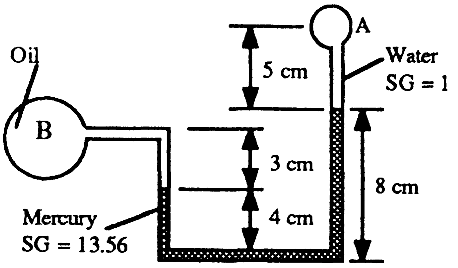


Fig. FE-2.3

FE-2.4 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point B is 14 psia, what is the absolute pressure at point B?

(a) 11 kPa (b) 41 kPa (c) 86 kPa **(d) 91 kPa** (e) 101 kPa

FE-2.5 A tank of water (SG = 1.0) has a gate in its vertical wall 5 m high and 3 m wide. The top edge of the gate is 2 m below the surface. What is the hydrostatic force on the gate?

(a) 147 kN (b) 367 kN (c) 490 kN **(d) 661 kN** (e) 1028 kN

FE-2.6 In Prob. FE-2.5 above, how far below the surface is the center of pressure of the hydrostatic force?

(a) 4.50 m (b) 5.46 m (c) 6.35 m (d) 5.33 m **(e) 4.96 m**

FE-2.7 A solid 1-m-diameter sphere floats at the interface between water (SG = 1.0) and mercury (SG = 13.56) such that 40% is in the water. What is the specific gravity of the sphere?

(a) 6.02 (b) 7.28 (c) 7.78 **(d) 8.54** (e) 12.56

FE-2.8 A 5-m-diameter balloon contains helium at 125 kPa absolute and 15°C, moored in sea-level standard air. If the gas constant of helium is 2077 m^2/(s^2·K) and balloon material weight is neglected, what is the net lifting force of the balloon?

(a) 67 N (b) 134 N (c) 522 N **(d) 653 N** (e) 787 N

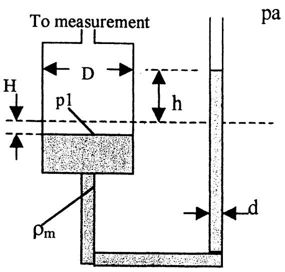
FE-2.9 A square wooden (SG = 0.6) rod, 5 cm by 5 cm by 10 m long, floats vertically in water at 20°C when 6 kg of steel (SG = 7.84) are attached to the lower end. How high above the water surface does the wooden end of the rod protrude?

(a) 0.6 m (b) 1.6 m **(c) 1.9 m** (d) 2.4 m (e) 4.0 m

FE-2.10 A floating body will always be stable when its

(a) CG is above the center of buoyancy (b) center of buoyancy is below the waterline
(c) center of buoyancy is above its metacenter (d) metacenter is above the center of buoyancy
(e) metacenter is above the CG

COMPREHENSIVE PROBLEMS



C2.1 Some manometers are constructed as in the figure at right, with one large reservoir and one small tube open to the atmosphere. We can then neglect movement of the reservoir level. If the reservoir is not large, its level will move, as in the figure. Tube height *h* is measured from the zero-pressure level, as shown.
(a) Let the reservoir pressure be high, as in the Figure, so its level goes down. Write an exact Expression for p_{1gage} as a function of h, d, D, and gravity *g*. (b) Write an approximate expression for p_{1gage}, neglecting the movement of the reservoir. (c) Suppose h = 26 cm, p_a = 101 kPa, and ρ_m = 820 kg/m^3. Estimate the ratio (D/d) required to keep the error in (b) less than 1.0% and also < 0.1%. Neglect surface tension.

Solution: Let *H* be the downward movement of the reservoir. If we neglect air density, the pressure difference is $p_1 - p_a = \rho_m g(h + H)$. But volumes of liquid must balance:

$$\frac{\pi}{4}D^2H = \frac{\pi}{4}d^2h, \quad \text{or:} \quad H = (d/D)^2 h$$

Then the pressure difference (exact except for air density) becomes

$$\boldsymbol{p_1 - p_a = p_{1gage} = \rho_m gh(1 + d^2/D^2)} \quad \textit{Ans.}\ \text{(a)}$$

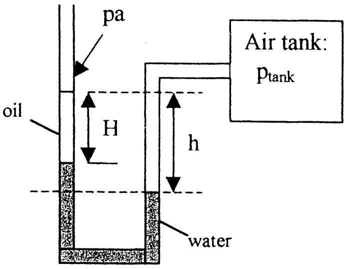
If we ignore the displacement *H*, then $\boldsymbol{p_{1gage} \approx \rho_m gh}$ *Ans.* (b)

(c) For the given numerical values, h = 26 cm and ρ_m = 820 kg/m^3 are irrelevant, all that matters is the ratio d/D. That is,

$$Error\ E = \frac{\Delta p_{exact} - \Delta p_{approx}}{\Delta p_{exact}} = \frac{(d/D)^2}{1 + (d/D)^2}, \quad or: \quad D/d = \sqrt{(1-E)/E}$$

For E = 1% or 0.01, D/d = $[(1 - 0.01)/0.01]^{1/2} \geq$ **9.95** *Ans.* (c-1%)
For E = 0.1% or 0.001, D/d = $[(1 - 0.001)/0.001]^{1/2} \geq$ **31.6** *Ans.* (c-0.1%)

C2.2 A prankster has added oil, of specific gravity SG_o, to the left leg of the manometer at right. Nevertheless, the U-tube is still to be used to measure the pressure in the air tank. (a) Find an expression for *h* as a function of *H* and other parameters in the problem.
(b) Find the special case of your result when $p_{tank} = p_a$.
(c) Suppose H = 5 cm, p_a = 101.2 kPa, SG_o = 0.85, and p_{tank} is 1.82 kPa higher than p_a. Calculate *h* in cm, ignoring surface tension and air density effects.



Solution: Equate pressures at level *i* in the tube (the right hand water level):

$$p_i = p_a + \rho g H + \rho_w g(h - H) = p_{tank},$$

$$\rho = SG_o \rho_w \quad \text{(ignore the column of air in the right leg)}$$

$$\text{Solve for:} \quad \boldsymbol{h = \frac{p_{tk} - p_a}{\rho_w g} + H(1 - SG_o)} \quad \textit{Ans.}\ \text{(a)}$$

If $p_{tank} = p_a$, then

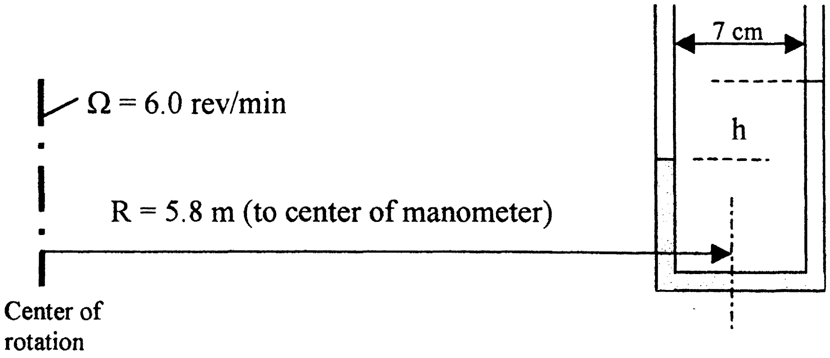
$$\boldsymbol{h = H(1 - SG_o)} \quad \textit{Ans.}\ \text{(b)}$$

(c) For the particular numerical values given above, the answer to (a) becomes

$$h = \frac{1820\ Pa}{998(9.81)} + 0.05(1 - 0.85) = 0.186 + 0.0075 = 0.193\ m = \boldsymbol{19.3\ cm} \quad \textit{Ans.}\ \text{(c)}$$

Note that this result is not affected by the actual value of atmospheric pressure.

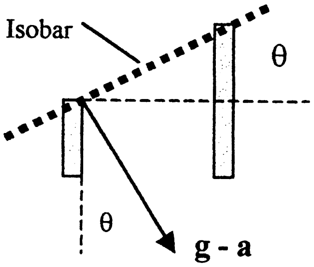
C2.3 Professor F. Dynamics, riding the merry-go-round with his son, has brought along his U-tube manometer. (You never know when a manometer might come in handy.) As shown in Fig. C2.3, the merry-go-round spins at constant angular velocity and the manometer legs are 7 cm apart. The manometer center is 5.8 m from the axis of rotation. Determine the height difference *h* in two ways: (a) approximately, by assuming rigid body translation with **a** equal to the average manometer acceleration; and (b) exactly, using rigid-body rotation theory. How good is the approximation?



Solution: (a) Approximate: The average acceleration of the manometer is $R_{avg}\Omega^2 = 5.8[6(2\pi/60)]^2 = 2.29$ rad/s toward the center of rotation, as shown. Then

$$\tan(\theta) = a/g = 2.29/9.81 = h/(7\text{ cm}) = 0.233$$

$$\text{Solve for}\quad h = \mathbf{1.63\ cm} \quad \textit{Ans. (a)}$$



(b) Exact: The isobar in the figure at right would be on the parabola $z = C + r^2\Omega^2/(2g)$, where C is a constant. Apply this to the left leg (z_1) and right leg (z_2). As above, the rotation rate is $\Omega = 6.0{*}(2\pi/60) = 0.6283$ rad/s. Then

$$h = z_2 - z_1 = \frac{\Omega^2}{2g}(r_2^2 - r_1^2) = \frac{(0.6283)^2}{2(9.81)}[(5.8+0.035)^2 - (5.8-0.035)^2]$$

$$= \mathbf{0.0163\ m} \quad \textit{Ans. (b)}$$

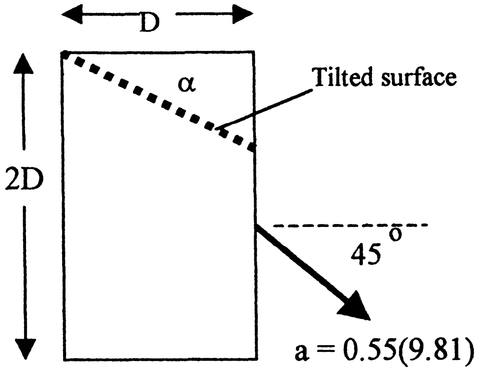
This is nearly identical to the approximate answer (a), because R >> Δr.

C2.4 A student sneaks a glass of cola onto a roller coaster ride. The glass is cylindrical, twice as tall as it is wide, and filled to the brim. He wants to know what percent of the cola he should drink before the ride begins, so that none of it spills during the big drop, in which the roller coaster achieves 0.55-g acceleration at a 45° angle below the horizontal. Make the calculation for him, neglecting sloshing and assuming that the glass is vertical at all times.

Solution: We have both horizontal and ver-tical acceleration. Thus the angle of tilt α is

$$\tan\alpha = \frac{a_x}{g + a_z} = \frac{0.55g\cos 45^\circ}{g - 0.55g\sin 45^\circ} = 0.6364$$

Thus $\alpha = 32.47°$. The tilted surface strikes the centerline at $R\tan\alpha = 0.6364R$ below the top. So the student should drink the cola until its rest position is 0.6364R below the top. The percentage drop in liquid level (and therefore liquid volume) is



$$\%\ removed = \frac{0.6364R}{4R} = 0.159 \quad or\text{:} \quad \mathbf{15.9\%} \quad Ans.$$

C2.5 Dry adiabatic lapse rate is defined as DALR = –dT/dz when T and p vary isentropically. Assuming $T = Cp^a$, where $a = (\gamma - 1)/\gamma$, $\gamma = c_p/c_v$, (a) show that DALR = $g(\gamma - 1)/(\gamma R)$, R = gas constant; and (b) calculate DALR for air in units of °C/km.

Solution: Write T(p) in the form $T/T_o = (p/p_o)^a$ and differentiate:

$$\frac{dT}{dz} = T_o a\left(\frac{p}{p_o}\right)^{a-1}\frac{1}{p_o}\frac{dp}{dz}, \quad \text{But for the hydrostatic condition: } \frac{dp}{dz} = -\rho g$$

Substitute $\rho = p/RT$ for an ideal gas, combine above, and rewrite:

$$\frac{dT}{dz} = -\frac{T_o}{p_o}a\left(\frac{p}{p_o}\right)^{a-1}\frac{p}{RT}g = -\frac{ag}{R}\left(\frac{T_o}{T}\right)\left(\frac{p}{p_o}\right)^{a}. \quad \text{But: } \frac{T_o}{T}\left(\frac{p}{p_o}\right)^{a} = 1\ (isentropic)$$

Therefore, finally,

$$-\frac{dT}{dz} = DALR = \frac{ag}{R} = \frac{\mathbf{(\gamma - 1)g}}{\boldsymbol{\gamma}\mathbf{R}} \quad Ans.\ \text{(a)}$$

(b) Regardless of the actual air temperature and pressure, the DALR for **air** equals

$$DALR = -\frac{dT}{dz}\Big|_s = \frac{(1.4-1)(9.81\ m/s^2)}{1.4(287\,m^2/s^2/°C)} = 0.00977\ \frac{°C}{m} = \mathbf{9.77}\ \frac{\mathbf{°C}}{\mathbf{km}} \quad Ans.\ \text{(b)}$$

C2.6 Use the approximate pressure-density relation for a "soft" liquid,

$$dp = a^2 d\rho, \quad \text{or} \quad p = p_o + a^2(\rho - \rho_o)$$

where *a* is the speed of sound and (ρ_o, p_o) are the conditions at the liquid surface *z* = 0. Use this approximation to derive a formula for the density distribution $\rho(z)$ and pressure distribution p(z) in a column of soft liquid. Then find the force F on a vertical wall of width b, extending from z = 0 down to z = –h, and compare with the incompressible result $F = \rho_o g h^2 b/2$.

Solution: Introduce this p(ρ) relation into the hydrostatic relation (2.18) and integrate:

$$dp = a^2 d\rho = -\gamma\, dz = -\rho g\, dz, \quad \text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\int_0^z \frac{g\,dz}{a^2}, \quad \text{or:} \quad \boldsymbol{\rho = \rho_o e^{-gz/a^2}} \quad \textit{Ans.}$$

assuming constant a^2. Substitute into the p(ρ) relation to obtain the pressure distribution:

$$p \approx p_o + a^2\rho_o[e^{-gz/a^2} - 1] \tag{1}$$

Since p(z) increases with z at a greater than linear rate, the center of pressure will always be *a little lower than predicted by linear theory* (Eq. 2.44). Integrate Eq. (1) above, neglecting p_o, into the pressure force on a vertical plate extending from z = 0 to z = –h:

$$F = -\int_0^{-h} pb\,dz = \int_{-h}^{0} a^2\rho_o(e^{-gz/a^2} - 1)b\,dz = \mathbf{ba^2\rho_o}\left[\frac{\mathbf{a^2}}{\mathbf{g}}(\mathbf{e^{gh/a^2} - 1}) - \mathbf{h}\right] \quad \textit{Ans.}$$

In the limit of small depth change relative to the "softness" of the liquid, $h \ll a^2/g$, this reduces to the linear formula $F = \rho_o g h^2 b/2$ by expanding the exponential into the first three terms of its series. For "hard" liquids, the difference in the two formulas is negligible. For example, for water (a ≈ 1490 m/s) with h = 10 m and b = 1 m, the linear formula predicts F = 48950**0** N while the exponential formula predicts F = 48950**7** N.

C2.7 Venice, Italy is slowly sinking, so now, especially in winter, plazas and walkways are flooded. The proposed solution is the floating levee of Fig. P2.7. When filled with air, it rises to block off the sea. The levee is 30 m high and 5 m wide. Assume a uniform density of 300 kg/m^3 when floating. For the 1-meter Sea-Lagoon difference shown, estimate the angle at which the levee floats.

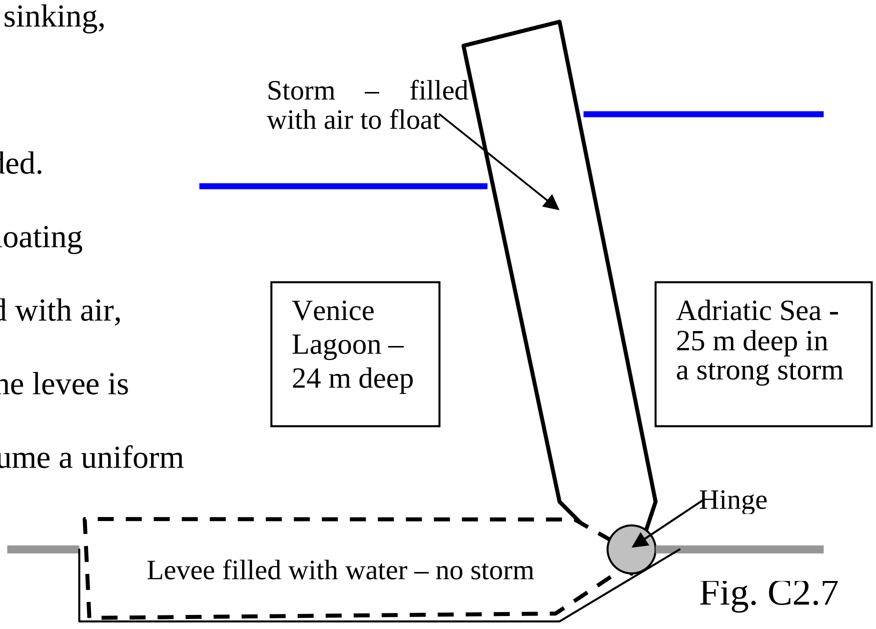
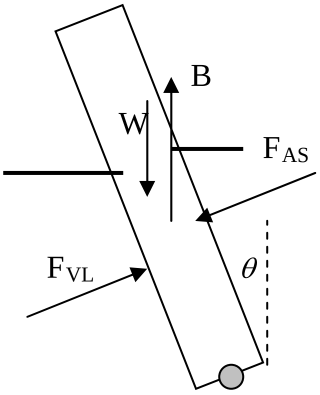

Storm – filled with air to float
Venice Lagoon – 24 m deep
Adriatic Sea - 25 m deep in a strong storm
Hinge
Levee filled with water – no storm

Fig. C2.7

Solution: The writer thinks this problem is rather laborious. Assume ρ_{seawater} = 1025 kg/m^3. There are 4 forces: the hydrostatic force F_{AS} on the Adriatic side, the hydrostatic force F_{VL} on the lagoon side, the weight W of the levee, and the buoyancy B of the submerged part of the levee. On the Adriatic side, $25/\cos\theta$ meters are submerged. On the lagoon side, $24/\cos\theta$ meters are submerged. For buoyancy, average the two depths, (25+24)/2 = 24.5 m.



For weight, the whole length of 30 m is used. Compute the four forces per unit width into the paper (since this width *b* will cancel out of all moments):

$$F_{AS} = \rho g h_{AS} L_{submerged} = (1025)(9.81)(25/2)(25/\cos\theta) = 3.142E6/\cos\theta$$

$$F_{VL} = \rho g h_{VL} L_{submerged} = (1025)(9.81)(24/2)(24/\cos\theta) = 2.896E6/\cos\theta$$

$$W = \rho_{levee} g L(\text{levee width}) = (300)(9.81)(30)(5) = 441500\ N/m$$

$$B = \rho g L_{sub-average}(\text{levee width}) = (1025)(9.81)(24.5)(5) = 1.232E6\ N/m$$

The hydrostatic forces have CP two-thirds of the way down the levee surfaces. The weight CG is in the center of the levee (15 m above the hinge). The buoyancy center is halfway down from the surface, or about (24.5)/2 m. The moments about the hinge are

$$\Sigma M_{hinge} = F_{AS}(\frac{25/\cos\theta}{3}m) + W(15m)\sin\theta - F_{VL}(\frac{24/\cos\theta}{3}m) - B(\frac{24.5}{2}m)\sin\theta = 0$$

where the forces are listed above and are not retyped here. Everything is known except the listing angle θ (measured from the vertical). Some iteration is required, say, on Excel, or, for a good initial guess (about θ = 15-30°), EES converges nicely to

$$\boldsymbol{\theta \approx 23.1°} \quad \textit{Ans.}$$

C2.8 What is the uncertainty is using pressure measurement as an altimeter? A gage on the side of an airplane measures a local pressure of 54 kPa, with an uncertainty of 3 kPa. The estimated lapse rate that day is 0.0070 K/m, with an uncertainty of 0.001 K/m. Effective sea-level temperature is 10°C, with an uncertainty of 4°C. Effective sea-level pressure is 100 kPa, with an uncertainty of 3 kPa. Estimate the airplane's altitude and its uncertainty.

Solution: We are dealing with the troposphere pressure variation formula, Eq. (2.20):

$$\frac{p}{p_o} = \left(1-\frac{Bz}{T_o}\right)^{g/RB} \quad ; \quad \text{Invert}: \quad z = \frac{T_o}{B}\left[1-\left(\frac{p}{p_o}\right)^{RB/g}\right]$$

To estimate the plane's altitude, just insert the given data for pressure, temperature, etc.:

$$z = \frac{283K}{0.0070K/m}\left[1-\left(\frac{54kPa}{100kPa}\right)^{(287)(0.0070)/9.81}\right] \approx \mathbf{4800\,m} \quad \textit{Ans.}$$

To evaluate the overall uncertainty in *z*, we have to compute four derivatives:

$$\delta z = \left[\left(\frac{\partial z}{\partial p}\delta p\right)^2 + \left(\frac{\partial z}{\partial p_o}\delta p_o\right)^2 + \left(\frac{\partial z}{\partial T_o}\delta T_o\right)^2 + \left(\frac{\partial z}{\partial B}\delta B\right)^2\right]^{1/2}$$

where we are given δp = 3 kPa, δp_o = 3 kPa, δT_o = 4°C, and δB = 0.001. Typing out those four derivatives is a nightmare for the writer, so we will just give the four results:

$$\frac{\partial z}{\partial p}\delta p = -404\,m \; ; \; \frac{\partial z}{\partial p_o}\delta p_o = 218\,m \; ; \; \frac{\partial z}{\partial T_o}\delta T_o = 68\,m \; ; \; \frac{\partial z}{\partial B}\delta B = 42\,m$$

$$\text{whence} \quad \delta z \approx [(404m)^2 + (218m)^2 + (68m)^2 + (42m)^2]^{1/2} = \mathbf{466\,m} \quad \textit{Ans.}$$

The overall uncertainty is about ±10%. The largest effect is the 5.6% uncertainty in pressure, *p*, which has a strong effect on the altitude formula.

C2.9 The deep submersible vehicle ALVIN in the chapter-opener photo has a hollow titanium sphere of inside diameter 78.08 inches and thickness 1.93 in. If the vehicle is submerged to a depth of 3,850 m in the ocean, estimate (*a*) the water pressure outside the sphere; (*b*) the maximum elastic stress in the sphere, in lbf/in^2; and (*c*) the factor of safety of the titanium alloy (6% aluminum, 4% vanadium).

Solution: This problem requires you to know (or read about) some solid mechanics!
(*a*) The hydrostatic (gage) pressure outside the submerged sphere would be

$$p_{water} \approx \rho_{water}\, g\, h = (1025\, kg/m^3)(9.81 m/s^2)(3850\, m) \approx 3.87\text{E}7\ Pa = 5600\, Psi$$

If we corrected for water compressibility, the result would increase by the small amount of 0.9%, giving as final estimate of $p_{\text{water}} = 3.90\text{E}7$ Pa ≈ **5665 lbf/in^2**. *Ans.*(*a*)

(*b*) From any textbook on elasticity or strength of materials, the maximum elastic stress in a hollow sphere under external pressure is *compression* and occurs at the inside surface. If *a* is the inside radius (39.04 in) and *b* the outside radius, 39.04+1.93in = 40.97 in, the formula for maximum stress is

$$\sigma_{\max} = p_{water}\frac{3b^3}{2(b^3 - a^3)} = (3.90E7)\frac{3(40.97\, in)^3}{2(40.97^3 - 39.04^3)} = 4.34\text{E}8\, Pa \approx \mathbf{63{,}000\, psi} \quad Ans.(b)$$

Various references found by the writer give the ultimate tensile strength of titanium alloys as 130,000 to 160,000 psi. Thus the factor of safety, based on *tensile* strength, is approximately

2.1 to 2.5. *Ans.*(*c*)

NOTE: For titanium, the ultimate compressive strength should be similar to the tensile strength. FURTHER NOTE: It is better to base the factor of safety on *yield* strength.

Chapter 3 • Integral Relations for a Control Volume

P3.1 Discuss Newton's second law (the linear momentum relation) in these three forms:

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum \mathbf{F} = \frac{d}{dt}(m\mathbf{V}) \qquad \sum \mathbf{F} = \frac{d}{dt}\left(\int_{system} \mathbf{V}\rho\, d\upsilon\right)$$

Solution: These questions are just to get the students thinking about the basic laws of mechanics. They are valid and equivalent for constant-mass systems, and we can make use of all of them in certain fluids problems, e.g. the #1 form for small elements, #2 form for rocket propulsion, but the #3 form is control-volume related and thus the most popular in this chapter.

P3.2 Consider the angular-momentum relation in the form

$$\sum \mathbf{M}_O = \frac{d}{dt}\left[\int_{system} (\mathbf{r}\times\mathbf{V})\rho\, d\upsilon\right]$$

What does **r** mean in this relation? Is this relation valid in both solid and fluid mechanics? Is it related to the *linear*-momentum equation (Prob. 3.1)? In what manner?

Solution: These questions are just to get the students thinking about angular momentum versus linear momentum. One might forget that **r** is the position vector from the moment-center O to the elements $\rho\, d\upsilon$ where momentum is being summed. Perhaps $\mathbf{r}_O$ is a better notation.

P3.3 For steady laminar flow through a long tube (see Prob. 1.12), the axial velocity distribution is given by $u = C(R^2 - r^2)$, where R is the tube outer radius and C is a constant. Integrate $u(r)$ to find the total volume flow Q through the tube.

Solution: The area element for this axisymmetric flow is $dA = 2\pi r\, dr$. From Eq. (3.7),

$$Q = \int u\, dA = \int_0^R C(R^2 - r^2)2\pi r\, dr = \frac{\boldsymbol{\pi}}{\mathbf{2}}\mathbf{CR^4} \quad \textit{Ans.}$$

P3.4 A fire hose has a 5-inch inside diameter and is flowing at 600 gal/min. The flow exits through a nozzle contraction at a diameter D_n. For steady flow, what should D_n be, in inches, to create an exit velocity of 25 m/s?

Solution: This is a straightforward one-dimensional steady-flow continuity problem. Some unit conversions are needed:

600 gal/min = 1.337 ft^3/s; 25 m/s = 82.02 ft/s ; 5 inches = 0.4167 ft

The hose diameter (5 in) would establish a hose average velocity of 9.8 ft/s, but we don't really need this. Go directly to the volume flow:

$$Q = 1.337\frac{ft^3}{s} = A_n V_n = \frac{\pi}{4}D_n^2(82.02\frac{ft}{s})^2 \;\; ; \;\; \text{Solve for} \quad D_n = 0.144\ ft = \mathbf{1.73\,in} \quad Ans.$$

P3.5 Water at 20°C flows through a 5-inch-diameter smooth pipe at a high Reynolds number, for which the velocity profile is given by $u \approx U_o(y/R)^{1/8}$, where U_o is the centerline velocity, R is the pipe radius, and y is the distance measured from the wall toward the centerline. If the centerline velocity is 25 ft/s, estimate the volume flow rate in gallons per minute.

Solution: The formula for average velocity in this power-law case was given in Example 3.4:

$$V_{av} = U_o\frac{2}{(1+m)(2+m)} = U_o\frac{2}{(1+1/8)(2+1/8)} = 0.837U_o = 0.837(25) = 20.92\ \frac{ft}{s}$$

$$Thus \quad Q = V_{av}\,A_{pipe} = [20.92\frac{ft}{s}]\ \pi(\frac{2.5}{12}\ ft)^2 = 2.85\frac{ft^3}{s} \approx \mathbf{1280}\ \frac{gal}{min} \quad Ans.$$

P3.6 When a gravity-driven liquid jet issues from a slot in a tank, as in Fig. P3.6, an approximation for the exit velocity distribution is $u \approx \sqrt{2g(h-z)}$, where h is the depth of the jet centerline. Near the slot, the jet is horizontal, two-dimensional, and of thickness $2L$, as shown. Find a general expression for the total volume flow Q issuing from the slot; then take the limit of your result if $L \ll h$.

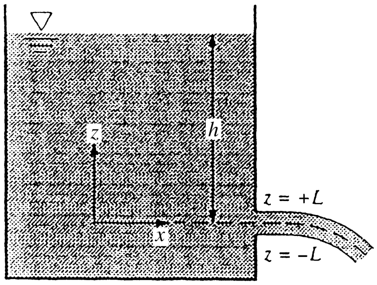


Fig. P3.6

Solution: Let the slot width be *b* into the paper. Then the volume flow from Eq. (3.7) is

$$Q = \int u\,dA = \int_{-L}^{+L} [2g(h-z)]^{1/2}\,b\,dz = \frac{\mathbf{2b}}{\mathbf{3}}\mathbf{\sqrt{(2g)}[(h+L)^{3/2} - (h-L)^{3/2}]} \quad \textit{Ans.}$$

In the limit of $L \ll h$, this formula reduces to $\mathbf{Q \approx (2Lb)\sqrt{(2gh)}}$ *Ans.*

P3.7 A spherical tank, of diameter 35 cm, is leaking air through a 5-mm-diameter hole in its side. The air exits the hole at 360 m/s and a density of 2.5 kg/m^3. Assuming uniform mixing, (*a*) find a formula for the rate of change of average density in the tank; and (*b*) calculate a numerical value for ($d\rho/dt$) in the tank for the given data.

Solution: If the control volume surrounds the tank and cuts through the exit flow,

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}(\rho_{\text{tank}}\upsilon_{\text{tank}}) + \dot{m}_{out} = \upsilon_{\text{tank}}\frac{d}{dt}(\rho_{\text{tank}}) + (\rho AV)_{out}$$

$$\text{Solve for} \quad \frac{d}{dt}(\rho_{\text{tank}}) = -\frac{(\rho AV)_{out}}{\upsilon_{\text{tank}}} \qquad \textit{Ans.(a)}$$

(b) For the given data, we calculate

$$\frac{d\rho_{\text{tank}}}{dt} = -\frac{(2.5\,kg/m^3)[(\pi/4)(0.005m)^2](360\,m/s)}{(\pi/6)(0.35m)^3} = \mathbf{-0.79\ \frac{kg/m^3}{s}} \quad Ans.(b)$$

P3.8 Three pipes steadily deliver water at 20°C to a large exit pipe in Fig. P3.8. The velocity V_2 = 5 m/s, and the exit flow rate Q_4 = 120 m^3/h. Find (a) V_1; (b) V_3; and (c) V_4 if it is known that increasing Q_3 by 20% would increase Q_4 by 10%.

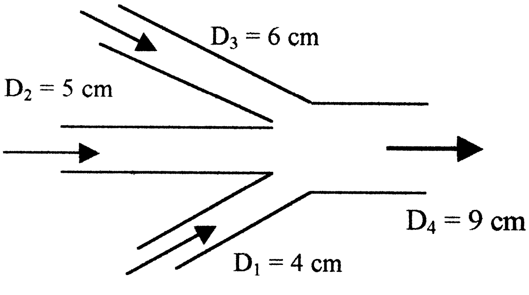


Fig. P3.8

Solution: (a) For steady flow we have Q1 + Q2 + Q3 = Q4, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

Since 0.2Q3 = 0.1Q4, and Q4 = (120 m^3/h)(1 h/3600 s) = 0.0333 m^3/s,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333\text{ m}^3/\text{s})}{\frac{\pi}{2}(0.06^2)} = \mathbf{5.89\ m/s} \quad Ans.\text{ (b)}$$

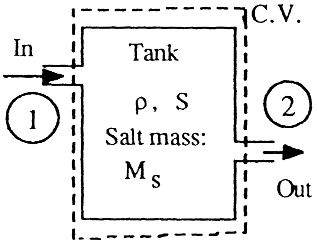
Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad \mathbf{V_1 = 5.45\ m/s} \quad Ans.\text{ (a)}$$

From mass conservation, Q4 = V4A4

$$(0.0333\text{ m}^3/\text{s}) = \text{V}_4(\pi)(0.06^2)/4 \qquad \mathbf{V_4 = 5.24\ m/s} \quad Ans.\text{ (c)}$$

P3.9 A laboratory test tank contains seawater of salinity S and density ρ. Water enters the tank at conditions (S_1, ρ_1, A_1, V_1) and is assumed to mix immediately in the tank. Tank water leaves through an outlet A_2 at velocity V_2. If salt is a "conservative" property (neither created nor destroyed), use the Reynolds transport theorem to find an expression for the rate of change of salt mass M_{salt} within the tank.



Solution: By definition, salinity S = ρ_{salt}/ρ. Since salt is a "conservative" substance (not consumed or created in this problem), the appropriate control volume relation is

$$\left.\frac{dM_{salt}}{dt}\right|_{system} = \frac{d}{dt}\left(\int_{CV} \rho_s \, d\upsilon\right) + S\dot{m}_2 - S_1\dot{m}_1 = 0$$

or: $$\left.\frac{\mathbf{dM_s}}{\mathbf{dt}}\right|_{CV} = \mathbf{S_1\rho_1A_1V_1 - S\rho A_2V_2} \quad \textit{Ans.}$$

3.10 Water flowing through an 8-cm-diameter pipe enters a porous section, as in Fig. P3.10, which allows a uniform radial velocity v_w through the wall surfaces for a distance of 1.2 m. If the entrance average velocity V_1 is 12 m/s, find the exit velocity V_2 if (a) v_w = 15 cm/s out of the pipe walls; (b) v_w = 10 cm/s into the pipe. (c) What value of v_w will make V_2 = 9 m/s?

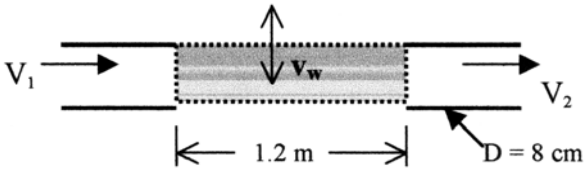


Fig. P3.10

Solution: (a) For a suction velocity of v_w = 0.15 m/s, and a cylindrical suction surface area,

$$A_w = 2\pi(0.04)(1.2) = 0.3016 \text{ m}^2$$

$$Q_1 = Q_w + Q_2$$

$$(12)(\pi)(0.08^2)/4 = (0.15)(0.3016) + V_2(\pi)(0.08^2)/4 \qquad \mathbf{V_2 = 3\ m/s} \quad \textit{Ans.}\text{ (a)}$$

(b) For an injection velocity into the pipe, $v_W = 0.10$ m/s, $Q_1 + Q_w = Q_2$, or:

$$(12)(\pi)(0.08^2)/4 + (0.10)(0.3016) = V_2(\pi)(0.08^2)/4 \qquad \mathbf{V_2 = 18\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

(c) Setting the outflow V_2 to 9 m/s, the wall suction velocity is,

$$(12)(\pi)(0.08^2)/4 = (v_w)(0.3016) + (9)(\pi)(0.08^2)/4 \qquad \mathbf{v_w = 0.05\ m/s = 5\ cm/s}\ \boldsymbol{out}$$

P3.11 The inlet section of a vacuum cleaner is a rectangle, 1 inch by 5 inches. The blower is able to provide suction at 25 cubic feet per minute. (*a*) What is the average velocity at the inlet, in m/s? (*b*) If conditions are sea level standard, what is the mass flow of air, in kg/s?

Solution: (*a*) Convert 25 ft^3/min to 25/60 = 0.417 ft^3/s. Then the inlet velocity is

$$V_{inlet} = \frac{Q}{A_{inlet}} = \frac{0.417\ ft^3/s}{(1/12\ ft)(5/12\ ft)} = 12.0\frac{ft}{s} \times 0.3048\frac{m}{ft} = \mathbf{3.66}\ \frac{m}{s} \qquad Ans.(a)$$

(*b*) At sea level, $\rho_{air} = 1.2255$ kg/m^3. Convert 25 ft^3/min to 0.0118 m^3/s. Then

$$\dot{m}_{air} = \rho_{air} Q = (1.2255\frac{kg}{m^3})(0.0118\frac{m^3}{s}) = \mathbf{0.0145}\ \frac{kg}{s} \qquad Ans.(b)$$

P3.12 The pipe flow in Fig. P3.12 fills a cylindrical tank as shown. At time t = 0, the water depth in the tank is 30 cm. Estimate the time required to fill the remainder of the tank.

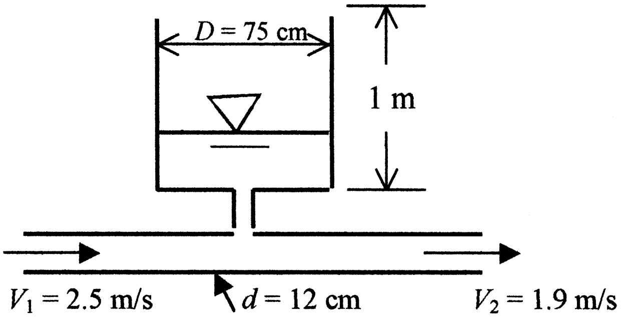


Fig. P3.12

Solution: For a control volume enclosing the tank and the portion of the pipe below the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\rho\pi R^2 \frac{dh}{dt} + (\rho AV)_{out} - (\rho AV)_{in} = 0$$

$$\frac{dh}{dt} = \frac{4}{998(\pi)(0.75^2)}\left[998\left(\frac{\pi}{4}\right)(0.12^2)(2.5-1.9)\right] = 0.0153\ m/s,$$

$$\Delta t = 0.7/0.0153 = 46\ \ s \quad \textit{Ans.}$$

P3.13 The cylindrical container in Fig. P3.13 is 20 cm in diameter and has a conical contraction at the bottom with an exit hole 3 cm in diameter. The tank contains fresh water at standard sea-level conditions. If the water surface is falling at the nearly steady rate $dh/dt \approx -0.072$ m/s, estimate the average velocity V from the bottom exit.

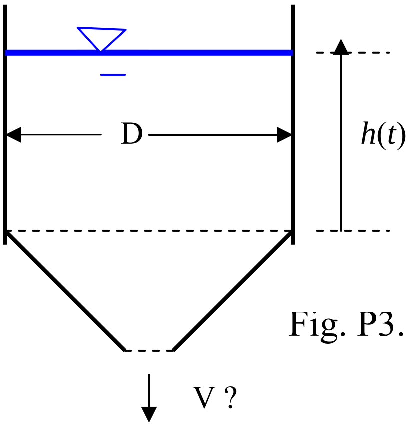


Fig. P3.13

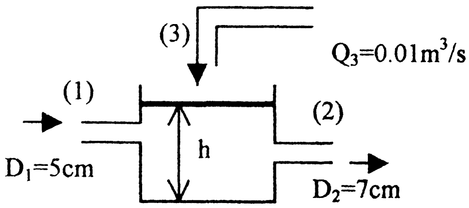
Solution: We could simply note that *dh/dt* is the same as the water *velocity* at the surface and use $Q_1 = Q_2$, or, more instructive, approach it as a control volume problem. Let the control volume encompass the entire container. Then the mass relation is

$$\left.\frac{dm}{dt}\right|_{system} = 0 = \frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \dot{m}_{out} = \frac{d}{dt}\left(\cancel{\upsilon_{cone}} + \rho\frac{\pi}{4}D^2 h\right) \,|\, + \rho\frac{\pi}{4}D_{exit}^2 V\,,$$

$$\text{or: } \rho\frac{\pi}{4}D^2\frac{dh}{dt} + \rho\frac{\pi}{4}D_{exit}^2 V = 0 \quad \text{Cancel } \rho\frac{\pi}{4}: \; V = \left(\frac{D}{D_{exit}}\right)^2\left(-\frac{dh}{dt}\right)$$

$$\text{Introduce the data: } V = \left(\frac{20\,cm}{3\,cm}\right)^2\left[-\left(-0.072\frac{m}{s}\right)\right] = \mathbf{3.2\frac{m}{s}} \qquad \textit{Ans.}$$

P3.14 The open tank in the figure contains water at 20°C. For incompressible flow, (a) derive an analytic expression for *dh/dt* in terms of (Q1, Q2, Q3). (b) If *h* is constant, determine V2 for the given data if V1 = 3 m/s and Q3 = 0.01 m^3/s.



Solution: For a control volume enclosing the tank,

$$\frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \rho(Q_2 - Q_1 - Q_3) = \rho\frac{\pi d^2}{4}\frac{dh}{dt} + \rho(Q_2 - Q_1 - Q_3),$$

$$\textit{solve}\quad \mathbf{\frac{dh}{dt}=\frac{Q_1+Q_3-Q_2}{(\pi d^2/4)}}\quad \textit{Ans.}\ \text{(a)}$$

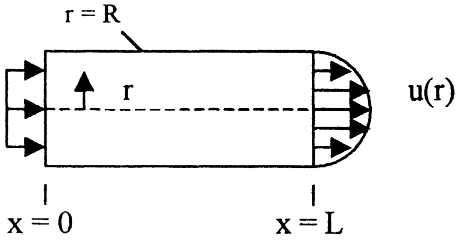
If *h* is constant, then

$$Q_2=Q_1+Q_3=0.01+\frac{\pi}{4}(0.05)^2(3.0)=0.0159=\frac{\pi}{4}(0.07)^2V_2,$$

$$\textit{solve}\quad \mathbf{V_2=4.13\ m/s}\quad \textit{Ans.}\ \text{(b)}$$

P3.15 Water flows steadily through the round pipe in the figure. The entrance velocity is Vo. The exit velocity approximates turbulent flow, $u = u_{max}(1 - r/R)^{1/7}$. Determine the ratio U_o/u_{max} for this incompressible flow.

Solution: Inlet and outlet flow must balance:



$$Q_1=Q_2,\quad \textit{or:}\quad \int_0^R U_o 2\pi r\,dr=\int_0^R u_{max}\left(1-\frac{r}{R}\right)^{1/7}2\pi r\,dr,\quad \textit{or:}\quad U_o\pi R^2=u_{max}\frac{49\pi}{60}R^2$$

Cancel and rearrange for this assumed incompressible pipe flow:

$$\frac{U_o}{u_{\max}}=\mathbf{\frac{49}{60}}\quad \textit{Ans.}$$

P3.16 An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.16, with a uniform inlet profile *u* = *U*o and a cubic polynomial exit profile

$$u\approx U_o\left(\frac{3\eta-\eta^3}{2}\right)\quad \text{where } \eta=\frac{y}{\delta}$$

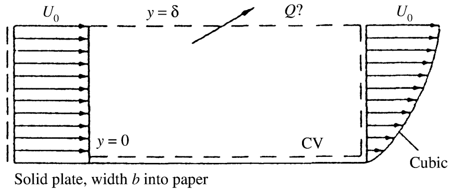


Fig. P3.16

Compute the volume flow *Q* across the top surface of the control volume.

Solution: For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^{\delta} U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right) b\,dy - \int_0^{\delta} U_o b\,dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } \mathbf{Q = \frac{3}{8}U_o b\delta} \quad \textit{Ans.}$$

P3.17 Incompressible steady flow in the inlet between parallel plates in Fig. P3.17 is uniform, *u* = *U*o = 8 cm/s, while downstream the flow develops into the parabolic laminar profile *u* = *az*(*z*o − *z*), where *a* is a constant. If *z*o = 4 cm and the fluid is SAE 30 oil at 20°C, what is the value of *u*max in cm/s?

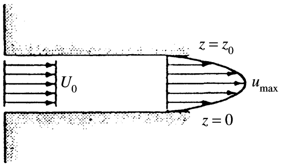


Fig. P3.17

Solution: Let *b* be the plate width into the paper. Let the control volume enclose the inlet and outlet. The walls are solid, so no flow through the wall. For incompressible flow,

$$0 = Q_{out} - Q_{in} = \int_0^{z_o} az(z_o - z)b\,dz - \int_0^{z_o} U_o b\,dz = abz_o^3/6 - U_o bz_o = 0, \quad \text{or:} \quad a = 6U_o/z_o^2$$

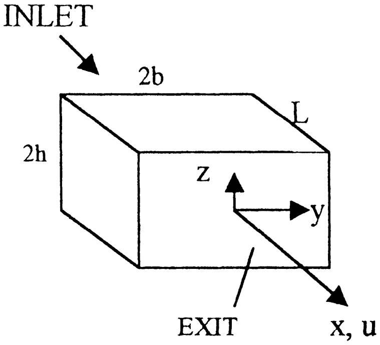
Thus continuity forces the constant *a* to have a particular value. Meanwhile, *a* is also related to the maximum velocity, which occurs at the center of the parabolic profile:

$$\text{At } z = z_o/2: \quad u = u_{max} = a\left(\frac{z_o}{2}\right)\left(z_o - \frac{z_o}{2}\right) = az_o^2/4 = (6U_o/z_o^2)(z_o^2/4)$$

$$\text{or:} \quad u_{max} = \frac{3}{2}U_o = \frac{3}{2}(8\text{ cm/s}) = \mathbf{12\,\frac{cm}{s}} \quad \textit{Ans.}$$

Note that the result is independent of zo or of the particular fluid, which is SAE 30 oil.

P3.18 An incompressible fluid flows steadily through the rectangular duct in the figure. The exit velocity profile is given by $u \approx u_{max}(1 - y^2/b^2)(1 - z^2/h^2)$. (a) Does this profile satisfy the correct boundary conditions for viscous fluid flow? (b) Find an analytical expression for the volume flow Q at the exit. (c) If the inlet flow is 300 ft^3/min, estimate u_{max} in m/s.



Solution: (a) The *fluid should not slip* at any of the duct surfaces, which are defined by $y = \pm b$ and $z = \pm h$. From our formula, we see **u ≡ 0 at all duct surfaces**, OK. *Ans.* (a)
(b) The exit volume flow Q is defined by the integral of u over the exit plane area:

$$Q = \iint u\,dA = \int_{-h}^{+h}\int_{-b}^{+b} u_{max}\left(1-\frac{y^2}{b^2}\right)\left(1-\frac{z^2}{h^2}\right) dy\,dz = u_{max}\left(\frac{4b}{3}\right)\left(\frac{4h}{3}\right)$$

$$= \mathbf{\frac{16bhu_{max}}{9}} \quad \textit{Ans.}\text{ (b)}$$

(c) Given $Q = 300$ ft^3/min $= 0.1416$ m^3/s and $b = h = 10$ cm, the maximum exit velocity is

$$Q = 0.1416\ \frac{m^3}{s} = \frac{16}{9}(0.1\text{ m})(0.1\text{ m})u_{max}, \quad \textit{solve for}\ \mathbf{u_{max} = 7.96\ m/s} \quad \textit{Ans.}\text{ (c)}$$

P3.19 Water from a storm drain flows over an outfall onto a porous bed which absorbs the water at a uniform vertical velocity of 8 mm/s, as shown in Fig. P3.19. The system is 5 m deep into the paper. Find the length L of bed which will completely absorb the storm water.

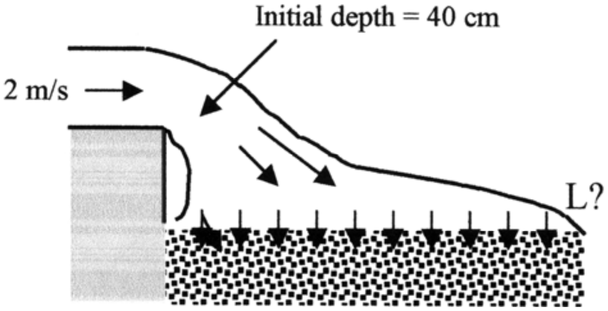


Fig. P3.19

Solution: For the bed to completely absorb the water, the flow rate over the outfall must equal that into the porous bed,

$$Q_1 = Q_{PB}; \quad \text{or} \quad (2\text{ m/s})(0.2\text{ m})(5\text{ m}) = (0.008\text{ m/s})(5\text{ m})L \quad \mathbf{L \approx 50\ m} \quad \textit{Ans.}$$

P3.20 Oil (SG-0.91) enters the thrust bearing at 250 N/hr and exits radially through the narrow clearance between thrust plates. Compute (a) the outlet volume flow in mL/s, and (b) the average outlet velocity in cm/s.

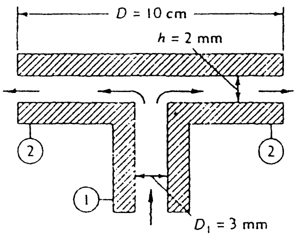


Fig. P3.20

Solution: The specific weight of the oil is (0.91)(9790) = 8909 N/m^3. Then

$$Q_2 = Q_1 = \frac{250/3600\text{ N/s}}{8909\text{ N/m}^3} = 7.8\times 10^{-6}\ \frac{\text{m}^3}{\text{s}} = \mathbf{7.8\ \frac{mL}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{But also} \quad Q_2 = V_2\,\pi(0.1\text{ m})(0.002\text{ m}) = 7.8\times 10^{-6}, \quad \text{solve for } V_2 = \mathbf{1.24\ \frac{cm}{s}} \quad \textit{Ans.}\text{ (b)}$$

P3.21 For the two-port tank in Fig. E3.5, let the dimensions remain the same, but assume $V_2 = 3$ ft/s and that V_1 is unknown. If the water surface is rising at a rate of 1 in/s, (*a*) determine the average velocity at section 1. (*b*) Is the flow at section 1 in or out?

Solution: Simply modify the calculations of Ex. 3.5 to match the new data. Assuming the water density is constant, the mass balance reduces to a set of volume flows:

$$A_1 V_1 + A_2 V_2 = A_t \frac{dh}{dt}, \quad \text{or:} \quad \frac{\pi}{4}\left(\frac{1}{12}\,ft\right)^2 V_1 + \frac{\pi}{4}\left(\frac{3}{12}\,ft\right)^2\left(3\frac{ft}{s}\right) = (2\,ft^2)\left(\frac{1}{12}\frac{ft}{s}\right)$$

$$(0.00545\,ft^2)V_1 + \left(0.1473\frac{ft^3}{s}\right) = \left(0.1667\frac{ft^3}{s}\right) \quad \textit{Solve for} \quad V_1 = \mathbf{3.56}\frac{ft}{s}\ \mathbf{(\textit{flow}\ in)} \quad \textit{Ans.}$$

P3.22 The converging-diverging nozzle shown in Fig. P3.22 expands and accelerates dry air to supersonic speeds at the exit, where $p2 = 8$ kPa and $T2 = 240$ K. At the throat, $p1 = 284$ kPa, $T1 = 665$ K, and $V1 = 517$ m/s. For steady compressible flow of an ideal gas, estimate (a) the mass flow in kg/h, (b) the velocity $V2$, and (c) the Mach number Ma2.

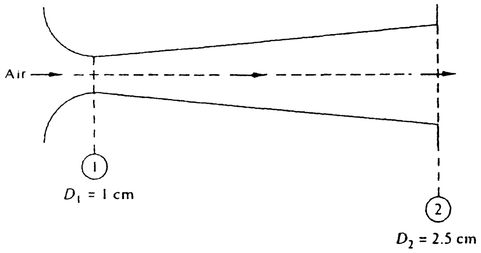


Fig. P3.22

Solution: The mass flow is given by the throat conditions:

$$\dot{m} = \rho_1 A_1 V_1 = \left[\frac{284000}{(287)(665)}\frac{\text{kg}}{\text{m}^3}\right]\frac{\pi}{4}(0.01\text{ m})^2\left(517\ \frac{\text{m}}{\text{s}}\right) = \mathbf{0.0604\ \frac{kg}{s}} \quad \textit{Ans. (a)}$$

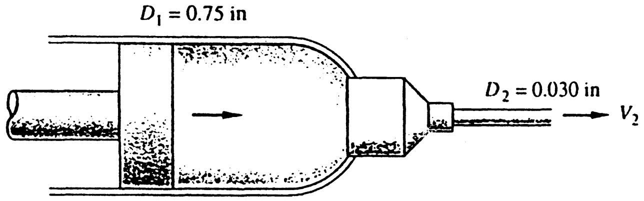
For steady flow, this must equal the mass flow at the exit:

$$0.0604\ \frac{\text{kg}}{\text{s}} = \rho_2 A_2 V_2 = \left[\frac{8000}{287(240)}\right]\frac{\pi}{4}(0.025)^2 V_2, \quad \text{or} \quad V_2 \approx \mathbf{1060\ \frac{m}{s}} \quad \textit{Ans. (b)}$$

Recall from Eq. (1.39) that the speed of sound of an ideal gas = $(kRT)^{1/2}$. Then

$$\text{Mach number at exit:} \quad \text{Ma} = V_2/a_2 = \frac{1060}{[1.4(287)(240)]^{1/2}} \approx \mathbf{3.41} \quad \textit{Ans. (c)}$$

P3.23 The hypodermic needle in the figure contains a liquid (SG = 1.05). If the serum is to be injected steadily at 6 cm^3/s, how fast should the plunger be advanced (a) if leakage in the plunger clearance is neglected; and (b) if leakage is 10 percent of the needle flow?



Solution: (a) For incompressible flow, the volume flow is the same at piston and exit:

$$Q = 6\ \frac{cm^3}{s} = 0.366\ \frac{in^3}{s} = A_1V_1 = \frac{\pi}{4}(0.75\ in)^2V_1,\quad solve\ V_{piston} = \mathbf{0.83\ \frac{in}{s}}\quad Ans.\ \text{(a)}$$

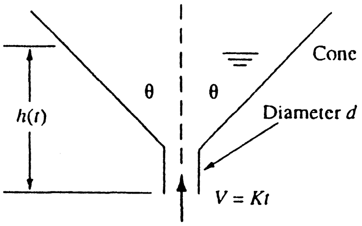
(b) If there is 10% leakage, the piston must deliver both needle flow and leakage:

$$A_1V_1 = Q_{needle} + Q_{clearance} = 6 + 0.1(6) = 6.6\ \frac{cm^3}{s} = 0.403\ \frac{in^3}{s} = \frac{\pi}{4}(0.75)^2V_1,$$

$$V_1 = \mathbf{0.91\ \frac{in}{s}}\quad Ans.\ \text{(b)}$$

P3.24 Water enters the bottom of the cone in the figure at a uniformly increasing average velocity $V = Kt$. If d is very small, derive an analytic formula for the water surface rise $h(t)$, assuming $h = 0$ at $t = 0$.

Solution: For a control volume around the cone, the mass relation becomes



$$\frac{d}{dt}\left(\int \rho\, d\upsilon\right) - \dot{m}_{in} = 0 = \frac{d}{dt}\left[\rho\frac{\pi}{3}(h\tan\theta)^2 h\right] - \rho\frac{\pi}{4}d^2Kt$$

$$Integrate\!:\quad \rho\frac{\pi}{3}h^3\tan^2\theta = \rho\frac{\pi}{8}d^2Kt^2$$

$$Solve\ for\quad \mathbf{h(t)} = \left[\frac{\mathbf{3}}{\mathbf{8}}\mathbf{Kt^2d^2\cot^2\theta}\right]^{1/3}\quad Ans.$$

P3.25 As will be discussed in Chaps. 7 and 8, the flow of a stream U_o past a blunt flat plate creates a broad low-velocity wake behind the plate. A simple model is given in Fig. P3.25, with only half of the flow shown due to symmetry. The velocity profile behind the plate is idealized as "dead air" (near-zero velocity) behind the plate, plus a higher velocity, decaying vertically above the wake according to the variation $u \approx U_o + \Delta U e^{-z/L}$, where L is the plate height and $z = 0$ is the top of the wake. Find ΔU as a function of stream speed U_o.

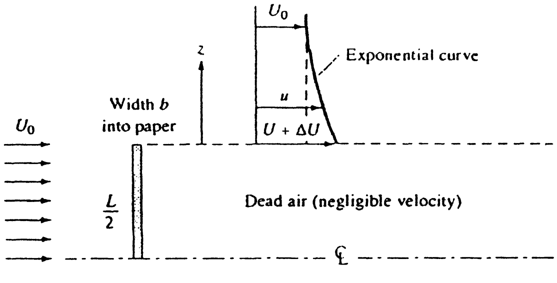


Fig. P3.25

Solution: For a control volume enclosing the upper half of the plate and the section where the exponential profile applies, extending upward to a large distance H such that exp(–H/L) ≈ 0, we must have inlet and outlet volume flows the same:

$$Q_{in} = \int_{-L/2}^{H} U_o\,dz = Q_{out} = \int_{0}^{H} (U_o + \Delta U\,e^{-z/L})\,dz, \quad \text{or:} \quad U_o\left(H + \frac{L}{2}\right) = U_oH + \Delta UL$$

$$\text{Cancel } U_oH \text{ and solve for } \quad \Delta U \approx \mathbf{\frac{1}{2}U_o} \quad \textit{Ans.}$$

P3.26 A thin layer of liquid, draining from an inclined plane, as in the figure, will have a laminar velocity profile $u = U_o(2y/h - y^2/h^2)$, where U_o is the surface velocity. If the plane has width b into the paper, (a) deter-mine the volume rate of flow of the film. (b) Suppose that $h = 0.5$ in and the flow rate per foot of channel width is 1.25 gal/min. Estimate U_o in ft/s.

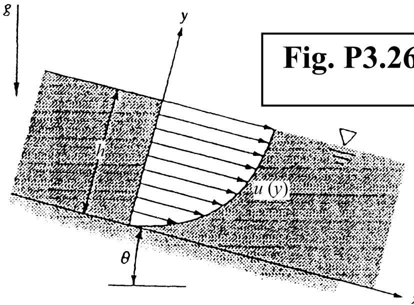


Fig. P3.26

Solution: (a) The total volume flow is computed by integration over the flow area:

$$Q = \int V_n\,dA = \int_0^h U_o\left(\frac{2y}{h} - \frac{y^2}{h^2}\right) b\,dy = \mathbf{\frac{2}{3}U_obh} \quad \textit{Ans.}\ \text{(a)}$$

(b) Evaluate the above expression for the given data:

$$Q = 1.25\ \frac{gal}{\min} = 0.002785\ \frac{ft^3}{s} = \frac{2}{3}U_o bh = \frac{2}{3}U_o(1.0\ ft)\left(\frac{0.5}{12}\ ft\right),$$

$$solve\ for\quad U_o = \mathbf{0.10}\ \frac{\mathbf{ft}}{\mathbf{s}}\quad Ans.\ \text{(b)}$$

P3.27 Consider a highly pressurized air tank at conditions (p_o, ρ_o, T_o) and volume υ_o. In Chap. 9 we will learn that, if the tank is allowed to exhaust to the atmosphere through a well-designed converging nozzle of exit area *A*, the outgoing mass flow rate will be

$$\dot{m}\ =\ \frac{\alpha\, p_o\, A}{\sqrt{RT_o}}\quad ,\qquad \text{where}\quad \alpha\ \approx\ 0.685\quad \text{for air}$$

This rate persists as long as p_o is at least twice as large as the atmospheric pressure. Assuming constant T_o and an ideal gas, (a) derive a formula for the change of density ρ_o(t) within the tank. (*b*) Analyze the time Δt required for the density to decrease by 25%.

Solution: First convert the formula to reflect tank *density* instead of pressure:

$$\dot{m}\ =\ \frac{\alpha\, p_o\, A}{\sqrt{RT_o}}\ =\ \frac{\alpha\,(\rho_o RT_o)\,A}{\sqrt{RT_o}}\ =\ \alpha\,\rho_o\,A\sqrt{RT_o}$$

(a) Now apply a mass balance to a control volume surrounding the tank:

$$\frac{dm}{dt}\Big|_{system} = 0\ =\ \frac{d}{dt}(\rho_o\upsilon_o) + \dot{m}_{out}\ =\ \upsilon_o\frac{d\rho_o}{dt}\ +\ \alpha\rho_o A\sqrt{RT_o}$$

$$\text{Separate variables:}\quad \frac{d\rho_o}{\rho_o}\ =\ -\,\alpha A\sqrt{RT_o}\ dt$$

$$\text{Integrate from state 1 to state 2:}\quad \frac{\rho_{o2}}{\rho_{o1}}\ =\ \exp\!\left[\frac{-\,\alpha A\sqrt{RT_o}}{\upsilon_o}(t_2 - t_1)\right]\qquad Ans.(a)$$

(*b*) If the density drops by 25%, then we compute

$$\frac{\alpha A\sqrt{RT_o}}{\upsilon_o}(t_2 - t_1)\ =\ -\ln(0.75)\ =\ 0.288\ ;\quad \text{Thus}\quad \Delta t\ =\ \frac{0.288\,\upsilon_o}{\alpha A\sqrt{RT_o}}\qquad Ans.(b)$$

P3.28 Air, assumed to be a perfect gas from Table A.4, flows through a long, 2-cm-diameter insulated tube. At section 1, the pressure is 1.1 MPa and the temperature is 345 K. At section 2, 67 meters further downstream, the density is 1.34 kg/m^3, the temperature 298 K, and the Mach number is 0.90. For one-dimensional flow, calculate (*a*) the mass flow; (*b*) p_2; (c) V_2; and (*d*) the change in entropy between 1 and 2. (*e*) How do you explain the entropy change?

Solution: For air, $k = 1.40$ and $R = 287$ m^2/s^2-K, hence $c_p = kR/(k-1) = 1005$ m^2/s^2-K. (*a, c)* We have enough information at section 2 to calculate the velocity, hence the mass flow:

$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287)(298K)} = 346\frac{m}{s}, \text{ thus } V_2 = Ma_2\, a_2 = (0.9)(346) = \mathbf{311}\frac{m}{s} \quad Ans.(c)$$

$$\text{Then } \dot{m} = \rho_2 A_2 V_2 = (1.34\frac{kg}{m^3})[\frac{\pi}{4}(0.02\,m)^2](311\frac{m}{s}) = \mathbf{0.131}\frac{kg}{s} \quad Ans.(a)$$

(*b*) The pressure at section 2 follows from the perfect gas law:

$$p_2 = \rho_2 RT_2 = (1.34\frac{kg}{m^3})(287\frac{N-m}{kg\ K})(298\,K) = 115{,}000\frac{N}{m^2} = \mathbf{115{,}000}\ Pa \quad Ans.(b)$$

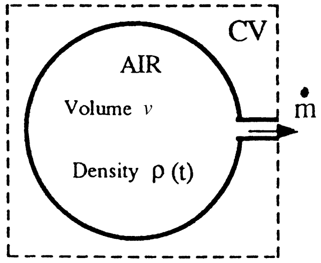
(*d*) For a perfect gas with constant specific heats, the entropy change is

$$s_2 - s_1 = c_p \ln(\frac{T_2}{T_1}) - R\ln(\frac{p_2}{p_1}) = (1005)\ln(\frac{298K}{345K}) - (287)\ln(\frac{115\,kPa}{1100\,kPa})$$

$$= -147 - (-648) = +\mathbf{501}\ \frac{J}{kg-K} \qquad Ans.(d)$$

(*e*) The entropy has *increased*, yet there is no heat transfer (insulated pipe). The answer is *irreversibility*. ***Friction*** in the long pipe has caused viscous dissipation in the fluid. NOTE: These numbers are not just made up. They represent a typical case of *compressible flow of air in a long pipe with friction*, to be studied in Chapter 9.

P3.29 In elementary compressible-flow theory (Chap. 9), compressed air will exhaust from a small hole in a tank at the mass flow rate $\dot{m} \approx C\rho$, where ρ is the air density in the tank and C is a constant. If ρ_o is the initial density in a tank of volume v, derive a formula for the density change $\rho(t)$ after the hole is opened. Apply your formula to the following case: a spherical tank of diameter 50 cm, with initial pressure 300 kPa and temperature 100°C, and a hole whose initial exhaust rate is 0.01 kg/s. Find the time required for the tank density to drop by 50 percent.



Solution: For a control volume enclosing the tank and the exit jet, we obtain

$$0 = \frac{d}{dt}\left(\int \rho dv\right) + \dot{m}_{out}, \quad \text{or:} \quad v\frac{d\rho}{dt} = -\dot{m}_{out} = -C\rho,$$

$$\text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\frac{C}{v}\int_0^t dt, \quad \text{or:} \quad \frac{\rho}{\rho_o} \approx \mathbf{exp}\left[-\frac{\mathbf{C}}{\boldsymbol{v}}\mathbf{t}\right] \quad \textit{Ans.}$$

Now apply this formula to the given data. If po = 300 kPa and To = 100°C = 373°K, then ρ_o = p/RT = (300,000)/[287(373)] ≈ 2.80 kg/m^3. This establishes the constant "C":

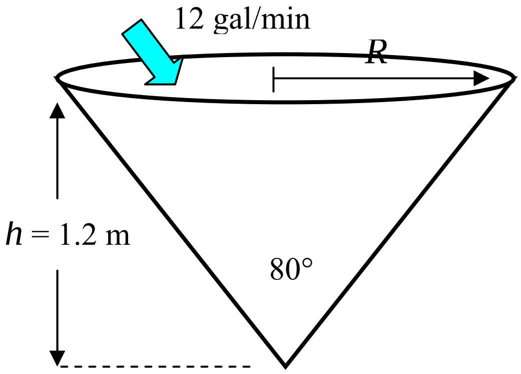
$$\dot{m}_o = C\rho_o = 0.01\ \frac{kg}{s} = C\left(2.80\ \frac{kg}{m^3}\right), \quad \text{or} \quad C \approx 0.00357\ \frac{m^3}{s} \text{ for this hole.}$$

The tank volume is $v = (\pi/6)D^3 = (\pi/6)(0.5\ m)^3 \approx 0.00654\ m^3$. Then we require

$$\rho/\rho_o = 0.5 = \exp\left[-\frac{0.00357}{0.00654}t\right] \quad \text{if } t \approx \mathbf{1.3\ s} \quad \textit{Ans.}$$

P3.30 A hollow conical container, standing point-down, is 1.2 m high and has a total included cone angle of 80°. It is being filled with water from a hose at 50 gallons per minute. How long will it take to fill the cone?

Solution: The control volume, of course, surrounds the cone with one inlet, no exits. We don't need any complicated integral mass relations, for the flow rate is known, as is the cone volume. The radius of the upper "base" of the cone is



$$R = h\tan(40^\circ) = (1.2\,m)(0.839) = 1.007\,m$$

The volume of the cone is

$$\upsilon = \frac{\pi}{3}R^2 h = \frac{\pi}{3}(1.007m)^2(1.2m) = 3.82\,m^3 \times 264.2\frac{gal}{m^3} = 1010\ gal$$

Clearly, then, the time to fill the cone is (1010 gal)/(50gal/min) = **20.2** minutes. *Ans.*

P3.31 A bellows may be modeled as a deforming wedge-shaped volume as in Fig. P3.31. The check valve on the left (pleated) end is closed during the stroke. If *b* is the bellows width into the paper, derive an expression for outlet mass flow $\dot{m}_o$ as a function of stroke $\theta(t)$.

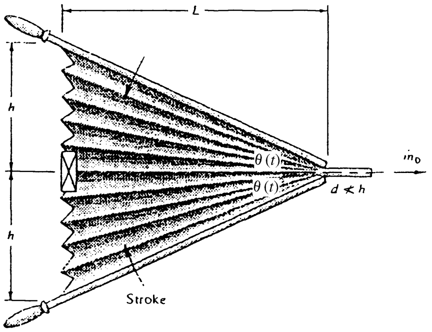


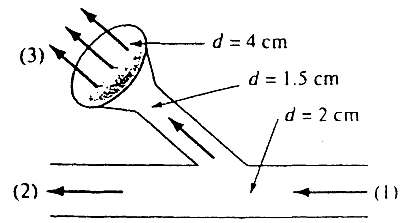
Fig. P3.31

Solution: For a control volume enclosing the bellows and the outlet flow, we obtain

$$\frac{d}{dt}(\rho\upsilon) + \dot{m}_{out} = 0, \quad \text{where } \upsilon = bhL = bL^2\tan\theta$$

$$\text{since L is constant, solve for} \quad \dot{m}_o = -\frac{d}{dt}(\rho bL^2\tan\theta) = \boldsymbol{-\rho bL^2\sec^2\theta\frac{d\theta}{dt}} \quad \textit{Ans.}$$

P3.32 Water at 20°C flows through the piping junction in the figure, entering section 1 at 20 gal/min. The average velocity at section 2 is 2.5 m/s. A portion of the flow is diverted through the showerhead, which contains 100 holes of 1-mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead jets.



Solution: A control volume around sections (1, 2, 3) yields

$$Q_1 = Q_2 + Q_3 = 20 \text{ gal/min} = 0.001262 \text{ m}^3/\text{s}.$$

Meanwhile, with $V_2 = 2.5$ m/s known, we can calculate Q_2 and then Q_3:

$$Q_2 = V_2 A_2 = (2.5 \text{ m})\frac{\pi}{4}(0.02 \text{ m})^2 = 0.000785 \ \frac{\text{m}^3}{\text{s}},$$

$$\textit{hence} \quad Q_3 = Q_1 - Q_2 = 0.001262 - 0.000785 = 0.000476 \ \frac{\text{m}^3}{\text{s}}$$

$$\textit{Each hole carries} \quad Q_3/100 = 0.00000476 \ \frac{\text{m}^3}{\text{s}} = \frac{\pi}{4}(0.001)^2 V_{jet},$$

$$\textit{solve} \quad V_{jet} = \mathbf{6.06} \ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}$$

P3.33 In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. P3.33 contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is $V_r = 8$ m/s, and the test-section entrance velocity is $V_1 = 35$ m/s. Assuming incompressible steady flow of air at 20°C, compute (a) V_o, (b) V_2, and (c) V_f, in m/s.

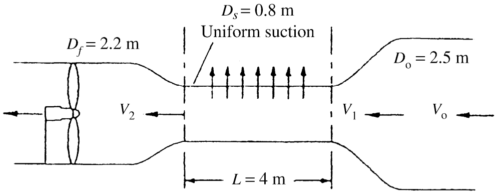


Fig. P3.33

Solution: The test section wall area is (π)(0.8 m)(4 m) = 10.053 m^2, hence the total number of holes is (1200)(10.053) = 12064 holes. The total suction flow leaving is

$$Q_{suction} = NQ_{hole} = (12064)(\pi/4)(0.005\text{ m})^2(8\text{ m/s}) \approx 1.895\text{ m}^3/\text{s}$$

$$\text{(a)}\quad \text{Find } V_o:\quad Q_o = Q_1 \quad \text{or} \quad V_o\frac{\pi}{4}(2.5)^2 = (35)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for}\quad V_o \approx \mathbf{3.58\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\quad Q_2 = Q_1 - Q_{suction} = (35)\frac{\pi}{4}(0.8)^2 - 1.895 = V_2\frac{\pi}{4}(0.8)^2,$$

$$\text{or:}\quad V_2 \approx \mathbf{31.2\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

$$\text{(c)}\quad \text{Find } V_f:\quad Q_f = Q_2 \quad \text{or} \quad V_f\frac{\pi}{4}(2.2)^2 = (31.2)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for}\quad V_f \approx \mathbf{4.13\ \frac{m}{s}} \quad \textit{Ans.}\text{ (c)}$$

P3.34 A rocket motor is operating steadily, as shown in Fig. P3.34. The products of combustion flowing out the exhaust nozzle approximate a perfect gas with a molecular weight of 28. For the given conditions calculate $V2$ in ft/s.

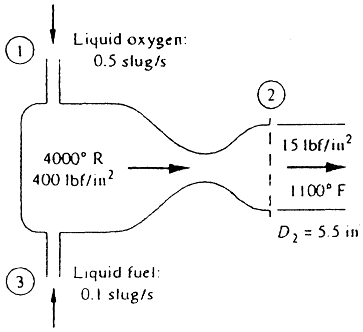


Fig. P3.34

Solution: Exit gas: Molecular weight = 28, thus $R_{gas} = 49700/28 = 1775$ ft²/(s²·°R). Then,

$$\rho_{\text{exit gas}} = \frac{p}{RT} = \frac{15(144)\text{ psf}}{(1775)(1100+460)} \approx 0.000780\text{ slug/ft}^2$$

For mass conservation, the exit mass flow must equal fuel + oxygen entering = 0.6 slug/s:

$$\dot{m}_{\text{exit}} = 0.6\,\frac{\text{slug}}{\text{s}} = \rho_e A_e V_e = (0.00078)\frac{\pi}{4}\left(\frac{5.5}{12}\right)^2 V_e, \quad \text{solve for } V_e \approx \mathbf{4660\ \frac{ft}{s}} \quad \textit{Ans.}$$

P3.35 In contrast to the liquid rocket in Fig. P3.34, the solid-propellant rocket in Fig. P3.35 is self-contained and has no entrance ducts. Using a control-volume analysis for the conditions shown in Fig. P3.35, compute the rate of mass loss of the propellant, assuming that the exit gas has a molecular weight of 28.

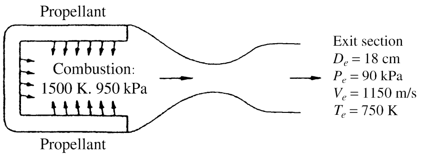


Fig. P3.35

Solution: With M = 28, R = 8313/28 = 297 m²/(s²·K), hence the exit gas density is

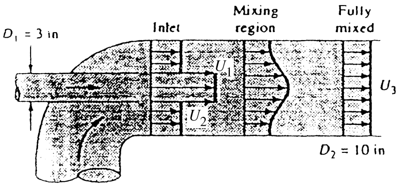
$$\rho_{\text{exit}} = \frac{p}{RT} = \frac{90{,}000\text{ Pa}}{(297)(750\text{ K})} = 0.404\text{ kg/m}^3$$

For a control volume enclosing the rocket engine and the outlet flow, we obtain

$$\frac{d}{dt}(m_{CV}) + \dot{m}_{\text{out}} = 0,$$

$$\text{or:}\quad \frac{d}{dt}(m_{\text{propellant}}) = -\dot{m}_{\text{exit}} = -\rho_e A_e V_e = -(0.404)(\pi/4)(0.18)^2(1150) \approx \mathbf{-11.8\ \frac{kg}{s}} \quad \textit{Ans.}$$

P3.36 The jet pump in Fig. P3.36 injects water at $U_1 = 40$ m/s through a 3-in pipe and entrains a secondary flow of water $U_2 = 3$ m/s in the annular region around the small pipe. The two flows become fully mixed down-stream, where U_3 is approximately constant. For steady incompressible flow, compute U_3 in m/s.



Solution: First modify the units: $D_1 = 3$ in $= 0.0762$ m, $D_2 = 10$ in $= 0.254$ m. For incompressible flow, the volume flows at inlet and exit must match:

$$Q_1 + Q_2 = Q_3, \quad \text{or:} \quad \frac{\pi}{4}(0.0762)^2(40) + \frac{\pi}{4}[(0.254)^2 - (0.0762)^2](3) = \frac{\pi}{4}(0.254)^2 U_3$$

Solve for $\mathbf{U_3 \approx 6.33\ m/s}$ *Ans.*

P3.37 If the rectangular tank full of water, in Fig. P3.37, has its right-hand wall lowered by an amount δ, as shown, water will flow out as it would over a weir or dam. In Prob. P1.14 we deduced that the outflow Q would be given by

$$Q = C b g^{1/2} \delta^{3/2}$$

where b is the tank width into the paper, g is the acceleration of gravity, and C is a dimensionless constant. Assume that the water surface is horizontal, not slightly curved as in the figure. Let the initial excess water level be δ_o. Derive a formula for the time required to reduce the excess water level to (*a*) $\delta_o/10$; and (*b*) to zero.

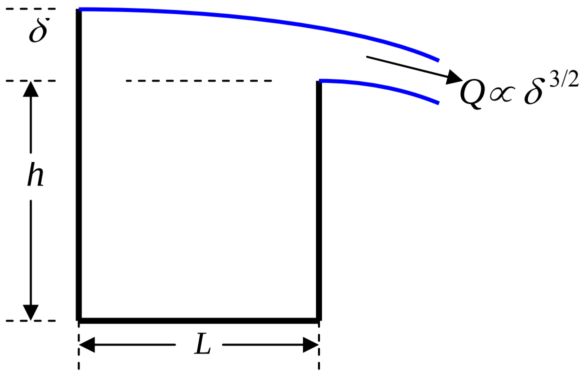


Fig. P3.37

Solution: The control volume encloses the tank and cuts through the outlet flow. From Eq. (3.20),

$$\frac{d}{dt}\left(\int \rho\, d\upsilon\right) + \rho Q_{out} = \frac{d}{dt}[\rho L b(h+\delta)] + \rho C b\, g^{1/2}\,\delta^{3/2}\ \text{, cancel } \rho \text{ and } b;$$

$$L\frac{d\delta}{dt} = -C\,g^{1/2}\,\delta^{3/2}\ \text{. Separate variables:}\ \int_{\delta_o}^{\delta}\frac{d\delta}{\delta^{3/2}} = -\int_0^t \frac{C\,g^{1/2}}{L}dt$$

where δ is the instantaneous excess water level. The integrated result for water level $\delta(t)$ is

$$\frac{1}{\delta^{1/2}} = \frac{1}{\delta_o^{1/2}} + \frac{C\,g^{1/2}}{2L}t$$

The two specific results requested are:

$$(a)\ \delta = \frac{\delta_o}{10}:\ t = \frac{4.32L}{C\,g^{1/2}\,\delta_o^{1/2}}\quad Ans.(a)\quad ;\quad (b)\ \delta = 0:\ t = \infty\quad Ans.(b)$$

It doesn't really take infinitely long to reach the final level, because surface tension comes into play, at the lip of the dam, as δ becomes very small.

P3.38 An incompressible fluid is squeezed between two disks by downward motion V₀ of the upper disk. Assuming 1-dimensional radial outflow, find the velocity V(r).

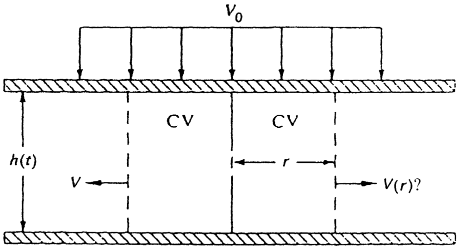


Fig. P3.38

Solution: Let the CV enclose the disks and have an upper surface moving down at speed V_o. There is no inflow. Thus

$$\frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \int_{CS} \rho V_{out}\, dA = 0 = \frac{d}{dt}(\rho\pi r^2 h) + \rho 2\pi r h\, V,$$

$$\text{or:}\quad r^2\frac{dh}{dt} + 2rhV = 0, \quad \text{but} \quad \frac{dh}{dt} = -V_o\text{(the disk velocity)}$$

As the disk spacing drops, $h(t) \approx h_o - V_o t$, the outlet velocity is $\mathbf{V = V_o\, r/(2h)}$. *Ans.*

P3.39 A wedge splits a sheet of 20°C water, as shown in Fig. P3.39. Both wedge and sheet are very long into the paper. If the force required to hold the wedge stationary is *F* = 126 N per meter of depth into the paper, what is the angle θ of the wedge?

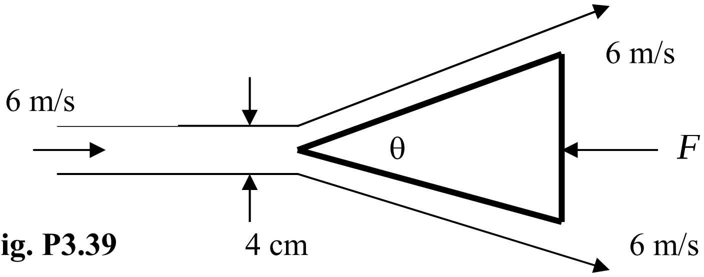


Fig. P3.39

Solution: For water take ρ = 998 kg/m³. First compute the mass flow per unit depth:

$$\dot{m}/b = \rho V h = (998\,kg/m^3)(6\,m/s)(0.04\,m) = 239.5\,kg/s-m$$

The mass flow (and velocity) are the same entering and leaving. Let the control volume surround the wedge. Then the *x*-momentum integral relation becomes

$$\Sigma F_x = -F = \dot{m}(u_{out} - u_{in}) = \dot{m}(V\cos\frac{\theta}{2} - V) = \dot{m}V(\cos\frac{\theta}{2} - 1)$$

$$or: \quad -124\,N/m = (239.5\,kg/s-m)(6\,m/s)(\cos\frac{\theta}{2} - 1)$$

$$\text{Solve} \quad \cos\frac{\theta}{2} = 0.9137\ ,\ \ \frac{\theta}{2} = 24^o\ ,\ \ \theta = \mathbf{48^o} \quad \textit{Ans.}$$

P3.40 The water jet in Fig. P3.40 strikes normal to a fixed plate. Neglect gravity and friction, and compute the force *F* in newtons required to hold the plate fixed.

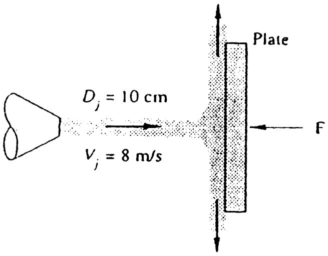


Fig. P3.40

Solution: For a CV enclosing the plate and the impinging jet, we obtain:

$$\sum F_x = -F = \dot{m}_{up}u_{up} + \dot{m}_{down}u_{down} - \dot{m}_j u_j$$

$$= -\dot{m}_j u_j, \quad \dot{m}_j = \rho A_j V_j$$

$$\text{Thus} \quad F = \rho A_j V_j^2 = (998)\pi(0.05)^2(8)^2 \approx \mathbf{500\ N} \leftarrow \quad \textit{Ans.}$$

P3.41 In Fig. P3.41 the vane turns the water jet completely around. Find the maximum jet velocity Vo for a force Fo.

Solution: For a CV enclosing the vane and the inlet and outlet jets,

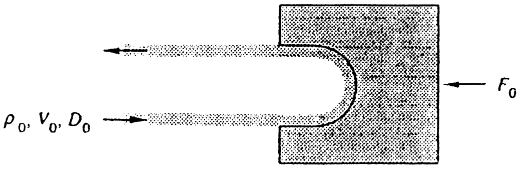


Fig. P3.41

$$\sum F_x = -F_o = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}_{jet}(-V_o) - \dot{m}_{jet}(+V_o)$$

$$\text{or:}\quad F_o = 2\rho_o A_o V_o^2, \quad \text{solve for } V_o = \sqrt{\frac{\mathbf{F_o}}{\mathbf{2\rho_o(\pi/4)D_o^2}}} \quad \textit{Ans.}$$

P3.42 A liquid of density ρ flows through the sudden contraction in Fig. P3.42 and exits to the atmosphere. Assume uniform conditions ($p1$, $V1$, $D1$) at section 1 and ($p2$, $V2$, $D2$) at section 2. Find an expression for the force F exerted by the fluid on the contraction.

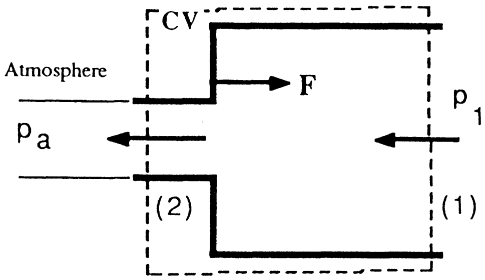


Fig. P3.42

Solution: Since the flow exits directly to the atmosphere, the exit pressure equals atmospheric: $p2 = pa$. Let the CV enclose sections 1 and 2, as shown. Use our trick (page 129 of the text) of subtracting pa everywhere, so that the only non-zero pressure on the CS is at section 1, $p = p1 - pa$. Then write the linear momentum relation with *x* to the right:

$$\sum F_x = F - (p_1 - p_a)A_1 = \dot{m}_2 u_2 - \dot{m}_1 u_1, \quad \text{where } \dot{m}_2 = \dot{m}_1 = \rho_1 A_1 V_1$$

$$\text{But } u_2 = -V_2 \quad \text{and} \quad u_1 = -V_1. \quad \text{Solve for } F_{\underline{on}\text{ fluid}} = (p_1 - p_a)A_1 + \rho_1 A_1 V_1(-V_2 + V_1)$$

Meanwhile, from continuity, we can relate the two velocities:

$$Q_1 = Q_2, \quad \text{or} \quad (\pi/4)D_1^2 V_1 = (\pi/4)D_2^2 V_2, \quad \text{or:} \quad V_2 = V_1(D_1^2/D_2^2)$$

Finally, the force of the fluid on the wall is equal and opposite to $F_{\underline{\text{on fluid}}}$, to the *left*:

$$F_{\text{fluid on wall}} = (p_1 - p_a)A_1 - \rho_1 A_1 V_1^2\left[\left(D_1^2/D_2^2\right) - 1\right], \quad A_1 = \frac{\pi}{4}D_1^2 \quad \textit{Ans.}$$

The pressure term is larger than the momentum term, thus F > 0 and acts to the left.

P3.43 Water at 20°C flows through a 5-cm-diameter pipe which has a 180° vertical bend, as in Fig. P3.43. The total length of pipe between flanges 1 and 2 is 75 cm. When the weight flow rate is 230 N/s, $p1$ = 165 kPa, and $p2$ = 134 kPa. Neglecting pipe weight, determine the total force which the flanges must withstand for this flow.

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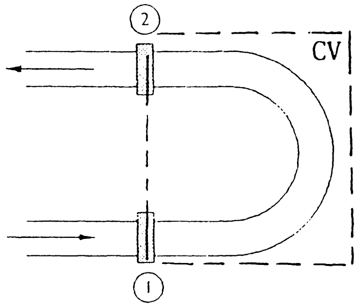


Fig. P3.43

Solution: Let the CV cut through the flanges and surround the pipe bend. The mass flow rate is (230 N/s)/(9.81 m/s^2) = 23.45 kg/s. The volume flow rate is Q = 230/9790 = 0.0235 m^3/s. Then the pipe inlet and exit velocities are the same magnitude:

$$V_1 = V_2 = V = Q/A = \frac{0.0235\ \text{m}^3/\text{s}}{(\pi/4)(0.05\ \text{m})^2} \approx 12.0\ \frac{\text{m}}{\text{s}}$$

Subtract p_a everywhere, so only p_1 and p_2 are non-zero. The horizontal force balance is:

$$\sum F_x = F_{x,\text{flange}} + (p_1 - p_a)A_1 + (p_2 - p_a)A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1$$

$$= F_{x,\text{fl}} + (64000)\frac{\pi}{4}(0.05)^2 + (33000)\frac{\pi}{4}(0.05)^2 = (23.45)(-12.0 - 12.0\ \text{m/s})$$

$$\text{or:}\quad F_{x,\text{flange}} = -126 - 65 - 561 \approx \mathbf{-750\ N} \quad \textit{Ans.}$$

The total *x*-directed force on the flanges acts to the left. The vertical force balance is

$$\sum F_y = F_{y,\text{flange}} = W_{\text{pipe}} + W_{\text{fluid}} = 0 + (9790)\frac{\pi}{4}(0.05)^2(0.75) \approx \mathbf{14\ N} \quad \textit{Ans.}$$

Clearly the fluid weight is pretty small. The largest force is due to the 180° turn.

P3.44 Consider uniform flow past a cylinder with a V-shaped *wake*, as shown. Pressures at (1) and (2) are equal. Let *b* be the width into the paper. Find a formula for the force *F* on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 Lb)$.

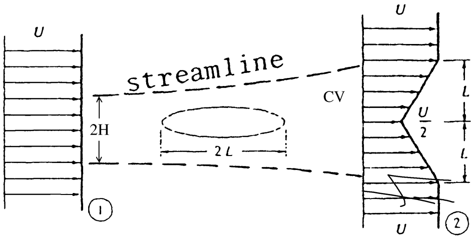


Fig. P3.44

Solution: The proper CV is the entrance (1) and exit (2) plus *streamlines* above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

$$0 = \int_2 \rho u\, dA - \int_1 \rho u\, dA = 2\int_0^L \rho \frac{U}{2}\left(1+\frac{y}{L}\right) b\, dy - 2\rho UbH,$$

where 2H is the inlet height. Solve for H = 3L/4.

Now the linear momentum relation is used. Note that the drag force F is to the right (force of the fluid on the body) thus the force F *of the body on fluid is to the left.* We obtain,

$$\sum F_x = 0 = \int_2 u\rho u\, dA - \int_1 u\rho u\, dA = 2\int_0^L \frac{U}{2}\left(1+\frac{y}{L}\right)\rho\frac{U}{2}\left(1+\frac{y}{L}\right) b\, dy - 2H\rho U^2 b = -F_{drag}$$

$$\text{Use}\quad H = \frac{3L}{4},\quad \text{then}\ F_{drag} = \frac{3}{2}\rho U^2 Lb - \frac{7}{6}\rho U^2 Lb \approx \boldsymbol{\frac{1}{3}\rho U^2 Lb} \quad \textit{Ans.}$$

The dimensionless force, or drag coefficient $F/(\rho U^2 Lb)$, equals $\mathbf{C_D = 1/3}$. *Ans.*

P3.45 A 12-cm-diameter pipe, containing water flowing at 200 N/s, is capped by an orifice plate, as in Fig. P3.45. The exit jet is 25 mm in diameter. The pressure in the pipe at section 1 is 800 kPa-gage. Calculate the force F required to hold the orifice plate.

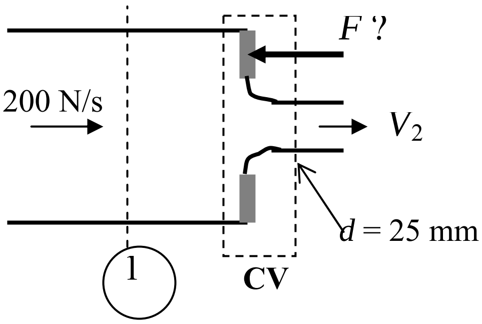


Fig. P3.45

Solution: For water take $\rho = 998$ kg/m^3. This is a straightforward x-momentum problem. First evaluate the mass flow and the two velocities:

$$\dot{m} = \frac{\dot{w}}{g} = \frac{200\,N/s}{9.81\,m/s^2} = 20.4\frac{kg}{s}\ ;\ V_1 = \frac{\dot{m}}{\rho A_1} = \frac{20.4\,kg/s}{(998kg/m^3)(\pi/4)(0.12m)^2} = 1.81\frac{m}{s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{20.4\,kg/s}{(998kg/m^3)(\pi/4)(0.025m)^2} = 41.6\frac{m}{s}$$

Now apply the x-momentum relation for a control volume surrounding the plate:

$$\Sigma F_x = -F + p_{1,gage}A_1 = \dot{m}(V_2 - V_1)\ ,\quad \text{or:}$$

$$F = (800000Pa)\frac{\pi}{4}(0.12m)^2 - (20.4\frac{kg}{s})(41.6 - 1.81\frac{m}{s}) = 9048 - 812 = \mathbf{8240\,N}\quad \textit{Ans.}$$

P3.46 When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes αQ at (2) and (1 – α)Q at (3), as shown. Find α, assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

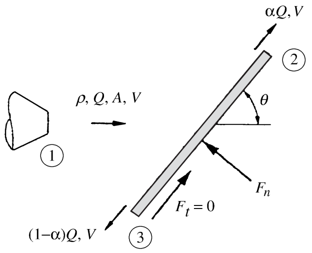


Fig. P3.46

Solution: Let the CV enclose all three jets and the surface of the plate. Analyze the force and momentum balance *tangential* to the plate:

$$\sum F_t = F_t = 0 = \dot{m}_2 V + \dot{m}_3(-V) - \dot{m}_1 V\cos\theta$$

$$= \alpha\dot{m}V - (1-\alpha)\dot{m}V - \dot{m}V\cos\theta = 0, \quad \text{solve for } \alpha = \mathbf{\frac{1}{2}(1+\cos\theta)} \quad \textit{Ans.}$$

The jet mass flow cancels out. Jet (3) has a fractional flow $(1-\alpha) = (1/2)(1-\cos\theta)$.

P3.47 A liquid jet Vj of diameter Dj strikes a fixed cone and deflects back as a conical sheet at the same velocity. Find the cone angle θ for which the restraining force $F = (3/2)\rho A_j V_j^2$.

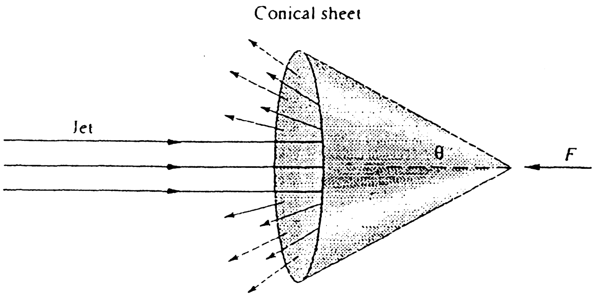


Fig. P3.47

Solution: Let the CV enclose the cone, the jet, and the sheet. Then,

$$\sum F_x = -F = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}(-V_j\cos\theta) - \dot{m}V_j, \quad \text{where } \dot{m} = \rho A_j V_j$$

$$\text{Solve for} \quad F = \rho A_j V_j^2(1+\cos\theta) = \frac{3}{2}\rho A_j V_j^2 \quad \text{if } \cos\theta = \frac{1}{2} \quad \text{or} \quad \theta = \mathbf{60°} \quad \textit{Ans.}$$

P3.48 The small boat is driven at steady speed Vo by compressed air issuing from a 3-cm-diameter hole at Ve = 343 m/s and pe = 1 atm, Te = 30°C. Neglect air drag. The hull drag is kV_o^2, where $k = 19\ \mathrm{N\cdot s^2/m^2}$. Estimate the boat speed Vo.

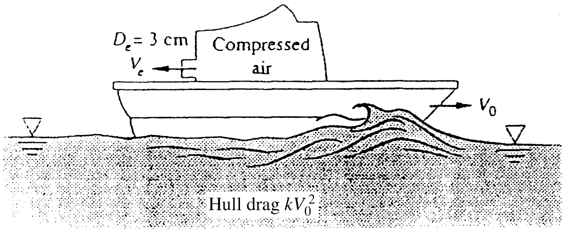


Fig. P3.48

Solution: For a CV enclosing the boat and *moving to the right at boat speed* V_o, the air appears to leave the left side at speed ($V_o + V_e$). The air density is $p_e/RT_e \approx 1.165\ kg/m^3$. The only mass flow across the CS is the air moving to the left. The force balance is

$$\sum F_x = -\text{Drag} = -kV_o^2 = \dot{m}_{out}u_{out} = [\rho_e A_e(V_o + V_e)](-V_o - V_e),$$

$$\text{or:}\quad \rho_e A_e(V_o + V_e)^2 = kV_o^2,\quad (1.165)(\pi/4)(0.03)^2(V_o + 343)^2 = 19V_o^2$$

work out the numbers: $(V_o + 343) = V_o\sqrt{(23060)}$, solve for $\mathbf{V_o = 2.27\ m/s}$ *Ans.*

P3.49 The horizontal nozzle in Fig. P3.49 has D_1 = 12 in, D_2 = 6 in, with p_1 = 38 psia and V_2 = 56 ft/s. For water at 20°C, find the force provided by the flange bolts to hold the nozzle fixed.

Solution: For an open jet, $p_2 = p_a$ = 15 psia. Subtract p_a everywhere so the only nonzero pressure is p_1 = 38 − 15 = 23 psig.

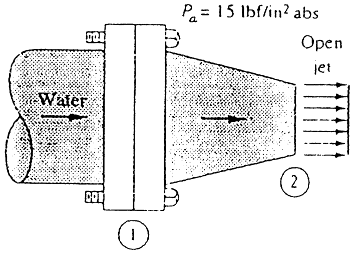
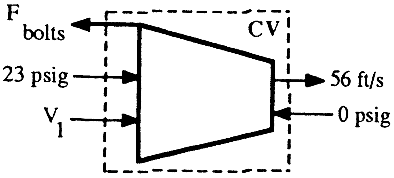


Fig. P3.49

The mass balance yields the inlet velocity:

$$V_1\frac{\pi}{4}(12)^2 = (56)\frac{\pi}{4}(6)^2, \quad V_1 = 14\ \frac{\text{ft}}{\text{s}}$$



The density of water is 1.94 slugs per cubic foot. Then the horizontal force balance is

$$\sum F_x = -F_{bolts} + (23\text{ psig})\frac{\pi}{4}(12\text{ in})^2 = \dot{m}_2u_2 - \dot{m}_1u_1 = \dot{m}(V_2 - V_1)$$

$$\text{Compute}\quad F_{bolts} = 2601 - (1.94)\frac{\pi}{4}(1\text{ ft})^2\left(14\ \frac{\text{ft}}{\text{s}}\right)\left(56 - 14\ \frac{\text{ft}}{\text{s}}\right) \approx \mathbf{1700\ lbf} \quad \textit{Ans.}$$

P3.50 The jet engine in Fig. P3.50 admits air at 20°C and 1 atm at (1), where A1 = 0.5 m^2 and V1 = 250 m/s. The fuel-air ratio is 1:30. The air leaves section (2) at 1 atm, V2 = 900 m/s, and A2 = 0.4 m^2. Compute the test stand support reaction Rx needed.

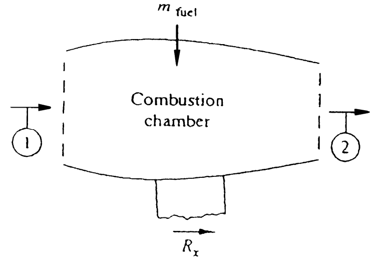


Fig. P3.50

Solution: ρ1 = p/RT = 101350/[287(293)] = 1.205 kg/m^3. For a CV enclosing the engine,

$$\dot{m}_1 = \rho_1A_1V_1 = (1.205)(0.5)(250) = 151\text{ kg/s}, \quad \dot{m}_2 = 151\left(1 + \frac{1}{30}\right) = 156\text{ kg/s}$$

$$\sum F_x = R_x = \dot{m}_2u_2 - \dot{m}_1u_1 - \dot{m}_{fuel}u_{fuel} = 156(900) - 151(250) - 0 \approx \mathbf{102{,}000\ N} \quad \textit{Ans.}$$

P3.51 A liquid jet of velocity Vj and area Aj strikes a single 180° bucket on a turbine wheel rotating at angular velocity Ω. Find an expression for the power *P* delivered. At what Ω is the power a maximum? How does the analysis differ if there are many buckets, so the jet continually strikes at least one?

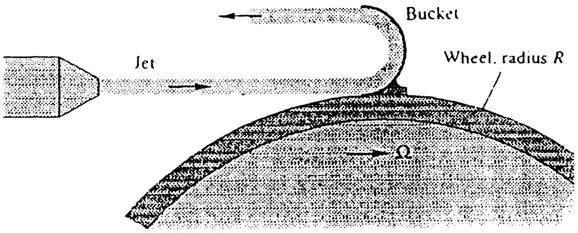
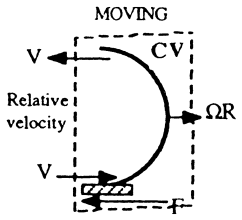


Fig. P3.51

Solution: Let the CV enclose the bucket and jet and let it move to the right at bucket velocity V = ΩR, so that the jet enters the CV at relative speed (Vj – ΩR). Then,

$$\sum F_x = -F_{bucket} = \dot{m}u_{out} - \dot{m}u_{in}$$
$$= \dot{m}[-(V_j - \Omega R)] - \dot{m}[V_j - \Omega R]$$



$$\text{or:}\quad F_{bucket} = 2\dot{m}(V_j - \Omega R) = 2\rho A_j(V_j - \Omega R)^2,$$

$$\text{and the power is}\quad \mathbf{P} = \Omega R F_{bucket} = \mathbf{2\rho A_j \Omega R(V_j - \Omega R)^2} \quad \textit{Ans.}$$

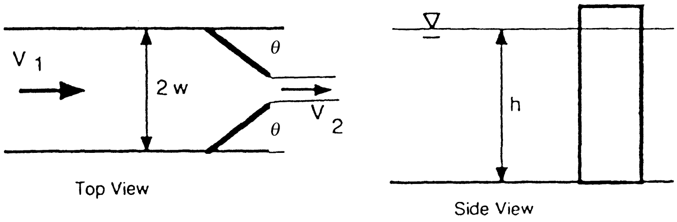
Maximum power is found by differentiating this expression:

$$\frac{dP}{d\Omega} = 0 \quad \text{if } \mathbf{\Omega R = \frac{V_j}{3}} \quad \textit{Ans.} \quad \left(\text{whence } P_{max} = \frac{8}{27}\rho A_j V_j^3\right)$$

If there were many buckets, then the *full* jet mass flow would be available for work:

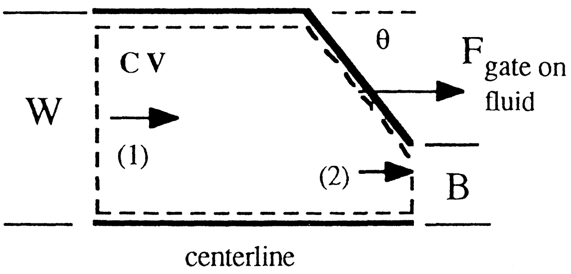
$$\dot{m}_{available} = \rho A_j V_j,\quad P = 2\rho A_j V_j \Omega R(V_j - \Omega R),\quad \mathbf{P_{max} = \frac{1}{2}\rho A_j V_j^3 \quad at \quad \Omega R = \frac{V_j}{2}} \quad \textit{Ans.}$$

P3.52 The vertical gate in a water channel is partially open, as in Fig. P3.52. Assuming no change in water level and a hydrostatic pressure distribution, derive an expression for the streamwise force *Fx* on one-half of the gate as a function of (ρ, *h*, *w*, θ, *V*1). Apply your result to the case of water at 20°C, *V*1 = 0.8 m/s, *h* = 2 m, *w* = 1.5 m, and θ = 50°.



Solution: Let the CV enclose sections (1) and (2), the centerline, and the inside of the gate, as shown. The volume flows are

$$V_1 Wh = V_2 Bh, \quad \text{or:} \quad V_2 = V_1 \frac{W}{B} = V_1 \frac{1}{1-\sin\theta}$$



since B = W − W sinθ. The problem is unrealistically idealized by letting the water depth remain *constant*, whereas actually the depth would decrease at section 2. Thus we have no net hydrostatic pressure force on the CV in this model! The force balance reduces to

$$\sum F_x = F_{\text{gate on fluid}} = \dot{m}V_2 - \dot{m}V_1, \quad \text{where } \dot{m} = \rho W h V_1 \quad \text{and} \quad V_2 = V_1/(1-\sin\theta)$$

$$\text{Solve for} \quad F_{\underline{\text{fluid on gate}}} = -\boldsymbol{\rho W h V_1^2}\left[\frac{\mathbf{1}}{\mathbf{(1-\sin\theta)}} - \mathbf{1}\right] \quad \textbf{(\textit{to the left})} \quad \textit{Ans.}$$

This is unrealistic—the pressure force would turn this gate force around to the right. For the particular data given, W = 1.5 m, θ = 50°, B = W(1 − sinθ) = 0.351 m, V_1 = 0.8 m/s, thus $V_2 = V_1/(1 - \sin 50°)$ = 3.42 m/s, ρ = 998 kg/m^3, h = 2 m. Thus compute

$$F_{\text{fluid on gate}} = (998)(2)(1.5)(0.8)^2\left[\frac{1}{1-\sin 50°} - 1\right] \approx \mathbf{6300\ N} \leftarrow \quad \textit{Ans.}$$

P3.53 Consider incompressible flow in the entrance of a circular tube, as in Fig. P3.53. The inlet flow is uniform, $u_1 = U_o$. The flow at section 2 is developed pipe flow. Find the wall drag force *F* as a function of (p_1, p_2, ρ, U_o, *R*) if the flow at section 2 is

(a) Laminar: $u_2 = u_{\max}\left(1 - \dfrac{r^2}{R^2}\right)$

(b) Turbulent: $u_2 \approx u_{\max}\left(1 - \dfrac{r}{R}\right)^{1/7}$

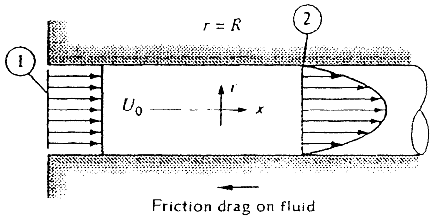


Fig. P3.53

Solution: The CV encloses the inlet and outlet and is just inside the walls of the tube. We don't need to establish a relation between umax and Uo by integration, because the results for these two profiles are given in the text. Note that Uo = uav at section (2). Now use these results as needed for the balance of forces:

$$\sum F_x = (p_1 - p_2)\pi R^2 - F_{drag} = \int_0^R u_2(\rho u_2\, 2\pi r\, dr) - U_o(\rho\pi R^2 U_o) = \rho\pi R^2 U_o^2(\beta_2 - 1)$$

We simply insert the appropriate momentum-flux factors β from p. 136 of the text:

(a) Laminar: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - (1/3)\rho\pi R^2 U_o^2}$ *Ans.* (a)

(b) Turbulent, $\beta_2 \approx 1.020$: $\mathbf{F_{drag} = (p_1 - p_2)\pi R^2 - 0.02\rho\pi R^2 U_o^2}$ *Ans.* (b)

P3.54 For the pipe-flow reducing section of Fig. P3.54, D1 = 8 cm, D2 = 5 cm, and p2 = 1 atm. All fluids are at 20°C. If V1 = 5 m/s and the manometer reading is *h* = 58 cm, estimate the total horizontal force resisted by the flange bolts.

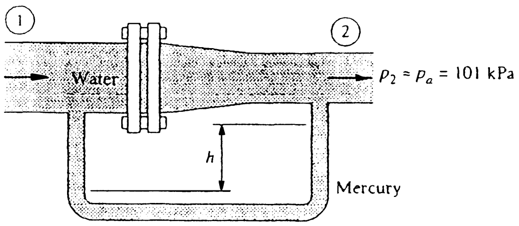


Fig. P3.54

Solution: Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58 \text{ m}) \approx 71300 \text{ Pa (gage)}$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5 \text{ m/s})(\pi/4)(0.08 \text{ m})^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8 \text{ m/s}$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx \mathbf{163\ N} \quad \textit{Ans.}$$

P3.55 In Fig. P3.55 the jet strikes a vane which moves to the right at constant velocity *Vc* on a frictionless cart. Compute (a) the force *Fx* required to restrain the cart and (b) the power *P* delivered to the cart. Also find the cart velocity for which (c) the force *Fx* is a maximum and (d) the power *P* is a maximum.

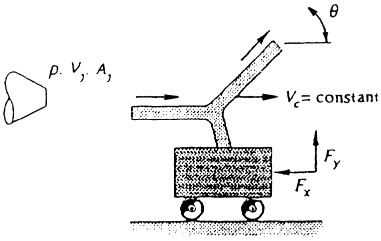


Fig. P3.55

Solution: Let the CV surround the vane and cart and move to the right at cart speed. The jet strikes the vane at *relative* speed Vj − Vc. The cart does not accelerate, so the horizontal force balance is

$$\sum F_x = -F_x = [\rho A_j(V_j - V_c)](V_j - V_c)\cos\theta - \rho A_j(V_j - V_c)^2$$

$$\text{or:}\quad F_x = \boldsymbol{\rho A_j(V_j - V_c)^2(1-\cos\theta)} \quad \textit{Ans.}\ \text{(a)}$$

The power delivered is $P = V_cF_x = \boldsymbol{\rho A_j V_c(V_j - V_c)^2(1-\cos\theta)}$ *Ans.* (b)

The maximum force occurs when the cart is fixed, or: $V_c = \mathbf{0}$ *Ans.* (c)

The maximum power occurs when $dP/dV_c = 0$, or: $V_c = \mathbf{V_j/3}$ *Ans.* (d)

P3.56 Water at 20°C flows steadily through the box in Fig. P3.56, entering station (1) at 2 m/s. Calculate the (a) horizontal; and (b) vertical forces required to hold the box stationary against the flow momentum.

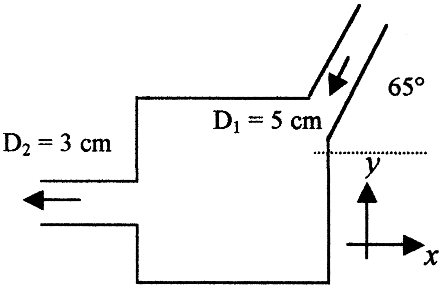


Fig. P3.56

Solution: (a) Summing horizontal forces,

$$\sum F_x = R_x = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in}$$

$$R_x = (998)\left[\left(\frac{\pi}{4}\right)(0.03^2)(5.56)\right](-5.56) - (998)\left[\left(\frac{\pi}{4}\right)(0.05^2)(2)\right](-2)(\cos 65^\circ)$$

$$= \mathbf{-18.46}\ \boldsymbol{N} \quad \textit{Ans.}$$

$$Rx = \mathbf{18.5}\ \boldsymbol{N} \quad \textit{to the left}$$

$$\sum F_y = R_y = -\dot{m}_{in}u_{in} = -(998)\left(\frac{\pi}{4}\right)(0.05^2)(2)(-2\ \sin 65^\circ) = \mathbf{7.1}\ \boldsymbol{N} \quad \textit{up}$$

P3.57 Water flows through the duct in Fig. P3.57, which is 50 cm wide and 1 m deep into the paper. Gate BC completely closes the duct when $\beta = 90°$. Assuming one-dimensional flow, for what angle β will the force of the exit jet on the plate be 3 kN?

Solution: The steady flow equation applied to the duct, $Q_1 = Q_2$, gives the jet velocity as $V_2 = V_1(1 - \sin\beta)$. Then for a force summation for a control volume around the jet's impingement area,

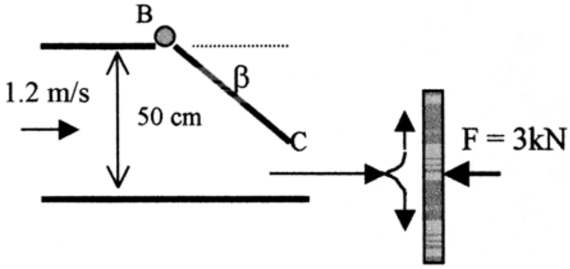


Fig. P3.57

$$\sum F_x = F = \dot{m}_j V_j = \rho(h_1 - h_1 \sin\beta)(D)\left[\frac{1}{1-\sin\beta}\right]^2 (V_1^2)$$

$$\beta = \sin^{-1}\left[1 - \frac{\rho h_1 D V_1^2}{F}\right] = \sin^{-1}\left[1 - \frac{(998)(0.5)(1)(1.2)^2}{3000}\right] = \mathbf{49.5°} \quad \textit{Ans.}$$

P3.58 The water tank in Fig. P3.58 stands on a frictionless cart and feeds a jet of diameter 4 cm and velocity 8 m/s, which is deflected 60° by a vane. Compute the tension in the supporting cable.

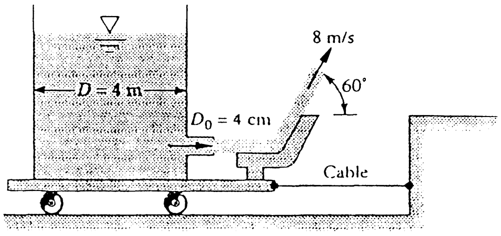


Fig. P3.58

Solution: The CV should surround the tank and wheels and cut through the cable and the exit water jet. Then the horizontal force balance is

$$\sum F_x = T_{cable} = \dot{m}_{out} u_{out} = (\rho A V_j) V_j \cos\theta = 998\left(\frac{\pi}{4}\right)(0.04)^2(8)^2 \cos 60° = \mathbf{40\ N} \quad \textit{Ans.}$$

P3.59 A pipe flow expands from (1) to (2), causing eddies as shown. Using the given CV and assuming p = p1 on the corner annular ring, show that the downstream pressure is given by, neglecting wall friction,

$$p_2 = p_1 + \rho V_1^2\left(\frac{A_1}{A_2}\right)\left(1-\frac{A_1}{A_2}\right)$$

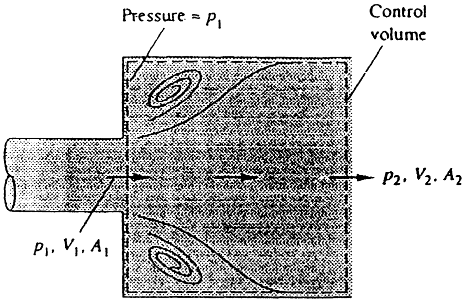


Fig. P3.59

Solution: From mass conservation, V1A1 = V2A2. The balance of x-forces gives

$$\sum F_x = p_1A_1 + p_{wall}(A_2 - A_1) - p_2A_2 = \dot{m}(V_2 - V_1), \quad \text{where } \dot{m} = \rho A_1V_1, \quad V_2 = V_1A_1/A_2$$

If $p_{wall} = p_1$ as given, this reduces to $\mathbf{p_2 = p_1 + \rho\frac{A_1}{A_2}V_1^2\left(1-\frac{A_1}{A_2}\right)}$ *Ans.*

P3.60 Water at 20°C flows through the elbow in Fig. P3.60 and exits to the atmosphere. The pipe diameter is D_1 = 10 cm, while D_2 = 3 cm. At a weight flow rate of 150 N/s, the pressure p_1 = 2.3 atm (gage). Neglect-ing the weight of water and elbow, estimate the force on the flange bolts at section 1.

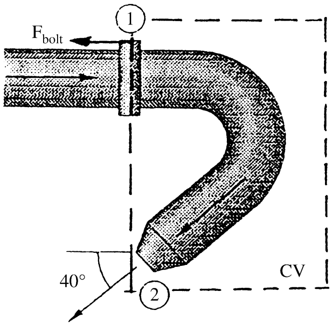


Fig. P3.60

Solution: First, from the weight flow, compute Q = (150 N/s)/(9790 N/m^3) = 0.0153 m^3/s. Then the velocities at (1) and (2) follow from the known areas:

$$V_1 = \frac{Q}{A_1} = \frac{0.0153}{(\pi/4)(0.1)^2} = 1.95\ \frac{m}{s}; \qquad V_2 = \frac{Q}{A_2} = \frac{0.0153}{(\pi/4)(0.03)^2} = 21.7\ \frac{m}{s}$$

The mass flow is $\rho A_1 V_1 = (998)(\pi/4)(0.1)^2(1.95) \approx 15.25$ kg/s. Then the balance of forces in the *x*-direction is:

$$\sum F_x = -F_{bolts} + p_1 A_1 = \dot{m}u_2 - \dot{m}u_1 = \dot{m}(-V_2\cos 40° - V_1)$$

$$\text{solve for}\quad F_{bolts} = (2.3 \times 101350)\frac{\pi}{4}(0.1)^2 + 15.25(21.7\cos 40° + 1.95) \approx \mathbf{2100\ N} \quad \textit{Ans.}$$

P3.61 A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If $\theta = 30°$, estimate the horizontal force F needed to hold the tank stationary.

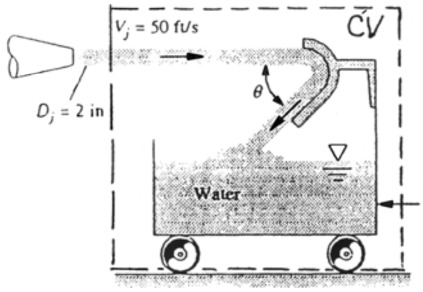


Fig. P3.61

Solution: The CV surrounds the tank and wheels and cuts through the jet, as shown. We should *assume that the splashing into the tank does not increase the x-momentum of the water in the tank*. Then we can write the CV horizontal force relation:

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\, d\upsilon\right)_{tank} - \dot{m}_{in}u_{in} = 0 - \dot{m}V_{jet} \ \text{ independent of } \theta$$

$$\text{Thus}\quad F = \rho A_j V_j^2 = \left(1.94\ \frac{slug}{ft^3}\right)\frac{\pi}{4}\left(\frac{2}{12}\ ft\right)^2\left(50\ \frac{ft}{s}\right)^2 \approx \mathbf{106\ lbf} \quad \textit{Ans.}$$

P3.62 Water at 20°C exits to the standard sea-level atmosphere through the split nozzle in Fig. P3.62. Duct areas are $A_1 = 0.02\ m^2$ and $A_2 = A_3 = 0.008\ m^2$. If $p_1 = 135$ kPa (absolute) and the flow rate is $Q_2 = Q_3 = 275\ m^3/h$, compute the force on the flange bolts at section 1.

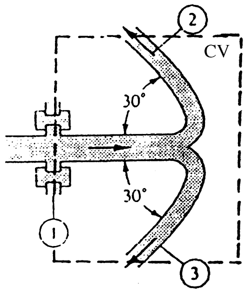


Fig. P3.62

Solution: With the known flow rates, we can compute the various velocities:

$$V_2 = V_3 = \frac{275/3600\ \mathrm{m^3/s}}{0.008\ \mathrm{m^2}} = 9.55\ \frac{\mathrm{m}}{\mathrm{s}};\quad V_1 = \frac{550/3600}{0.02} = 7.64\ \frac{\mathrm{m}}{\mathrm{s}}$$

The CV encloses the split nozzle and cuts through the flange. The balance of forces is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \rho Q_2(-V_2\cos 30°) + \rho Q_3(-V_3\cos 30°) - \rho Q_1(+V_1),$$

$$\text{or:}\quad F_{bolts} = 2(998)\left(\frac{275}{3600}\right)(9.55\cos 30°) + 998\left(\frac{550}{3600}\right)(7.64) + (135000 - 101350)(0.02)$$

$$= 1261 + 1165 + 673 \approx \mathbf{3100\ N} \quad \textit{Ans.}$$

P3.63 A steady two-dimensional water jet, 4 cm thick with a weight flow rate of 1960 N/s, strikes an angled barrier as in Fig. P3.63. Pressure and water velocity are constant everywhere. Thirty percent of the jet passes through the slot. The rest splits symmetrically along the barrier.

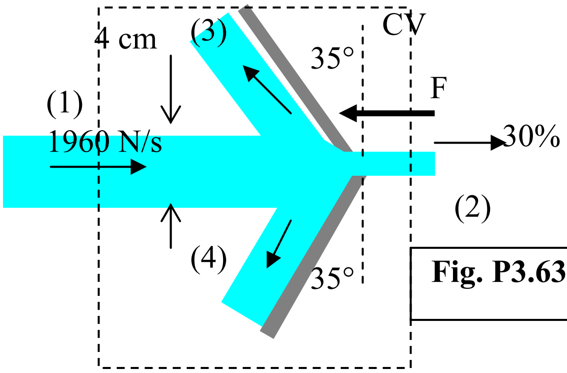


Fig. P3.63

Calculate the horizontal force *F* needed, per unit thickness into the paper, to hold the barrier stationary.

Solution: For water take ρ = 998 kg/m. The control volume (see figure) cuts through all four jets, which are numbered. The velocity of all jets follows from the weight flow at (1):

$$V_{1,2,3,4} = V_1 = \frac{\dot{w}_1}{\rho g A_1} = \frac{1960\,N/s}{(9.81m/s^2)(998kg/m^3)(0.04m)(1m)} = 5.0\frac{m}{s}$$

$$\dot{m}_1 = \frac{\dot{w}_1}{g} = \frac{1960N/s-m}{9.81N/s^2} = 200\frac{kg}{s-m};\ \dot{m}_2 = 0.3\dot{m}_1 = 60\frac{kg}{s-m};\ \dot{m}_3 = \dot{m}_4 = 70\frac{kg}{s-m}$$

Then the *x*-momentum relation for this control volume yields

$$\Sigma F_x = -F = \dot{m}_2 u_2 + \dot{m}_3 u_3 + \dot{m}_4 u_4 - \dot{m}_1 u_1 =$$

$$-F = (60)(5.0) + (70)(-5.0\cos 55^\circ) + (70)(-5.0\cos 55^\circ) - 200(5.0)\text{, or:}$$

$$F = 1000 + 201 + 201 - 300 \approx \mathbf{1100\ N}\ \text{per meter of width} \quad \textit{Ans.}$$

P3.64 The 6-cm-diameter 20°C water jet in Fig. P3.64 strikes a plate containing a hole of 4-cm diameter. Part of the jet passes through the hole, and part is deflected. Determine the horizontal force required to hold the plate.

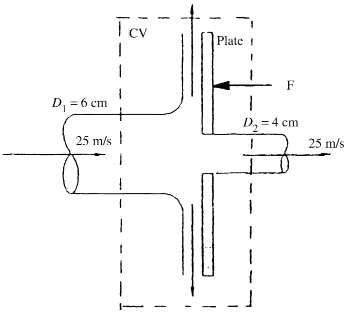


Fig. P3.64

Solution: First determine the incoming flow and the flow through the hole:

$$Q_{in} = \frac{\pi}{4}(0.06)^2(25) = 0.0707\ \frac{m^3}{s}, \quad Q_{hole} = \frac{\pi}{4}(0.04)^2(25) = 0.0314\ \frac{m^2}{s}$$

Then, for a CV enclosing the plate and the two jets, the horizontal force balance is

$$\sum F_x = -F_{plate} = \dot{m}_{hole}u_{hole} + \dot{m}_{upper}u_{upper} + \dot{m}_{lower}u_{lower} - \dot{m}_{in}u_{in}$$

$$= (998)(0.0314)(25) + 0 + 0 - (998)(0.0707)(25)$$

$$= 784 - 1764, \quad \text{solve for } \mathbf{F \approx 980\ N\ (to\ left)} \quad \textit{Ans.}$$

P3.65 The box in Fig. P3.65 has three 0.5-in holes on the right side. The volume flows of 20°C water shown are steady, but the details of the interior are not known. Compute the force, if any, which this water flow causes on the box.

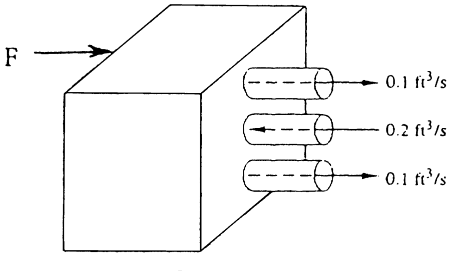


Fig. P3.65

Solution: First we need to compute the velocities through the various holes:

$$V_{top} = V_{bottom} = \frac{0.1\ ft^3/s}{(\pi/4)(0.5/12)^2} = 73.3\ ft/s; \quad V_{middle} = 2V_{top} = 146.6\ ft/s$$

Pretty fast, but do-able, I guess. Then make a force balance for a CV enclosing the box:

$$\sum F_x = F_{box} = -\dot{m}_{in}u_{in} + 2\dot{m}_{top}u_{top}, \quad \text{where} \quad u_{in} = -V_{middle} \quad \text{and} \quad u_{top} = V_{top}$$

$$\text{Solve for } F_{box} = (1.94)(0.2)(146.6) + 2(1.94)(0.1)(73.3) \approx \mathbf{85\ lbf} \quad \textit{Ans.}$$

P3.66 The tank in Fig. P3.66 weighs 500 N empty and contains 600 L of water at 20°C. Pipes 1 and 2 have D = 6 cm and Q = 300 m^3/hr. What should the scale reading W be, in newtons?

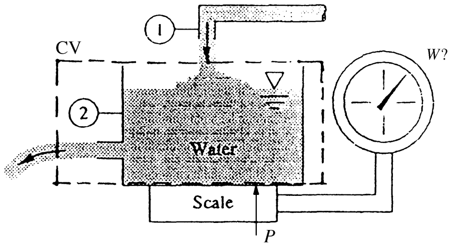


Fig. P3.66

Solution: Let the CV surround the tank, cut through the two jets, and slip just under the tank bottom, as shown. The relevant jet velocities are

$$V_1 = V_2 = \frac{Q}{A} = \frac{(300/3600)\ \text{m}^3/\text{s}}{(\pi/4)(0.06\ \text{m})^2} \approx 29.5\ \text{m/s}$$

The scale reads force “P” on the tank bottom. Then the vertical force balance is

$$\sum F_z = P - W_{tank} - W_{water} = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}[0 - (-V_1)]$$

$$\text{Solve for} \quad \mathbf{P} = 500 + 9790(0.6\ \text{m}^3) + 998\left(\frac{300}{3600}\right)(29.5) \approx \mathbf{8800\ N} \quad \textit{Ans.}$$

P3.67 Gravel is dumped from a hopper, at a rate of 650 N/s, onto a moving belt, as in Fig. P3.67. The gravel then passes off the end of the belt. The drive wheels are 80 cm in diameter and rotate clockwise at 150 r/min. Neglecting system friction and air drag, estimate the power required to drive this belt.

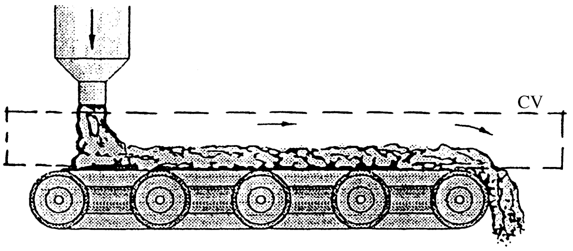


Fig. P3.67

Solution: The CV goes under the gravel on the belt and cuts through the inlet and outlet gravel streams, as shown. The no-slip belt velocity must be

$$V_{belt} = V_{outlet} = \Omega R_{wheel} = \left[150\frac{rev}{min}2\pi\frac{rad}{rev}\frac{1}{60}\frac{min}{s}\right](0.4\ m) \approx 6.28\ \frac{m}{s}$$

Then the belt applies tangential force *F* to the gravel, and the force balance is

$$\sum F_x = F_{on\ belt} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in}, \quad \text{but } u_{in} = 0.$$

$$\text{Then} \quad F_{belt} = \dot{m}V_{out} = \left(\frac{650}{9.81}\ \frac{kg}{s}\right)\left(6.28\ \frac{m}{s}\right) = 416\ N$$

The power required to drive the belt is $\boldsymbol{P} = FV_{belt} = (416)(6.28) \approx \mathbf{2600\ W}$ *Ans.*

P3.68 The rocket in Fig. P3.68 has a super-sonic exhaust, and the exit pressure *pe* is not necessarily equal to *pa*. Show that the force *F* required to hold this rocket on the test stand is $F = \rho e A e V e^2 + Ae(pe - pa)$. Is this force *F* what we term the *thrust* of the rocket?

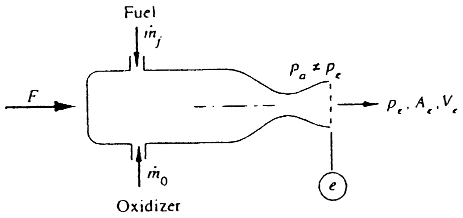


Fig. P3.68

Solution: The appropriate CV surrounds the entire rocket and cuts through the exit jet. Subtract pa everywhere so only exit pressure ≠ 0. The horizontal force balance is

$$\sum F_x = F - (p_e - p_a)A_e = \dot{m}_e u_e - \dot{m}_f u_f - \dot{m}_o u_o, \quad \text{but } u_f = u_o = 0, \quad \dot{m}_e = \rho_e A_e V_e$$

$$\text{Thus} \quad \mathbf{F = \rho_e A_e V_e^2 + (p_e - p_a)A_e} \quad (\text{yes, the } \underline{\textbf{thrust}}) \quad \textit{Ans.}$$

P3.69 A uniform rectangular plate, 40 cm long and 30 cm deep into the paper, hangs in air from a hinge at its top, 30-cm side. It is struck in its center by a horizontal 3-cm-diameter jet of water moving at 8 m/s. If the gate has a mass of 16 kg, estimate the angle at which the plate will hang from the vertical.

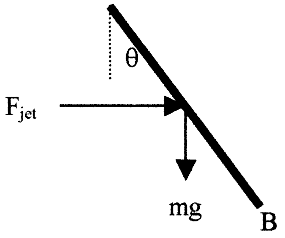


Fig. P3.69

Solution: The plate orientation can be found through force and moment balances. Find the force normal to the plate:

$$\sum F_n = F_n = \dot{m}_{jet} u_n = \rho A V u_n = \ (998)\left(\frac{\pi}{4}\right)(0.03^2)(8)(8\cos\theta) = 45.1\ \cos\theta \quad \textit{Newtons}$$

$$\sum M_{hinge} = 0 = -(45.1\cos\theta)(0.2\text{m}) + [(16)(9.81)\text{N}](0.2m)(\sin\theta);\ \tan\theta = 0.287, \quad \boldsymbol{\theta = 16°}$$

If the force and weight are centered in the plate, and the weight and jet flow are constant, the answer is independent of the length (40 cm) of the plate.

P3.70 The dredger in Fig. P3.70 is loading sand (SG = 2.6) onto a barge. The sand leaves the dredger pipe at 4 ft/s with a weight flux of 850 lbf/s. Estimate the tension on the mooring line caused by this loading process.

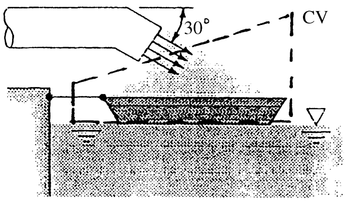


Fig. P3.70

Solution: The CV encloses the boat and cuts through the cable and the sand flow jet. Then,

$$\sum F_x = -T_{cable} = -\dot{m}_{sand}u_{sand} = -\dot{m}\,V_{sand}\cos\theta,$$

$$\text{or:}\quad T_{cable} = \left(\frac{850}{32.2}\ \frac{slug}{s}\right)\left(4\ \frac{ft}{s}\right)\cos 30° \approx \mathbf{91\ lbf} \quad \textit{Ans.}$$

P3.71 Suppose that a deflector is deployed at the exit of the jet engine of Prob. 3.50, as shown in Fig. P3.71. What will the reaction *Rx* on the test stand be now? Is this reaction sufficient to serve as a braking force during airplane landing?

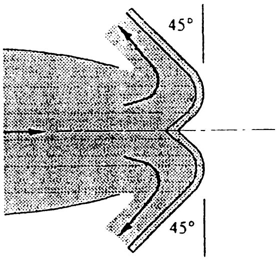


Fig. P3.71

Solution: From Prob. 3.50, recall that the essential data was

$$V_1 = 250\ \text{m/s},\quad V_2 = 900\ \text{m/s},\quad \dot{m}_1 = 151\ \text{kg/s},\quad \dot{m}_2 = 156\ \text{kg/s}$$

The CV should enclose the entire engine and also the deflector, cutting through the support and the 45° exit jets. Assume (unrealistically) that the exit velocity is *still* 900 m/s. Then,

$$\sum F_x = R_x = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in},\quad \text{where}\ u_{out} = -V_{out}\cos 45° \quad \text{and} \quad u_{in} = V_1$$

$$\text{Then}\quad R_x = -156(900\cos 45°) - 151(250) = -137{,}000\ \text{N}$$

The support reaction is to the left and equals 137 kN *Ans.*

P3.72 A thick elliptical cylinder immersed in a water stream creates the idealized wake shown. Upstream and downstream pressures are equal, and Uo = 4 m/s, L = 80 cm. Find the drag force on the cylinder per unit width into the paper. Also compute the dimensionless *drag coefficient* CD = 2F/(ρ Uo^{2}bL).

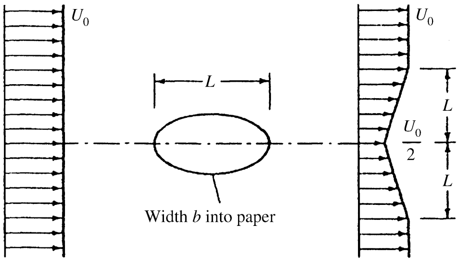


Fig. P3.72

Solution: This is a 'numerical' version of the "analytical" body-drag Prob. 3.44. The student still must make a CV analysis similar to Prob. P3.44 of this Manual. The wake is exactly the same shape, so the result from Prob. 3.44 holds here also:

$$F_{drag} = \frac{1}{3}\rho U_o^2 Lb = \frac{1}{3}(998)(4)^2(0.8)(1.0) \approx \mathbf{4260\ N} \quad \textit{Ans.}$$

The drag coefficient is easily calculated from the above result: **C_D = 2/3**. *Ans.*

P3.73 A pump in a tank of water directs a jet at 45 ft/s and 200 gal/min against a vane, as shown in the figure. Compute the force F to hold the cart stationary if the jet follows (a) path A; or (b) path B. The tank holds 550 gallons of water at this instant.

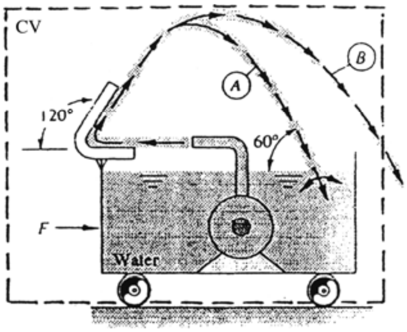


Fig. P3.73

Solution: The CV encloses the tank and passes through jet B.

(a) For jet path **A**, *no momentum flux crosses the CV*, therefore **F = 0** *Ans.* (a)

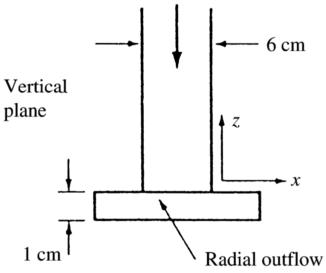
(b) For jet path B, there *is* momentum flux, so the *x*-momentum relation yields:

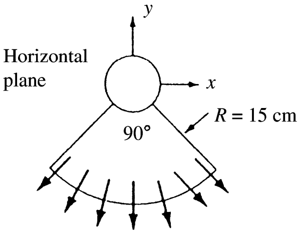
$$\sum F_x = F = \dot{m}_{out}u_{out} = \dot{m}_{jet}u_B$$

Now we don't really *know* u_B exactly, but we make the reasonable assumption that the jet trajectory is *frictionless* and maintains its horizontal velocity component, that is, $u_B \approx V_{jet}\cos 60°$. Thus we can estimate

$$F = \dot{m}u_B = \left(1.94\ \frac{slug}{ft^3}\right)\left(\frac{200}{448.8}\ \frac{ft^3}{s}\right)(45\cos 60°) \approx \mathbf{19.5\ lbf} \quad \textit{Ans.}\ \text{(b)}$$

P3.74 Water at 20°C flows down a vertical 6-cm-diameter tube at 300 gal/min, as in the figure. The flow then turns horizontally and exits through a 90° radial duct segment 1 cm thick, as shown. If the radial outflow is uniform and steady, estimate the forces (F_x, F_y, F_z) required to support this system against fluid momentum changes.





Solution: First convert 300 gal/min = 0.01893 m^3/s, hence the mass flow is ρQ = 18.9 kg/s. The vertical-tube velocity (down) is $V_{tube} = 0.01893/[(\pi/4)(0.06)^2] = -6.69\,\mathbf{k}$ m/s. The exit tube area is $(\pi/2)R\Delta h = (\pi/2)(0.15)(0.01) = 0.002356\ m^2$, hence $V_{exit} = Q/A_{exit}$ = 0.01893/0.002356 = *8.03* m/s. Now estimate the force components:

$$\Sigma F_x = \mathbf{F}_x = \int u_{out}\, d\dot{m}_{out} = \int_{-45°}^{45°} -V_{exit}\sin\theta\,\rho\,V_{exit}\,\Delta h\,R\,d\theta = 0 \quad Ans.(a)$$

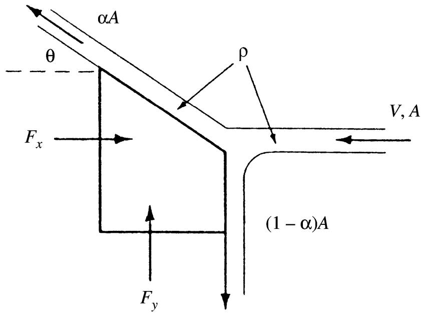
$$\Sigma F_y = \mathbf{F}_y = \int v_{out}\, d\dot{m}_{out} - \dot{m}\,v_{in} = \int_{-45°}^{45°} -V_{exit}\cos\theta\,\rho\,V_{exit}\,\Delta h\,R\,d\theta - 0$$

$$= -V_{exit}^2\,\rho\,\Delta h\,R\,\sqrt{2} \quad Ans.(b)$$

$$or:\ \mathbf{F}_y = -(8.03)^2(998)(0.01)(0.15)\sqrt{2} = \mathbf{-137\ N} \quad Ans.(b)$$

$$\sum F_z = \mathbf{F_z} = \dot{m}(w_{out} - w_{in}) = (18.9\ kg/s)[0-(-6.69\ m/s)] \approx \mathbf{+126\ N} \quad Ans.\ (c)$$

P3.75 A liquid jet of density r and area A strikes a block and splits into two jets, as shown in the figure. All three jets have the same velocity V. The upper jet exits at angle θ and area αA, the lower jet turns down at 90° and area (1 – α)A. (a) Derive a formula for the forces (Fx,Fy) required to support the block against momentum changes. (b) Show that Fy = 0 only if $\alpha \geq 0.5$. (c) Find the values of α and θ for which both Fx and Fy are zero.



Solution: (a) Set up the *x*- and *y*-momentum relations:

$$\sum F_x = F_x = \alpha\dot{m}(-V\cos\theta) - \dot{m}(-V) \quad where\ \ \dot{m} = \rho AV\ \ of\ \ the\ \ inlet\ jet$$

$$\sum F_y = F_y = \alpha\dot{m}V\sin\theta + (1-\alpha)\dot{m}(-V)$$

Clean this up for the final result:

$$F_x = \mathbf{\dot{m}V(1-\boldsymbol{\alpha}\cos\boldsymbol{\theta})}$$

$$F_y = \mathbf{\dot{m}V(\boldsymbol{\alpha}\sin\boldsymbol{\theta}+\boldsymbol{\alpha}-1)} \quad Ans.\ (a)$$

(b) Examining F_y above, we see that it can be zero only when,

$$\sin\theta = \frac{1-\alpha}{\alpha}$$

But this makes no sense if $\alpha < 0.5$, hence **$F_y = 0$ only if $\alpha \ge 0.5$**. *Ans.* (b)
(c) Examining F_x, we see that it can be zero only if $\cos\theta = 1/\alpha$, which makes no sense unless $\alpha = 1$, $\theta = 0°$. This situation also makes $F_x = 0$ above ($\sin\theta = 0$). Therefore the only scenario for which both forces are zero is the trivial case for which all the flow goes horizontally across a flat block:

$$F_x = F_y = 0 \quad \text{only if: } \alpha = \mathbf{1},\ \theta = \mathbf{0°} \quad \textit{Ans. } (c)$$

P3.76 A two-dimensional sheet of water, 10 cm thick and moving at 7 m/s, strikes a fixed wall inclined at 20° with respect to the jet direction. Assuming frictionless flow, find (a) the normal force on the wall per meter of depth, and the widths of the sheet deflected (b) upstream, and (c) downstream along the wall.

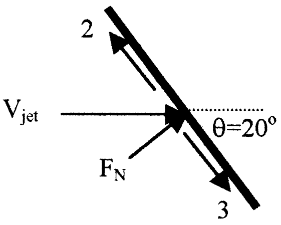


Fig. P3.76

Solution: (a) The force normal to the wall is due to the jet's momentum,

$$\sum F_N = -\dot{m}_{in}u_{in} = -(998)(0.1)(7^2)(\cos 70°) = \mathbf{1670\ N/m} \quad \textit{Ans.}$$

(b) Assuming $V_1 = V_2 = V_3 = V_{jet}$, $V_jA_1 = V_jA_2 + V_jA_3$ where,

$$A_2 = A_1\sin\theta = (0.1)(1)(\sin 20°) = 0.034\text{ m} \approx \mathbf{3\ cm} \quad \textit{Ans.}$$

(c) Similarly, $A_3 = A_1\cos\theta = (0.1)(1)(\cos 20°) = 0.094\text{ m} \approx \mathbf{9.4\ cm}$ *Ans.*

P3.77 Water at 20°C flows steadily through a reducing pipe bend, as in Fig. P3.77. Known conditions are p_1 = 350 kPa, D_1 = 25 cm, V_1 = 2.2 m/s, p_2 = 120 kPa, and D_2 = 8 cm. Neglecting bend and water weight, estimate the total force which must be resisted by the flange bolts.

Solution: First establish the mass flow and exit velocity:

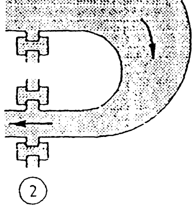


Fig. P3.77

$$\dot{m} = \rho_1 A_1 V_1 = 998\left(\frac{\pi}{4}\right)(0.25)^2(2.2) = 108\ \frac{kg}{s} = 998\left(\frac{\pi}{4}\right)(0.08)^2 V_2, \quad \text{or} \quad V_2 = 21.5\ \frac{m}{s}$$

The CV surrounds the bend and cuts through the flanges. The force balance is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 + p_{2,gage}A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1, \quad \text{where } u_2 = -V_2 \quad \text{and} \quad u_1 = V_1$$

$$\text{or} \quad F_{bolts} = (350000-100000)\frac{\pi}{4}(0.25)^2 + (120000-100000)\frac{\pi}{4}(0.08)^2 + 108(21.5+2.2)$$

$$= 12271 + 101 + 2553 \approx \mathbf{14900\ N} \quad \textit{Ans.}$$

P3.78 A fluid jet of diameter *D*1 enters a cascade of moving blades at absolute velocity *V*1 and angle *β*1, and it leaves at absolute velocity *V*1 and angle *β*2, as in Fig. P3.78. The blades move at velocity *u*. Derive a formula for the power *P* delivered to the blades as a function of these parameters.

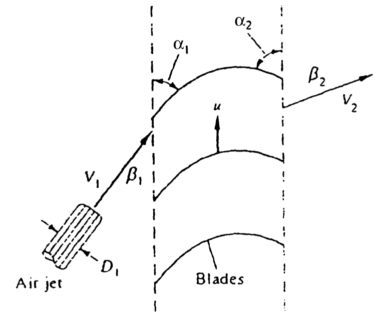
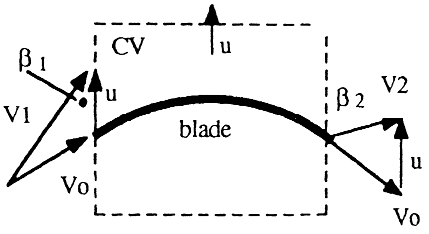


Fig. P3.78

Solution: Let the CV enclose the blades and move upward at speed *u*, so that the flow appears steady in that frame, as shown at right. The relative velocity Vo may be eliminated by the law of cosines:



$$V_o^2 = V_1^2 + u^2 - 2V_1 u\cos\beta_1$$

$$= V_2^2 + u^2 - 2V_2 u\cos\beta_2$$

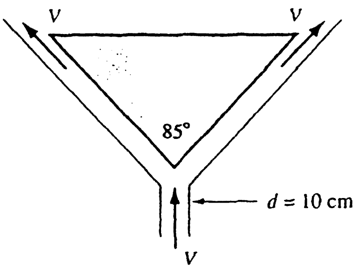
$$\text{solve for} \quad u = \frac{(1/2)\left(V_1^2 - V_2^2\right)}{V_1\cos\beta_1 - V_2\cos\beta_2}$$

Then apply momentum in the direction of blade motion:

$$\sum F_y = F_{vanes} = \dot{m}_{jet}(V_{o1y} - V_{o2y}) = \dot{m}(V_1\cos\beta_1 - V_2\cos\beta_2), \quad \dot{m} = \rho A_1 V_1$$

The power delivered is P = Fu, which causes the parenthesis "cosβ" terms to cancel:

$$\mathbf{P = Fu = \frac{1}{2}\dot{m}_{jet}\left(V_1^2 - V_2^2\right)} \quad \textit{Ans.}$$



P3.79 Air at 20°C and 1 atm enters the bottom of an 85° conical flowmeter duct at a mass flow rate of 0.3 kg/s, as shown in the figure. It supports a centered conical body by steady annular flow around the cone and exits at the same velocity as it enters. Estimate the weight of the body in newtons.

Solution: First estimate the velocity from the known inlet duct size:

$$\rho_{air} = \frac{p}{RT} = \frac{101350}{287(293)} = 1.205\ \frac{\text{kg}}{\text{m}^3},$$

$$\textit{thus}\quad \dot{m} = 0.3 = \rho AV = (1.205)\frac{\pi}{4}(0.1)^2 V,\quad \textit{solve}\ \mathbf{V = 31.7\ \frac{m}{s}}$$

Then set up the vertical momentum equation, the unknown is the body weight:

$$\sum F_z = -W = \dot{m}V\cos 42.5° - \dot{m}V = \dot{m}V(\cos 42.5° - 1)$$

$$\textit{Thus}\quad \mathbf{W_{cone}} = (0.3)(31.7)(1-\cos 42.5°) = \mathbf{2.5\ N}\quad \textit{Ans.}$$

P3.80 A river (1) passes over a “drowned” weir as shown, leaving at a new condition (2). Neglect atmospheric pressure and assume hydrostatic pressure at (1) and (2). Derive an expression for the force F exerted by the river on the obstacle. Neglect bottom friction.

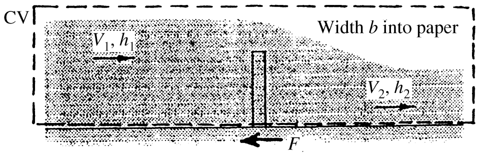


Fig. P3.80

Solution: The CV encloses (1) and (2) and cuts through the gate along the bottom, as shown. The volume flow and horizontal force relations give

$$V_1 b h_1 = V_2 b h_2$$

$$\sum F_x = -F_{weir} + \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = (\rho h_1 b V_1)(V_2 - V_1)$$

Note that, except for the different geometry, the analysis is exactly the same as for the sluice gate in Ex. 3.10. The force result is the same, also:

$$\mathbf{F_{weir}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\rho}\mathbf{gb}\left(\mathbf{h_1^2 - h_2^2}\right) - \boldsymbol{\rho}\mathbf{h_1 b V_1^2}\left(\frac{\mathbf{h_1}}{\mathbf{h_2}} - \mathbf{1}\right)\quad \textit{Ans.}$$

P3.81 Torricelli's idealization of efflux from a hole in the side of a tank is $V \approx \sqrt{2gh}$, as shown in Fig. P3.81. The tank weighs 150 N when empty and contains water at 20°C. The tank bottom is on very smooth ice (static friction coefficient $\zeta \approx 0.01$). For what water depth h will the tank just begin to move to the right?

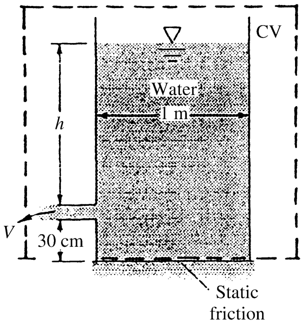


Fig. P3.81

Solution: The hole diameter is 9 cm. The CV encloses the tank as shown. The coefficient of static friction is $\zeta = 0.01$. The x-momentum equation becomes

$$\sum F_x = -\zeta W_{tank} = \dot{m}u_{out} = -\dot{m}\,V_{hole} = -\rho A V^2 = -\rho A(2gh)$$

$$\text{or:}\quad 0.01\left[(9790)\frac{\pi}{4}(1\text{ m})^2(h+0.3+0.09)+150\right] = 998\left(\frac{\pi}{4}\right)(0.09)^2(2)(9.81)h$$

Solve for **h ≈ 0.66 m** *Ans.*

P3.82 The model car in Fig. P3.82 weighs 17 N and is to be accelerated from rest by a 1-cm-diameter water jet moving at 75 m/s. Neglecting air drag and wheel friction, estimate the velocity of the car after it has moved forward 1 m.

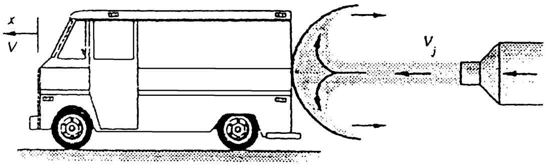


Fig. P3.82

Solution: The CV encloses the car, moves to the left at *accelerating* car speed V(t), and cuts through the inlet and outlet jets, which leave the CS at *relative* velocity Vj − V. The force relation is Eq. (3.50):

$$\sum F_x - \int a_{rel}\,dm = 0 - m_{car}a_{car} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = -2\dot{m}_{jet}(V_j - V),$$

$$\text{or:}\quad m_{car}\frac{dV}{dt} = 2\rho A_j(V_j - V)^2$$

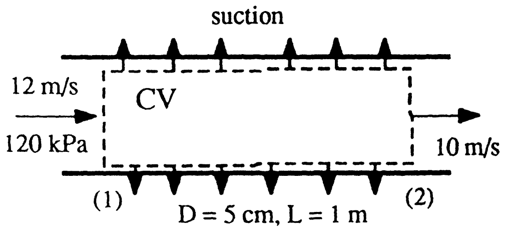
Except for the factor of "2," this is identical to the "cart" analysis of Example 3.12 on page 140 of the text. The solution, for V = 0 at t = 0, is given there:

$$V = \frac{V_j^2 Kt}{1+V_j Kt},\quad \text{where } K = \frac{2\rho A_j}{m_{car}} = \frac{2(998)(\pi/4)(0.01)^2}{(17/9.81)} = 0.0905\text{ m}^{-1}$$

$$\text{Thus} \quad V\ (\text{in m/s}) = \frac{509t}{1+6.785t} \quad \text{and then compute distance} \quad S = \int_0^t V\,dt$$

The initial acceleration is 509 m/s^2, quite large. Assuming the jet can follow the car without dipping, the car reaches $\underline{S = 1\text{ m}}$ at t ≈ 0.072 s, where **V ≈ 24.6 m/s**. *Ans.*

P3.83 Gasoline at 20°C is flowing at V1 = 12 m/s in a 5-cm-diameter pipe when it encounters a 1-m length of uniform radial wall suction. After the suction, the velocity has dropped to 10 m/s. If p1 = 120 kPa, estimate p2 if wall friction is neglected.



Solution: The CV cuts through sections 1 and 2 and the inside of the walls. We compute the mass flow at each section, taking $\rho \approx 680$ kg/m^3 for gasoline:

$$\dot{m}_1 = 680\left(\frac{\pi}{4}\right)(0.05)^2(12) = 16.02\ \frac{\text{kg}}{\text{s}}; \quad \dot{m}_2 = 680\left(\frac{\pi}{4}\right)(0.05)^2(10) = 13.35\ \frac{\text{kg}}{\text{s}}$$

The difference, 16.02 − 13.35 = 2.67 kg/s, is sucked through the walls. If wall friction is neglected, the force balance (taking the momentum correction factors $\beta \approx 1.0$) is:

$$\sum F_x = p_1A_1 - p_2A_2 = \dot{m}_2V_2 - \dot{m}_1V_1 = (120000 - p_2)\frac{\pi}{4}(0.05)^2$$

$$= (13.35)(10) - (16.02)(12), \quad \text{solve for } p_2 \approx \mathbf{150\ kPa} \quad \textit{Ans.}$$

P3.84 Air at 20°C and 1 atm flows in a 25-cm-diameter duct at 15 m/s, as in Fig. P3.84. The exit is choked by a 90° cone, as shown. Estimate the force of the airflow on the cone.

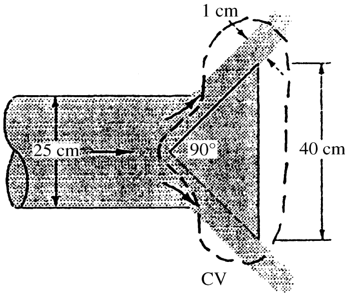


Fig. P3.84

Solution: The CV encloses the cone, as shown. We need to know exit velocity. The exit area is approximated as a ring of diameter 40.7 cm and thickness 1 cm:

$$Q = A_1V_1 = \frac{\pi}{4}(0.25)^2(15) = 0.736\ \frac{\text{m}^3}{\text{s}} = A_2V_2 \approx \pi(0.407)(0.01)V_2, \quad \text{or} \quad V_2 \approx 57.6\ \frac{\text{m}}{\text{s}}$$

The air density is $\rho = p/RT = (101350)/[287(293)] \approx 1.205\ \text{kg/m}^3$. We are not given any pressures on the cone so we consider momentum only. The force balance is

$$\sum F_x = F_{cone} = \dot{m}(u_{out} - u_{in}) = (1.205)(0.736)(57.6\cos 45° - 15) \approx \mathbf{22.8\ N} \quad \textit{Ans.}$$

The force on the cone is *to the right* because we neglected pressure forces.

P3.85 The thin-plate orifice in Fig. P3.85 causes a large pressure drop. For 20°C water flow at 500 gal/min, with pipe *D* = 10 cm and orifice *d* = 6 cm, $p_1 - p_2 \approx 145$ kPa. If the wall friction is negligible, estimate the force of the water on the orifice plate.

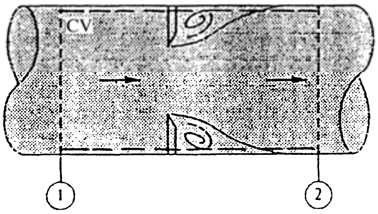


Fig. P3.85

Solution: The CV is inside the pipe walls, cutting through the orifice plate, as shown. At least to one-dimensional approximation, $V_1 = V_2$, so there is no momentum change. The force balance yields the force of the plate on the fluid:

$$\sum F_x = -F_{\text{plate on fluid}} + p_1A_1 - p_2A_2 - \tau_{wall}A_{wall} = \dot{m}(V_2 - V_1) \approx 0$$

$$\text{Since } \tau_{wall} \approx 0, \quad \text{we obtain } F_{plate} = (145000)\frac{\pi}{4}(0.1)^2 \approx \mathbf{1140\ N} \quad \textit{Ans.}$$

The force of the fluid on the plate is opposite to the sketch, or to the right.

P3.86 For the water-jet pump of Prob. 3.36, add the following data: $p_1 = p_2 = 25$ lbf/in^2, and the distance between sections 1 and 3 is 80 in. If the average wall shear stress between sections 1 and 3 is 7 lbf/ft^2, esti-mate the pressure p_3. Why is it higher than p_1?

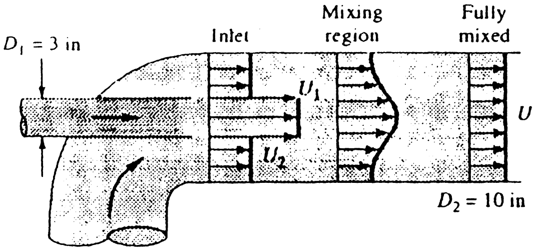


Fig. P3.36

Solution: The CV cuts through sections 1, 2, 3 and along the inside pipe walls. Recall from Prob. 3.36 that mass conservation led to the calculation $\mathbf{V_3 \approx 6.33\ m/s}$. Convert data to SI units: L = 80 in = 2.032 m, $p_1 = p_2$ = 25 psi = 172.4 kPa, and τ_{wall} = 7 psf = 335 Pa. We need mass flows for each of the three sections:

$$\dot{m}_1 = 998\left(\frac{\pi}{4}\right)(0.0762)^2(40) \approx 182\ \frac{kg}{s};$$

$$\dot{m}_2 = 998\left(\frac{\pi}{4}\right)[(0.254)^2 - (0.0762)^2](3) \approx 138\ \frac{kg}{s} \quad \text{and} \quad \dot{m}_3 \approx 182 + 138 \approx 320\ \frac{kg}{s}$$

Then the horizontal force balance will yield the (high) downstream pressure:

$$\sum F_x = p_1(A_1 + A_2) - p_3A_3 - \tau_{wall}\pi D_2 L = \dot{m}_3 V_3 - \dot{m}_2 V_2 - \dot{m}_1 V_1$$

$$= (172400 - p_3)\frac{\pi}{4}(0.254)^2 - 335\pi(0.254)(2.032) = 320(6.33) - 138(3) - 182(40)$$

$$\text{Solve for} \quad p_3 \approx 274000\ Pa \ = \ \mathbf{39.7}\ lbf/in^2 \quad \textit{Ans.}$$

The pressure is high because the primary inlet kinetic energy at section (1) is converted by viscous mixing to pressure-type energy at the exit.

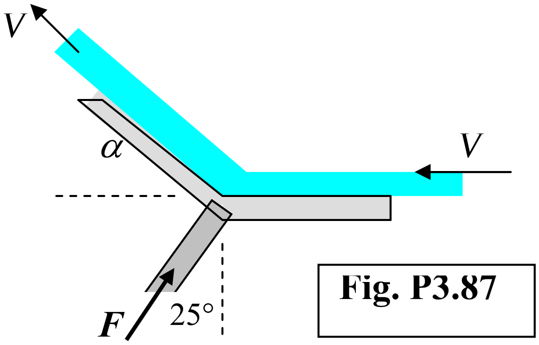


Fig. P3.87

P3.87 A vane turns a water jet through an angle α, as shown in Fig. P3.87. Neglect friction on the vane walls. (*a*) What is the angle α for the support force to be in pure compression? (*b*) Calculate this compression force if the water velocity is 22 ft/s and the jet cross-section is 4 in^2.

Solution: (*a*) From the solution to Example 3.8, the support will be in pure compression (aligned with ***F***) if the vane angle is twice the support angle.

$$\text{Therefore} \quad \alpha = 2(25°) = \mathbf{50°} \quad \text{Ans.}(a)$$

(*b*) The mass flow of the jet is

$$\dot{m} = \rho A_{jet} V_{jet} = (1.94\frac{slug}{ft^3})\frac{\pi}{4}(\frac{4}{12}\ ft)^2(22\frac{ft}{s}) = 3.72\ \frac{slug}{s}$$

Then, also from Example 3.8, the magnitude of the support force is

$$F = 2\dot{m}V\sin\frac{\alpha}{2} = 2(3.72\frac{slug}{s})(22\frac{ft}{s})\sin(\frac{50^o}{2}) = \mathbf{69}\ lbf \quad \textit{Ans.}(b)$$

P3.88 The boat in Fig. P3.88 is jet-propelled by a pump which develops a volume flow rate Q and ejects water out the stern at velocity V_j. If the boat drag force is $F = kV^2$, where k is a constant, develop a formula for the steady forward speed V of the boat.

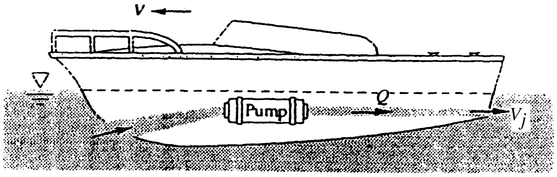


Fig. P3.88

Solution: Let the CV move to the left at boat speed V and enclose the boat and the pump's inlet and exit. Then the momentum relation is

$$\sum F_x = kV^2 = \dot{m}_{pump}(V_j + V - V_{inlet}) \approx \rho Q(V_j + V) \quad \text{if we assume } V_{inlet} << V_j$$

If, further, $V << V_j$, then the approximate solution is: $\mathbf{V \approx (\rho Q V_j/k)^{1/2}}$ *Ans.*
If V and Vj are comparable, then we solve a quadratic equation:

$$V \approx \zeta + [\zeta^2 + 2\zeta V_j]^{1/2}, \quad \text{where } \zeta = \frac{\rho Q}{2k} \quad \textit{Ans.}$$

P3.89 Consider Fig. P3.36 as a general problem for analysis of a mixing ejector pump. If all conditions (p, ρ, V) are known at sections 1 and 2 and if the wall friction is negligible, derive formulas for estimating (a) V_3 and (b) p_3.

Solution: Use the CV in Prob. 3.86 but use symbols throughout. For volume flow,

$$V_1\frac{\pi}{4}D_1^2 + V_2\frac{\pi}{4}\left(D_2^2 - D_1^2\right) = V_3\frac{\pi}{4}D_2^2, \quad \text{or:} \quad V_3 = V_1\alpha + V_2(1-\alpha), \quad \alpha = (D_1/D_2)^2 \qquad \text{(A)}$$

Now apply x-momentum, assuming (quite reasonably) that $p_1 = p_2$:

$$(p_1 - p_3)\frac{\pi}{4}D_2^2 - \tau_w \pi D_2 L = \rho\frac{\pi}{4}D_2^2V_3^2 - \rho\frac{\pi}{4}\left(D_2^2 - D_1^2\right)V_2^2 - \rho\frac{\pi}{4}D_1^2V_1^2$$

$$\text{Clean up:} \quad \mathbf{p_3 = p_1 - \frac{4L\tau_w}{D_2} + \rho\left[\alpha V_1^2 + (1-\alpha)V_2^2 - V_3^2\right]} \quad \text{where } \boldsymbol{\alpha} = \left(\frac{\mathbf{D_1}}{\mathbf{D_2}}\right)^2 \quad \textit{Ans.}$$

You have to insert V_3 into this answer from Eq. (A) above, but the algebra is messy.

P3.90 As shown in Fig. P3.90, a liquid column of height h is confined in a vertical tube of cross-sectional area A by a stopper. At $t = 0$ the stopper is suddenly removed, exposing the bottom of the liquid to atmospheric pressure. Using a control-volume analysis of mass and vertical momentum, derive the differential equation for the downward motion *V*(*t*) of the liquid. Assume one-dimensional, incompressible, frictionless flow.

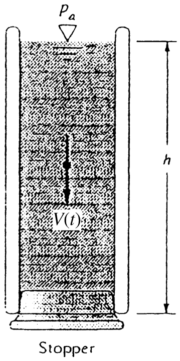


Fig. P3.90

Solution: Let the CV enclose the cylindrical blob of liquid. With density, area, and the blob volume constant, mass conservation requires that V = V(t) only. The CV accelerates downward at blob speed V(t). Vertical (downward) force balance gives

$$\sum F_{down} - \int a_{rel}\, dm = \frac{d}{dt}\left(\int V_{down}\rho\, d\upsilon\right) + \dot{m}_{out}V_{out} - \dot{m}_{in}V_{in} = 0$$

$$\text{or:}\quad m_{blob}g + \Delta p\, A - \tau_w A_w - a m_{blob} = 0$$

$$\text{Since }\ \Delta p = 0 \quad\text{and}\quad \tau = 0,\quad \text{we are left with }\ \mathbf{a_{blob} = \frac{dV}{dt} = g} \quad \textit{Ans.}$$

P3.91 Extend Prob. 3.90 to include a linear (laminar) average wall shear stress of the form $\tau \approx cV$, where c is a constant. Find V(t), assuming that the wall area remains constant.

Solution: The downward momentum relation from Prob. 3.90 above now becomes

$$0 = m_{blob}g - \tau_w \pi DL - m_{blob}\frac{dV}{dt},\quad \text{or}\quad \frac{dV}{dt} + \zeta V = g,\quad \text{where }\ \zeta = \frac{c\pi DL}{m_{blob}}$$

where we have inserted the laminar shear $\tau = cV$. The blob mass equals $\rho(\pi/4)D^2L$. For V = 0 at t = 0, the solution to this equation is

$$\mathbf{V = \frac{g}{\zeta}(1 - e^{-\zeta t}),}\quad \text{where }\ \boldsymbol{\zeta = \frac{c\pi DL}{m_{blob}} = \frac{4c}{\rho D}} \quad \textit{Ans.}$$

P3.92 A more involved version of Prob. 3.90 is the elbow-shaped tube in Fig. P3.92, with constant cross-sectional area *A* and diameter *D* << *h*, *L*. Assume incompressible flow, neglect friction, and derive a differential equation for *dV/dt* when the stopper is opened. *Hint:* Combine two control volumes, one for each leg of the tube.

Solution: Use two CV's, one for the vertical blob and one for the horizontal blob, connected as shown by pressure.

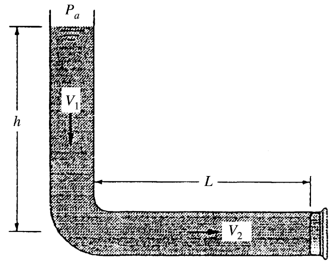
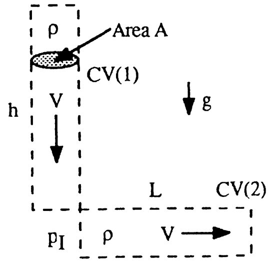


Fig. P3.92



From mass conservation, V1 = V2 = V(t). For CV's #1 and #2,

$$\Sigma F_{down} - \int a_{rel}\, dm = \Delta(\dot{m}v) = 0 = (p_{atm} - p_I)A + \rho gAh - m_1\frac{dV}{dt} \qquad \text{(No. 1)}$$

$$\Sigma F_x - \int a_{rel}\, dm = \Delta(\dot{m}u) = 0 = (p_I - p_{atm})A + 0 - m_2\frac{dV}{dt} \qquad \text{(No. 2)}$$

Add these two together. The pressure terms cancel, and we insert the two blob masses:

$$\rho gAh - (\rho Ah + \rho AL)\frac{dV}{dt} = 0, \quad \text{or:} \quad \mathbf{\frac{dV}{dt} = g\frac{h}{L+h}} \quad \textit{Ans.}$$

P3.93 According to Torricelli's theorem, the velocity of a fluid draining from a hole in a tank is $V \approx (2gh)^{1/2}$, where h is the depth of water above the hole, as in Fig. P3.93. Let the hole have area A_o and the cylindrical tank have cross-section area A_b. Derive a formula for the time to drain the tank from an initial depth h_o.

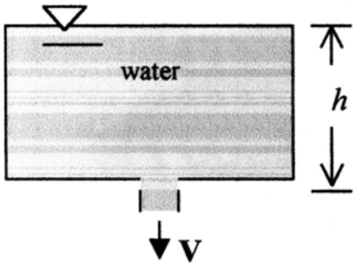


Fig. P3.93

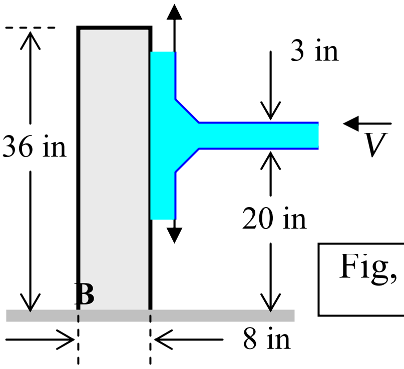
Solution: For a control volume around the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} = 0$$

$$\rho A_b \frac{dh}{dt} = -\dot{m}_{out} \approx -\rho A_o \sqrt{2gh}$$

$$\int_{h_o}^{0} \frac{dh}{\sqrt{h}} = \int_0^t \frac{A_o\sqrt{2g}}{A_b}\, dt; \qquad \boldsymbol{t = \frac{A_b}{A_o}\sqrt{\frac{h_o}{2g}}} \quad \textit{Ans.}$$

P3.94 A water jet 3 inches in diameter strikes a concrete (SG = 2.3) slab which rests freely on a level floor. If the slab is 1 ft wide into the paper, calculate the jet velocity which will just begin to tip the slab over.



Fig, P3.94

Solution: For water let $\rho = 1.94\ \text{slug/ft}^3$. Find the water force and then take moments about the lower left corner of the slab, point B. A control volume around the water flow yields

$$\sum F_x = F_{on\,jet} = \Sigma \dot{m}_{out} u_{out} - \Sigma \dot{m}_{in} u_{in} = \dot{m}_{out}(0) - \rho A V(-V), \quad F = \rho A V^2$$

$$\sum M_B = (\rho A V^2)\left(\frac{21.5}{12} ft\right) - W_{slab}\left(\frac{4}{12} ft\right), \quad W_{slab} = (2.3\times 62.4)\left(\frac{8}{12} ft\right)(3\,ft)(1\,ft) = 287\ lbf$$

Thus $(1.94)\frac{\pi}{4}\left(\frac{3}{12} ft\right)^2 V^2\left(\frac{21.5}{12} ft\right) = (287\,lbf)\left(\frac{4}{12} ft\right)$, solve for $V_{jet} = \mathbf{23.7\frac{ft}{s}}$ *Ans.*

P3.95 A cylindrical water tank discharges through a well-rounded orifice to hit a plate, as in Fig. P3.95. Use the Torricelli formula of Prob. P3.81 to estimate the exit velocity. (*a*) If, at this instant, the force *F* required to hold the plate is 40 N, what is the depth *h* ? (*b*) If the tank surface is dropping at the rate of 5 cm every 2 seconds, what is the tank diameter *D*?

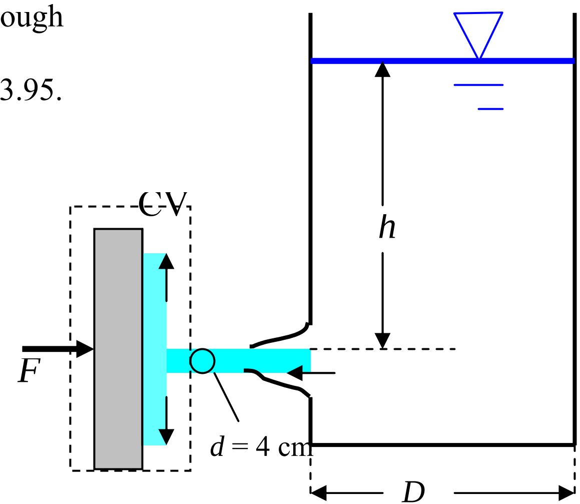


Fig. P3.95

Solution: For water take $\rho = 998\ \text{kg/m}^3$. The control volume surrounds the plate and yields

$$\Sigma F_x = F = -\dot{m}_{in} u_{in} = -\dot{m}_{jet}(-V_{jet}) = \rho A_{jet} V_{jet}(V_{jet}) = \rho \frac{\pi}{4} d^2 V_{jet}^2$$

But Torricelli says $V_{jet}^2 = 2gh$; Thus $h = \dfrac{F}{\rho(\pi/4)d^2(2g)}$

Given data: $h = \dfrac{40\,N}{(998\,kg/m^3)(\pi/4)(0.04\,m)^2(2)(9.81\,m/s^2)} = \mathbf{1.63\,m}$ *Ans.*(*a*)

(b) In 2 seconds, h drops from 1.63m to 1.58m, not much change. So, instead of a laborious calculus solution, find $Q_{\text{jet,av}}$ for an average depth $h_{\text{av}} = (1.63+1.58)/2 = 1.605$ m:

$$Q_{av} = A_{jet}\sqrt{2gh_{av}} = \frac{\pi}{4}(0.04m)^2\sqrt{2(9.81m/s^2)(1.605m)} \approx 0.00705\,m^3/s$$

$$\text{Equate } Q\Delta t = A_{\text{tank}}\Delta h\text{ , or : } D = \sqrt{\frac{Q\Delta t}{(\pi/4)\Delta h}} = \sqrt{\frac{(0.00705)(2s)}{(\pi/4)(0.05m)}} \approx \mathbf{0.60\,m} \quad \textit{Ans.}(b)$$

P3.96 Extend Prob. 3.90 to the case of the liquid motion in a frictionless U-tube whose liquid column is displaced a distance Z upward and then released, as in Fig. P3.96. Neglect the short horizontal leg and combine control-volume analyses for the left and right legs to derive a single differential equation for *V*(*t*) of the liquid column.

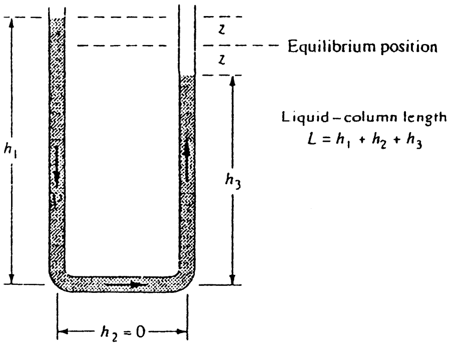
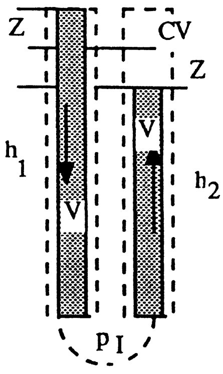


Fig. P3.96

Solution: As in Prob. 3.92, break it up into two moving CV's, one for each leg, as shown. By mass conservation, the velocity V(t) is the same in each leg. Let pI be the bottom pressure in the (very short) cross-over leg. Neglect wall shear stress. Now apply vertical momentum to each leg:



$$\text{Leg\#1:}\quad \sum F_{down} - \int a_{rel}\,dm$$

$$= (p_a - p_I)A + \rho g A h_1 - m_1\frac{dV}{dt} = 0$$

$$\text{Leg\#2:}\quad \sum F_{up} - \int a_{rel}\,dm = (p_I - p_a)A - \rho g A h_2 - m_2\frac{dV}{dt} = 0$$

Add these together. The pressure terms will cancel. Substitute for the h's as follows:

$$\rho g A(h_1 - h_2) = \rho g A(2Z) = (m_1 + m_2)\frac{dV}{dt} = \rho A(h_1 + h_2)\frac{dV}{dt} = \rho A L\frac{dV}{dt}$$

Since $V = -\frac{dZ}{dt}$, we arrive at, finally, $\mathbf{\frac{d^2Z}{dt^2} + \frac{2g}{L}Z = 0}$ *Ans.*

The solution is a simple harmonic oscillation: $Z = C\cos\left[t\sqrt{(2g/L)}\right] + D\sin\left[t\sqrt{(2g/L)}\right]$.

P3.97 Extend Prob. 3.96 to include a linear (laminar) average wall shear stress resistance of the form $\tau \approx 8\mu V/D$, where μ is the fluid viscosity. Find the differential equation for *dV/dt* and then solve for *V*(*t*), assuming an initial displacement $z = z_o$, $V = 0$ at $t = 0$. The result should be a damped oscillation tending toward $z = 0$.

Solution: The derivation now includes wall shear stress on each leg (see Prob. 3.96):

$$\text{Leg\#1:}\quad \sum F_{down} - \int a_{rel}\,dm = \Delta p\,A + \rho g A h_1 - \tau_w \pi D h_1 - m_1\frac{dV}{dt} = 0$$

$$\text{Leg\#2:}\quad \sum F_{up} - \int a_{rel}\,dm = -\Delta p\,A - \rho g A h_2 - \tau_w \pi D h_2 - m_2\frac{dV}{dt} = 0$$

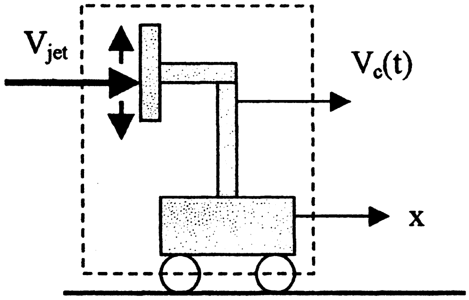
Again add these two together: the pressure terms cancel, and we obtain, if A = $\pi D^2/4$,

$$\mathbf{\frac{d^2Z}{dt^2} + \frac{4\tau_w}{\rho D} + \frac{2g}{L}Z = 0,}\quad \text{where}\ \tau_w = \frac{\mathbf{8\mu V}}{\mathbf{D}}\quad \textit{Ans.}$$

The shear term is equal to the linear damping term $\mathbf{(32\mu/\rho D^2)(dZ/dt)}$. If we assume an initial static displacement Z = Zo, V = 0, at t = 0, we obtain the damped oscillation

$$\mathbf{Z = Z_o e^{-t/t^*}\cos(\omega t),}\quad \text{where}\ \mathbf{t^* = \frac{\rho D^2}{16\mu}}\quad \text{and}\quad \mathbf{\omega = \sqrt{2g/L}}\quad \textit{Ans.}$$

P3.98 As an extension of Ex. 3.9, let the plate and cart be unrestrained, with frictionless wheels. Derive (a) the equation of motion for cart velocity Vc(t); and (b) the time required for the cart to accelerate to 90% of jet velocity. (c) Compute numerical values for (b) using the data from Ex. 3.9 and a cart mass of 2 kg.



Solution: (a) Use Eq. (3.49) with **a**rel equal to the cart acceleration and $\sum F_x = 0$:

$$\sum F_x - \cancelto{}{a}_{x,rel}\,m = \int u\rho \mathbf{V}\cdot\mathbf{n}\,dA = -m_c\frac{dV_c}{dt} = \boldsymbol{-\rho_j A_j (V_j - V_c)^2}\quad \textit{Ans.}\ \text{(a)}$$

The above 1st-order differential equation can be solved by separating the variables:

$$\int_0^{V_c}\frac{dV_c}{(V_j - V_c)^2} = K\int_0^t dt,\quad \textit{where}\ K = \frac{\rho A_j}{m_c}$$

$$\textit{Solve for:}\quad \frac{V_c}{V_j} = \frac{V_j K t}{1 + V_j K t} = 0.90\quad \textit{if}\quad \mathbf{t_{90\%}} = \frac{9}{KV_j} = \frac{\mathbf{9m_c}}{\boldsymbol{\rho}\mathbf{A_j V_j}}\quad \textit{Ans.}\ \text{(b)}$$

$$\textit{For the Example } 3.10\ \textit{data},\ \mathbf{t_{90\%}} = \frac{9(2\ kg)}{(1000\ \text{kg/m}^3)(0.0003\ \text{m}^2)(20\ \text{m/s})} \approx \mathbf{3.0\ s}\quad \textit{Ans.}\ \text{(c)}$$

P3.99 Let the rocket of Fig. E3.12 start at z = 0, with constant exit velocity and exit mass flow, and rise vertically with zero drag. (a) Show that, as long as fuel burning continues, the vertical height *S*(*t*) reached is given by

$$S = \frac{V_e M_o}{\dot{m}}[\zeta \ln\zeta - \zeta + 1], \quad where\ \ \zeta = 1 - \frac{\dot{m}t}{M_o}$$

(b) Apply this to the case V_e = 1500 m/s and M_o = 1000 kg to find the height reached after a burn of 30 seconds, when the final rocket mass is 400 kg.

Solution: (a) Ignoring gravity effects, integrate the equation of the projectile's velocity (from E3.12):

$$S(t) = \int V(t)\,dt = \int_0^t \left[-V_e \ln\left(1 - \frac{\dot{m}t}{M_o}\right)\right] dt$$

$$Let\ \ \zeta = 1 - \frac{\dot{m}t}{M_o}, \quad then\ \ d\zeta = -\frac{\dot{m}}{M_o}\,dt \quad and\ the\ integral\ becomes,$$

$$S(t) = (-V_e)\left[\frac{-M_o}{\dot{m}}\right]\int_1^{\zeta}(\zeta\ln\zeta)\,d\zeta = \left(\frac{V_e M_o}{\dot{m}}\right)[\zeta\ln\zeta - \zeta]_1^{\zeta} = \left(\frac{V_e M_o}{\dot{m}}\right)[\zeta\ln\zeta - \zeta + 1]$$

(b) Substituting the numerical values given,

$$\dot{m} = \frac{\Delta M}{\Delta t} = \frac{M_f - M_o}{\Delta t} = \frac{1000\text{ kg} - 400\text{ kg}}{30\text{ s}} = 20\text{ kg/s} \quad and \quad \zeta = 1 - \frac{(20\text{ kg/s})(30\text{ s})}{1000\text{ kg}} = 0.40$$

$$S(t = 30\text{ s}) = \frac{(1500\text{ m/s})(1000\text{ kg})}{(20\text{ kg/s})}[0.4\ \ln(0.4) - (0.4) + 1] = \mathbf{17{,}500}\text{ m} \quad Ans.$$

P3.100 Suppose that the solid-propellant rocket of Prob. 3.35 is built into a missile of diameter 70 cm and length 4 m. The system weighs 1800 N, which includes 700 N of propellant. Neglect air drag. If the missile is fired vertically from rest at sea level, estimate (a) its velocity and height at fuel burnout and (b) the maximum height it will attain.

Solution: The theory of Example 3.12 holds until burnout. Now M_o = 1800/9.81 = 183.5 kg, and recall from Prob. 3.35 that V_e = 1150 m/s and the exit mass flow is 11.8 kg/s. The fuel mass is 700/9.81 = 71.4 kg, so burnout will occur at $t_{burnout}$ = 71.4/11.8 = 6.05 s. Then Example 3.12 predicts the velocity at burnout:

$$V_b = -1150\ \ln\left(1 - \frac{11.8(6.05)}{183.5}\right) - 9.81(6.05) \approx \mathbf{507\ \frac{m}{s}} \quad Ans.\text{ (a)}$$

Meanwhile, Prob. 3.99 gives the formula for altitude reached at burnout:

$$S_b = \frac{183.5(1150)}{11.8}[1+(0.611)\{\ln(0.611)-1\}] - \frac{1}{2}(9.81)(0.605)^2 \approx \mathbf{1393\ m} \quad \textit{Ans. (a)}$$

where "0.611" = 1 – 11.8(6.05)/183.5, that is, the mass ratio at burnout. After burnout, with drag neglected, the missile moves as a falling body. Maximum height occurs at

$$\Delta t = \frac{V_o}{g} = \frac{507}{9.81} = 51.7\ \text{s, whence}$$

$$S = S_o + \frac{1}{2}g\Delta t^2 = 1393 + (1/2)(9.81)(51.7)^2 \approx \mathbf{14500\ m} \quad \textit{Ans. (b)}$$

P3.101 Water at 20°C flows steadily through the tank in Fig. P3.101. Known conditions are D_1 = 8 cm, V_1 = 6 m/s, and D_2 = 4 cm. A rightward force F = 70 N is required to keep the tank fixed.

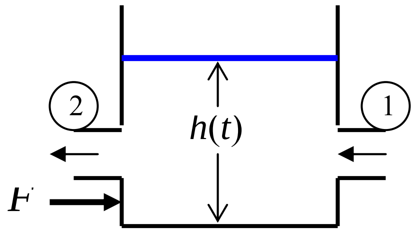


Fig. P3.101

(*a*) What is the velocity leaving section 2?

(*b*) If the tank cross-section is 1.2 m^2, how fast is the water surface $h(t)$ rising or falling?

Solution: First, for water at 20°C, ρ = 998 kg/m^3. (*a*) For a control volume around the tank,

$$\sum F_x = F = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}_2(-V_2) - \dot{m}_1(-V_1)$$

$$\text{or:}\quad 70\,N = [(998)\frac{\pi}{4}(0.04m)^2 V_2](-V_2) + [(998)\frac{\pi}{4}(0.08m)^2(6\frac{m}{s})](6\frac{m}{s})$$

$$= -1.254\,V_2^2 + 180.6\,N\ ,\ \textit{solve}\ V_2 = \sqrt{\frac{70-180.6}{-1.254}} = \mathbf{9.39}\,\frac{m}{s} \quad \textit{Ans.(a)}$$

(*b*) The mass flows at 1 and 2 are *not* equal. The difference in volume flow moves the surface:

$$A_{\text{tank}}\frac{dh}{dt} = Q_1 - Q_2 = \frac{\pi}{4}(0.08m)^2(6\frac{m}{s}) - \frac{\pi}{4}(0.04m)^2(9.39\frac{m}{s})$$

$$\text{or: } (1.2m^2)\frac{dh}{dt} = 0.0302 - 0.0118 = 0.0184\frac{m^3}{s}, \text{ solve } \frac{dh}{dt} \approx +\mathbf{0.0153}\frac{m}{s}\uparrow \quad \textit{Ans.(b)}$$

P3.102 As can often be seen in a kitchen sink when the faucet is running, a high-speed channel flow (V_1, h_1) may "jump" to a low-speed, low-energy condition (V_2, h_2) as in Fig. P3.102. The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to find h_2 and V_2 in terms of (h_1, V_1).

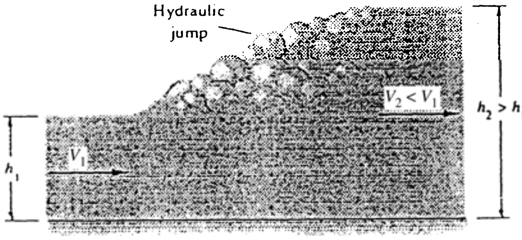


Fig. P3.102

Solution: The CV cuts through sections 1 and 2 and surrounds the jump, as shown. Wall shear is neglected. There are no obstacles. The only forces are due to hydrostatic pressure:

$$\sum F_x = 0 = \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = \dot{m}(V_2 - V_1),$$

$$\text{where } \dot{m} = \rho V_1 h_1 b = \rho V_2 h_2 b$$

$$\text{Solve for } V_2 = \mathbf{V_1 h_1/h_2} \quad \text{and} \quad h_2/h_1 = -\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{1 + 8V_1^2/(gh_1)}} \quad \textit{Ans.}$$

P3.103 Suppose that the solid-propellant rocket of Prob. 3.35 is mounted on a 1000-kg car to propel it up a long slope of 15°. The rocket motor weighs 900 N, which includes 500 N of propellant. If the car starts from rest when the rocket is fired, and if air drag and wheel friction are neglected, estimate the maximum distance that the car will travel up the hill.

Solution: This is a variation of Prob. 3.100, except that "*g*" is now replaced by "*g* sinθ." Recall from Prob. 3.35 that the rocket mass flow is 11.8 kg/s and its exit velocity is 1150 m/s. The rocket fires for $t_b = (500/9.81)/11.8 = 4.32$ sec, and the initial mass is $M_o = (1000 + 900/9.81) = 1092$ kg. Then the differential equation for uphill powered motion is

$$m\frac{dV}{dt} = \dot{m}V_e - mg\sin\theta, \quad m = M_o - \dot{m}t$$

This integrates to: $V(t) = -V_e\ln(1 - \dot{m}t/M_o) - gt\sin\theta$ for $t \le 4.32$ s.

After burnout, the rocket coasts uphill with the usual falling-body formulas with "*g* sinθ." The distance travelled during rocket power is modified from Prob. 3.99:

$$S = (M_oV_e/\dot{m})[1 + (1 - \dot{m}t/M_o)\{\ln(1 - \dot{m}t/M_o) - 1\}] - \frac{1}{2}gt^2\sin\theta$$

Apply these to the given data at burnout to obtain

$$V_{burnout} = -1150\ln(0.9533) - \frac{1}{2}(9.81)\sin 15°(4.32) \approx 44.0 \text{ m/s}$$

$$S_{burnout} = \frac{1092(1150)}{11.8}[1 + 0.9533\{\ln(0.9533) - 1\}] - \frac{1}{2}(9.81)\sin 15°(4.32)^2 \approx 94 \text{ m}$$

The rocket then coasts uphill a distance ΔS such that $V_b^2 = 2g\Delta S\sin\theta$, or $\Delta S = (44.0)^2/[2(9.81)\sin 15°] \approx 381$ m. The total distance travelled is 381 + 94 ≈ **475 m** *Ans.*

P3.104 A rocket is attached to a rigid horizontal rod hinged at the origin as in Fig. P3.104. Its initial mass is M_o, and its exit properties are $\dot{m}$ and V_e relative to the rocket. Set up the differential equation for rocket motion, and solve for the angular velocity $\omega(t)$ of the rod. Neglect gravity, air drag, and the rod mass.

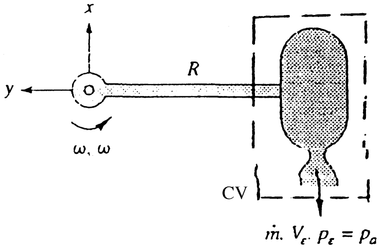


Fig. P3.104

Solution: The CV encloses the rocket and moves at (accelerating) rocket speed Ω(t). The rocket arm is free to rotate, there is no force parallel to the rocket motion. Then we have

$$\sum F_{tangent} = 0 - \int a_{rel}\, dm = \dot{m}(-V_e), \quad \text{or} \quad mR\frac{d\Omega}{dt} = \dot{m}V_e, \quad \text{where } m = M_o - \dot{m}t$$

$$\text{Integrate, with } \Omega = 0 \text{ at } t = 0, \text{ to obtain} \quad \boldsymbol{\Omega = -\frac{V_e}{R}\ln\left(1 - \frac{\dot{m}t}{M_o}\right)} \quad \textit{Ans.}$$

P3.105 Extend Prob. 3.104 to the case where the rocket has a linear air drag force *F* = *cV*, where *c* is a constant. Assuming no burnout, solve for *ω*(*t*) and find the *terminal* angular velocity, i.e., the final motion when the angular acceleration is zero. Apply to the case *M*o = 6 kg, *R* = 3 m, *m* = 0.05 kg/s, *V*e = 1100 m/s, and *c* = 0.075 N·s/m to find the angular velocity after 12 s of burning.

Solution: If linear resistive drag is added to Prob. 3.104, the equation of motion becomes

$$m\frac{d\Omega}{dt} = \frac{\dot{m}V_e}{R} - C\Omega, \quad \text{where } m = M_o - \dot{m}t, \text{ with } \Omega = 0 \text{ at } t = 0$$

The solution is found by separation of variables:

$$\text{If } B = \dot{m}V_e/R, \quad \text{then} \int_0^{\Omega}\frac{d\Omega}{B - C\Omega} = \int_0^{t}\frac{dt}{M_o - \dot{m}t}, \quad \text{or:} \quad \boldsymbol{\Omega = \frac{B}{C}\left[1 - \left(1 - \frac{\dot{m}t}{M_o}\right)^{C/\dot{m}}\right]} \quad \textit{Ans.}\text{ (a)}$$

Strictly speaking, there is no terminal velocity, but if we set the acceleration equal to zero in the basic differential equation, we obtain an estimate $\boldsymbol{\Omega_{term} = mV_e/(RC)}$. *Ans.* (b)

For the given data, at t = 12 s, we obtain the angular velocity

$$\text{At } t = 12 \text{ s:} \quad \Omega = \frac{(0.05)(1100)}{(3.0)(0.075)}\left[1 - \left(1 - \frac{0.05(12)}{6.0}\right)^{\frac{0.075}{0.05}}\right] \approx \mathbf{36}\ \boldsymbol{\frac{rad}{sec}} \quad \textit{Ans.}\text{ (c)}$$

P3.106 Actual air flow past a parachute creates a variable distribution of velocities and directions. Let us model this as a circular air jet, of diameter half the parachute diameter, which is turned completely around by the parachute, as in Fig. P3.106.

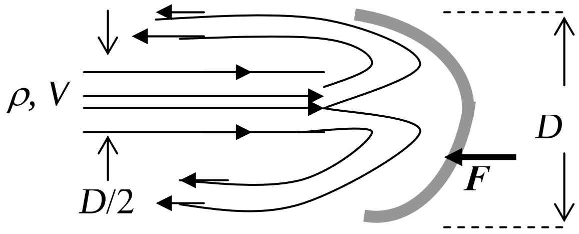


Fig. P3.106

(*a*) Find the force *F* required to support the chute.

(*b*) Express this force as a dimensionless

drag coefficient, $C_D = F/[(1/2)\rho V^2(\pi/4)D^2]$ and compare with Table 7.3.

Solution: This model is crude, compared to velocity-field theory, but gives the right order of magnitude. (*a*) Let the control volume surround the parachute and cut through the oncoming and leaving air streams:

$$\sum F_x = -F = \dot{m}u_{out} - \dot{m}u_{in} = \dot{m}(-V) - \dot{m}(+V) \ ,$$

$$\text{or: } F = 2\,\dot{m}\,V = 2(\rho A_{jet} V)V = 2\rho[\frac{\pi}{4}(\frac{D}{2})^2]V^2 = \boldsymbol{\frac{\pi}{8}\rho D^2 V^2} \qquad Ans.(a)$$

(*b*) Express this approximate result as a dimensionless drag coefficient:

$$C_D = \frac{F}{(1/2)\rho V^2(\pi/4)D^2} = \frac{(\pi/8)\rho V^2 D}{(1/2)\rho V^2(\pi/4)D^2} \approx \mathbf{1.0} \qquad Ans.(b)$$

From Table 7.3, actual measurements show a parachute drag coefficient of about **1.2**. Not bad!

P3.107 The cart in Fig. P3.107 moves at constant velocity $V_o = 12$ m/s and takes on water with a scoop 80 cm wide which dips $h = 2.5$ cm into a pond. Neglect air drag and wheel friction. Estimate the force required to keep the cart moving.

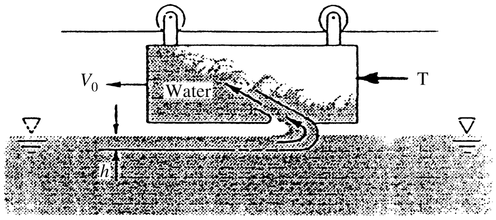


Fig. P3.107

Solution: The CV surrounds the cart and scoop and moves to the left at cart speed V_o. Momentum *within* the cart fluid is neglected. The horizontal force balance is

$$\sum F_x = -\text{Thrust} = -\dot{m}_{scoop}V_{inlet}, \quad \text{but } V_{inlet} = V_o \text{ (water motion relative to scoop)}$$

$$\text{Therefore} \quad \text{Thrust} = \dot{m}V_o = [998(0.025)(0.8)(12)](12) \approx \mathbf{2900\ N} \quad \textit{Ans.}$$

P3.108 A rocket sled of mass *M* is to be decelerated by a scoop, as in Fig. P3.108, which has width *b* into the paper and dips into the water a depth *h*, creating an upward jet at 60°. The rocket thrust is *T* to the left. Let the initial velocity be V_o, and neglect air drag and wheel friction. Find an expression for *V*(*t*) of the sled for (a) $T = 0$ and (b) finite $T \neq 0$.

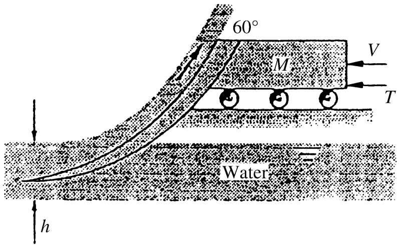


Fig. P3.108

Solution: The CV surrounds the sled and scoop and moves to the *left* at sled speed V(t). Let *x* be positive to the left. The horizontal force balance is

$$\sum F_x = T - M\frac{dV}{dt} = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}(-V\cos\theta) - \dot{m}(-V), \quad \dot{m} = \rho bhV$$

$$\text{or:} \quad M_{sled}\frac{dV}{dt} = T - CV^2, \quad C = \rho bh(1 - \cos\theta)$$

Whether or not thrust T = 0, the variables can be separated and integrated:

$$\text{(a) T = 0:}\quad \int_{V_o}^{V} \frac{dV}{V^2} = -\frac{C}{M}\int_0^t dt,\quad \text{or:}\quad \mathbf{V = \frac{V_o}{1 + CV_o t/M}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) T > 0:}\quad \int_{V_o}^{V} \frac{M\,dV}{T - CV^2} = \int_0^t dt,\quad \text{or:}\quad \mathbf{V = V_{final}\tanh[\boldsymbol{\alpha} t + \boldsymbol{\phi}]} \quad \textit{Ans.}\text{ (b)}$$

where $\quad V_{final} = [T/\rho bg(1 - \cos\theta)]^{1/2}, \quad \alpha = [T\rho bh(1 - \cos\theta)]^{1/2}/M, \quad \phi = \tanh^{-1}(V_o/V_f)$

This solution only applies when $V_o < V_{final}$, which may not be the case for a speedy sled.

P3.109 For the boundary layer flow in Fig. 3.10, let the exit velocity profile, at x = *L* , simulate turbulent flow, $u \approx U_o(y/\delta)^{1/7}$. (*a*) Find a relation between *h* and *δ*. (*b*) Find an expression for the drag force *F* on the plate between 0 and *L*.

Solution: (*a*) Since the upper and lower boundaries of the control volume are streamlines, the mass flow in, at *x*=0, must equal the mass flow out, at *x*=*L*.

$$\int d\dot{m}_{in} = \int_0^h \rho U_o b\,dy \;=\; \rho U_o b\,h = \int d\dot{m}_{out} \;=\; \int_0^\delta \rho U_o(\frac{y}{\delta})^{1/7} b\,dy \;=\; \frac{7}{8}\rho U_o b\,\delta$$

$$\text{after cancellation},\qquad h \;=\; \frac{7}{8}\delta \qquad\qquad \textit{Ans.}(a)$$

(*b*) Instead of blindly using Karman's formula from Example 3.10, derive the drag force in straightforward control volume momentum-integral fashion:

$$\sum F_x \;=-F\;=\;\int u_{out}\,d\dot{m}_{out} \;-\; \int u_{in}\,d\dot{m}_{in} \;=\; \int_0^\delta U_o(\frac{y}{\delta})^{1/7}[\rho U_o(\frac{y}{\delta})^{1/7}]b\,dy - \int_0^h U_o \rho U_o\,b\,dy$$

$$\text{or}:\quad F \;=\; \rho U_o^2\,b\,\delta(\frac{7}{8} - \frac{7}{9}) \quad=\quad \frac{7}{72}\rho U_o^2\,b\,\delta \qquad\qquad \textit{Ans.}(b)$$

P3.110 Repeat Prob. 3.49 by assuming that p_1 is unknown and using Bernoulli's equation with no losses. Compute the new bolt force for this assumption. What is the head loss between 1 and 2 for the data of Prob. 3.49?

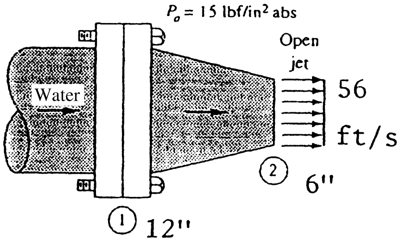


Fig. P3.49

Solution: Use one-dimensional, incompressible continuity to find V_1:

$$V_1 A_1 = V_1 (\pi/4)(1\,ft)^2 = V_2 A_2 = (56\ ft/s)(0.5\ ft)^2\quad,\ \text{or}\quad V_1 = 14\ ft/s$$

Bernoulli's equation with no losses to estimate p_1 with $\Delta z = 0$:

$$\frac{p_1}{\gamma} + \frac{(14)^2}{2(32.2)} \approx \frac{15(144)}{62.4} + \frac{(56)^2}{2(32.2)},\quad \text{solve for } p_{1,\text{ideal}} \approx \mathbf{34.8\ psia}$$

From the x-momentum CV analysis of Prob. 3.49, the bolt force is given by

$$F_{\text{bolts}} = p_{2,\text{gage}} A_2 - \dot{m}(V_2 - V_1)$$

$$= (34.8 - 15)(144)\frac{\pi}{4}(1\text{ ft})^2 - 1.94\left(\frac{\pi}{4}\right)(1\text{ ft})^2(14)(56 - 14) \approx \mathbf{1340\ lbf}\quad \textit{Ans.}$$

We can estimate the friction head loss in Prob. 3.49 from the steady flow energy equation, with p_1 taken to be the value of 38 psia given in that problem:

$$\frac{38(144)}{62.4} + \frac{(14)^2}{2(32.2)} = \frac{15(144)}{62.4} + \frac{(56)^2}{2(32.2)} + h_f,\quad \text{solve for } h_f \approx \mathbf{7.4\ ft}\quad \textit{Ans.}$$

P3.111 Extend the siphon analysis of Ex. 3.22 as follows. Let p_1 = 1 atm and let the fluid be hot water at 60°C. Let $z_{1,2,4}$ be the same, with z_3 unknown. Find the value of z_3 for which the water might begin to vaporize.

Solution: Given p_1 = 101350 Pa and recall that z_1 = 60 cm, z_2 = -25 cm, and z_4 was not needed. Then note that, because of steady-flow one-dimensional continuity, from Ex. 3.22,

$$V_3 = V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2(9.81)[0.6-(-0.25)]} = 4.08\,m/s$$

For cavitation, p_3 should drop down to the vapor pressure of water at 60°C, which from Table A.5 is 19.92 kPa. And, from Table A.3, the density of water at 60°C is 983 kg/m^3. Now write Bernoulli from point 1 to point 3 at the top of the siphon:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3$$

$$\frac{101350}{983} + \frac{0^2}{2} + (9.81)(0.6m) = \frac{19920\,Pa}{983\,kg/m^3} + \frac{(4.08m/s)^2}{2} + 9.81z_3$$

$$103.1 + 0 + 5.9 = 20.3 + 8.3 + 9.81z_3 \;,\quad Solve \quad z_3 \approx \frac{80.4}{9.81} = \mathbf{8.2\ m} \quad Ans.$$

That's pretty high, so the writer does not think cavitation is a problem with this siphon.

P3.112 A jet of alcohol strikes the vertical plate in Fig. P3.112. A force $F \approx 425$ N is required to hold the plate stationary. Assuming there are no losses in the nozzle, estimate (a) the mass flow rate of alcohol and (b) the absolute pressure at section 1.

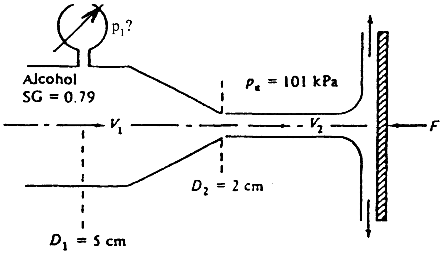


Fig. 3.112

Solution: A momentum analysis of the plate (e.g. Prob. 3.40) will give

$$F = \dot{m}V_2 = \rho A_2 V_2^2 = 0.79(998)\frac{\pi}{4}(0.02)^2 V_2^2 = 425\text{ N},$$

$$\text{solve for } V_2 \approx 41.4\text{ m/s}$$

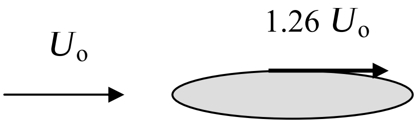
whence $\dot{m} = 0.79(998)(\pi/4)(0.02)^2(41.4) \approx \mathbf{10.3\ kg/s}$ *Ans.* (a)

We find V_1 from mass conservation and then find p_1 from Bernoulli with no losses:

$$\text{Incompressible mass conservation:}\quad V_1 = V_2(D_2/D_1)^2 = (41.4)\left(\frac{2}{5}\right)^2 \approx 6.63 \text{ m/s}$$

$$\text{Bernoulli, } z_1 = z_2:\quad p_1 = p_2 + \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = 101000 + \frac{0.79(998)}{2}[(41.4)^2 - (6.63)^2]$$
$$\approx \mathbf{760,000\ Pa} \quad \textit{Ans. } \text{(b)}$$

P3.113 An airplane is flying at 300 mi/h at 4000 m standard altitude. As is typical, the air velocity relative to the upper surface of the wing, near its maximum thickness, is 26 percent higher than the plane's velocity. Using Bernoulli's equation, calculate the absolute pressure at this point on the wing. Neglect elevation changes and compressibility.



Solution: Fix the frame of steady flow relative to the wing. Let point 1 be the oncoming stream and point 2 be the maximum thickness point. From Table A.5, at 4000 m, p = 61,633 Pa, and ρ = 0.8191 kg/m^3. Convert U_1 = 300 mi/h to 134 m/s. Then the velocity U_2 at the max thickness point is 1.26(134) = 169 m/s. Then, from the figure,

$$p_1 + \frac{1}{2}\rho U_1^2 \approx (61633) + \frac{1}{2}(0.8191)(134)^2 = p_2 + \frac{1}{2}\rho U_2^2 = p_2 + \frac{1}{2}(0.8191)(169)^2$$

$$\text{Solve for } \quad p_2 = \mathbf{57,300}\ Pa \text{ on the upper surface} \qquad \textit{Ans.}$$

If the elevation change were accounted for, the answer would differ by *less than one* Pascal.

P3.114 Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as in Fig. P3.114. The force required to hold the plate steady is 70 N. Assuming frictionless one-dimensional flow, estimate (a) the velocities at sections (1) and (2); (b) the mercury manometer reading *h*.

Solution: (a) First examine the momentum of the jet striking the plate,

$$\sum F = F = -\dot{m}_{in}u_{in} = -\rho A_2 V_2^2$$

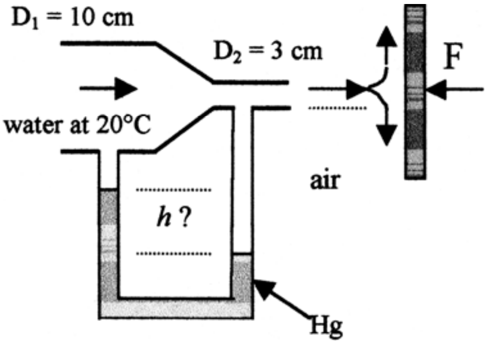


Fig. P3.114

$$70\ N = -(998)\left(\frac{\pi}{4}\right)(0.03^2)(V_2^2) \quad \mathbf{V_2 = 9.96\ m/s} \quad \textit{Ans. (a)}$$

$$\textit{Then} \quad V_1 = \frac{V_2 A_2}{A_1} = \frac{(9.96)\left(\frac{\pi}{4}\right)(0.03^2)}{\frac{\pi}{4}(0.1^2)} \quad \textit{or} \quad \mathbf{V_1 = 0.9\ m/s} \quad \textit{Ans. (a)}$$

(b) Applying Bernoulli,

$$p_2 - p_1 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \frac{1}{2}(998)(9.96^2 - 0.9^2) = 49{,}100\ Pa$$

And from our manometry principles,

$$h = \frac{\Delta p}{\Delta(\rho g)} = \frac{49{,}100}{(133{,}100 - 9790)} \approx \mathbf{0.40\ m} \quad \textit{Ans. (b)}$$

P3.115 A free liquid jet, as in Fig. P3.115, has constant ambient pressure and small losses; hence from Bernoulli's equation $z + V^2/(2g)$ is constant along the jet. For the fire nozzle in the figure, what are (a) the minimum and (b) the maximum values of θ for which the water jet will clear the corner of the building? For which case will the jet velocity be higher when it strikes the roof of the building?

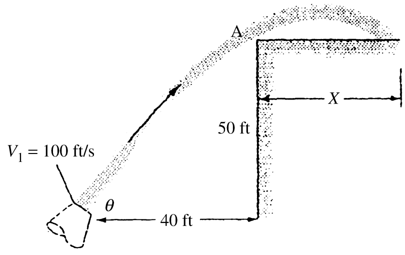


Fig. P3.115

Solution: The two extreme cases are when the jet just touches the corner A of the building. For these two cases, Bernoulli's equation requires that

$$V_1^2 + 2gz_1 = (100)^2 + 2g(0) = V_A^2 + 2gz_A = V_A^2 + 2(32.2)(50), \quad \text{or:} \quad V_A = 82.3\ \frac{\text{ft}}{\text{s}}$$

The jet moves like a frictionless particle as in elementary particle dynamics:

$$\text{Vertical motion:} \quad z = (V_1 \sin\theta)\,t - \frac{1}{2}gt^2; \quad \text{Horizontal motion:} \quad x = (V_1 \cos\theta)\,t$$

Eliminate "t" between these two and apply the result to point A:

$$z_A = 50 = x_A \tan\theta - \frac{g x_A^2}{2V_1^2 \cos^2\theta} = 40 \tan\theta - \frac{(32.2)(40)^2}{2(100)^2 \cos^2\theta}; \quad \text{clean up and rearrange:}$$

$$\tan\theta = 1.25 + 0.0644 \sec^2\theta, \quad \text{solve for } \theta = \mathbf{85.94°} \quad \textit{Ans.}\ \text{(a)} \quad \text{and} \quad \mathbf{55.40°} \quad \textit{Ans.}\ \text{(b)}$$

Path (b) is shown in the figure, where the jet just grazes the corner A and goes over the top of the roof. Path (a) goes nearly straight up, to z = 155 ft, then falls down to pt. A. In both cases, the velocity when the jet strikes point A is the same, 82.3 ft/s.

P3.116 For the container of Fig. P3.116 use Bernoulli's equation to derive a formula for the distance *X* where the free jet leaving horizontally will strike the floor, as a function of *h* and *H*. For what ratio *h/H* will *X* be maximum? Sketch the three trajectories for *h/H* = 0.25, 0.5, and 0.75.

Solution: The velocity out the hole and the time to fall from hole to ground are given by

$$V_o = \sqrt{2g(H-h)} \qquad t_{fall} = \sqrt{2h/g}$$

Then the distance travelled horizontally is

$$X = V_o t_{fall} = \mathbf{2\sqrt{h(H-h)}} \quad \textit{Ans.}$$

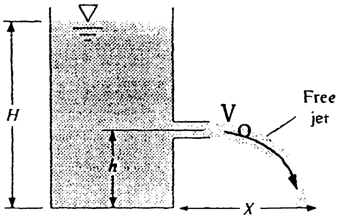
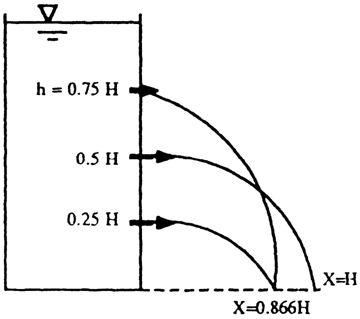
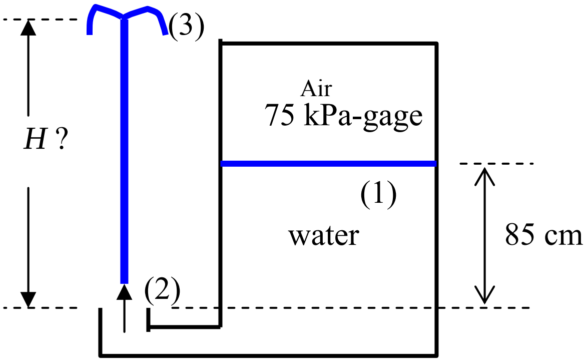


Fig. P3.116



Maximum X occurs at **h = H/2**, or **Xmax = H**. When h = 0.25H or 0.75H, the jet travels out to X = 0.866H. These three trajectories are shown in the sketch on the previous page.



P3.117 Water at 20°C, in the pressurized tank of Fig. P3.117, flows out and creates a vertical jet as shown. Assuming steady frictionless flow, determine the height *H* to which the jet rises.

Fig. P3.117

Solution: This is a straightforward Bernoulli problem. Let the water surface be (1), the exit plane be (2), and the top of the vertical jet be (3). Let $z_2 = 0$ for convenience. If we are clever, we can bypass (2) and write Bernoulli directly from (1) to (3):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + z_3 \quad , \quad \text{or:}$$

$$\frac{75000}{(9.81)(998)} + 0 - 0.85m = 0 + 0 + H$$

$$\textit{Solve} \qquad H = 7.66m + 0.85m = \mathbf{8.51\,m} \qquad \textit{Ans.}$$

If we took an intermediate step from (1) to (2), we would find $V_2^2/2g = 8.51$ m, and then going from (2) to (3) would convert the velocity head into pure elevation, because $V_3 = 0$.

P3.118 Bernoulli's 1738 treatise *Hydrodynamica* contains many excellent sketches of flow patterns. One, however, redrawn here as Fig. P3.118, seems physically misleading. What is wrong with the drawing?

Solution: If friction is neglected and the exit pipe is fully open, then pressure in the closed "piezometer" tube would be atmospheric and the fluid would not rise at all in the tube. The open jet coming from the hole in the tube would have $V \approx \sqrt{(2gh)}$ and would rise up to nearly the same height as the water in the tank.

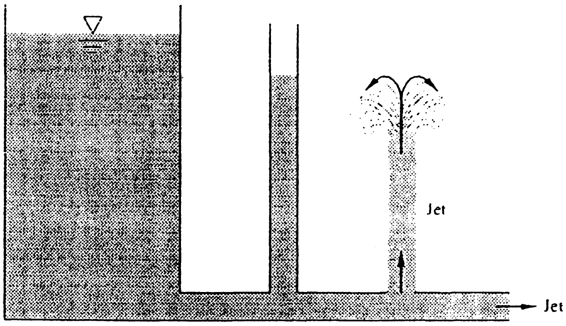
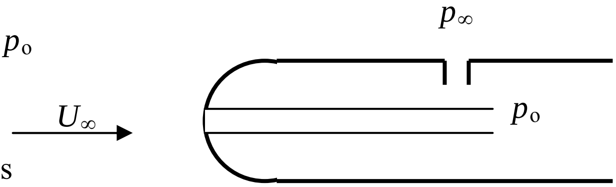


Fig. P3.118

P3.119 A long fixed tube with a rounded nose, aligned with an oncoming flow, can be used to measure velocity. Measurements are made of the pressure at (1) the front nose and (2) a hole in the side of the tube further along, where the pressure nearly equals stream pressure. (*a*) Make a sketch of this device and show how the velocity is calculated. (*b*) For a particular sea-level air flow, the difference between nose pressure and side pressure is 1.5 lbf/in^2. What is the air velocity, in mi/h?

Solution: (*a*) The front nose measures p_o and the side hole measures the stream pressure p_∞. With no elevation changes, the Bernoulli equation predicts



$$U_\infty = \sqrt{\frac{2(p_o - p_\infty)}{\rho}} \qquad \textit{Ans.}(a)$$

The device is, of course, called a *Pitot-static tube* and is in wide use in fluids engineering. (*b*) For sea-level conditions, take $\rho_{air} = 1.2255$ kg/m^3. Convert 1.5 lbf/in^2 to 10342 Pa. Work the whole problem in SI units and convert at the end:

$$U_\infty = \sqrt{\frac{2(p_o - p_\infty)}{\rho}} = \sqrt{\frac{2(10342 Pa)}{1.2255\, kg/m^3}} = 130\, m/s = \mathbf{291}\ \frac{mi}{h} \quad Ans.(b)$$

P3.120 The manometer fluid in Fig. P3.120 is mercury. Estimate the volume flow in the tube if the flowing fluid is (a) gasoline and (b) nitrogen, at 20°C and 1 atm.

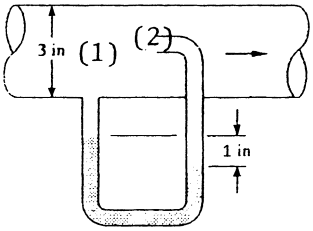


Fig. P3.120

Solution: For gasoline (a) take ρ = 1.32 slug/ft^3. For nitrogen (b), R ≈ 297 J/kg·°C and ρ = p/RT = (101350)/[(297)(293)] ≈ 1.165 kg/m^3 = 0.00226 slug/ft^3. For mercury, take ρ ≈ 26.34 slug/ft^3. The pitot tube (2) reads stagnation pressure, and the wall hole (1) reads static pressure. Thus Bernoulli's relation becomes, with $\Delta z = 0$,

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2, \quad \text{or} \quad V_1 = \sqrt{2(p_2 - p_1)/\rho}$$

The pressure difference is found from the manometer reading, for each fluid in turn:

$$\text{(a) Gasoline:} \quad \Delta p = (\rho_{Hg} - \rho)\gamma\eta = (26.34 - 1.32)(32.2)(1/12\ \text{ft}) \approx 67.1\ \text{lbf/ft}^2$$

$$V_1 = [2(67.1)/1.32]^{1/2} = 10.1\ \frac{\text{ft}}{\text{s}}, \quad Q = V_1A_1 = (10.1)\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2 = \mathbf{0.495}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b) N}_2\text{:} \quad \Delta p = (\rho_{Hg} - \rho)gh = (26.34 - 0.00226)(32.2)(1/12) \approx 70.7\ \text{lbf/ft}^2$$

$$V_1 = [2(70.7)/0.00226]^{1/2} = 250\ \frac{\text{ft}}{\text{s}}, \quad Q = V_1A_1 = (250)\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2 \approx \mathbf{12.3}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(b)}$$

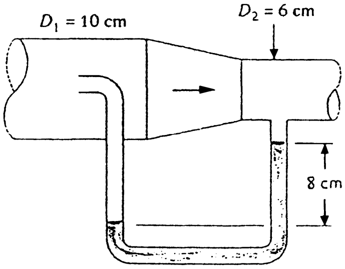


Fig. P3.121

P3.121 In Fig. P3.121 the flowing fluid is CO2 at 20°C. Neglect losses. If p_1 = 170 kPa and the manometer fluid is Meriam red oil (SG = 0.827), estimate (a) p_2 and (b) the gas flow rate in m^3/h.

Solution: Estimate the CO2 density as $\rho = p/RT = (170000)/[189(293)] \approx 3.07\ \text{kg/m}^3$. The manometer reading gives the downstream pressure:

$$p_1 - p_2 = (\rho_{oil} - \rho_{CO_2})gh = [0.827(998) - 3.07](9.81)(0.08) \approx 645\ \text{Pa}$$

$$\text{Hence}\quad p_2 = 170{,}000 - 645 \approx \mathbf{169400\ Pa} \quad \textit{Ans.}\ \text{(a)}$$

Now use Bernoulli to find V2, assuming p1 ≈ stagnation pressure (V1 = 0):

$$p_1 + \frac{1}{2}\rho(0)^2 \approx p_2 + \frac{1}{2}\rho V_2^2,$$

$$\text{or:}\quad V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} = \sqrt{\frac{2(645)}{3.07}} \approx 20.5\ \frac{m}{s}$$

$$\text{Then}\quad Q = V_2 A_2 = (20.5)(\pi/4)(0.06)^2 = 0.058\ m^3/s \approx \mathbf{209\ \frac{m^3}{hr}} \quad \textit{Ans.}\ \text{(b)}$$

P3.122 The cylindrical water tank in Fig. P3.122 is being filled at a volume flow Q_1 = 1.0 gal/min, while the water also drains from a bottom hole of diameter d = 6 mm. At time t = 0, h = 0. Find (a) an expression for $h(t)$ and (b) the eventual maximum water depth h_{max}. Assume that Bernoulli's steady-flow equation is valid.

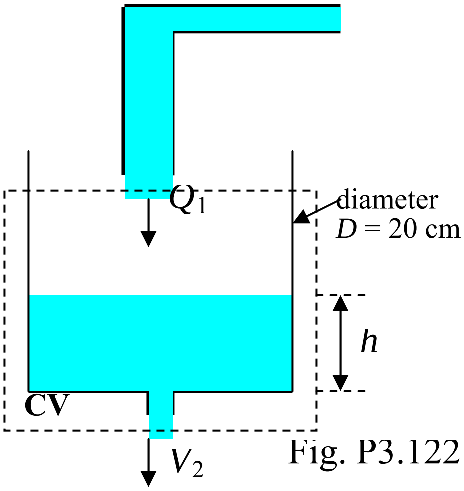


Fig. P3.122

Solution: Bernoulli predicts that $V_2 \approx \sqrt{(2gh)}$.
Convert Q_1 = 6.309E-5 m^3/s. A control volume around the tank gives the mass balance:

$$\frac{dm}{dt}\Big|_{system} = 0 = \frac{d}{dt}(Ah) - Q_1 + A_2\sqrt{2gh}\ , \text{ where } A = \frac{\pi}{4}D^2 \text{ and } A_2 = \frac{\pi}{4}d^2$$

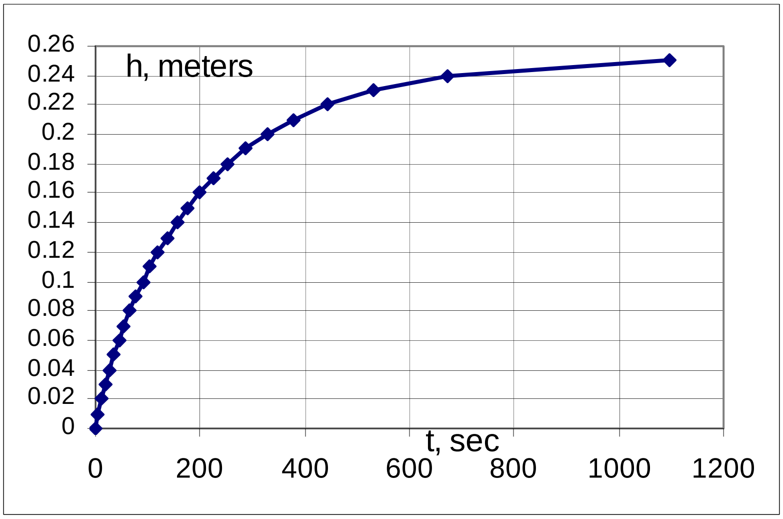
Rearrange, separate the variables, and integrate:

$$\int_0^{h(t)} \frac{dh}{Q_1 - A_2\sqrt{2gh}} = \frac{1}{A}\int_0^t dt$$

(*a*) The integration is a bit tricky and laborious. Here is the writer's result:

$$t = \frac{2Q_1 A}{\alpha^2}\ln\left(\frac{Q_1}{Q_1 - \alpha\sqrt{h}}\right) - \frac{2A\sqrt{h}}{\alpha}\ , \quad \text{where } \alpha = A_2\sqrt{2g} \qquad \textit{Ans.}(a)$$

(*a*) A graph of *h* versus *t* for the particular given data is as follows:



(*b*) The water level rises fast and then slower and is asymptotic to the value h_{max} = 0.254 m. This is when the outflow through the hole exactly equals the inflow from the pipe:

$$Q_1 = A_2\sqrt{2gh_{max}}\ , \quad or: \quad 6.309E-5\frac{m^3}{s} = \frac{\pi}{4}(0.006m)^2\sqrt{2(9.81)h_{max}}$$

$$\textit{Solve for } h_{max} = \mathbf{0.254\,m} \qquad \textit{Ans.}(b)$$

P3.123 The air-cushion vehicle in Fig. P3.123 brings in sea-level standard air through a fan and discharges it at high velocity through an annular skirt of 3-cm clearance. If the vehicle weighs 50 kN, estimate (a) the required airflow rate and (b) the fan power in kW.

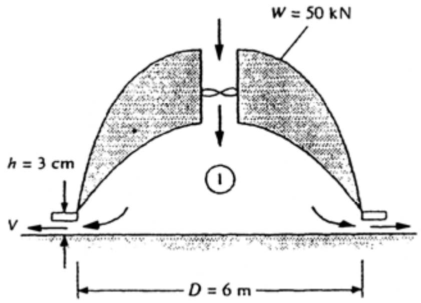


Fig. P3.123

Solution: The air inside at section 1 is nearly stagnant (V ≈ 0) and supports the weight and also drives the flow out of the interior into the atmosphere:

$$p_1 \approx p_{o1}:\quad p_{o1} - p_{atm} = \frac{\text{weight}}{\text{area}} = \frac{50{,}000\text{ N}}{\pi(3\text{ m})^2} = \frac{1}{2}\rho V_{exit}^2 = \frac{1}{2}(1.205)V_{exit}^2 \approx 1768\text{ Pa}$$

$$\text{Solve for}\quad V_{exit} \approx 54.2\text{ m/s, whence}\quad Q_e = A_eV_e = \pi(6)(0.03)(54.2) = 30.6\ \frac{m^3}{s}$$

Then the power required by the fan is $P = Q_e\Delta p = (30.6)(1768) \approx$ **54000 W** *Ans.*

P3.124 A necked-down section in a pipe flow, called a *venturi*, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P3.124. Using Bernoulli's equation with no losses, derive an expression for the velocity V_1 which is just sufficient to bring reservoir fluid into the throat.

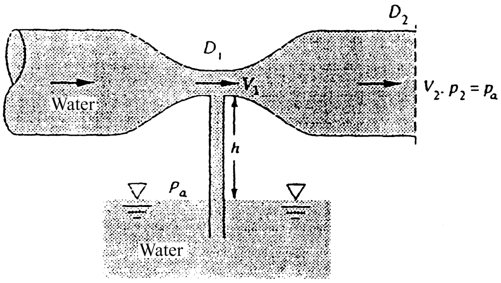


Fig. P3.124

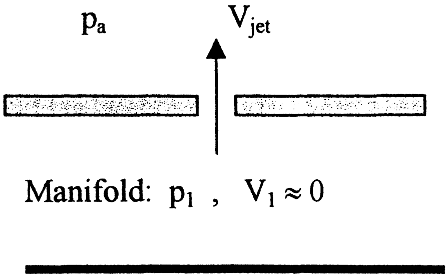
Solution: Water will begin to aspirate into the throat when $p_a - p_1 = \rho gh$. Hence:

$$\text{Volume flow: } V_1 = V_2(D_2/D_1)^2; \quad \text{Bernoulli } (\Delta z = 0)\text{: } p_1 + \frac{1}{2}\rho V_1^2 \approx p_{atm} + \frac{1}{2}\rho V_2^2$$

$$\text{Solve for } p_a - p_1 = \frac{\rho}{2}(\alpha^4 - 1)V_2^2 \ge \rho gh, \quad \alpha = \frac{D_2}{D_1}, \quad \text{or: } \mathbf{V_2 \ge \sqrt{\frac{2gh}{\alpha^4 - 1}}} \quad \textit{Ans.}$$

$$\textit{Similarly, } \mathbf{V_{1,\,min}} = \boldsymbol{\alpha^2 V_{2,\,min}} = \mathbf{\sqrt{\frac{2gh}{1-(D_1/D_2)^4}}} \quad \textit{Ans.}$$

P3.125 Suppose you are designing a 3 × 6-ft air-hockey table, with 1/16-inch-diameter holes spaced every inch in a rectangular pattern (2592 holes total), the required jet speed from each hole is 50 ft/s. You must select an appropriate blower. Estimate the volumetric flow rate (in ft^3/min) and pressure rise (in psi) required. *Hint*: Assume the air is stagnant in the large manifold under the table surface, and neglect frictional losses.



Solution: Assume an air density of about sea-level, **0.00235 slug/ft^3**. Apply Bernoulli's equation through any single hole, as in the figure:

$$p_1 + \cancel{\frac{\rho}{2}V_1^2} = p_a + \frac{\rho}{2}V_{jet}^2, \quad or:$$

$$\Delta p_{required} = p_1 - p_a = \frac{\rho}{2}V_{jet}^2 = \frac{0.00235}{2}(50)^2 = 2.94\ \frac{lbf}{ft^2} = \mathbf{0.0204\ \frac{lbf}{in^2}} \quad \textit{Ans.}$$

The total volume flow required is

$$Q = VA_{1-hole}(\#\ of\ holes) = \left(50\ \frac{ft}{s}\right)\frac{\pi}{4}\left(\frac{1/16}{12}\ ft\right)^2(2592\ holes)$$

$$= 2.76\ \frac{ft^3}{s} = \mathbf{166\ \frac{ft^3}{min}} \quad \textit{Ans.}$$

It wasn't asked, but the power required would be $P = Q\Delta p = (2.76\ \text{ft}^3/\text{s})(2.94\ \text{lbf/ft}^2) =$ 8.1 ft·lbf/s, or about 11 watts.

P3.126 The liquid in Fig. P3.126 is kerosine at 20°C. Estimate the flow rate from the tank for (a) no losses and (b) pipe losses $h_f \approx 4.5V^2/(2g)$.

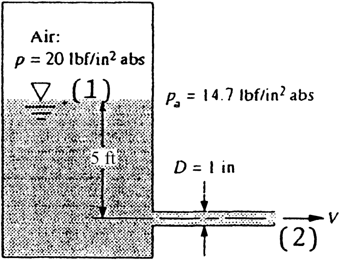


Fig. P3.126

Solution: For kerosine let $\gamma = 50.3$ lbf/ft³. Let (1) be the surface and (2) the exit jet:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f, \quad \text{with } z_2 = 0 \text{ and } V_1 \approx 0,\ h_f = K\frac{V_2^2}{2g}$$

$$\text{Solve for } \frac{V_2^2}{2g}(1+K) = z_1 + \frac{p_1 - p_2}{\gamma} = 5 + \frac{(20-14.7)(144)}{50.3} \approx 20.2 \text{ ft}$$

We are asked to compute two cases (a) no losses; and (b) substantial losses, K ≈ 4.5:

$$\text{(a) } K = 0: \quad V_2 = \left[\frac{2(32.2)(20.2)}{1+0}\right]^{1/2} = 36.0\ \frac{\text{ft}}{\text{s}}, \quad Q = 36.0\frac{\pi}{4}\left(\frac{1}{12}\right)^2 \approx \mathbf{0.197}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans. (a)}$$

$$\text{(b) } K = 4.5: \quad V_2 = \sqrt{\frac{2(32.2)(23.0)}{1+4.5}} = 16.4\ \frac{\text{ft}}{\text{s}}, \quad Q = 16.4\frac{\pi}{4}\left(\frac{1}{12}\right)^2 \approx \mathbf{0.089}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans. (b)}$$

P3.127 An open water jet exits from a nozzle into sea-level air, as shown, and strikes a stagnation tube. If the centerline pressure at section (1) is 110 kPa and losses are neglected, estimate (a) the mass flow in kg/s; and (b) the height *H* of the fluid in the tube.

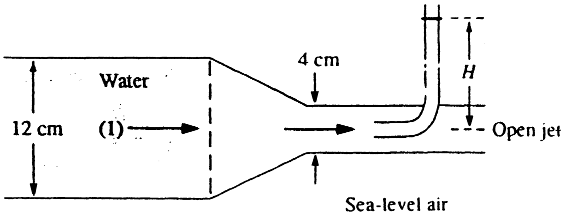


Fig. P3.127

Solution: Writing Bernoulli and continuity between pipe and jet yields jet velocity:

$$p_1 - p_a = \frac{\rho}{2}V_{jet}^2\left[1-\left(\frac{D_{jet}}{D_1}\right)^4\right] = 110000 - 101350 = \frac{998}{2}V_{jet}^2\left[1-\left(\frac{4}{12}\right)^4\right],$$

$$solve \quad V_{jet} = \mathbf{4.19}\ \frac{\mathrm{m}}{\mathrm{s}}$$

Then the mass flow is $\dot{\mathbf{m}} = \rho A_{jet}V_{jet} = 998\frac{\pi}{4}(0.04)^2(4.19) = \mathbf{5.25}\ \frac{\mathbf{kg}}{\mathbf{s}}$ *Ans.* (a)

(b) The water in the stagnation tube will rise above the jet surface by an amount equal to the stagnation pressure head of the jet:

$$\mathbf{H} = R_{jet} + \frac{V_{jet}^2}{2g} = 0.02\ \mathrm{m} + \frac{(4.19)^2}{2(9.81)} = 0.02 + 0.89 = \mathbf{0.91\ m} \quad \textit{Ans.}\ \text{(b)}$$

P3.128 A *venturi meter*, shown in Fig. P3.128, is a carefully designed constriction whose pressure difference is a measure of the flow rate in a pipe. Using Bernoulli's equation for steady incompressible flow with no losses, show that the flow rate Q is related to the manometer reading h by

$$Q = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2gh(\rho_M-\rho)}{\rho}}$$

where ρM is the density of the manometer fluid.

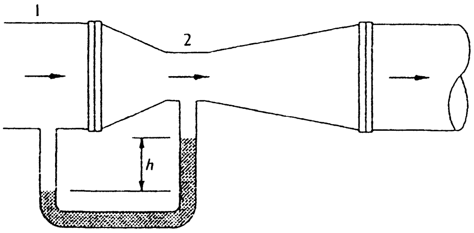


Fig. P3.128

Solution: First establish that the manometer reads the pressure difference between 1 and 2:

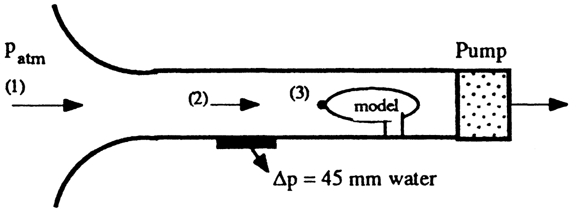
$$\mathrm{p}_1 - \mathrm{p}_2 = (\rho_\mathrm{M} - \rho)\mathrm{gh} \qquad (1)$$

Then write incompressible Bernoulli's equation and continuity between (1) and (2):

$$(\Delta z = 0):\quad \frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} \quad \text{and} \quad V_2 = V_1(D_1/D_2)^2, \quad Q = A_1V_1 = A_2V_2$$

Eliminate V_2 and $(p_1 - p_2)$ from (1) above: $\mathbf{Q} = \dfrac{\mathbf{A_2\sqrt{2gh(\rho_M - \rho)/\rho}}}{\mathbf{\sqrt{1-(D_2/D_1)^4}}}$ *Ans.*

P3.129 A wind tunnel draws in sea-level standard air from the room and accelerates it into a 1-m by 1-m test section. A pressure transducer in the test section wall measures Δp = 45 mm water between inside and outside. Estimate (a) the test section velocity in mi/hr; and (b) the absolute pressure at the nose of the model.



Solution: (a) First apply Bernoulli from the atmosphere (1) to (2), using the known Δp:

$$p_a - p_2 = 45\ \text{mm H}_2\text{O} = 441\ \text{Pa};\quad \rho_a = 1.225\ \text{kg/m}^3;\quad p_1 + \frac{\rho}{2}V_1^2 \approx p_2 + \frac{\rho}{2}V_2^2$$

Since $V_1 \approx 0$ and $p_1 = p_a$, we obtain $V_2 = \sqrt{\dfrac{2\,\Delta p}{\rho}} = \sqrt{\dfrac{2(441)}{1.225}} = 26.8\,\dfrac{m}{s} = \mathbf{60\,\dfrac{mi}{hr}}$ *Ans.* (a)

(b) Bernoulli from 1 to 3: both velocities = 0, so $p_{nose} = p_a \approx$ **101350 Pa**. *Ans.* (b)

P3.130 In Fig. P3.130 the fluid is gasoline at 20°C at a weight flux of 120 N/s. Assuming no losses, estimate the gage pressure at section 1.

Solution: For gasoline, $\rho = 680$ kg/m^3. Compute the velocities from the given flow rate:

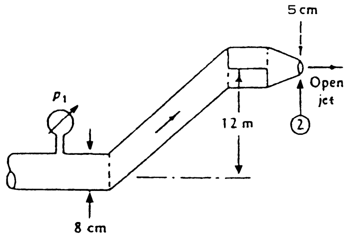


Fig. P3.130

$$Q = \frac{\dot{W}}{\rho g} = \frac{120\text{ N/s}}{680(9.81)} = 0.018\ \frac{\text{m}^3}{\text{s}},$$

$$V_1 = \frac{0.018}{\pi(0.04)^2} = 3.58\ \frac{\text{m}}{\text{s}};\quad V_2 = \frac{0.018}{\pi(0.025)^2} = 9.16\ \frac{\text{m}}{\text{s}}$$

Now apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2,\quad \text{or:}\quad \frac{p_1}{\rho} + \frac{(3.58)^2}{2} + 0 \approx \frac{0(\text{gage})}{680} + \frac{(9.16)^2}{2} + 9.81(12)$$

$$\text{Solve for}\quad p_1 \approx \mathbf{104{,}000\ Pa\,(gage)}\quad \textit{Ans.}$$

P3.131 In Fig. P3.131 both fluids are at 20°C. If V_1 = 1.7 ft/s and losses are neglected, what should the manometer reading *h* ft be?

Solution: By continuity, establish V_2:

$$V_2 = V_1(D_1/D_2)^2 = 1.7(3/1)^2 = 15.3\ \frac{\text{ft}}{\text{s}}$$

Now apply Bernoulli between 1 and 2 to establish the pressure at section 2:

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2,$$

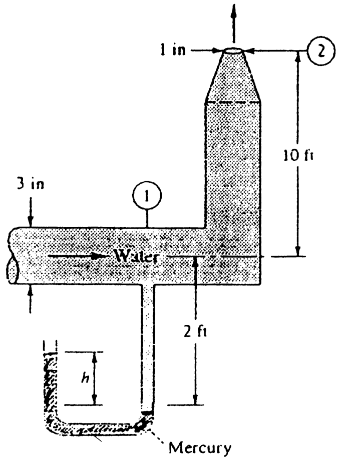


Fig. P3.131

$$\text{or:}\quad p_1 + (1.94/2)(1.7)^2 + 0 \approx 0 + (1.94/2)(15.3)^2 + (62.4)(10), \quad p_1 = 848 \text{ psf}$$

This is gage pressure. Now the manometer *reads* gage pressure, so

$$p_1 - p_a = 848\ \frac{\text{lbf}}{\text{ft}^2} = (\rho_{\text{merc}} - \rho_{\text{water}})gh = (846 - 62.4)h, \quad \text{solve for } h \approx \mathbf{1.08\ ft} \quad \textit{Ans.}$$

P3.132 Extend the siphon analysis of Ex. 3.22 to account for friction in the tube, as follows. Let the friction head loss in the tube be correlated as $5.4(V_{\text{tube}})^2/(2g)$, which approximates turbulent flow in a 2-m-long tube. Calculate the exit velocity in m/s and the volume flow rate in cm^3/s. We repeat the sketch of Ex. 3.22 for convenience.

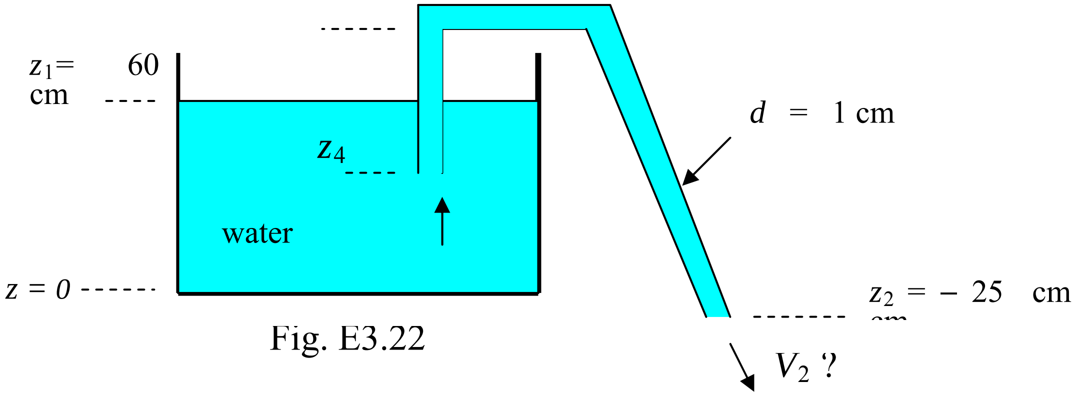


Fig. E3.22

Solution: Write the steady flow energy equation from the water surface (1) to the exit (2):

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f\ ,\quad \text{where}\ h_f=5.4\frac{V_{tube}^2}{2g}$$

The tube area is constant, hence $V_{\text{tube}} = V_2$. Also, $p_1 = p_2$ and $V_1 \approx 0$. Thus we obtain

$$z_1-z_2=0.6m-(-0.25m)=0.85m=\frac{V_2^2}{2g}(1+5.4)$$

$$\textit{Solve}\quad V_2=\sqrt{\frac{2(9.81m/s^2)(0.85m)}{1+5.4}}=\mathbf{1.61\frac{m}{s}}\qquad \textit{Ans.}$$

$$\textit{and}\quad Q=V_2A_2=(1.61\frac{m}{s})\frac{\pi}{4}(0.01m)^2=0.000167\frac{m^3}{s}=\mathbf{127\frac{cm^3}{s}}\quad \textit{Ans.}$$

Tube friction has reduced the flow rate by more than 60%.

P3.133 If losses are neglected in Fig. P3.133, for what water level *h* will the flow begin to form vapor cavities at the throat of the nozzle?

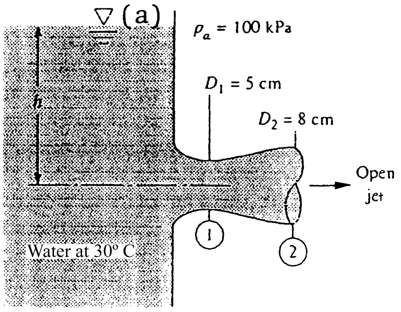


Fig. P3.133

Solution: Applying Bernoulli from (a) to (2) gives Torricelli's relation: $V2 = \surd(2gh)$. Also,

$$V_1 = V_2(D_2/D_1)^2 = V_2(8/5)^2 = 2.56V_2$$

Vapor bubbles form when p_1 reaches the vapor pressure at 30°C, $p_{vap} \approx 4242$ Pa (from Table A.5), while $\rho \approx 996\ \text{kg/m}^3$ at 30°C (Table A.1). Apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1 \approx \frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2, \quad \text{or:} \quad \frac{4242}{996}+\frac{(2.56V_2)^2}{2}+0 \approx \frac{100000}{996}+\frac{V_2^2}{2}+0$$

$$\text{Solve for} \quad V_2^2 = 34.62 = 2gh, \quad \text{or} \quad h = 34.62/[2(9.81)] \approx \mathbf{1.76\ m} \quad \textit{Ans.}$$

P3.134 For the 40°C water flow in Fig. P3.134, estimate the volume flow through the pipe, assuming no losses; then explain what is wrong with this seemingly innocent question. If the actual flow rate is Q = 40 m^3/h, compute (a) the head loss in ft and (b) the constriction diameter D which causes cavitation, assuming that the throat divides the head loss equally and that changing the constriction causes no additional losses.

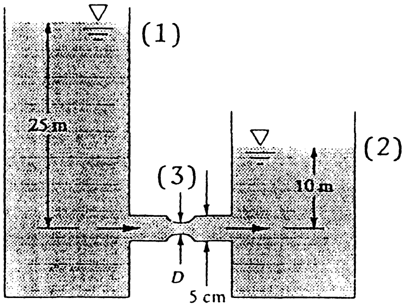


Fig. P3.134

Solution: Apply Bernoulli between 1 and 2:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \approx \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2, \quad \text{or:} \quad 0+0+25 \approx 0+0+10, \quad \text{or:} \quad \mathbf{25 = 10\ ??}$$

This is madness, what happened? The answer is that this problem cannot be free of losses. There is a **15-m loss** as the pipe-exit jet dissipates into the downstream reservoir. *Ans.* (a)
(b) Examining analysis (a) shows that the head loss is 15 meters. For water at 40°C, the vapor pressure is 7375 Pa (Table A.5), and the density is 992 kg/m^3 (Table A.1). Now write Bernoulli between (1) and (3), assuming a head loss of 15/2 = 7.5 m:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1 = \frac{p_3}{\rho}+\frac{V_3^2}{2}+gz_3+\frac{g}{2}h_{f,\text{total}}, \quad \text{where } V_3 = \frac{Q}{A_3} = \frac{40/3600}{(\pi/4)D^2} = \frac{0.0141}{D^2}$$

$$\text{Thus} \quad \frac{101350}{992}+0+9.81(25) \approx \frac{7375}{992}+\frac{(0.0141/D^2)^2}{2}+0+(9.81)(7.5)$$

$$\text{Solve for} \quad D^4 \approx 3.75\text{E–}7\ \text{m}^4, \quad \text{or} \quad D \approx 0.0248\ m \approx 25\ mm \quad \textit{Ans.(b)}$$

This corresponds to $V_3 \approx 23$ m/s.

P3.135 The 35°C water flow of Fig. P3.135 discharges to sea-level standard atmosphere. Neglecting losses, for what nozzle diameter *D* will cavitation begin to occur? To avoid cavitation, should you increase or decrease *D* from this critical value?

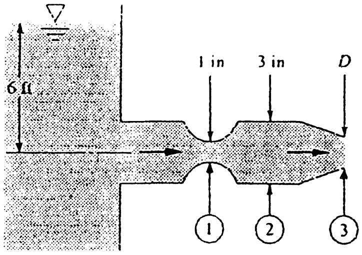


Fig. P3.135

Solution: At 35°C the vapor pressure of water is approximately 5600 Pa (Table A.5). Bernoulli from the surface to point 3 gives the Torricelli result $V_3 = \sqrt{(2gh)} = \sqrt{[2(32.2)(6)]} \approx 19.66$ ft/s. We can ignore section 2 and write Bernoulli from (1) to (3), with $p_1 = p_{vap}$ and $\Delta z = 0$:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2} \approx \frac{p_2}{\rho}+\frac{V_2^2}{2}, \quad \text{or:} \quad \frac{117}{1.93}+\frac{V_1^2}{2} \approx \frac{2116}{1.93}+\frac{V_3^2}{2},$$

$$\text{but also } V_1 = V_3\left(\frac{D}{1/12}\right)^2$$

Eliminate V_1 and introduce $V_3 = 19.66\ \frac{ft}{s}$ to obtain $D^4 = 3.07E{-}4$, $\mathbf{D \approx 0.132\ ft}$ *Ans.*

To avoid cavitation, we would keep **D < 0.132 ft**, which will keep $p_1 > p_{vapor}$.

P3.136 Air, assumed frictionless, flows through a tube, exiting to sea-level atmosphere. Diameters at 1 and 3 are 5 cm, while D_2 = 3 cm. What mass flow of air is required to suck water

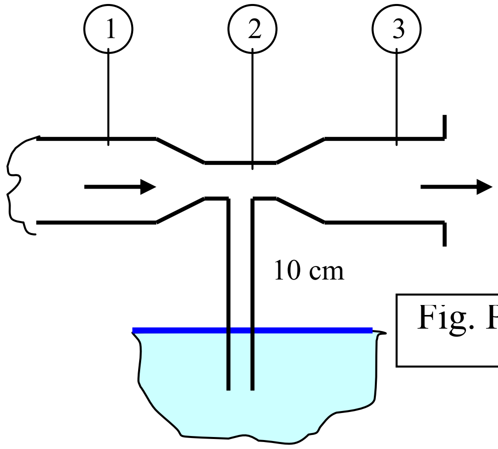


Fig. P3.136

up 10 cm into section 2 of Fig. P3.136?

Solution: For sea-level, take $\rho_{air} = 1.2255\ \text{kg/m}^3$. Section 2 must be less than atmospheric. How much less? Determine the pressure change for 10 cm of water:

$$\Delta p = p_3 - p_2 = \rho_{water}\, g\, h = (998\, kg/m^3)(9.81\, m/s^2)(0.1\ m) = 979\ Pa$$

This must be the pressure difference between sections 2 and 3. From Bernoulli's equation,

$$p_3 - p_2 = \frac{\rho}{2}(V_2^2 - V_3^2) \quad \text{plus continuity}:\ V_3 = \frac{A_2}{A_3}V_2 = \frac{D_2^2}{D_3^2}V_2 = \frac{9}{25}V_2$$

$$\text{or}:\ 979\, Pa = \frac{1.2255}{2}(V_2^2)[1-(\frac{9}{25})^2]\ ;\ Solve\ for\ V_2 = 42.8\ \frac{m}{s}\ ,\ V_3 = 15.4\ \frac{m}{s}$$

$$\text{Finally},\ \dot{m}_{air} = \rho A_3 V_3 = (1.2255\frac{kg}{m^3})\frac{\pi}{4}(0.05\, m)^2(15.4\frac{m}{s}) = \mathbf{0.037}\ \frac{kg}{s}\quad Ans.$$

P3.137 In Fig. P3.137 the piston drives water at 20°C. Neglecting losses, estimate the exit velocity $V2$ ft/s. If $D2$ is further constricted, what is the maximum possible value of $V2$?

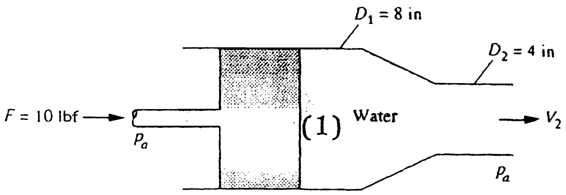


Fig. P3.137

Solution: Find p1 from a freebody of the piston:

$$\sum F_x = F + p_a A_1 - p_1 A_1,\quad \text{or:}\quad p_1 - p_a = \frac{10.0\ \text{lbf}}{(\pi/4)(8/12)^2} \approx 28.65\ \frac{\text{lbf}}{\text{ft}^2}$$

Now apply continuity and Bernoulli from 1 to 2:

$$V_1 A_1 = V_2 A_2,\quad \text{or}\quad V_1 = \frac{1}{4}V_2;\quad \frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_a}{\rho} + \frac{V_2^2}{2}$$

Introduce $p_1 - p_a$ and substitute for V_1 to obtain $V_2^2 = \dfrac{2(28.65)}{1.94(1-1/16)}$,

$$V_2 = \mathbf{5.61}\ \frac{\mathbf{ft}}{\mathbf{s}}\quad Ans.$$

If we reduce section 2 to a pinhole, V2 will drop off slowly until V1 vanishes:

$$\text{Severely constricted section 2:}\quad V_2 = \sqrt{\frac{2(28.65)}{1.94(1-0)}} \approx \mathbf{5.43\ \frac{ft}{s}} \quad \textit{Ans.}$$

P3.138 For the sluice gate flow of Example 3.10, use Bernoulli's equation, along the surface, to estimate the flow rate *Q* as a function of the two water depths. Assume constant width *b*.

Solution: Along surface 1, down the inside of the gate, and along surface 2 is a streamline of the flow. Therefore Bernoulli applies if we neglect friction, and we can use continuity also:

$$\text{Continuity:}\quad Q_1 = V_1 h_1 b = Q_2 = V_2 h_2 b \;,\;\; or:\; V_1 = V_2 (h_2/h_1)$$

$$\text{Bernoulli:}\quad \frac{p_1}{\rho} + \frac{V_1^2}{2g} + h_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + h_2$$

$$\text{But}\;\; p_1 = p_2 = p_{atmosphere}\,,\quad \text{thus}\;\; V_2^2 - V_1^2 = 2g(h_1 - h_2)$$

Eliminate V_1 from continuity and solve for V_2, hence solve for *Q*:

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1-(h_2/h_1)^2}}\;;\;\; Q = b\,h_2\sqrt{\frac{2g(h_1 - h_2)}{1-(h_2/h_1)^2}} = b\,h_2\,h_1\sqrt{\frac{2g}{h_1 + h_2}} \quad Ans$$

P3.139 In the spillway flow of Fig. P3.139, the flow is assumed uniform and hydrostatic at sections 1 and 2. If losses are neglected, compute (a) V_2 and (b) the force per unit width of the water on the spillway.

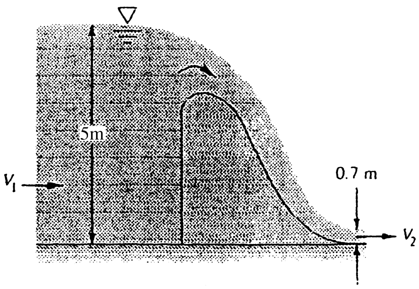


Fig. P3.139

Solution: For mass conservation,

$$V_2 = V_1 h_1/h_2 = \frac{5.0}{0.7}V_1 = 7.14V_1$$

(a) Now apply Bernoulli from 1 to 2:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+h_1 \approx \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+h_2; \quad \text{or:} \quad 0+\frac{V_1^2}{2g}+5.0 \approx 0+\frac{(7.14V_1)^2}{2g}+0.7$$

$$\text{Solve for } V_1^2 = \frac{2(9.81)(5.0-0.7)}{[(7.14)^2-1]}, \text{ or } V_1 = \mathbf{1.30}\ \frac{\mathbf{m}}{\mathbf{s}}, \quad V_2 = 7.14V_1 = \mathbf{9.28}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (a)}$$

(b) To find the force on the spillway (F ←), put a CV around sections 1 and 2 to obtain

$$\sum F_x = -F + \frac{\gamma}{2}h_1^2 - \frac{\gamma}{2}h_2^2 = \dot{m}(V_2 - V_1), \quad \text{or, using the given data,}$$

$$F = \frac{1}{2}(9790)[(5.0)^2-(0.7)^2] - 998[(1.30)(5.0)](9.28-1.30) \approx \mathbf{68300}\ \frac{\mathbf{N}}{\mathbf{m}} \quad \textit{Ans.}\text{ (b)}$$

P3.140 For the water-channel flow of.Fig. P3.140, h_1 = 1.5 m, H = 4 m, and V_1 = 3 m/s. Neglecting losses and assuming uniform flow at sections 1 and 2, find the downstream depth h_2, and show that *two* realistic solutions are possible.

Solution: Combine continuity and Bernoulli between 1 and 2:

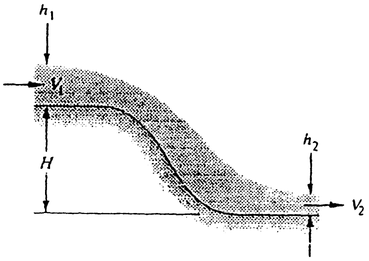


Fig. P3.140

$$V_2 = V_1\frac{h_1}{h_2} = \frac{3(1.5)}{h_2}; \quad \frac{V_1^2}{2g}+h_1+H \approx \frac{V_2^2}{2g}+h_2 = \frac{V_1^2}{2(9.81)}+1.5+4 \approx \frac{(4.5/h_2)^2}{2(9.81)}+h_2$$

Combine into a cubic equation: $h_2^3 - 5.959\,h_2^2 + 1.032 = 0$. The three roots are:

$h_2 = -0.403$ m (impossible); $h_2 = $ **+5.93 m** (subcritical);
$h_2 = $ **+0.432 m** (supercritical) *Ans.*

P3.141 For the water channel flow of Fig. P3.141, h1 = 0.45 ft, H = 2.2 ft, and V1 = 16 ft/s. Neglecting losses and assuming uniform flow at sections 1 and 2, find the downstream depth h2. Show that *two* realistic solutions are possible.

Solution: The analysis is quite similar to Prob. 3.177 - continuity + Bernoulli:

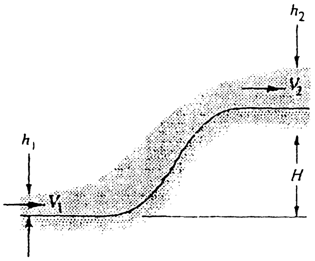


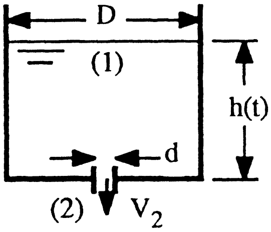
Fig. P3.141

$$V_2 = V_1\frac{h_1}{h_2} = \frac{16(0.45)}{h_2}; \quad \frac{V_1^2}{2g}+h_1 = \frac{V_2^2}{2g}+h_2+H = \frac{V_1^2}{2(32.2)}+0.45 = \frac{(7.2/h_2)^2}{2(32.2)}+h_2+2.2$$

Combine into a cubic equation: $h_2^3 - 2.225\,h_2^2 + 0.805 = 0$. The three roots are:

$h_2 = -0.540$ ft (impossible); $h_2 = $ **+2.03 ft** (subcritical);
$h_2 = $ **+0.735 ft** (supercritical) *Ans.*

P3.142 A cylindrical tank of diameter *D* contains liquid to an initial height *h*o. At time *t* = 0 a small stopper of diameter *d* is removed from the bottom. Using Bernoulli's equation with no losses, derive (a) a differential equation for the free-surface height *h*(*t*) during draining and (b) an expression for the time *t*o to drain the entire tank.



Solution: Write continuity and the unsteady Bernoulli relation from 1 to 2:

$$\int_1^2 \frac{\partial V}{\partial t}\,ds+\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2=\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1;\quad \text{Continuity: } V_2=V_1\frac{A_1}{A_2}=V_1\left(\frac{D}{d}\right)^2$$

The integral term $\int \frac{\partial V}{\partial t}\,ds \approx \frac{dV_1}{dt}h$ is very small and will be neglected, and $p_1 = p_2$.. Then

$V_1 \approx \left[\frac{2gh}{\alpha-1}\right]^{1/2}$, where $\alpha = (D/d)^4$; but also $V_1 = -\frac{dh}{dt}$, separate and integrate:

$$\int_{h_o}^{h}\frac{dh}{h^{1/2}}=-\left[\frac{2g}{\alpha-1}\right]^{1/2}\int_o^t dt,\quad \text{or:}\quad \mathbf{h}=\left[\mathbf{h_o^{1/2}}-\left\{\frac{\mathbf{g}}{\mathbf{2(\alpha-1)}}\right\}^{\mathbf{1/2}}\mathbf{t}\right]^{\mathbf{2}},\quad \alpha=\left(\frac{D}{d}\right)^4\quad \textit{Ans.}\text{ (a)}$$

(b) the tank is empty when [] = 0 in (a) above, or $\mathbf{t_o = [2(\alpha - 1)g/h_o]^{1/2}}$. *Ans.* (b)

P3.143 The large tank of incompressible liquid in Fig. P3.143 is at rest when, at *t* = 0, the valve is opened to the atmosphere. Assuming *h* ≈ constant (negligible velocities and accelerations in the tank), use the unsteady frictionless Bernoulli equation to derive and solve a differential equation for *V*(*t*) in the pipe.

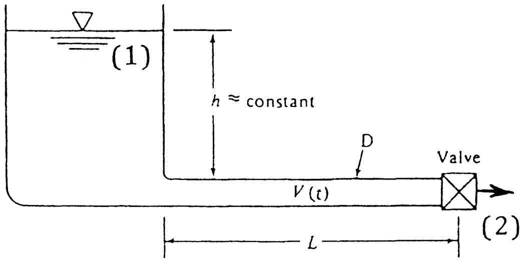


Fig. P3.143

Solution: Write unsteady Bernoulli from 1 to 2:

$$\int_1^2 \frac{\partial V}{\partial t}\,ds + \frac{V_2^2}{2} + gz_2 \approx \frac{V_1^2}{2} + gz_1, \quad \text{where } p_1 = p_2,\ V_1 \approx 0,\ z_2 \approx 0, \text{ and } z_1 = h = \text{const}$$

The integral approximately equals $\frac{dV}{dt}L$, so the diff. eqn. is $2L\frac{dV}{dt} + V^2 = 2gh$

This first-order ordinary differential equation has an exact solution for V = 0 at t = 0:

$$\mathbf{V = V_{final}\tanh\left(\frac{V_{final}t}{2L}\right)}, \quad \text{where } \mathbf{V_{final} = \sqrt{2gh}} \quad \textit{Ans.}$$

P3.144 A 6-cm-diameter free water jet, in sea-level air at 101,350 Pa, strikes perpendicular to a flat wall. If the water stagnation pressure at the wall is 213,600 Pa, estimate the force required to support the wall against jet momentum.

Solution: For standard conditions, take $\rho_{water} = 998\ kg/m^3$. The moving free jet must be at sea-level pressure; hence the stagnation pressure value allows us to compute the jet velocity:

$$p_o = 213{,}600Pa = p_{jet} + \frac{\rho}{2}V_{jet}^2 = 101{,}350 + \frac{998}{2}V_{jet}^2$$

$$Solve\ for\ V_{jet} = \sqrt{225m^2/s^2} = 15\frac{m}{s}$$

From many other momentum problems of this jet-striking-wall type, we found that

$$F_{support} = \rho A_{jet} V_{jet}^2 = (998)[\frac{\pi}{4}(0.06m)^2](15\frac{m}{s})^2 \approx \mathbf{635\ N} \quad \textit{Ans.}$$

P3.145 The incompressible-flow form of Bernoulli's relation, Eq. (3.54), is accurate only for Mach numbers less than about 0.3. At higher speeds, variable density must be accounted for. The most common assumption for compressible fluids is *isentropic flow of an ideal gas*, or $p = C\rho^k$, where $k = c_p/c_v$. Substitute this relation into Eq. (3.52), integrate, and eliminate the constant *C*. Compare your compressible result with Eq. (3.54) and comment.

Solution: We are to integrate the differential Bernoulli relation with variable density:

$$\mathrm{p = C\rho^{k}, \quad so \quad dp = kC\rho^{k-1}d\rho, \quad k = c_p/c_v}$$

Substitute this into the Bernoulli relation:

$$\mathrm{\frac{dp}{\rho} + V\,dV + g\,dz = \frac{kC\rho^{k-1}d\rho}{\rho} + V\,dV + g\,dz = 0}$$

$$\text{Integrate:} \quad \int \mathrm{kC\rho^{k-2}d\rho} + \int \mathrm{V\,dV} + \int \mathrm{g\,dz} = \int 0 = \text{constant}$$

The first integral equals $\mathrm{kC\rho^{k-1}/(k-1) = kp/[\rho(k-1)]}$ from the isentropic relation. Thus the compressible isentropic Bernoulli relation can be written in the form

$$\mathbf{\frac{kp}{(k-1)\rho} + \frac{V^2}{2} + gz = constant} \quad \textit{Ans.}$$

It looks quite different from the incompressible relation, which only has "$\mathrm{p/\rho}$." It becomes more clear when we make the ideal-gas substitution $\mathrm{p/\rho = RT}$ and $\mathrm{c_p = kR/(k-1)}$. Then we obtain the equivalent of the adiabatic, no-shaft-work energy equation:

$$\mathbf{c_pT + \frac{V^2}{2} + gz = constant} \quad \textit{Ans.}$$

P3.146 The pump in Fig. P3.146 draws gasoline at 20°C from a reservoir. Pumps are in big trouble if the liquid vaporizes (cavitates) before it enters the pump.
(a) Neglecting losses and assuming a flow rate of 65 gal/min, find the limitations on (*x*, *y*, *z*) for avoiding cavitation. (b) If pipe-friction losses are included, what additional limitations might be important?

Solution: (a) From Table A.3, $\rho = 680\ \mathrm{kg/m^3}$ and $\mathrm{p_v} = 5.51\mathrm{E}{+}4$.

$$\mathrm{z_2 - z_1} = y + z = \frac{p_1 - p_2}{\rho g} = \frac{(p_a + \rho g y) - p_v}{\rho g}$$

$$y + z = \frac{(100{,}000 - 55{,}100)}{(680)(9.81)} + y \quad z = 6.73\ m$$

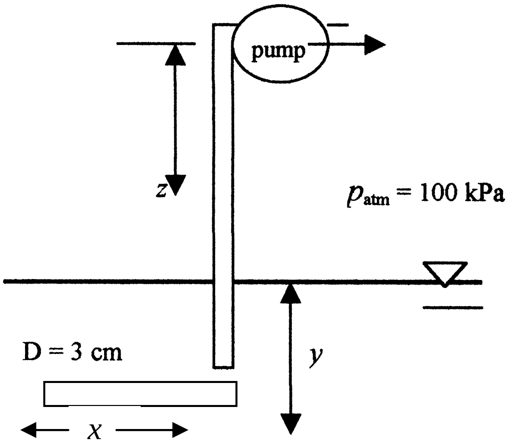


Fig. P3.146

Thus make length z appreciably less than 6.73m (25% less), or **z < 5 m**. *Ans.* (a)

(b) **Total pipe length (*x* + *y* + *z*) restricted by friction losses**. *Ans.* (b)

P3.147 For the system of Prob. 3.146, let the pump exhaust gasoline at 65 gal/min to the atmosphere through a 3-cm-diameter opening, with no cavitation, when $x = 3$ m, $y = 2.5$ m, and $z = 2$ m. If the friction head loss is $h_{loss} \approx 3.7(V^2/2g)$, where V is the average velocity in the pipe, estimate the horsepower required to be delivered by the pump.

Solution: Since power is a function of h$_{pump}$, Bernoulli is required. Thus calculate the velocity,

$$V = \frac{Q}{A} = \frac{(65\ \text{gal/min})\left(6.3083E{-}5\,\dfrac{\text{m}^3/\text{s}}{\text{gal/min}}\right)}{\dfrac{\pi}{4}(0.03^2)} = 5.8\ \text{m/s}$$

The pump head may then be found,

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_f - h_p + \frac{V_j^2}{2g}$$

$$\frac{100{,}000 + (680)(9.81)(2.5)}{(680)(9.81)} - 2.5 = \frac{100{,}000}{(680)(9.81)} + 2 + \frac{3.7(5.8^2)}{2(9.81)} - h_p + \frac{(5.8^2)}{2(9.81)}$$

$$h_p = 10.05\ m$$

$$P = \gamma Q h_p = (680)(9.81)(0.0041)(10.05) \quad \mathbf{P = 275\ W = 0.37\ hp} \quad \textit{Ans.}$$

P3.148 By neglecting friction, (*a*) use the Bernoulli equation between surfaces 1 and 2 to estimate the volume flow through the orifice, whose diameter is 3 cm. (*b*) Why is the result to part (*a*) absurd? (*c*) Suggest a way to resolve this paradox and find the true flow rate.

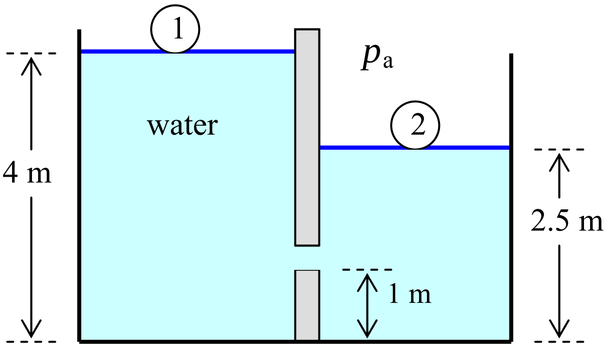


Fig. P3.148

Solution: (*a*) The incompressible Bernoulli equation between surfaces 1 and 2 yields

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_a}{9790} + \frac{0^2}{2(9..81)} + 4 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_a}{9790} + \frac{0^2}{2(9..81)} + 2.5$$

This gives the absurd result $\mathbf{4\,m = 2.5\,m}$ **?** *Ans.*(*a*)

(*b*) The absurd result arises because the flow is *not frictionless*. The jet of water passing through the orifice loses all of its kinetic energy by viscous dissipation in the right-side tank. (*c*) As we shall see in Chap. 6, we add an orifice-exit *head loss* equal to the jet kinetic energy:

$$z_1 - z_2 = h_{f,jet} = \frac{V_{jet}^2}{2g}, \quad \text{or} \quad V_{jet} = \sqrt{2(9.81\frac{m}{s^2})(4m - 2.5m)}$$

Solve for $V_{jet} = 5.42\frac{m}{s}$, $Q = AV = \frac{\pi}{4}(0.03m)^2(5.42\frac{m}{s}) \approx \mathbf{0.0038\ \frac{m^3}{2}}$ *Ans.*(*c*)

P3.149 The horizontal lawn sprinkler in Fig. P3.149 has a water flow rate of 4.0 gal/min introduced vertically through the center. Estimate (a) the retarding torque required to keep the arms from rotating and (b) the rotation rate (r/min) if there is no retarding torque.

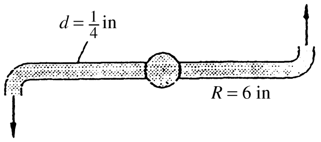


Fig. P3.149

Solution: The flow rate is 4 gal/min = 0.008912 ft^3/s, and ρ = 1.94 $slug/ft^3$. The velocity issuing from each arm is $V_o = (0.008912/2)/[(\pi/4)(0.25/12\text{ ft})^2] \approx 13.1$ ft/s. Then:

(a) From Example 3.15, $\omega = \dfrac{V_o}{R} - \dfrac{T_o}{\rho Q R^2}$ and, if there is no motion ($\omega = 0$),

$$T_o = \rho Q R V_o = (1.94)(0.008912)(6/12)(13.1) \approx \mathbf{0.113\ ft\text{-}lbf} \quad \textit{Ans.}\text{ (a)}$$

(b) If $T_o = 0$, then $\omega_{\text{no friction}} = V_o/R = \dfrac{13.1\text{ ft/s}}{6/12\text{ ft}} = 26.14\ \dfrac{\text{rad}}{\text{s}} \approx \mathbf{250\ \dfrac{rev}{min}}$ *Ans.* (b)

P3.150 In Prob. 3.60 find the torque caused around flange 1 if the center point of exit 2 is 1.2 m directly below the flange center.

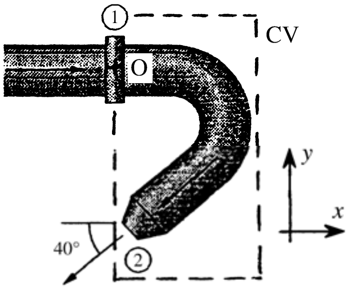


Fig. P3.60

Solution: The CV encloses the elbow and cuts through flange (1). Recall from Prob. 3.60 that D_1 = 10 cm, D_2 = 3 cm, weight flow = 150 N/s, whence V_1 = 1.95 m/s and V_2 = 21.7 m/s. Let "O" be in the center of flange (1). Then $\mathbf{r}_{O2} = -1.2\mathbf{j}$ and $\mathbf{r}_{O1} = 0$.

The pressure at (1) passes through O, thus causes no torque. The moment relation is

$$\sum M_O = \mathbf{T}_O = \dot{m}[(\mathbf{r}_{O2}\times\mathbf{V}_2)-(\mathbf{r}_{O1}\times\mathbf{V}_1)] = \left(\frac{150}{9.81}\ \frac{\text{kg}}{\text{s}}\right)[(-1.2\mathbf{j})\times(-16.6\mathbf{i}-13.9\mathbf{j})]$$

$$\text{or:}\quad \mathbf{T}_O = -305\,\mathbf{k}\ \text{N}\cdot\text{m} \quad \textit{Ans.}$$

P3.151 The wye joint in Fig. P3.151 splits the pipe flow into equal amounts *Q*/2, which exit, as shown, a distance *R*o from the axis. Neglect gravity and friction. Find an expression for the torque *T* about the *x* axis required to keep the system rotating at angular velocity Ω.

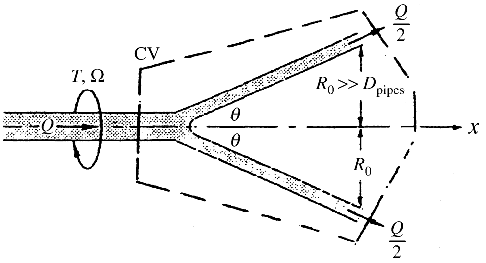


Fig. P3.151

Solution: Let the CV enclose the junction, cutting through the inlet pipe and thus exposing the required torque *T*. If *y* is "up" in the figure, the absolute exit velocities are

$$\mathbf{V}_{\text{upper}} = V_o\cos\theta\,\mathbf{i} + V_o\sin\theta\,\mathbf{j} + R_o\Omega\,\mathbf{k};\quad \mathbf{V}_{\text{lower}} = V_o\cos\theta\,\mathbf{i} - V_o\sin\theta\,\mathbf{j} - R_o\Omega\,\mathbf{k}$$

where Vo = Q/(2A) is the exit velocity relative to the pipe walls. Then the moments about the *x* axis are related to angular momentum fluxes by

$$\sum \mathbf{M}_{\text{axis}} = T\,\mathbf{i} = (\rho Q/2)(R_o\mathbf{j})\times\mathbf{V}_{\text{upper}} + (\rho Q/2)(-R_o\mathbf{j})\times\mathbf{V}_{\text{lower}} - \rho Q(\mathbf{r}_{\text{inlet}}\mathbf{V}_{\text{inlet}})$$

$$= \frac{\rho Q}{2}\left(R_o^2\Omega\,\mathbf{i} - R_oV_o\Omega\,\mathbf{k}\right) + \frac{\rho Q}{2}\left(R_o^2\Omega\,\mathbf{i} + R_oV_o\Omega\,\mathbf{k}\right) - \rho Q(0)$$

Each arm contributes to the torque via relative velocity (ΩRo). Other terms with Vo cancel.

$$\text{Final torque result:}\quad T = \rho QR_o^2\Omega = \dot{\mathbf{m}}\mathbf{R}_o^2\,\boldsymbol{\Omega} \quad \textit{Ans.}$$

P3.152 Modify Ex. 3.19 so that the arm starts up from rest and spins up to its final rotation speed. The moment of inertia of the arm about O is Io. Neglect air drag. Find dω/dt and integrate to determine ω(t), assuming $\omega = 0$ at t = 0.

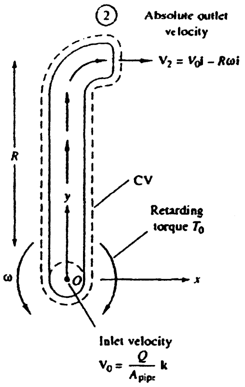


Fig. 3.16 View from above of a single arm of a rotating lawn sprinkler.

Solution: The CV is shown. Apply clockwise moments:

$$\sum \mathbf{M}_o - \int (\mathbf{r} \times \mathbf{a}_{rel})\, dm = \int_{CS} (\mathbf{r} \times \mathbf{V})\, d\dot{m},$$

$$\text{or:}\quad -T_o - I_o \frac{d\omega}{dt} = \rho Q(R^2\omega - RV_o),$$

$$\text{or:}\quad \frac{d\omega}{dt} + \frac{\rho QR^2}{I_o}\omega = \frac{\rho QRV_o - T_o}{I_o}$$

Integrate this first-order linear differential equation, with $\omega = 0$ at t = 0. The result is:

$$\omega = \left(\frac{\mathbf{V_o}}{\mathbf{R}} - \frac{\mathbf{T_o}}{\rho \mathbf{QR^2}}\right)\left[\mathbf{1 - e^{-\rho QR^2 t/I_o}}\right] \quad \textit{Ans.}$$

P3.153 The 3-arm lawn sprinkler of Fig. P3.153 receives 20°C water through the center at 2.7 m^3/hr. If collar friction is neglected, what is the steady rotation rate in rev/min for (a) $\theta = 0°$; (b) $\theta = 40°$?

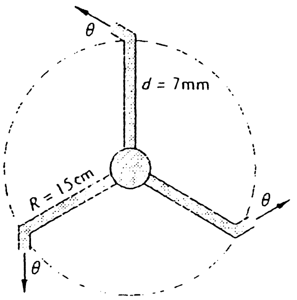


Fig. P3.153

Solution: The velocity exiting each arm is

$$V_o = \frac{Q/3}{(\pi/4)d^2} = \frac{2.7/[(3600)(3)]}{(\pi/4)(0.007)^2} = 6.50\ \frac{m}{s}$$

With negligible air drag and bearing friction, the steady rotation rate (Example 3.15) is

$$\omega_{\text{final}} = \frac{V_o\cos\theta}{R} \quad \text{(a) } \theta = 0°: \quad \omega = \frac{(6.50)\cos 0°}{0.15\ \text{m}} = 43.3\ \frac{\text{rad}}{\text{s}} = \mathbf{414}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans. } \text{(a)}$$

$$\text{(b) } \theta = 40°: \quad \omega = \omega_o\cos\theta = (414)\cos 40° = \mathbf{317}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans. } \text{(b)}$$

P3.154 Water at 20°C flows at 30 gal/min through the 0.75-in-diameter double pipe bend of Fig. P3.154. The pressures are p_1 = 30 lbf/in^2 and p_2 = 24 lbf/in^2. Compute the torque *T* at point *B* necessary to keep the pipe from rotating.

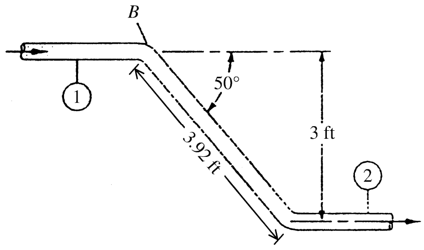


Fig. P3.154

Solution: This is similar to Example 3.13, of the text. The volume flow Q = 30 gal/min = 0.0668 ft^3/s, and ρ = 1.94 slug/ft^3. Thus the mass flow ρQ = 0.130 slug/s. The velocity in the pipe is

$$V_1 = V_2 = Q/A = \frac{0.0668}{(\pi/4)(0.75/12)^2} = 21.8\ \frac{ft}{s}$$

If we take torques about point B, then the distance "h1," from p. 143, = 0, and h2 = 3 ft. The final torque at point B, from "Ans. (a)" on p. 143 of the text, is

$$T_B = h_2(p_2A_2 + \dot{m}V_2) = (3\ \text{ft})[(24\ \text{psi})\frac{\pi}{4}(0.75\ \text{in})^2 + (0.130)(21.8)] \approx \mathbf{40\ ft\cdot lbf} \quad \textit{Ans.}$$

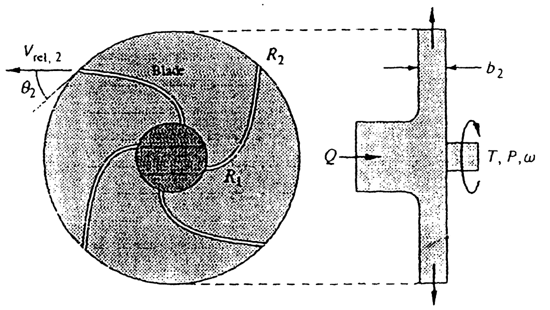
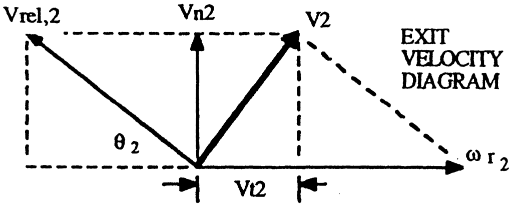


Fig. P3.155

P3.155 The centrifugal pump of Fig. P3.155 has a flow rate *Q* and exits the impeller at an angle θ_2 relative to the blades, as shown. The fluid enters axially at section 1. Assuming incompressible flow at shaft angular velocity ω, derive a formula for the power *P* required to drive the impeller.



Solution: Relative to the blade, the fluid exits at velocity Vrel,2 tangent to the blade, as shown in Fig. P3.116. But the Euler turbine formula, Ans. (a) from Example 3.14 of the text,

$$\text{Torque } T = \rho Q(r_2V_{t2} - r_1V_{t1})$$
$$\approx \rho Q r_2 V_{t2} \text{ (assuming } V_{t1} \approx 0)$$

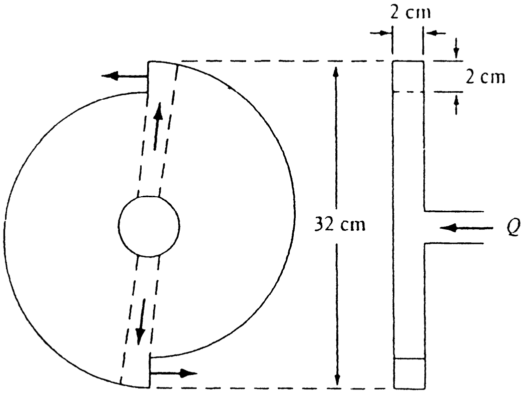
involves the *absolute fluid velocity tangential to the blade circle* (see Fig. 3.13). To derive this velocity we need the "velocity diagram" shown above, where absolute exit velocity V2 is found by adding blade tip rotation speed ωr2 to Vrel,2. With trigonometry,

$$V_{t2} = r_2\omega - V_{n2}\cot\theta_2, \quad \text{where } V_{n2} = Q/A_{exit} = \frac{Q}{2\pi r_2 b_2} \text{ is the normal velocity}$$

With torque T known, the power required is P = Tω. The final formula is:

$$\mathbf{P = \rho Q r_2 \omega\left[r_2\omega - \left(\frac{Q}{2\pi r_2 b_2}\right)\cot\theta_2\right]} \quad \textit{Ans.}$$

P3.156 A simple turbomachine is constructed from a disk with two internal ducts which exit tangentially through square holes, as in the figure. Water at 20C enters the disk at the center, as shown. The disk must drive, at 250 rev/min, a small device whose retarding torque is 1.5 N·m. What is the proper mass flow of water, in kg/s?



Solution: This problem is a disguised version of the lawn-sprinkler arm in Example 3.15. For that problem, the steady rotating speed, with retarding torque To, was

$$\omega = \frac{V_o}{R} - \frac{T_o}{\rho Q R^2}, \quad \textit{where } V_o \textit{ is the exit velocity and R is the arm radius.}$$

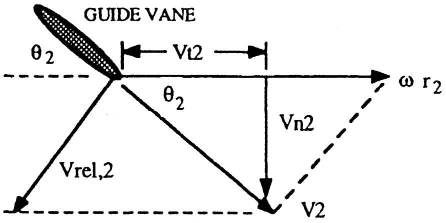
Enter the given data, noting that Q = $2V_o(\Delta L_{exit})^2$ is the total volume flow from the two arms:

$$\omega = 250\left(\frac{2\pi}{60}\right)\frac{rad}{s} = \frac{V_o}{0.16\text{ m}} - \frac{1.5\ N\cdot\text{m}}{998(2V_o)(0.02\text{ m})^2(0.16\text{ m})^2}, \quad solve\ V_o = \mathbf{6.11}\ \frac{\text{m}}{\text{s}}$$

The required mass flow is thus,

$$\dot{\mathbf{m}} = \rho Q = \left(998\ \frac{\text{kg}}{\text{m}^3}\right)\left(2\left(6.11\ \frac{\text{m}}{\text{s}}\right)\right)(0.02\text{ m})^2 = \mathbf{2.44}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}$$

P3.157 Reverse the flow in Fig. P3.155, so that the system operates as a radial-inflow *turbine*. Assuming that the outflow into section 1 has no tangential velocity, derive an expression for the power *P* extracted by the turbine.



Solution: The Euler turbine formula, "Ans. (a)" from Example 3.14 of the text, is valid in reverse, that is, for a turbine with inflow at section 2 and outflow at section 1. The torque developed is

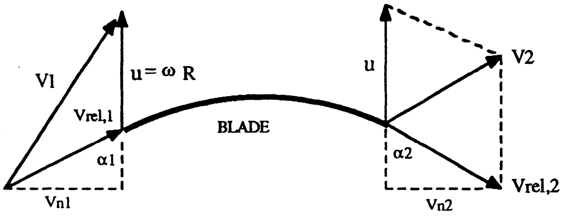
$$T_o = \rho Q(r_2 V_{t2} - r_1 V_{t1}) \approx \rho Q r_2 V_{t2} \quad \text{if } V_{t1} \approx 0$$

The velocity diagram is reversed, as shown in the figure. The fluid enters the turbine at angle θ_2, which can only be ensured by a guide vane set at that angle. The absolute tangential velocity component is directly related to inlet normal velocity, giving the final result

$$V_{t2} = V_{n2}\cot\theta_2, \quad V_{n2} = \frac{Q}{2\pi r_2 b_2},$$

$$\text{thus} \quad P = \omega T_o = \boldsymbol{\rho Q \omega r_2}\left(\frac{\mathbf{Q}}{\mathbf{2\pi r_2 b_2}}\right)\mathbf{cot}\boldsymbol{\theta_2} \quad \textit{Ans.}$$

P3.158 Revisit the turbine cascade system of Prob. 3.78, and derive a formula for the power *P* delivered, using the *angular*-momentum theorem of Eq. (3.59).



Solution: To use the angular momentum theorem, we need the inlet and outlet velocity diagrams, as in the figure. The Euler turbine formula becomes

$$T_o = \rho Q(r_1 V_{t1} - r_2 V_{t2}) \approx \rho Q R(V_{t1} - V_{t2})$$

since the blades are at nearly constant radius R. From the velocity diagrams, we find

$$V_{t1} = u + V_{n1}\cot\alpha_1;\quad V_{t2} = u - V_{n2}\cot\alpha_2,\quad \text{where } V_{n1} = V_{n2} = V_1\cos\beta_1$$

The normal velocities are equal by virtue of mass conservation across the blades. Finally,

$$P = \rho Q\omega R(V_{t1} - V_{t2}) = \boldsymbol{\rho Q u V_n(\cot\alpha_1 + \cot\alpha_2)} \quad \textit{Ans.}$$

P3.159 A centrifugal pump delivers 4000 gal/min of water at 20°C with a shaft rotating at 1750 rpm. Neglect losses. If r_1 = 6 in, r_2 = 14 in, $b_1 = b_2$ = 1.75 in, V_{t1} = 10 ft/s, and V_{t2} = 110 ft/s, compute the absolute velocities (a) V_1 and (b) V_2, and (c) the ideal horsepower required.

Solution: First convert 4000 gal/min = 8.91 ft^3/s and 1750 rpm = 183 rad/s. For water, take ρ = 1.94 slug/ft^3. The normal velocities are determined from mass conservation:

$$V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{8.91}{2\pi(6/12)(1.75/12)} = 19.5\ \frac{ft}{s};\quad V_{n2} = \frac{Q}{2\pi r_2 b_2} = 8.34\ \frac{ft}{s}$$

Then the desired absolute velocities are simply the resultants of V_t and V_n:

$$V_1 = [(10)^2 + (19.45)^2]^{1/2} = \mathbf{22\ \frac{ft}{s}} \qquad V_2 = [(110)^2 + (8.3)^2]^{1/2} = \mathbf{110\ \frac{ft}{s}} \quad \textit{Ans. (a, b)}$$

The ideal power required is given by Euler's formula:

$$P = \rho Q\omega(r_2 V_{t2} - r_1 V_{t1}) = (1.94)(8.91)(183)[(14/12)(110) - (6/12)(10)]$$

$$= 391{,}000\ \frac{\text{ft-lbf}}{s} \approx \mathbf{710\ hp} \quad \textit{Ans. (c)}$$

P3.160 The pipe bend of Fig. P3.160 has D_1 = 27 cm and D_2 = 13 cm. When water at 20°C flows through the pipe at 4000 gal/ min, p_1 = 194 kPa (gage). Compute the torque required at point *B* to hold the bend stationary.

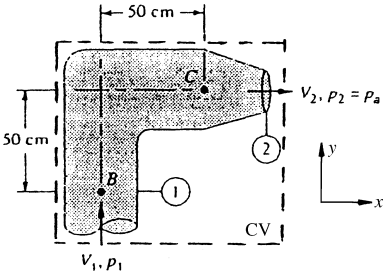


Fig. P3.160

Solution: First convert Q = 4000 gal/ min = 0.252 m^3/s. We need the exit velocity:

$$V_2 = Q/A_2 = \frac{0.252}{(\pi/4)(0.13)^2} = 19.0\ \frac{m}{s} \quad \text{Meanwhile, } V_1 = Q/A_1 = 4.4\ \frac{m}{s}$$

We don't really need V_1, because it passes through B and has no angular momentum. The angular momentum theorem is then applied to point B:

$$\sum \mathbf{M}_B = \mathbf{T}_B + \mathbf{r}_1 \times p_1A_1\mathbf{j} + \mathbf{r}_2 \times p_2A_2(-\mathbf{i}) = \dot{m}(\mathbf{r}_2 \times \mathbf{V}_2 - \mathbf{r}_1 \times \mathbf{V}_1)$$

But r_1 and p_2 are zero,

$$\text{hence} \quad \mathbf{T}_B = \dot{m}(\mathbf{r}_2 \times \mathbf{V}_2) = \rho Q[(0.5\mathbf{i} + 0.5\mathbf{j}) \times (19.0\mathbf{i})]$$

Thus, finally, $\mathbf{T}_B = (998)(0.252)(0.5)(19.0)(-\mathbf{k}) \approx \mathbf{-2400\ k}\ N\cdot m$ (clockwise) *Ans.*

P3.161 Extend Prob. 3.46 to the problem of computing the center of pressure *L* of the normal face F_n, as in Fig. P3.161. (At the center of pressure, no moments are required to hold the plate at rest.) Neglect friction. Express your result in terms of the sheet thickness h_1 and the angle θ between the plate and the oncoming jet 1.

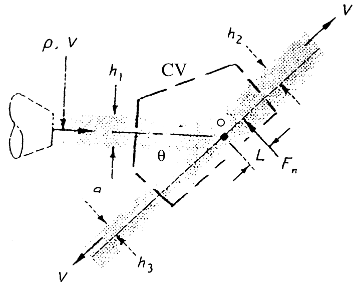


Fig. P3.161

Solution: Recall that in Prob. 3.46 of this Manual, we found h2 = (h1/2)(1 + cosθ) and that h3 = (h1/2)(1 − cosθ), where θ is the angle between the plate and the horizontal. . The force on the plate was Fn = ρQVsinθ. Take clockwise moments about O, where the jet strikes the plate, and use the angular momentum theorem:

$$\sum M_o = -F_n L = \dot{m}_2|\mathbf{r}_{2O}\times\mathbf{V}_2|_z + \dot{m}_3|\mathbf{r}_{3O}\times\mathbf{V}_3|_z - \dot{m}_1|\mathbf{r}_{1O}\times\mathbf{V}_1|_z$$

$$= \rho V h_2(h_2 V/2) + \rho V h_3(-h_3 V/2) - 0 = (1/2)\rho V^2\left(h_2^2 - h_3^2\right)$$

$$\text{Thus}\quad L = -\frac{(1/2)\rho V^2\left(h_2^2 - h_3^2\right)}{\rho V^2 h_1 \sin\theta} = -\frac{\left(h_2^2 - h_3^2\right)}{2h_1\sin\theta} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{h_1}\,\mathbf{cot}\boldsymbol{\theta}\quad \textit{Ans.}$$

The latter result follows from the (h1, h2, h3) relations in 3.46. The C.P. is below point O.

P3.162 The waterwheel in Fig. P3.162 is being driven at 200 r/min by a 150-ft/s jet of water at 20°C. The jet diameter is 2.5 in. Assuming no losses, what is the horsepower developed by the wheel? For what speed Ω r/min will the horsepower developed be a maximum? Assume that there are many buckets on the waterwheel.

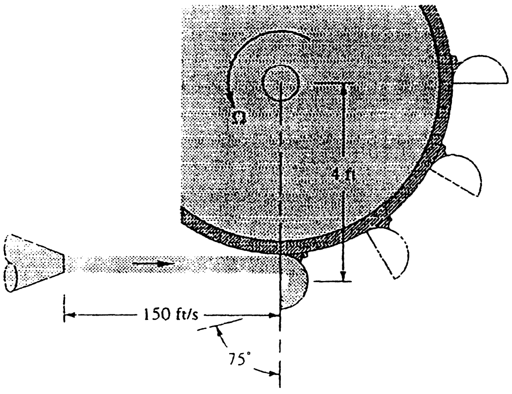


Fig. P3.162

Solution: First convert Ω = 200 rpm = 20.9 rad/s. The bucket velocity = Vb = ΩR = (20.9)(4) = 83.8 ft/s. From Prob. 3.51 of this Manual, if there are many buckets, the entire (absolute) jet mass flow does the work:

$$P = \dot{m}_{jet}V_b(V_{jet} - V_b)(1 - \cos 165°) = \rho A_{jet}V_{jet}V_b(V_{jet} - V_b)(1.966)$$

$$= (1.94)\frac{\pi}{4}\left(\frac{2.5}{12}\right)^2(150)(83.8)(150 - 83.8)(1.966)$$

$$= 108200\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{197\ hp}\quad \textit{Ans.}$$

Prob. 3.51: Max. power is for Vb = Vjet/2 = 75 ft/s, or Ω = 18.75 rad/s = **179 rpm** *Ans.*

P3.163 A rotating dishwasher arm delivers at 60°C to six nozzles, as in Fig. P3.163. The total flow rate is 3.0 gal/min. Each nozzle has a diameter of $\frac{3}{16}$ in. If the nozzle flows are equal and friction is neglected, estimate the steady rotation rate of the arm, in r/min.

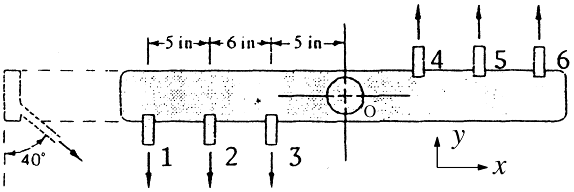


Fig. P3.163

Solution: First we need the mass flow and velocity from each hole "i," *i* = 1 to 6:

$$V_i = \frac{Q_i}{A_i} = \frac{(3.0/448.8)/6}{(\pi/4)\left(\frac{3/16}{12}\right)^2} \approx 5.81\ \frac{ft}{s} \qquad \dot{m}_i = \frac{\rho Q}{6} = 1.94\left(\frac{3/448.8}{6}\right) = 0.00216\ \frac{slug}{s}$$

Recall Example 3.15 from the text. For each hole, we need the absolute velocity, $V_i - \Omega r_i$. The angular momentum theorem is then applied to moments about point O:

$$\sum M_O = T_O = \sum \dot{m}_i(\mathbf{r}_{iO} \times \mathbf{V}_{i,abs}) - \dot{m}_{in}V_{in} = \sum \dot{m}_i r_i (V_i \cos 40° - \Omega r_i)$$

All the velocities and mass flows from each hole are equal. Then, if $T_O = 0$ (no friction),

$$\Omega = \frac{\sum \dot{m}_i r_i V_i \cos 40°}{\sum \dot{m}_i r_i^2} = V_i \cos 40° \frac{\sum r_i}{\sum r_i^2} = (5.81)(0.766)\frac{5.33}{5.58} = 4.25\ \frac{rad}{s} = \mathbf{41\ rpm} \quad \textit{Ans.}$$

P3.164 A liquid of density ρ flows through a 90° bend as in Fig. P3.164 and issues vertically from a uniformly porous section of length L. Neglecting weight, find a result for the support torque M required at point O.

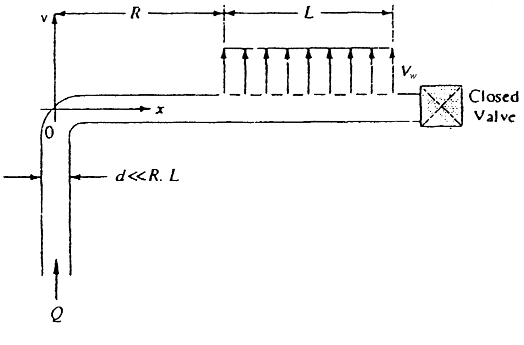


Fig. P3.164

Solution: Mass conservation requires

$$Q = \int_0^L V_w(\pi d)\,dx = V_w \pi d L, \quad \text{or:} \quad \frac{dQ}{dx} = \pi d V_w$$

Then the angular momentum theorem applied to moments about point O yields

$$\sum \mathbf{M}_O = \mathbf{T}_O = \int_{CS} (\mathbf{r_O} \times \mathbf{V})d\dot{m}_{out} = \square\int_0^L (R + x)V_w \rho\pi d V_w\, dx$$

$$= \frac{\square}{2}\rho\pi d V_w^2 [(R + x)^2 - R^2]\Big|_0^L$$

Substitute $V_w \pi d L = Q$ and clean up to obtain $\mathbf{T}_O = \rho Q V_w\left(R + \frac{L}{2}\right)\mathbf{k} \circlearrowleft$ *Ans.*

P3.165 Given a steady isothermal flow of water at 20°C through the device in Fig. P3.165. Heat-transfer, gravity, and temperature effects are negligible. Known data are D_1 = 9 cm, Q_1 = 220 m^3/h, p_1 = 150 kPa, D_2 = 7 cm, Q_2 = 100 m^3/h, p_2 = 225 kPa, D_3 = 4 cm, and p_3 = 265 kPa. Compute the rate of shaft work done for this device and its direction.

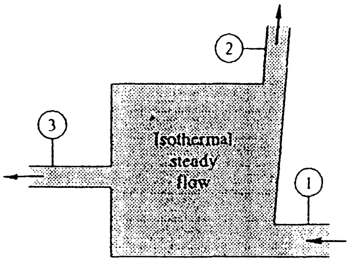


Fig. P3.165

Solution: For continuity, $Q_3 = Q_1 - Q_2 = 120\ \text{m}^3/\text{hr}$. Establish the velocities at each port:

$$V_1 = \frac{Q_1}{A_1} = \frac{220/3600}{\pi(0.045)^2} = 9.61\ \frac{m}{s};\quad V_2 = \frac{100/3600}{\pi(0.035)^2} = 7.22\ \frac{m}{s};\quad V_3 = \frac{120/3600}{\pi(0.02)^2} = 26.5\ \frac{m}{s}$$

With gravity and heat transfer and internal energy neglected, the energy equation becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_3\left(\frac{p_3}{\rho_3} + \frac{V_3^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho_1} + \frac{V_1^2}{2}\right),$$

$$\text{or:}\quad -\dot{W}_s/\rho = \frac{100}{3600}\left[\frac{225000}{998} + \frac{(7.22)^2}{2}\right] + \frac{120}{3600}\left[\frac{265000}{998} + \frac{(26.5)^2}{2}\right]$$
$$+\frac{220}{3600}\left[\frac{150000}{998} + \frac{(9.61)^2}{2}\right]$$

Solve for the shaft work: $\dot{W}_s = 998(-6.99 - 20.56 + 12.00) \approx \mathbf{-15500\ W}$ *Ans.*

(negative denotes work done on the fluid)

P3.166 A power plant on a river, as in Fig. P3.166, must eliminate 55 MW of waste heat to the river. The river conditions upstream are Q_1 = 2.5 m^3/s and T_1 = 18°C. The river is 45 m wide and 2.7 m deep. If heat losses to the atmosphere and ground are negligible, estimate the downstream river conditions (Q_0, T_0).

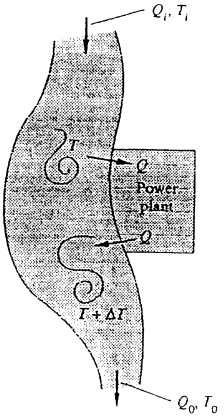


Fig. P3.166

Solution: For water, take $c_p \approx 4280$ J/kg·°C. For an overall CV enclosing the entire sketch,

$$\dot{Q} = \dot{m}_{out}(c_p T_{out}) - \dot{m}_{in}(c_p T_{in}),$$

$$\text{or:}\quad 55{,}000{,}000 \text{ W} \approx (998\times 2.5)[4280T_{out} - 4280(18)], \quad \text{solve for } T_{out} \approx \mathbf{23.15°C} \quad \textit{Ans.}$$

The power plant flow is "internal" to the CV, hence $Q_{out} = Q_{in} = \mathbf{2.5\ m^3/s}$. *Ans.*

P3.167 For the conditions of Prob. 3.166, if the power plant is to heat the nearby river water by no more than 12°C, what should be the minimum flow rate *Q*, in m^3/s, through the plant heat exchanger? How will the value of *Q* affect the downstream conditions (Q_o, T_o)?

Solution: Now let the CV only enclose the power plant, so that the flow going through the plant shows as an inlet and an outlet. The CV energy equation, with no work, gives

$$\dot{Q}_{plant} = \dot{m}_{out} c_p T_{out} - \dot{m}_{in} c_p T_{in} = (998)Q_{plant}(4280)(12°C) \quad \text{since } Q_{in} = Q_{out}$$

$$\text{Solve for}\quad Q_{plant} = \frac{55{,}000{,}000}{(998)(4280)(12)} \approx \mathbf{1.07\ m^3/s} \quad \textit{Ans.}$$

It's a lot of flow, but if the river water mixes well, the downstream flow is still the same.

P3.168 Multnomah Falls in the Columbia River Gorge has a sheer drop of 543 ft. Use the steady flow energy equation to estimate the water temperature rise, in °F, resulting.

Solution: For water, convert $c_p = 4200 \times 5.9798 = 25100$ ft·lbf/(slug·°F). Use the steady flow energy equation in the form of Eq. (3.70), with "1" upstream at the top of the falls:

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q$$

Assume adiabatic flow, *q* = 0 (although evaporation might be important), and neglect the kinetic energies, which are much smaller than the potential energy change. Solve for

$$\Delta h = c_p \Delta T \approx g(z_1 - z_2), \quad \text{or:}\quad \Delta T = \frac{32.2(543)}{25100} \approx 0.70°F \quad \textit{Ans.}$$

P3.169 When the pump in Fig. P3.169 draws 220 m^3/h of water at 20°C from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere Estimate the pump power in kW delivered to the water.

Solution: Let “1” be at the reservoir surface and “2” be at the nozzle exit, as shown. We need to know the exit velocity:

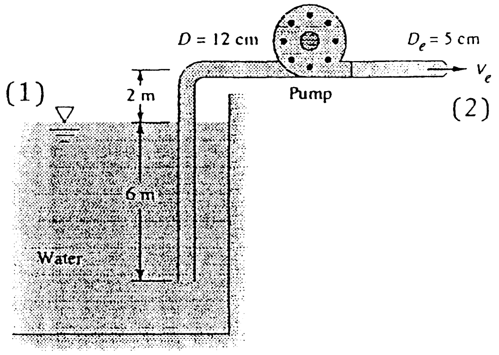


Fig. P3.169

$$V_2 = Q/A_2 = \frac{220/3600}{\pi(0.025)^2} = 31.12\ \frac{m}{s},\quad \text{while } V_1 \approx 0 \text{ (reservoir surface)}$$

Now apply the steady flow energy equation from (1) to (2):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0+0+0 = 0 + (31.12)^2/[2(9.81)] + 2 + 5 - h_p,\quad \text{solve for } h_p \approx 56.4 \text{ m.}$$

$$\text{The pump power}\quad P = \rho g Q h_p = (998)(9.81)(220/3600)(56.4)$$

$$= 33700 \text{ W} = \mathbf{33.7\ kW}\quad \textit{Ans.}$$

P3.170 A steam turbine operates steadily under the following conditions. At the inlet, p = 2.5 MPa, T = 450°C, and V = 40 m/s. At the outlet, p = 22 kPa, T = 70°C, and V = 225 m/s. (*a*) If we neglect elevation changes and heat transfer, how much work is delivered to the turbine blades, in kJ/kg? (*b*) If the mass flow is 10 kg/s, how much total power is delivered? (*c*) Is the steam wet as it leaves the exit?

Solution: This problem is made to order for the general steady-flow energy equation (3.70).

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q + w_s + w_v$$

The viscous work w_v is zero because the control volume has all no-slip surfaces. Look up the two enthalpies of steam, in the Steam Tables or with EES:

At 2.5 MPa and 450°C, h_1 = 3351 kJ/kg (or 3,351,000 J/kg or m^2/s^2)
At 22 kPa and 70°C, h_2 = 2628 kJ/kg (or 2,628,000 J/kg or m^2/s^2)

The energy equation thus becomes

$$3,351,000 + (1/2)(40m/s)^2 + 0 = 2,628,000 + (1/2)(225m/s)^2 - 0 + w_s + 0$$
$$\Delta h + \Delta(KE) = 723,100 - 24,500 = 698,600\ J/kg = \mathbf{+698.6}\ kJ/kg \quad Ans.(a)$$

(*b*) For the given mass flow of 10 kg/s of steam, the overall power extracted is

Power = (10 kg/s)(698.6 kJ/kg) = 6986 kJ/s ≈ **7.0 MW** *Ans.*(*b*)

(*c*) For the exit pressure of 22 kPa, EES states that the saturation temperature of steam is 62°C, less than the exit temperature of 70°C. The ***exit is just barely into the superheat region***. *Ans.*(*c*)

P3.171 Consider a turbine extracting energy from a penstock in a dam, as in the figure. For turbulent flow (Chap. 6) the friction head loss is $h_f = CQ^2$, where the constant C depends upon penstock dimensions and water physical properties. Show that, for a given penstock and river flow Q, the maximum turbine power possible is $P_{max} = 2\rho gHQ/3$ and occurs when $Q = (H/3C)^{1/2}$.

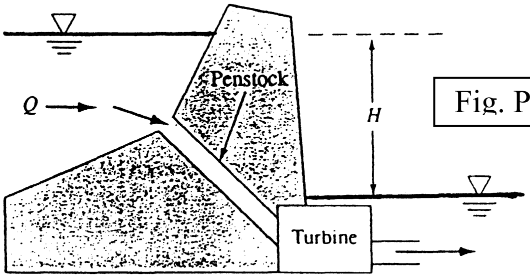


Fig. P3.171

Solution: Write the steady flow energy equation from point 1 on the upper surface to point 2 on the lower surface:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + H = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f + h_{turbine}$$

But $p_1 = p_2 = p_{atm}$ and $V_1 \approx V_2 \approx 0$. Thus the turbine head is given by

$$h_t = H - h_f = H - CQ^2,$$

$$or: \quad Power = P = \rho g Q h_t = \rho g Q H - \rho g C Q^3$$

Differentiate and set equal to zero for max power and appropriate flow rate:

$$\frac{dP}{dQ} = \rho g H - 3\rho g C Q^2 = 0 \quad if \quad \mathbf{Q = \sqrt{H/3C}} \quad Ans.$$

$$Insert\ Q\ in\ P\ to\ obtain\ \ \mathbf{P_{max}} = \rho \mathbf{gQ}\left(\frac{\mathbf{2H}}{\mathbf{3}}\right) \quad Ans.$$

P3.172 The long pipe in Fig. 3.172 is filled with water at 20°C. When valve *A* is closed, $p_1 - p_2 = 75$ kPa. When the valve is open and water flows at 500 m^3/h, $p_1 - p_2 = 160$ kPa. What is the friction head loss between 1 and 2, in m, for the flowing condition?

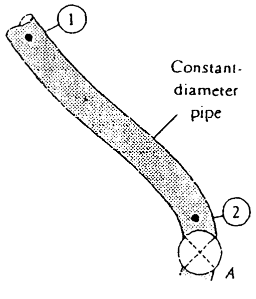


Fig. P3.172

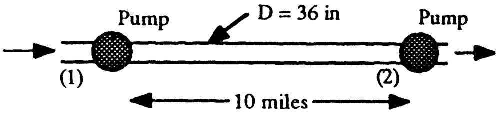
Solution: With the valve closed, there is no velocity or friction loss:

$$\frac{p_1}{\rho g}+z_1=\frac{p_2}{\rho g}+z_2,\quad \text{or:}\quad z_2-z_1=\frac{p_1-p_2}{\rho g}=\frac{75000}{998(9.81)}\approx 7.66\text{ m}$$

When the valve is open, the velocity is the same at (1) and (2), thus "d" is not needed:

$$\text{With flow:}\quad h_f=\frac{p_1-p_2}{\rho g}+\frac{V_1^2-V_2^2}{2g}+(z_1-z_2)=\frac{160000}{998(9.81)}+0-7.66\approx \mathbf{8.7\text{ m}}\quad \textit{Ans.}$$

P3.173 A 36-in-diameter pipeline carries oil (SG = 0.89) at 1 million barrels per day (bbl/day) (1 bbl = 42 U.S. gal). The friction head loss is 13 ft/1000 ft of pipe. It is planned to place pumping stations every 10 mi along the pipe. Estimate the horsepower which must be delivered to the oil by each pump.



Solution: Since ΔV and Δz are zero, the energy equation reduces to

$$h_f=\frac{\Delta p}{\rho g},\quad \text{and}\quad h_f=0.013\ \frac{\text{ft-loss}}{\text{ft-pipe}}(10\text{ mi})\left(5280\ \frac{\text{ft}}{\text{mi}}\right)\approx 686\text{ ft}$$

Convert the flow rate from 1E6 bbl/day to 29166 gal/min to **65.0** ft^3/s. Then the power is

$$P=Q\Delta p=\gamma Q h_f=(62.4)(65.0)(686)=2.78\text{E}6\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}}\approx \mathbf{5060\text{ hp}}\quad \textit{Ans.}$$

P3.174 The *pump-turbine* system in Fig. P3.174 draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoirs to restore the situation. For a design flow rate of 15,000 gal/min in either direction, the friction head loss is 17 ft. Estimate the power in kW (a) extracted by the turbine and (b) delivered by the pump.

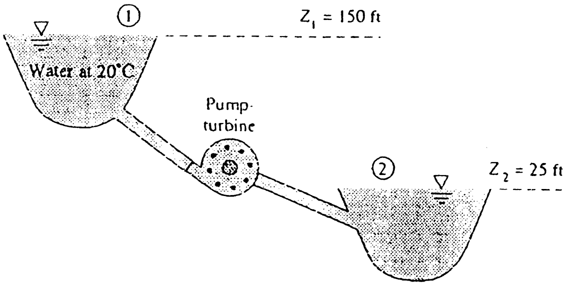


Fig. P3.174

Solution: (a) With the turbine, "1" is upstream:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f+h_t,$$

$$\text{or:}\quad 0+0+150=0+0+25+17=h_t$$

Solve for h_t = 108 ft. Convert Q = 15000 gal/min = **33.4** ft^3/s. Then the turbine power is

$$P=\gamma Qh_{turb}=(62.4)(33.4)(108)=225{,}000\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}}\approx\mathbf{410\,hp}\quad \textit{Ans.}\text{ (a)}$$

(b) For pump operation, point "2" is upstream:

$$\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2=\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_f-h_p,$$

$$\text{or:}\quad 0+0+25=0+0+150+17-h_p$$

$$\text{Solve for}\quad h_p\approx 142\text{ ft}$$

The pump power is $P_{pump}=\gamma Qh_p$ = (62.4)(33.4)(142) = 296000 ft·lbf/s = **540 hp**. *Ans.* (b)

P3.175 Water at 20°C is delivered from one reservoir to another through a long 8-cm-diameter pipe. The lower reservoir has a surface elevation z_2 = 80 m. The friction loss in the pipe is correlated by the formula $h_{loss}\approx 17.5(V^2/2g)$, where V is the average velocity in the pipe. If the steady flow rate through the pipe is 500 gallons per minute, estimate the surface elevation of the higher reservoir.

Solution: We may apply Bernoulli here. Convert 500 gal/min = 0.0315 m^3/s.

$$h_f=\frac{17.5V^2}{2g}=z_1-z_2$$

$$\frac{17.5}{2(9.81\text{ m/s}^2)}\left[\frac{0.0315\,m^3/s}{(\pi/4)(0.08m)^2}\right]^2=z_1-80\text{ m}$$

$$z_1\approx\mathbf{115\ m}\quad \textit{Ans.}$$

P3.176 A fireboat draws seawater (SG = 1.025) from a submerged pipe and discharges it through a nozzle, as in Fig. P3.176. The total head loss is 6.5 ft. If the pump efficiency is 75 percent, what horsepower motor is required to drive it?

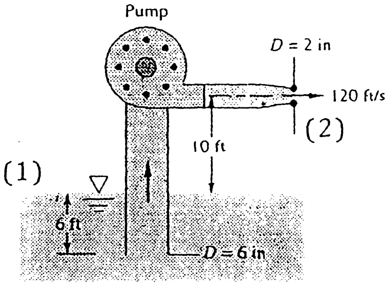


Fig. P3.176

Solution: For seawater, $\gamma = 1.025(62.4) = 63.96\ \text{lbf/ft}^3$. The energy equation becomes

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 0 = 0 + \frac{(120)^2}{2(32.2)} + 10 + 6.5 - h_p$$

Solve for $h_p = 240$ ft. The flow rate is $Q = V_2A_2 = (120)(\pi/4)(2/12)^2 = 2.62\ \text{ft}^3/\text{s}$. Then

$$P_{pump} = \frac{\gamma Q h_p}{\text{efficiency}} = \frac{(63.96)(2.62)(240)}{0.75} = 53600\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{97\ hp} \quad \textit{Ans.}$$

P3.177 A device for measuring liquid viscosity is shown in Fig. P3.177. With the parameters (ρ, L, H, d) known, the flow rate Q is measured and the viscosity calculated, assuming a laminar-flow pipe loss from Chap. 6, $h_f = (32\mu LV)/(\rho g d^2)$. Heat transfer and all other losses are negligible. (*a*) Derive a formula for the viscosity μ of the fluid. (*b*) Calculate μ for the case $d = 2$ mm, $\rho = 800\ \text{kg/m}^3$, $L = 95$ cm, $H = 30$ cm, and $Q = 760$

cm^3/h. (*c*) What is your guess of the fluid in part (*b*)? (*d*) Verify that the Reynolds number Re_d is less than 2000 (laminar pipe flow).

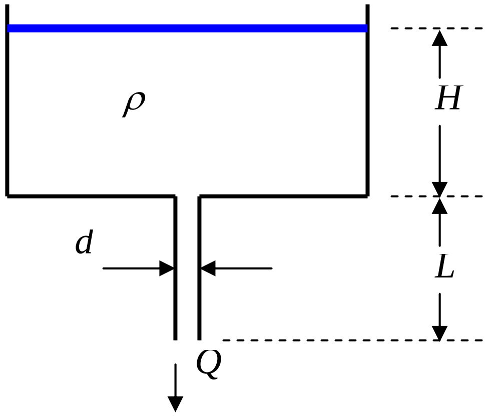


Fig. P3.177

Solution: Use energy Eq. (3.75) so we don't forget the *laminar* kinetic energy correction factor α:

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_{turbine} - h_{pump} + h_f$$

$$0 + 0 + H + L = 0 + \frac{\alpha_2 V_2^2}{2g} + 0 + 0 - 0 + \frac{32\mu L V_2}{\rho g d^2}$$

Introduce $Q = (\pi/4)(d^2)(V_2)$, rewrite, and solve for μ:

$$\mu = \frac{\pi \rho g d^4}{128 LQ}(H+L) - \frac{\alpha_2 \rho Q}{16\pi L} \qquad \textit{Ans.(a)}$$

(*b*) Introduce the given data and compute the viscosity of the liquid. Convert Q to $(760cm^3/h)/(3600s/h)/(1E6cm^3/m^3) = 2.11E\text{-}7\ m^3/s$. Recall that $\alpha_2 = 2.0$. Then

$$\mu = \frac{\pi(800)(9.81)(0.002)^4}{128(0.95)(2.11E-7)}(0.30+0.95) - \frac{(2.0)(800)(2.11E-7)}{16\pi(0.95)}$$

$$= 0.0192 - 0.000007 = \mathbf{0.0192}\ \frac{kg}{m-s} \qquad \textit{Ans.(b)}$$

(*c*) From Table A.4, both μ and ρ seem to fit **kerosene** very well. *Ans.*(*c*) (*d*) Check the diameter Reynolds number of this flow:

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi \mu d} = \frac{4(800)(2.11E-7)}{\pi(0.0192)(0.002)} = \mathbf{5.6} \ll 2000 \;\; \text{OK, laminar}$$

The flow is so slow (0.067 m/s) that the kinetic energy term is negligible.

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P3.178 The horizontal pump in Fig. P3.178 discharges 20°C water at 57 m^3/h. Neglecting losses, what power in kW is delivered to the water by the pump?

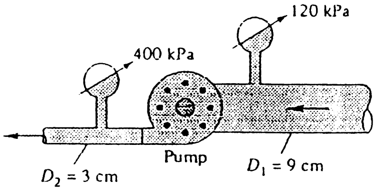


Fig. P3.178

Solution: First we need to compute the velocities at sections (1) and (2):

$$V_1 = \frac{Q}{A_1} = \frac{57/3600}{\pi(0.045)^2} = 2.49\ \frac{m}{s}; \quad V_2 = \frac{Q}{A_2} = \frac{57/3600}{\pi(0.015)^2} = 22.4\ \frac{m}{s}$$

Then apply the steady flow energy equation across the pump, neglecting losses:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad \frac{120000}{9790} + \frac{(2.49)^2}{2(9.81)} + 0 = \frac{400000}{9790} + \frac{(22.4)^2}{2(9.81)} + 0 + 0 - h_p, \quad \text{solve for } h_p \approx 53.85 \text{ m}$$

Then the pump power is $P_p = \gamma Q h_p = 9790\left(\frac{57}{3600}\right)(53.85) = 8350 \text{ W} = \mathbf{8.4\ kW}$ *Ans.*

P3.179 Steam enters a horizontal turbine at 350 lbf/in^2 absolute, 580°C, and 12 ft/s and is discharged at 110 ft/s and 25°C saturated conditions. The mass flow is 2.5 lbm/s, and the heat losses are 7 Btu/lb of steam. If head losses are negligible, how much horsepower does the turbine develop?

Solution: We have to use the Steam Tables to find the enthalpies. State (2) is *saturated* vapor at 25°C = 77°F, for which we find $h_2 \approx$ 1095.1 Btu/lbm $\approx$ 2.74E7 ft·lbf/slug. At state (1), 350 psia and 580°C = 1076°F, we find $h_1 \approx$ 1565.3 Btu/lbm $\approx$ 3.92E7 ft·lbf/slug. The heat loss is 7 Btu/lbm ≈ 1.75E5 ft·lbf/slug. The steady flow energy equation is best written on a per-mass basis:

$$q - w_s = h_2 + \frac{1}{2}V_2^2 - h_1 - \frac{1}{2}V_1^2, \quad \text{or:}$$

$$-1.75\text{E}5 - w_s = 2.74\text{E}7 + (110)^2/2 - 3.92\text{E}7 - (12)^2/2, \quad \text{solve for } w_s \approx 1.16\text{E}7\ \frac{\text{ft-lbf}}{\text{slug}}$$

The result is positive because work is done by the fluid. The turbine power at 100% is

$$P_{turb} = \dot{m}w_s = \left(\frac{2.5}{32.2}\ \frac{\text{slug}}{\text{s}}\right)\left(1.16\text{E}7\ \frac{\text{ft}\cdot\text{lbf}}{\text{slug}}\right) = 901000\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{1640\ hp} \quad \textit{Ans.}$$

P3.180 Water at 20°C is pumped at 1500 gal/ min from the lower to the upper reservoir, as in Fig. P3.180. Pipe friction losses are approximated by $h_f \approx 27V^2/(2g)$, where V is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?

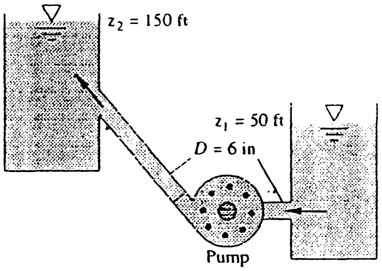


Fig. P3.180

Solution: First evaluate the average velocity in the pipe and the friction head loss:

$$Q = \frac{1500}{448.8} = 3.34\ \frac{\text{ft}^3}{\text{s}}, \quad \text{so } V = \frac{Q}{A} = \frac{3.34}{\pi(3/12)^2} = 17.0\ \frac{\text{ft}}{\text{s}} \quad \text{and} \quad h_f = 27\frac{(17.0)^2}{2(32.2)} \approx \mathbf{121\ ft}$$

Then apply the steady flow energy equation:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 50 = 0 + 0 + 150 + 121 - h_p$$

$$\text{Thus } h_p = 221\text{ ft}, \quad \text{so } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(62.4)(3.34)(221)}{0.75}$$

$$= 61600\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{112\ hp} \quad \textit{Ans.}$$

P3.181 A typical pump has a head which, for a given shaft rotation rate, varies with the flow rate, resulting in a *pump performance curve* as in Fig. P3.181. Suppose that this pump is 75 percent efficient and is used for the system in Prob. 3.180. Estimate (a) the flow rate, in gal/min, and (b) the horsepower needed to drive the pump.

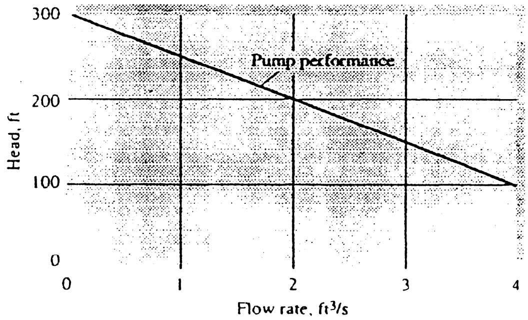


Fig. P3.181

Solution: This time we do not know the flow rate, but the pump head is $h_p \approx 300 - 50Q$, with Q in cubic feet per second. The energy equation directly above becomes,

$$0+0+50 = 0+0+150+(27)\frac{V^2}{2(32.2)} - (300-50Q), \quad \text{where } Q = V\frac{\pi}{4}(0.5\text{ ft})^2$$

This becomes the quadratic $Q^2 + 4.60Q - 18.4 = 0$, solve for $Q \approx 2.57\text{ ft}^3/\text{s}$ [NOTE: The second solution, $Q = 7.17\text{ ft}^3$/s, is far outside the range of Fig. P3.181.]

$$\text{Then the power is } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(62.4)(2.57)[300-50(2.57)]}{0.75}$$

$$= 36700\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{67\ hp} \quad \textit{Ans.}$$

P3.182 The insulated tank in Fig. P3.182 is to be filled from a high-pressure air supply. Initial conditions in the tank are $T = 20°C$ and $p = 200$ kPa. When the valve is opened, the initial mass flow rate into the tank is 0.013 kg/s. Assuming an ideal gas, estimate the initial rate of temperature rise of the air in the tank.

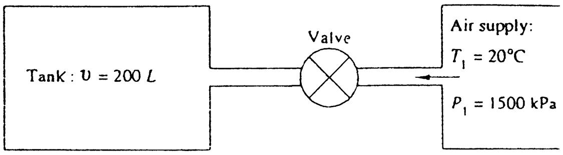


Fig. P3.182

Solution: For a CV surrounding the tank, with *unsteady* flow, the energy equation is

$$\frac{d}{dt}\left(\int e\rho d\upsilon\right)-\dot{m}_{in}\left(\hat{u}+\frac{p}{\rho}+\frac{V^2}{2}+gz\right)=\dot{Q}-\dot{W}_{shaft}=0, \quad \text{neglect } V^2/2 \text{ and gz}$$

$$\text{Rewrite as} \quad \frac{d}{dt}(\rho\upsilon c_v T)\approx \dot{m}_{in}c_p T_{in}=\rho\upsilon c_v\frac{dT}{dt}+c_v T\upsilon\frac{d\rho}{dt}$$

where ρ and T are the instantaneous conditions inside the tank. The CV mass flow gives

$$\frac{d}{dt}\left(\int \rho d\upsilon\right)-\dot{m}_{in}=0, \quad \text{or:} \quad \upsilon\frac{d\rho}{dt}=\dot{m}_{in}$$

Combine these two to eliminate $\upsilon(d\rho/dt)$ and use the given data for air:

$$\left.\frac{dT}{dt}\right|_{tank}=\frac{\dot{m}(c_p-c_v)T}{\rho\upsilon c_v}=\frac{(0.013)(1005-718)(293)}{\left[\frac{200000}{287(293)}\right](0.2\text{ m}^3)(718)}\approx \mathbf{3.2}\ \frac{\mathbf{°C}}{\mathbf{s}} \quad \textit{Ans.}$$

P3.183 The pump in Fig. P3.183 creates a 20°C water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?

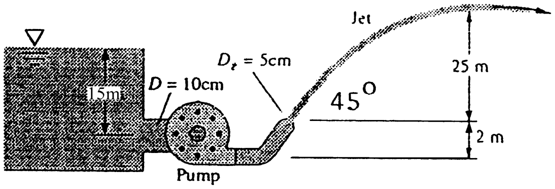


Fig. P3.183

Solution: For maximum travel, the jet must exit at $\theta = 45°$, and the exit velocity must be

$$V_2\sin\theta=\sqrt{2g\Delta z_{max}} \quad \text{or:} \quad V_2=\frac{[2(9.81)(25)]^{1/2}}{\sin 45°}\approx 31.32\ \frac{m}{s}$$

The steady flow energy equation for the piping system may then be evaluated:

$$p_1/\gamma+V_1^2/2g+z_1=p_2/\gamma+V_2^2/2g+z_2+h_f-h_p,$$

$$\text{or:}\quad 0+0+15=0+(31.32)^2/[2(9.81)]+2+6.5-h_p,\quad \text{solve for } h_p\approx 43.5\text{ m}$$

$$\text{Then}\quad P_{pump}=\gamma Q h_p=(9790)\left[\frac{\pi}{4}(0.05)^2(31.32)\right](43.5)\approx \mathbf{26200\ W}\quad \textit{Ans.}$$

P3.184 The large turbine in Fig. P3.184 diverts the river flow under a dam as shown. System friction losses are $h_f = 3.5V^2/(2g)$, where V is the average velocity in the supply pipe. For what river flow rate in m^3/s will the power extracted be 25 MW? Which of the *two* possible solutions has a better "conversion efficiency"?

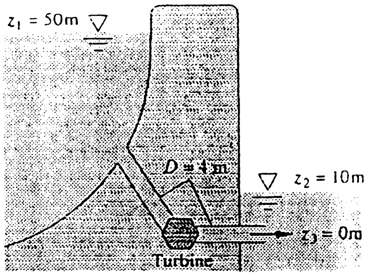


Fig. P3.184

Solution: The flow rate is the unknown, with the turbine power known:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_f+h_{turb},\quad \text{or:}\quad 0+0+50=0+0+10+h_f+h_{turb}$$

$$\text{where}\quad h_f=3.5V_{pipe}^2/(2g)\quad\text{and}\quad h_p=P_p/(\gamma Q)\quad\text{and}\quad V_{pipe}=\frac{Q}{(\pi/4)D_{pipe}^2}$$

Introduce the given numerical data (e.g. Dpipe = 4 m, Ppump = 25E6 W) and solve:

$$Q^3-35410Q+2.261E6=0,\quad \text{with roots}\ Q=\mathbf{+76.5},\ \mathbf{+137.9},\ \text{and}\ -214.4\text{ m}^3/\text{s}$$

The *negative* Q is nonsense. The large Q (137.9m^3/s) gives large friction loss, hf ≈ 21.5 m. The smaller Q (76.5 m^3/s) gives hf ≈ 6.6 m, about right. Select Qriver ≈ **76.5 m^3/s**. *Ans.*

P3.185 Kerosene at 20°C flows through the pump in Fig. P3.185 at 2.3 ft^3/s. Head losses between 1 and 2 are 8 ft, and the pump delivers 8 hp to the flow. What should the mercury-manometer reading *h* ft be?

Solution: First establish the two velocities:

$$V_1 = \frac{Q}{A_1} = \frac{2.3\ \mathrm{ft^3/s}}{(\pi/4)(3/12\ \mathrm{ft})^2}$$

$$= 46.9\ \frac{\mathrm{ft}}{\mathrm{s}};\quad V_2 = \frac{1}{4}V_1 = 11.7\ \frac{\mathrm{ft}}{\mathrm{s}}$$

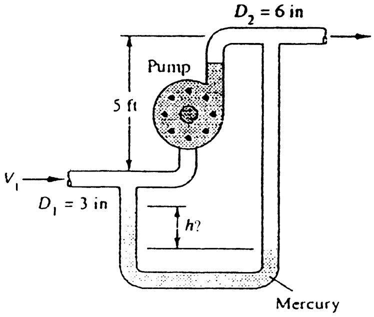


Fig. P3.185

For kerosene take ρ = 804 kg/m^3 = 1.56 slug/ft^3, or γ_k = 1.56(32.2) = 50.2 lbf/ft^3. For mercury take γ_m = 846 lbf/ft^3. Then apply a manometer analysis to determine the pressure difference between points 1 and 2:

$$p_2 - p_1 = (\gamma_m - \gamma_k)h - \gamma_k\Delta z = (846 - 50.2)h - \left(50.2\frac{lbf}{ft^3}\right)(5\ ft) = 796h - 251\ \frac{lbf}{ft^2}$$

Now apply the steady flow energy equation between points 1 and 2:

$$\frac{p_1}{\gamma_k} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_k} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,\quad \text{where } h_p = \frac{P}{\gamma_k Q} = \frac{8(550)\ ft{\cdot}lbf/s}{(50.2)(2.3\ ft^3/s)} = 38.1\ ft$$

$$\text{Thus:}\quad \frac{p_1}{50.2} + \frac{(46.9)^2}{2(32.2)} + 0 = \frac{p_2}{50.2} + \frac{(11.7)^2}{2(32.2)} + 5 + 8 - 38.1\ ft \quad \text{Solve } p_2 - p_1 = 2866\ \frac{lbf}{ft^2}$$

Now, with the pressure difference known, apply the manometer result to find *h*:

$$p_2 - p_1 = 2866 = 796h - 251,\quad \text{or:}\quad h = \frac{2866 + 251\ lbf/ft^2}{796\ lbf/ft^3} = \mathbf{3.92\ ft}\quad \textit{Ans.}$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE3.1 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the flow rate is 160 gal/min, what is the average velocity at section 1?

(a) 2.6 m/s (b) 0.81 m/s (c) 93 m/s (d) 23 m/s (e) 1.62 m/s

FE3.2 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the flow rate is 160 gal/min and friction is neglected, what is the gage pressure at section 1?

(a) 1.4 kPa (b) 32 kPa (c) 43 kPa **(d) 22 kPa** (e) 123 kPa

FE3.3 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the exit velocity is $V_2 = 8$ m/s and friction is neglected, what is the axial flange force required to keep the nozzle attached to pipe 1?

(a) 11 N **(b) 36 N** (c) 83 N (d) 123 N (e) 110 N

FE3.4 In Fig. FE3.1 water exits from a nozzle into atmospheric pressure of 101 kPa. If the manometer fluid has a specific gravity of 1.6 and $h = 66$ cm, with friction neglected, what is the average velocity at section 2?

(a) 4.55 m/s (b) 2.4 m/s **(c) 2.8 m/s** (d) 5.55 m/s (e) 3.4 m/s

FE3.5 A jet of water 3 cm in diameter strikes normal to a plate as in Fig. FE3.5. If the force required to hold the plate is 23 N, what is the jet velocity?

(a) 2.85 m/s **(b) 5.7 m/s** (c) 8.1 m/s (d) 4.0 m/s (e) 23 m/s

FE3.6 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If friction is neglected and the flow rate is 500 gal/min, how high will the outlet water jet rise?

(a) 2.0 m (b) 9.8 m **(c) 32 m** (d) 64 m (e) 98 m

FE3.7 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If friction is neglected and the pump increases the pressure at section 1 to 51 kPa (gage), what will be the resulting flow rate?

(a) 187 gal/min (b) 199 gal/min (c) 214 gal/min (d) 359 gal/min (e) 141 gal/min

FE3.8 A fireboat pump delivers water to a vertical nozzle with a 3:1 diameter ratio, as in Fig. FE3.6. If duct and nozzle friction are neglected and the pump provides 12.3 feet of head to the flow, what will be the outlet flow rate?

(a) 85 gal/min (b) 120 gal/min **(c) 154 gal/min** (d) 217 gal/min (e) 285 gal/min

FE3.9 Water flowing in a smooth 6-cm-diameter pipe enters a venturi contraction with a throat diameter of 3 cm. Upstream pressure is 120 kPa. If cavitation occurs in the throat at a flow rate of 155 gal/min, what is the estimated fluid vapor pressure, assuming ideal frictionless flow?

(a) 6 kPa (b) 12 kPa (c) 24 kPa **(d) 31 kPa** (e) 52 kPa

FE3.10 Water flowing in a smooth 6-cm-diameter pipe enters a venturi contraction with a throat diameter of 4 cm. Upstream pressure is 120 kPa. If the pressure in the throat is 50 kPa, what is the flow rate, assuming ideal frictionless flow?

(a) 7.5 gal/min (b) 236 gal/min **(c) 263 gal/min** (d) 745 gal/min (e) 1053 gal/min

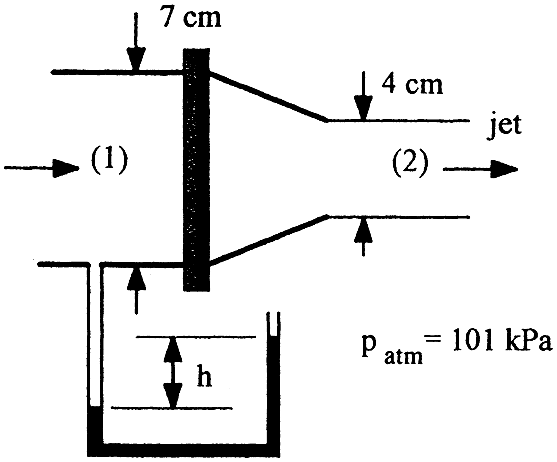


Fig. FE3.1

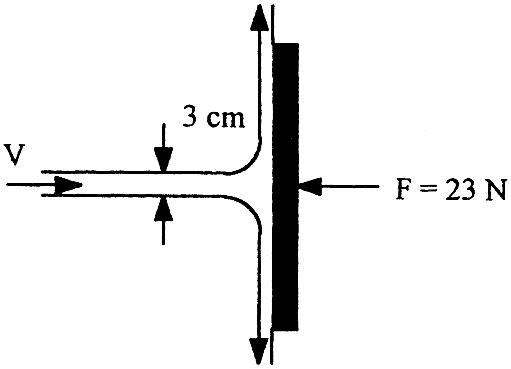


Fig. FE3.5

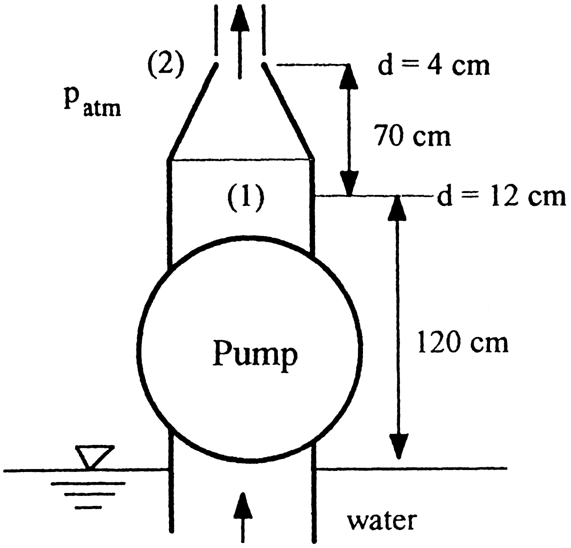
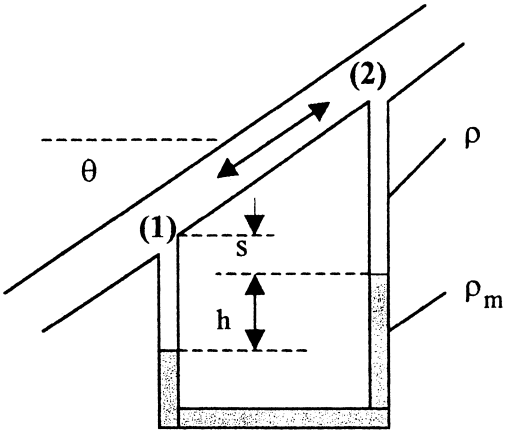


Fig. FE3.6

COMPREHENSIVE PROBLEMS

C3.1 In a certain industrial process, oil of density ρ flows through the inclined pipe in the figure. A U-tube manometer with fluid density ρ_m, measures the pressure difference between points 1 and 2, as shown. The flow is steady, so that fluids in the U-tube are stationary. (a) Find an analytic expression for $p_1 - p_2$ in terms of system parameters. (b) Discuss the conditions on *h* necessary for there to be no flow in the pipe. (c) What about flow *up*, from 1 to 2? (d) What about flow *down*, from 2 to 1?



Solution: (a) Start at 1 and work your way around the U-tube to point 2:

$$p_1 + \rho gs + \rho gh - \rho_m gh - \rho gs - \rho g\Delta z = p_2,$$

$$or: \ \boldsymbol{p_1 - p_2 = \rho g\Delta z + (\rho_m - \rho)gh} \quad \text{where } \Delta z = z_2 - z_1 \quad \textit{Ans. (a)}$$

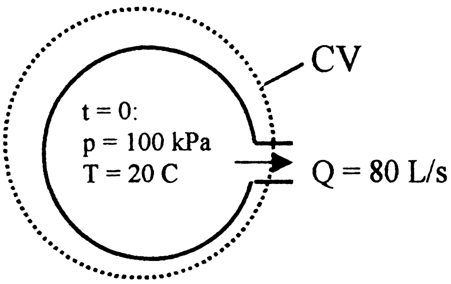
(b) If there is no flow, the pressure is entirely hydrostatic, therefore $\Delta p = \rho g$ and, since $\rho_m \neq \rho$, it follows from Ans. (a) above that **h = 0** *Ans.* (b)

(c) If *h* is positive (as in the figure above), p_1 is greater than it would be for no flow, because of head losses in the pipe. **Thus, if h > 0, flow is *up* from 1 to 2**. *Ans.* (c)

(d) If *h* is negative, p_1 is less than it would be for no flow, because the head losses act against hydrostatics. **Thus, if h < 0, flow is *down* from 2 to 1**. *Ans.* (d)

Note that *h* is a direct measure of flow, regardless of the angle θ of the pipe.

C3.2 A rigid tank of volume $\upsilon = 1.0\ \text{m}^3$ is initially filled with air at 20°C and p_o = 100 kPa. At time t = 0, a vacuum pump is turned on and evacuates air at a constant volume flow rate Q = 80 L/min (regardless of the pressure). Assume an ideal gas and an isothermal process. (a) Set up a differential equation for this flow. (b) Solve this equation for t as a function of (υ, Q, p, p_o). (c) Compute the time in minutes to pump the tank down to p = 20 kPa. [Hint: Your answer should lie between 15 and 25 minutes.]



Solution: The control volume encloses the tank, as shown. The CV mass flow relation becomes

$$\frac{d}{dt}\left(\int \rho d\upsilon\right)+\sum \dot{m}_{out}-\sum \cancel{\dot{m}_{in}}=0$$

Assuming that ρ is constant throughout the tank, the integral equals $\rho\upsilon$, and we obtain

$$\upsilon\frac{d\rho}{dt}+\rho Q=0,\quad or\text{:}\quad \int\frac{d\rho}{\rho}=-\frac{Q}{\upsilon}\int dt,\quad yielding\ \ln\left(\frac{\rho}{\rho_o}\right)=-\frac{Qt}{\upsilon}$$

Where ρ_o is the initial density. But, for an isothermal ideal gas, $\rho/\rho_o = p/p_o$. Thus the time required to pump the tank down to pressure p is given by

$$\boldsymbol{t=-\frac{\upsilon}{Q}\ln\left(\frac{p}{p_o}\right)}\quad \textit{Ans.}\text{ (a, b)}$$

(c) For our particular numbers, noting Q = 80 L/min = 0.080 m^3/min, the time to pump a 1 m^3 tank down from 100 to 20 kPa is

$$t=-\frac{1.0\text{ m}^3}{0.08\text{ m}^3\text{/min}}\ln\left(\frac{20}{100}\right)=\mathbf{20.1\ min}\quad \textit{Ans.}\text{ (c)}$$

C3.3 Suppose the same steady water jet as in Prob. 3.40 (jet velocity 8 m/s and jet diameter 10 cm) impinges instead on a cup cavity as shown in the figure. The water is turned 180° and exits, due to friction, at lower velocity, V_e = 4 m/s. (Looking from the left, the exit jet is a circular annulus of outer radius R and thickness *h*, flowing toward the viewer.) The cup has a radius of curvature of 25 cm. Find (a) the thickness *h* of the exit jet, and (b) the force *F* required to hold the cupped object in place. (c) Compare part (b) to Prob. 3.40, where F = 500 N, and give a physical explanation as to why F has changed.

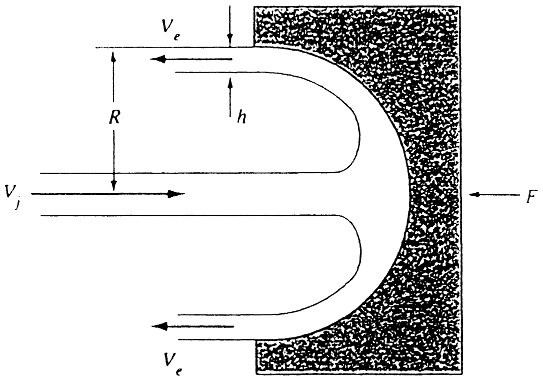


Fig. C3.3

Solution: For a steady-flow control volume enclosing the block and cutting through the jets, we obtain $\Sigma Q_{in} = \Sigma Q_{out}$, or:

$$V_j\frac{\pi}{4}D_j^2=V_e\pi[R^2-(R-h)^2],\quad or\text{:}\quad \boldsymbol{h=R-\sqrt{R^2-\frac{V_j}{V_e}\frac{D_j^2}{4}}}\quad \textit{Ans.}\text{ (a)}$$

For our particular numbers,

$$h = 0.25 - \sqrt{(0.25)^2 - \frac{8}{4}\frac{(0.1)^2}{4}} = 0.25 - 0.2398 = 0.0102 \text{ m} = \mathbf{1.02\ cm} \quad \textit{Ans.} \text{ (a)}$$

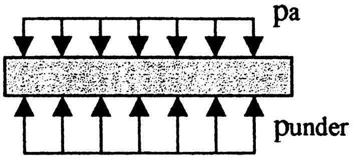
(b) Use the momentum relation, assuming no net pressure force except for F:

$$\sum F_x = -F = \dot{m}_{jet}(-V_e) - \dot{m}_{jet}(V_j), \quad or: \quad \boldsymbol{F = \rho V_j \frac{\pi}{4} D_j^2 (V_j + V_e)} \quad \textit{Ans.} \text{ (b)}$$

For our particular numbers:

$$F = 998(8)\frac{\pi}{4}(0.1)^2(8+4) = \mathbf{752\ N} \quad \textbf{to the left} \quad \textit{Ans.} \text{ (b)}$$

(c) The answer to Prob. 3.40 was 502 N. We get **50% more** because we turned through 180°, not 90°. *Ans.* (c)



C3.4 The air flow beneath an air hockey puck is very complex, especially since the air jets from the table impinge on the puck at various points asymmetrically. A reasonable approximation is that, at any given time, the gage pressure on the bottom of the puck is halfway between zero (atmospheric) and the stagnation pressure of the impinging jets, $p_o = 1/2\ \rho V_{jet}^2$. (a) Find the velocity V_{jet} required to support a puck of weight *W* and diameter *d*, with air density ρ as a parameter. (b) For $W = 0.05$ lbf and $d = 2.5$ inches, estimate the required jet velocity in ft/s.

Solution: (a) The puck has atmospheric pressure on the top and slightly higher on the bottom:

$$(p_{under} - p_a)A_{puck} = W = \frac{1}{2}\left(0 + \frac{\rho}{2}V_{jet}^2\right)\frac{\pi}{4}d^2, \quad Solve\ for \quad \boldsymbol{V_{jet} = \frac{4}{d}\sqrt{\frac{W}{\pi\rho}}} \quad \textit{Ans.} \text{ (a)}$$

For our particular numbers, W = 0.05 lbf and d = 2.5 inches, we assume sea-level air, $\rho = 0.00237$ slug/ft^3, and obtain

$$V_{jet} = \frac{4}{(2.5/12 \text{ ft})}\sqrt{\frac{0.05 \text{ lbf}}{\pi(0.00237 \text{ slug/ft}^3)}} = \mathbf{50\ ft/s} \quad \textit{Ans.} \text{ (b)}$$

C3.5 Neglecting friction sometimes leads to odd results. You are asked to analyze and discuss the following example in Fig. C3.5. A fan blows air vertically through a duct from section 1 to section 2, as shown. Assume constant air density ρ. Neglecting frictional losses, find a relation between the required fan head h_p and the flow rate and the elevation change. Then explain what may be an unexpected result.

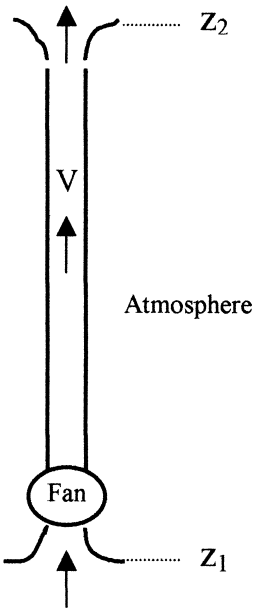


Fig. C3.5

Solution: Neglecting frictional losses, $h_f = 0$, and Bernoulli becomes,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}-h_p$$

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2+\rho g(z_1-z_2)}{\rho g}+\frac{V_2^2}{2g}+z_2-h_p$$

Since the fan draws from and exhausts to atmosphere, $V_1 = V_2 \approx 0$. Solving for h_p,

$$\boldsymbol{h_p=\rho g(z_1-z_2)+\rho g z_2-\rho g z_1=0} \quad \textit{Ans.}$$

Without friction, and with $V_1 = V_2$, the energy equation predicts that $\boldsymbol{h_p = 0}$**!** Because the air has insignificant weight, as compared to a heavier fluid such as water, the power input required to lift the air is also negligible.

Chapter 4 • Differential Relations for a Fluid Particle

P4.1 An idealized velocity field is given by the formula

$$\mathbf{V} = 4tx\mathbf{i} - 2t^2y\mathbf{j} + 4xz\mathbf{k}$$

Is this flow field steady or unsteady? Is it two- or three-dimensional? At the point $(x, y, z) = (-1, +1, 0)$, compute (a) the acceleration vector and (b) any unit vector normal to the acceleration.

Solution: (a) The flow is unsteady because time t appears explicitly in the components. (b) The flow is three-dimensional because all three velocity components are nonzero. (c) Evaluate, by laborious differentiation, the acceleration vector at (x, y, z) = (–1, +1, 0).

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 4x + 4tx(4t) - 2t^2y(0) + 4xz(0) = 4x + 16t^2x$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -4ty + 4tx(0) - 2t^2y(-2t^2) + 4xz(0) = -4ty + 4t^4y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + 4tx(4z) - 2t^2y(0) + 4xz(4x) = 16txz + 16x^2z$$

$$\text{or:} \quad \frac{d\mathbf{V}}{dt} = (4x + 16t^2x)\mathbf{i} + (-4ty + 4t^4y)\mathbf{j} + (16txz + 16x^2z)\mathbf{k}$$

$$\text{at } (x, y, z) = (-1, +1, 0), \text{ we obtain} \quad \frac{d\mathbf{V}}{dt} = -4(1 + 4t^2)\mathbf{i} - 4t(1 - t^3)\mathbf{j} + 0\mathbf{k} \quad \textit{Ans.}\text{ (c)}$$

(d) At (–1, +1, 0) there are many unit vectors normal to d**V**/dt. One obvious one is **k**. *Ans.*

P4.2 Flow through the converging nozzle in Fig. P4.2 can be approximated by the one-dimensional velocity distribution

$$u \approx V_0\left(1 + \frac{2x}{L}\right) \quad \upsilon \approx 0 \quad w \approx 0$$

(a) Find a general expression for the fluid acceleration in the nozzle. (b) For the specific case V_0 = 10 ft/s and L = 6 in, compute the acceleration, in g's, at the entrance and at the exit.

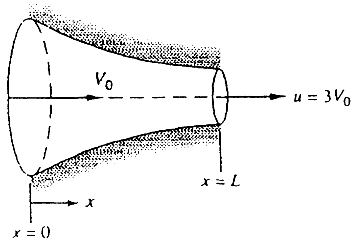


Fig. P4.2

Solution: Here we have only the single ‘one-dimensional’ convective acceleration:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} = \left[V_o\left(1+\frac{2x}{L}\right)\right]\frac{2V_o}{L} = \frac{\mathbf{2V_o^2}}{\mathbf{L}}\left(\mathbf{1}+\frac{\mathbf{2x}}{\mathbf{L}}\right) \quad \textit{Ans.}\text{ (a)}$$

$$For\ L = 6'' \ and\ V_o = 10\ \frac{ft}{s},\quad \frac{du}{dt} = \frac{2(10)^2}{6/12}\left(1+\frac{2x}{6/12}\right) = 400(1+4x),\ with\ x\ in\ feet$$

At x = 0, du/dt = **400** ft/s^2 (12 g’s); at x = L = 0.5 ft, du/dt = **1200** ft/s^2 (37 g’s). *Ans.* (b)

P4.3 A two-dimensional velocity field is given by

$$\mathbf{V} = (x^2 - y^2 + x)\mathbf{i} - (2xy + y)\mathbf{j}$$

in arbitrary units. At (*x*, *y*) = (1, 2), compute (a) the accelerations a_x and a_y, (b) the velocity component in the direction $\theta = 40°$, (c) the direction of maximum velocity, and (d) the direction of maximum acceleration.

Solution: (a) Do each component of acceleration:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (x^2 - y^2 + x)(2x+1) + (-2xy - y)(-2y) = a_x$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (x^2 - y^2 + x)(-2y) + (-2xy - y)(-2x-1) = a_y$$

At (x, y) = (1, 2), we obtain $\mathbf{a_x} = 18\mathbf{i}$ and $\mathbf{a_y} = 26\mathbf{j}$ *Ans.* (a)

(b) At (x, y) = (1, 2), $\mathbf{V} = -2\mathbf{i} - 6\mathbf{j}$. A unit vector along a 40° line would be $\mathbf{n} = \cos 40°\mathbf{i} + \sin 40°\mathbf{j}$. Then the velocity component along a 40° line is

$$V_{40°} = \mathbf{V}\cdot\mathbf{n_{40°}} = (-2\mathbf{i} - 6\mathbf{j})\cdot(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) \approx \mathbf{5.39\ units} \quad \textit{Ans.}\text{ (b)}$$

(c) The maximum acceleration is $\mathbf{a_{max}} = [18^2 + 26^2]^{1/2} = \mathbf{31.6}$ units at $\angle 55.3°$ *Ans.* (c, d)

P4.4 A simple flow model for a two-dimensional converging nozzle is the distribution

$$u = U_o(1+\frac{x}{L}) \qquad v = -U_o\frac{y}{L} \qquad w = 0$$

(*a*) Sketch a few streamlines in the region $0<x/L<1$ and $0<y/L<1$, using the method of Section 1.11. (*b*) Find expressions for the horizontal and vertical accelerations.

(*c*) Where is the largest resultant acceleration and its numerical value?

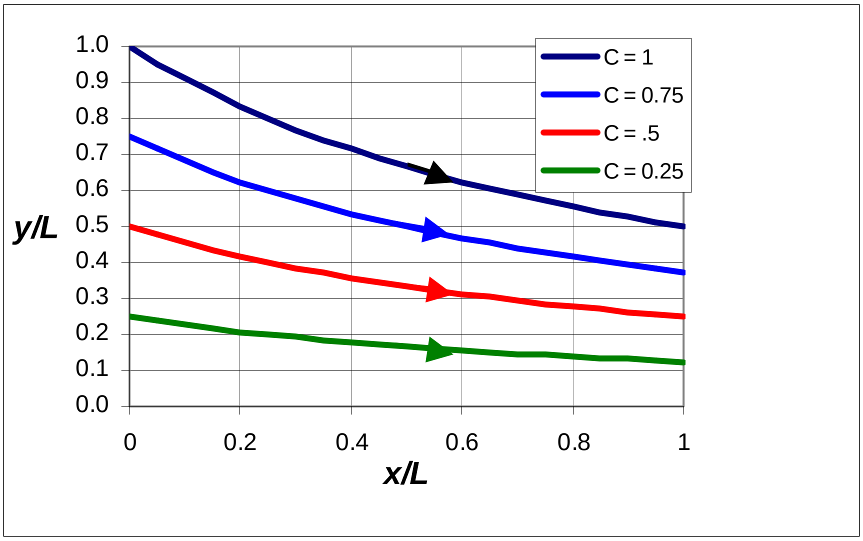
Solution: The streamlines are in the *x*-*y* plane and are found from the velocities:

$$\frac{dx}{u} = \frac{dy}{v} \quad \text{or integrate:} \quad \int \frac{dx}{U_o(1+x/L)} = -\int \frac{dy}{U_o y/L} \quad \text{Cancel } U_o$$

$$L\ln(1+x/L) = -L\ln(y/L) + const, \quad or: \ \ln[(y/L)(1+x/L)] = constant$$

$$\text{Finally the streamlines:} \quad \mathbf{\frac{y}{L} = \frac{C}{1+x/L}} \qquad \textit{Ans.}(a)$$

These may be plotted for various values of the dimensionless constant *C*, as shown:



The streamlines converge and the velocity increases to the right. *Ans.*(*a*)

(*b*) The accelerations are calculated from Eq. (4.2):

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = [U_o(1+x/L)](U_o/L) + 0 = \frac{U_o^2}{L}(1+\frac{x}{L})$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + (-U_o y/L)(-U_o/L) = \frac{U_o^2}{L}\frac{y}{L} \qquad \textit{Ans.}(b)$$

(*c*) Find the resultant of a_x and a_y from *Ans*.(*b*) above and introduce *y/L* from *Ans*.(*a*):

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{1+2\eta+\eta^2 + C^2/(1+\eta)^2}, \quad \text{where} \quad \eta = x/L \qquad \textit{Ans.}(c)$$

We observe that the resultant acceleration increases with *x* and is greatest at *x* = *L*, where its numerical value is $(U_o^2/L)\,[4 + C^2/4]^{1/2}$.

P4.5 The velocity field near a stagnation point (see Example 1.10) may be written in the form

$$u = \frac{U_o x}{L} \qquad v = \frac{-U_o y}{L} \qquad U_o \text{ and } L \text{ are constants}$$

(a) Show that the acceleration vector is purely radial. (b) For the particular case *L* = 1.5 m, if the acceleration at (*x*, *y*) = (1 m, 1 m) is 25 m/s^2, what is the value of *U*o?

Solution: (a) For two-dimensional steady flow, the acceleration components are

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(U_o\frac{x}{L}\right)\left(\frac{U_o}{L}\right) + \left(-U_o\frac{y}{L}\right)(0) = \frac{U_o^2}{L^2}x$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \left(U_o\frac{x}{L}\right)(0) + \left(-U_o\frac{y}{L}\right)\left(-\frac{U_o}{L}\right) = \frac{U_o^2}{L^2}y$$

Therefore the resultant $\mathbf{a} = (U_o^2/L^2)(x\mathbf{i} + y\mathbf{j}) = (U_o^2/L^2)\mathbf{r}$ (purely radial) *Ans.* (a)

(b) For the given resultant acceleration of 25 m/s^2 at (x, y) = (1 m, 1 m), we obtain

$$|a| = 25\ \frac{m}{s^2} = \frac{U_o^2}{L^2}|r| = \frac{U_o^2}{(1.5\text{ m})^2}\sqrt{2\text{ m}}, \quad \text{solve for } U_o = \mathbf{6.3}\ \frac{\mathbf{m}}{\mathbf{s}} \qquad \textit{Ans.}\text{ (b)}$$

P4.6 An incompressible plane flow has the velocity components $u = 2y$, $v = 8x$, $w = 0$. (*a*) Find the acceleration components. (*b*) Determine if the vector acceleration is radial.

(*c*) Which two streamlines of this flow are straight lines?

Solution: (*a*, *b*) With no *z* activity, we can stick to steady two-dimensional formulas:

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (2y)(0) + (8x)(2) = 16x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (2y)(8) + (8x)(0) = 16y \qquad \textit{Ans.}(a)$$

$$\mathbf{a} = \mathbf{i}a_x + \mathbf{j}a_y = 16\mathbf{i}x + 16\mathbf{j}y = 16(\mathbf{i}x + \mathbf{j}y) = 16\mathbf{r} \qquad \textbf{Radial}\text{, yes. } \textit{Ans.}(b)$$

(*c*) Find the streamlines in the manner suggested in Chapter 1, Eq. (1.41):

$$\frac{dx}{u} = \frac{dy}{v}, \quad \text{or: } \frac{dx}{2y} = \frac{dy}{8x}, \quad 2y\,dy = 8x\,dx$$

$$\text{Integrate: } \int 2y\,dy = \int 8x\,dx, \text{ or: } y^2 = 4x^2 + const$$

The streamlines are all *curved* except when the constant of integration is zero, for which there are two *straight* streamlines: $y = \pm 2x$. *Ans.*(*c*)

P4.7 Consider a sphere of radius *R* immersed in a uniform stream *U*o, as shown in Fig. P4.7. According to the theory of Chap. 8, the fluid velocity along streamline *AB* is given by

$$\mathbf{V} = u\mathbf{i} = U_o\left(1+\frac{R^3}{x^3}\right)\mathbf{i}$$

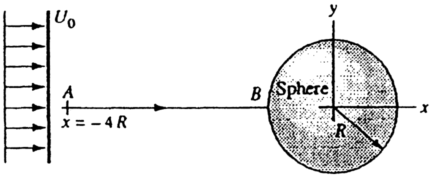


Fig. P4.7

Find (a) the position of maximum fluid acceleration along *AB* and (b) the time required for a fluid particle to travel from *A* to *B*. Note that *x* is negative along line AB.

Solution: (a) Along this streamline, the fluid acceleration is one-dimensional:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} = U_o(1+R^3/x^3)(-3U_oR^3/x^4) = -3U_oR^3(x^{-4}+R^3x^{-7}) \quad \text{for } x \le -R$$

The maximum occurs where $d(a_x)/dx = 0$, or at $x = -(7R^3/4)^{1/3} \approx$ $\mathbf{-1.205R}$ *Ans.* (a)
(b) The time required to move along this path from A to B is computed from

$$u = \frac{dx}{dt} = U_o(1+R^3/x^3), \quad \text{or:} \quad \int_{-4R}^{-R}\frac{dx}{1+R^3/x^3} = \int_0^t U_o\,dt,$$

$$\text{or:} \quad U_o t = \left[x - \frac{R}{6}\ln\frac{(x+R)^2}{x^2-Rx+R^2} + \frac{R}{\sqrt{3}}\tan^{-1}\left(\frac{2x-R}{R\sqrt{3}}\right)\right]_{-4R}^{-R} = \infty$$

It takes **an infinite time** to actually *reach* the stagnation point, where the velocity is zero. *Ans.* (b)

P4.8 When a valve is opened, fluid flows in the expansion duct of Fig. P4.8 according to the approximation

$$\mathbf{V} = \mathbf{i}U\left(1-\frac{x}{2L}\right)\tanh\frac{Ut}{L}$$

Find (a) the fluid acceleration at $(x, t) = (L, L/U)$ and (b) the time for which the fluid acceleration at $x = L$ is zero. Why does the fluid acceleration become negative after condition (b)?

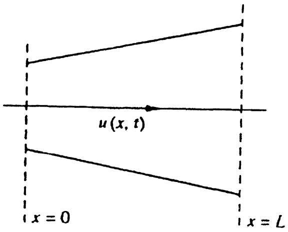


Fig. P4.8

Solution: This is a one-dimensional *unsteady* flow. The acceleration is

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = U\left(1 - \frac{x}{2L}\right)\frac{U}{L}\text{sech}^2\left(\frac{Ut}{L}\right) - U\left(1 - \frac{x}{2L}\right)\left(\frac{U}{2L}\right)\tanh\left(\frac{Ut}{L}\right)$$

$$= \frac{U^2}{L}(1 - \frac{x}{2L})[\text{sech}^2\left(\frac{Ut}{L}\right) - \frac{1}{2}\tanh^2\left(\frac{Ut}{L}\right)]$$

At $(x, t) = (L, L/U)$, $a_x = (U^2/L)(1/2)[\text{sech}^2(1) - 0.5\tanh^2(1)] \approx \mathbf{0.0650\ U^2/L}$ *Ans.* (a)

The acceleration becomes zero when

$$\text{sech}^2\left(\frac{Ut}{L}\right) = \frac{1}{2}\tanh^2\left(\frac{Ut}{L}\right), \qquad \text{or} \quad \sinh^2\left(\frac{Ut}{L}\right) = 2,$$

$$\text{or} \quad \frac{Ut}{L} = 1.146 \qquad \textit{Ans. (b)}$$

The acceleration starts off positive, then goes through zero and turns negative as the negative *convective* acceleration overtakes the decaying positive *local* acceleration.

P4.9 An idealized incompressible flow has the proposed three-dimensional velocity distribution

$$\mathbf{V} = 4xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$$

Find the appropriate form of the function $f(y)$ which satisfies the continuity relation.

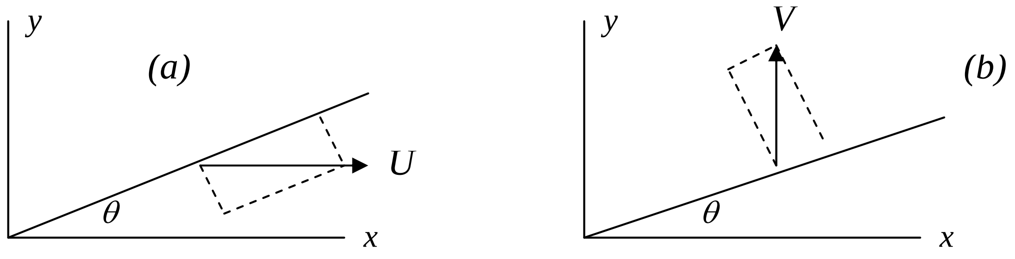
Solution: Simply substitute the given velocity components into the incompressible continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{\partial}{\partial x}(4xy^2)+\frac{\partial f}{\partial y}+\frac{\partial}{\partial z}(-zy^2)=4y^2+\frac{df}{dy}-y^2=0$$

$$or:\quad \frac{df}{dy}=-3y^2.\quad \text{Integrate: } f(y)=\int(-3y^2)dy=\mathbf{-y^3+constant}\quad \textit{Ans.}$$

P4.10 Consider the simple incompressible plane flow pattern $u = U$, $v = V$, and $w = 0$, where U and V are constants. (*a*) Convert these velocities into polar coordinate components, v_r and v_θ. [HINT: Make a sketch of the velocity components.] (*b*) Determine whether these new components satisfy the continuity equation in polar coordinates.

Solution: This is harder than it looks. Make a sketch of each separate cartesian component:



(*a*) We can resolve each figure into radial and circumferential components. For Figure (a), U has a radial component $U\cos\theta$ and a circumferential component $(-U\sin\theta)$. For Figure (b), V has a radial component $V\sin\theta$ and a circumferential component $V\cos\theta$. Combine these into the result

$$v_r = U\cos\theta + V\sin\theta \quad ; \quad v_\theta = -U\sin\theta + V\cos\theta \qquad \textit{Ans.}(a)$$

(*b*) The original (cartesian) distribution, being constant velocity, obviously satisfied continuity. The new version, in polar coordinates, requires some effort. From Eq. (4.9) for incompressible flow,

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) = \frac{1}{r}\frac{\partial}{\partial r}[\,r\,(U\cos\theta+V\sin\theta)] + \frac{1}{r}\frac{\partial}{\partial\theta}(-U\sin\theta+V\cos\theta)$$

$$= \frac{1}{r}(U\cos\theta+V\sin\theta) + \frac{1}{r}(-U\cos\theta-V\sin\theta) = \mathbf{0} \quad \text{Yes, satisfied.} \quad \textit{Ans.}(b)$$

P4.11 Derive Eq. (4.12*b*) for cylindrical coordinates by considering the flux of an incompressible fluid in and out of the elemental control volume in Fig. 4.2.

Solution: For the differential CV shown,

$$\frac{\partial\rho}{\partial t}\,d\upsilon ol + \sum d\dot{m}_{out} - \sum d\dot{m}_{in} = 0$$

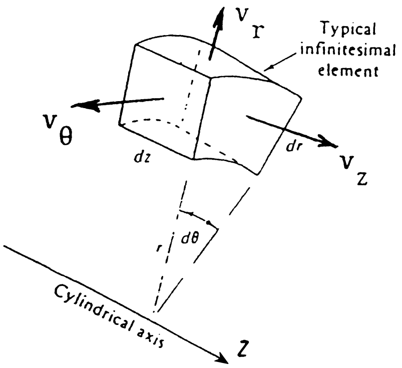


Fig. 4.2

$$\frac{\partial\rho}{\partial t}\left(r+\frac{dr}{2}\right)d\theta\,dr\,dz + \rho v_r r\,dz\,d\theta + \frac{\partial}{\partial r}(\rho v_r)dr(r+dr)dz\,d\theta + \rho v_\theta\,dz\,dr$$

$$+\frac{\partial}{\partial\theta}(\rho v_\theta)d\theta\,dz\,dr + \rho v_z\left(r+\frac{dr}{2}\right)d\theta\,dr + \frac{\partial}{\partial z}(\rho v_z)\left(r+\frac{dr}{2}\right)d\theta\,dr$$

$$-\rho v_r r\,dz\,d\theta - \rho v_\theta\,dz\,dr - \rho v_z\left(r+\frac{dr}{2}\right)d\theta\,dr = 0$$

Cancel ($d\theta\,dr\,dz$) and higher-order (4th-order) differentials such as ($dr\,d\theta\,dz\,dr$) and, finally, divide by *r* to obtain the final result:

$$\boldsymbol{\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z) = 0} \quad \textit{Ans.}$$

P4.12 Spherical polar coordinates (r, θ, ϕ) are defined in Fig. P4.12. The cartesian transformations are

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Do not show that the cartesian incompressible continuity relation (4.12*a*) can be transformed to the spherical polar form

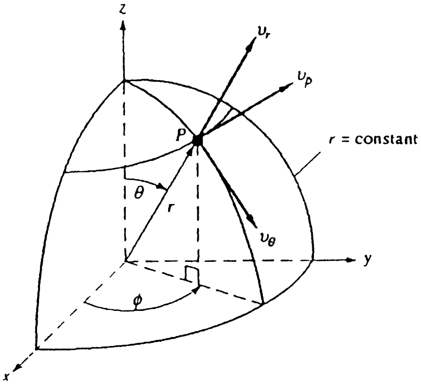


Fig. P4.12

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\upsilon_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\upsilon_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\upsilon_\phi) = 0$$

What is the most general form of υ_r when the flow is purely radial, that is, υ_θ and υ_ϕ are zero?

Solution: *Note to instructors*: **Do not assign the derivation of this continuity relation, it takes years to achieve, the writer can't do it successfully.** The problem is only meant to acquaint students with spherical coordinates.

If $\upsilon_\theta = \upsilon_\phi = 0$, then $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\upsilon_r) = 0$, so, in general, $\boldsymbol{\upsilon_r = \frac{1}{r^2}\,\mathrm{fcn}(\theta,\ \phi)}$ *Ans.*

P4.13 For an incompressible plane flow in polar coordinates, we are given

$$v_r = r^3\cos\theta + r^2\sin\theta$$

Find the appropriate form of circumferential velocity for which continuity is satisfied.

Solution: Substitute into continuity, Eq. (4.9), for incompressible flow:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) = \frac{1}{r}\frac{\partial}{\partial r}[r(r^3\cos\theta + r^2\sin\theta)] + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta},$$

$$\text{or}: \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} = -4r^2\cos\theta - 3r\sin\theta$$

$$\text{Integrate}: \quad v_\theta = -4r^3\sin\theta + 3r^2\cos\theta + f(r) \qquad \textit{Ans.}$$

We can’t determine the form of the “constant of integration” *f*(*r*) without further information.

P4.14 For incompressible polar-coordinate flow, what is the most general form of a purely circulatory motion, $\upsilon_\theta = \upsilon_\theta(r, \theta, t)$ and $\upsilon_r = 0$, which satisfies continuity?

Solution: If $v_r = 0$, the plane polar coordinate continuity equation reduces to:

$$\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} = 0, \quad \text{or:} \quad \boldsymbol{v_\theta = \mathbf{fcn(r)}} \text{ only} \quad \textit{Ans.}$$

P4.15 What is the most general form of a purely radial polar-coordinate incompressible-flow pattern, $\upsilon_r = \upsilon_r(r, \theta, t)$ and $\upsilon_\theta = 0$, which satisfies continuity?

Solution: If $v_\theta = 0$, the plane polar coordinate continuity equation reduces to:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) = 0, \quad \text{or:} \quad \boldsymbol{v_r = \frac{1}{r}\mathbf{fcn}(\theta)} \text{ only} \quad \textit{Ans.}$$

P4.16 Consider the plane polar coordinate velocity distribution

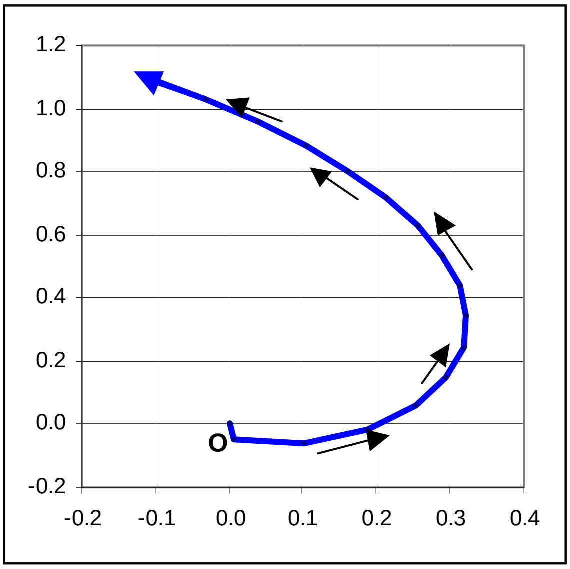
$$v_r = \frac{C}{r} \qquad v_\theta = \frac{K}{r} \qquad v_z = 0$$

where C and K are constants. (*a*) Determine if the incompressible equation of continuity is satisfied. (*b*) By sketching some velocity vector directions, plot a single streamline for $C = K$. What might this flow field simulate?

Solution: (*a*) Evaluate the incompressible continuity equation (4.12*b*) in polar coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C}{r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{K}{r}\right) = 0 + 0 = 0 \qquad \textit{Ans.(a)}$$

Incompressible continuity is indeed satisfied. (*b*) For $C = K$, we can plot a representative streamline by putting in some velocity vectors and sketching a line parallel to them:



The streamlines are logarithmic spirals moving out from the origin. [They have axisymmetry about O.] This simple distribution is often used to simulate a swirling flow such as a tornado.

P4.17 An excellent approximation for the two-dimensional incompressible laminar boundary layer on the flat surface in Fig. P4.17 is

$$u \approx U\left(2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right) \quad \text{for } y \le \delta \text{, where } \delta = C x^{1/2},\ C = \text{ constant}$$

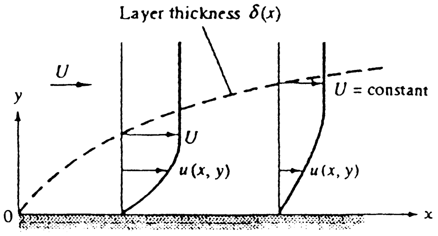


Fig. P4.17

(*a*) Assuming a no-slip condition at the wall, find an expression for the velocity component *v*(*x*, *y*) for $y \le \delta$. (*b*) Then find the maximum value of *v* at the station *x* = 1 m, for the particular case of airflow, when *U* = 3 m/s and δ = 1.1 cm.

Solution: (*a*) With *u* known, use the two-dimensional equation of continuity to find *v*:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -U\left(-\frac{2y}{\delta^2}\frac{d\delta}{dx} + \frac{6y^3}{\delta^4}\frac{d\delta}{dx} - \frac{4y^4}{\delta^5}\frac{d\delta}{dx}\right),$$

$$or:\ v = 2U\frac{d\delta}{dx}\int_0^y \left(\frac{y}{\delta^2} - \frac{3y^3}{\delta^4} + \frac{2y^4}{\delta^5}\right)dy = 2U\frac{d\delta}{dx}\left(\frac{y^2}{2\delta^2} - \frac{3y^4}{4\delta^4} + \frac{2y^5}{5\delta^5}\right) \qquad Ans.(a)$$

(*b*) First evaluate *C* from the given data at *x* = 1 m:

$$\delta = 0.011m = C(1m)^{1/2}\ , \quad \text{hence } C = 0.011\, m^{1/2}$$

$$\text{Or, alternately}, \ \frac{d\delta}{dx} = \frac{1}{2}C x^{-1/2} = \frac{1}{2}\left(\frac{\delta}{x^{1/2}}\right)x^{-1/2} = \frac{\delta}{2x}$$

Substitute this into *Ans.*(*a*) above and note that *v* rises monotonically with *y* to a **maximum** at the outer edge of the boundary layer, $y = \delta$. The maximum velocity *v* is thus

$$v_{\max} \approx 2U\frac{d\delta}{dx}\left(\frac{1}{2}-\frac{3}{4}+\frac{2}{5}\right) = 2\left(3\frac{m}{s}\right)\left[\frac{0.011m}{2(1m)}\right]\left(\frac{3}{20}\right) \approx \mathbf{0.0050\ \frac{m}{s}} \quad Ans.(b)$$

This is slightly smaller than the exact value of $v_{\max}$ from laminar boundary theory (Chap. 7).

P4.18 A piston compresses gas in a cylinder by moving at constant speed *V*, as in Fig. P4.18. Let the gas density and length at $t = 0$ be ρ_o and L_o, respectively. Let the gas velocity vary linearly from $u = V$ at the piston face to $u = 0$ at $x = L$. If the gas density varies only with time, find an expression for $\rho(t)$.

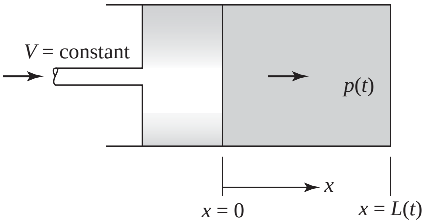


Fig. P4.18

Solution: The one-dimensional unsteady continuity equation reduces to

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=\frac{d\rho}{dt}+\rho\frac{\partial u}{\partial x}, \quad \text{where } u = V\left(1-\frac{x}{L}\right), \quad L = L_o - Vt, \quad \rho = \rho(t) \text{ only}$$

$$\text{Enter } \frac{\partial u}{\partial x} = -\frac{V}{L} \quad \text{and separate variables: } \int_{\rho_o}^{\rho}\frac{d\rho}{\rho} = V\int_{o}^{t}\frac{dt}{L_o - Vt}$$

$$\text{The solution is } \ln(\rho/\rho_o) = -\ln(1 - Vt/L_o), \quad \text{or: } \rho = \rho_o\left(\frac{\mathbf{L_o}}{\mathbf{L_o - Vt}}\right) \quad Ans.$$

P4.19 An incompressible flow field has the cylindrical velocity components $\upsilon_\theta = Cr$, $\upsilon_z = K(R^2 - r^2)$, $\upsilon_r = 0$, where *C* and *K* are constants and $r \le R$, $z \le L$. Does this flow satisfy continuity? What might it represent physically?

Solution: We check the incompressible continuity relation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial v_z}{\partial z} = 0 = 0+0+0 \quad \textbf{satisfied identically} \quad Ans.$$

This flow also satisfies (cylindrical) momentum and could represent laminar flow inside a tube of radius R whose outer wall (r = R) is rotating at uniform angular velocity.

P4.20 A two-dimensional incompressible velocity field has $u = K(1 - e^{-ay})$, for $x \le L$ and $0 \le y \le \infty$. What is the most general form of $v(x, y)$ for which continuity is satisfied and $v = v_o$ at $y = 0$? What are the proper dimensions for constants K and a?

Solution: We can find the appropriate velocity v from two-dimensional continuity:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}[K(1-e^{-ay})] = 0, \quad \text{or:} \quad v = \text{fcn}(x) \text{ only}$$

Since $v = v_o$ at y = 0 for all x, then it must be that $\boldsymbol{v = v_o = \text{const}}$ *Ans.*

The dimensions of K are $\mathbf{\{K\} = \{L/T\}}$ and the dimensions of a are $\mathbf{\{L^{-1}\}}$. *Ans.*

P4.21 Air flows under steady, approximately one-dimensional conditions through the conical nozzle in Fig. P4.21. If the speed of sound is approximately 340 m/s, what is the minimum nozzle-diameter ratio D_e/D_o for which we can safely neglect compressibility effects if V_o = (a) 10 m/s and (b) 30 m/s?

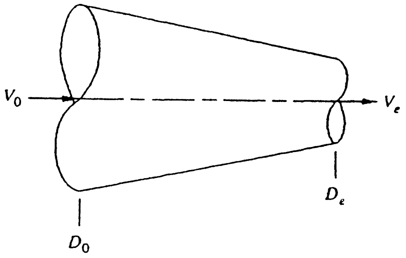


Fig. P4.21

Solution: If we apply one-dimensional continuity to this duct,

$$\rho_o V_o \frac{\pi}{4} D_o^2 = \rho_e V_e \frac{\pi}{4} D_e^2, \quad \text{or} \quad V_o \approx V_e (D_e/D_o)^2 \quad \text{if} \quad \rho_o \approx \rho_e$$

To avoid compressibility corrections, we require (Eq. 4.18) that Ma ≤ 0.3 or, in this case, the highest velocity (at the exit) should be $V_e \le 0.3(340) = 102$ m/s. Then we compute

$$(D_e/D_o)_{\min} = (V_o/V_e)^{1/2} = (V_o/102)^{1/2} = \mathbf{0.31} \quad \text{if } V_o = 10 \text{ m/s} \quad \textit{Ans. } (a)$$

$$= \mathbf{0.54} \quad \text{if } V_o = 30 \text{ m/s} \quad \textit{Ans. } (b)$$

P4.22 In an *axisymmetric* flow, nothing varies with θ; the only nonzero velocities are v_r and v_z (see Fig. 4.2 of the text). If the flow is steady and incompressible and $v_z = Bz$, where B is constant, find the most general form of v_r which satisfies continuity.

Solution: With no θ variation and no v_θ, the equation of continuity (4.9) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0 = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial}{\partial z}(Bz) ,$$

$$\text{or: } \frac{\partial}{\partial r}(r v_r) = -Br \text{ ; Integrate: } r v_r = -\frac{B}{2}r^2 + f(z)$$

$$\text{Finally, } \quad v_r = -\frac{B}{2}r + \frac{f(z)}{r} \qquad \textit{Ans.}$$

The "function of integration", $f(z)$, is arbitrary, at least until boundary conditions are set.

P4.23 A tank volume $\mathcal{V}$ contains gas at conditions (ρ_o, p_o, T_o). At time $t = 0$ it is punctured by a small hole of area A. According to the theory of Chap. 9, the mass flow out of such a hole is approximately proportional to A and to the tank pressure. If the tank temperature is assumed constant and the gas is ideal, find an expression for the variation of density within the tank.

Solution: This problem is a realistic approximation of the "blowdown" of a high-pressure tank, where the exit mass flow is choked and thus proportional to tank pressure. For a control volume enclosing the tank and cutting through the exit jet, the mass relation is

$$\frac{d}{dt}(m_{tank}) + \dot{m}_{exit} = 0, \quad \text{or:} \quad \frac{d}{dt}(\rho\upsilon) = -\dot{m}_{exit} = -C\,p\,A, \quad \text{where } C = \text{constant}$$

$$\text{Introduce} \quad \rho = \frac{p}{RT_o} \quad \text{and separate variables:} \quad \int_{p_o}^{p(t)} \frac{dp}{p} = -\frac{CRT_oA}{\upsilon}\int_0^t dt$$

The solution is an exponential decay of tank density: $\mathbf{p = p_o \exp(-CRT_oAt/\mathcal{V})}$. *Ans.*

P4.24 For incompressible laminar flow between parallel plates (see Fig. 4.12*b*), the flow is two-dimensional ($v \neq 0$) if the walls are porous. A special case solution is $u = (A - Bx)(h^2 - y^2)$, where A and B are constants. (*a*) Find a general formula for velocity v if $v = 0$ at $y = 0$. (*b*) What is the value of the constant B if $v = v_w$ at $y = +h$?

Solution: (*a*) Use the equation of continuity to find the velocity v:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -(-B)(h^2 - y^2)$$

$$\text{Integrate: } v = B\int(h^2 - y^2)dy = B(h^2 y - \frac{y^3}{3}) + f(x)$$

$$\text{If } v=0 \text{ at } y=0, \text{ then } f(x) = 0. \quad \therefore \quad v = B(h^2 y - \frac{y^3}{3}) \qquad Ans.(a)$$

(*b*) Just simply introduce this boundary condition into the answer to part (*a*):

$$v(y=h) = v_w = B(h^3 - \frac{h^3}{3}), \quad hence \quad B = \frac{3v_w}{2h^3} \qquad Ans.(b)$$

P4.25 An incompressible flow in polar coordinates is given by

$$v_r = K\cos\theta\left(1 - \frac{b}{r^2}\right)$$

$$v_\theta = -K\sin\theta\left(1 + \frac{b}{r^2}\right)$$

Does this field satisfy continuity? For consistency, what should the dimensions of constants *K* and *b* be? Sketch the surface where v*r* = 0 and interpret.

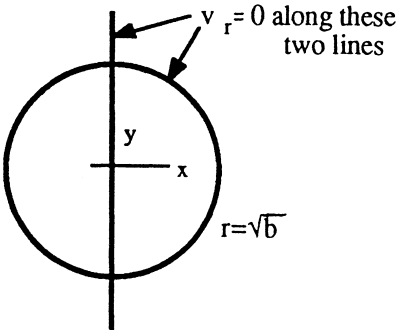


Fig. P4.25

Solution: Substitute into plane polar coordinate continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 \overset{?}{=} \frac{1}{r}\frac{\partial}{\partial r}\left[K\cos\theta\left(r - \frac{b}{r}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[-K\sin\theta\left(1 + \frac{b}{r^2}\right)\right] = 0 \quad \textit{Satisfied}$$

The dimensions of *K* must be velocity, {K} = {L/T}, and *b* must be area, {b} = {L^2}. The surfaces where $v_r = 0$ are the y-axis and the circle r = √b, as shown above. The pattern represents inviscid flow of a uniform stream past a circular cylinder (Chap. 8).

P4.26 Curvilinear, or streamline, coordinates are defined in Fig. P4.26, where n is normal to the streamline in the plane of the radius of curvature R. Show that Euler's frictionless momentum equation (4.36) in streamline coordinates becomes

$$\partial V/\partial t + V(\partial V/\partial s) = -(1/\rho)(\partial p/\partial s) + g_s \qquad (1)$$

$$-V\frac{\partial\theta}{\partial t}-\frac{V^2}{R}=-\frac{1}{\rho}\frac{\partial p}{\partial n}+g_n \qquad (2)$$

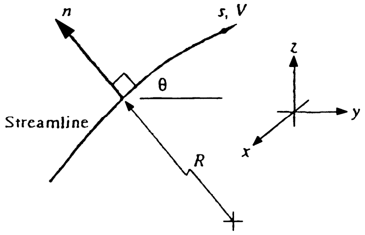


Fig. P4.26

Further show that the integral of Eq. (1) with respect to *s* is none other than our old friend Bernoulli's equation (3.76).

Solution: This is a laborious derivation, really, **the problem is only meant to *acquaint* the student with streamline coordinates.** The second part is not too hard, though. Multiply the streamwise momentum equation by *ds* and integrate:

$$\frac{\partial V}{\partial t}ds + V\,dV = -\frac{dp}{\rho_2} + g_s\,ds = -\frac{dp}{\rho} - g\sin\theta\,ds = -\frac{dp}{\rho} - g\,dz$$

Integrate from 1 to 2: $\mathbf{\int_1^2 \frac{\partial V}{\partial t}ds + \frac{V_2^2 - V_1^2}{2} + \int_1^2 \frac{dp}{\rho} + g(z_2 - z_1) = 0}$ (Bernoulli) *Ans.*

P4.27 A frictionless, incompressible steady-flow field is given by

$$\mathbf{V} = 2xy\mathbf{i} - y^2\mathbf{j}$$

in arbitrary units. Let the density be ρ_0 = constant and neglect gravity. Find an expression for the pressure gradient in the *x* direction.

Solution: For this (gravity-free) velocity, the momentum equation is

$$\rho\left(u\frac{\partial \mathbf{V}}{\partial x}+v\frac{\partial \mathbf{V}}{\partial y}\right)=-\nabla p, \quad \text{or:} \quad \rho_o[(2xy)(2y\mathbf{i})+(-y^2)(2x\mathbf{i}-2y\mathbf{j})]=-\nabla p$$

$$\text{Solve for} \quad \nabla p=-\rho_o(2xy^2\mathbf{i}+2y^3\mathbf{j}), \quad \text{or:} \quad \boldsymbol{\frac{\partial p}{\partial x}=-\rho_o 2xy^2} \quad \textit{Ans.}$$

P4.28 Consider the incompressible flow field of Prob. P4.6, with velocity components $u = 2y$, $v = 8x$, $w = 0$. Neglect gravity and assume constant viscosity. (*a*) Determine whether this flow satisfies the Navier-Stokes equations. (*b*) If so, find the pressure distribution $p(x, y)$ if the pressure at the origin is p_o.

Solution: In Prob. P4.6 we found the accelerations, so we can proceed to Navier-Stokes:

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) = \rho[0+(8x)(2)] = -\frac{\partial p}{\partial x} + \rho g_x + \mu\nabla^2 u = -\frac{\partial p}{\partial x}+0+0; \ \frac{\partial p}{\partial x} = -16\rho x$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}) = \rho[(2y)(8)+0] = -\frac{\partial p}{\partial y} + \rho g_y + \mu\nabla^2 v = -\frac{\partial p}{\partial y}+0+0; \ \frac{\partial p}{\partial y} = -16\rho y$$

Noting that

$\partial^2 p/(\partial x\partial y) = 0$ in both cases, we conclude **Yes, satisfies Navier-Stokes.** *Ans.*(*a*)

(*b*) The pressure gradients are simple, so we may easily integrate:

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy \ , \ \text{or:} \ p = \int -16\rho x\,dx + \int -16\rho y\,dy = -8\rho(x^2+y^2) + const$$

$$\text{If} \ p(0,0) = p_o \ , \ then \quad p = p_o - 8\rho(x^2+y^2) \qquad \textit{Ans.}(b)$$

This is an exact solution, but it is *not* Bernoulli’s equation. The flow is *rotational*.

P4.29 Consider a steady, two-dimensional, incompressible flow of a newtonian fluid with the velocity field $u = -2xy$, $v = y^2 - x^2$, and $w = 0$. (a) Does this flow satisfy conservation of mass? (b) Find the pressure field $p(x, y)$ if the pressure at point (x = 0, y = 0) is equal to p_a.

Solution: Evaluate and check the incompressible continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0=-2y+2y+0\equiv 0 \quad \textbf{Yes!} \quad \textit{Ans.}\text{ (a)}$$

(b) Find the pressure gradients from the Navier-Stokes *x*- and *y*-relations:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \quad or:$$

$$\rho[-2xy(-2y)+(y^2-x^2)(-2x)]=-\frac{\partial p}{\partial x}+\mu(0+0+0), \quad or: \quad \frac{\partial p}{\partial x}=-2\rho(xy^2+x^3)$$

and, similarly for the *y*-momentum relation,

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \quad or:$$

$$\rho[-2xy(-2x)+(y^2-x^2)(2y)]=-\frac{\partial p}{\partial y}+\mu(-2+2+0), \quad or: \quad \frac{\partial p}{\partial y}=-2\rho(x^2y+y^3)$$

The two gradients $\partial p/\partial x$ and $\partial p/\partial y$ may be integrated to find p(x, y):

$$p=\int\frac{\partial p}{\partial x}dx\Big|_{y=Const}=-2\rho\left(\frac{x^2y^2}{2}+\frac{x^4}{4}\right)+f(y), \quad then\ differentiate:$$

$$\frac{\partial p}{\partial y}=-2\rho(x^2y)+\frac{df}{dy}=-2\rho(x^2y+y^3), \quad whence\ \frac{df}{dy}=-2\rho y^3, \quad or: \quad f(y)=-\frac{\rho}{2}y^4+C$$

$$Thus: \quad p=-\frac{\rho}{2}(2x^2y^2+x^4+y^4)+C=p_a \quad at\ (x,y)=(0,0), \quad or: \quad \mathbf{C=p_a}$$

Finally, the pressure field for this flow is given by

$$\mathbf{p=p_a-\frac{1}{2}\rho(2x^2y^2+x^4+y^4)} \quad \textit{Ans.}\text{(b)}$$

P4.30 For the velocity distribution of Prob. P4.4, determine if (*a*) the equation of continuity and (*b*) the Navier-Stokes equation are satisfied. (*c*) If the latter is true, find the pressure distribution *p*(*x*,*y*) when the pressure at the origin equals p_o. Neglect gravity.

Solution: Recall that we were given $u = U_o(1+x/L)$ and $v = -U_o\, y/L$. (*a*) Test continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial}{\partial x}[U_o(1+\frac{x}{L})]+\frac{\partial}{\partial y}(-U_o\frac{y}{L}) = \frac{U_o}{L}-\frac{U_o}{L} \equiv 0 \quad \text{OK, satisfied.} \quad \textit{Ans.}(a)$$

(*b*) Now substitute these velocities into the *x*- and *y*- Navier-Stokes equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = U_o(1+\frac{x}{L})\frac{U_o}{L}+(-U_o\frac{y}{L})(0) = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u = -\frac{1}{\rho}\frac{\partial p}{\partial x}+0$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} = U_o(1+\frac{x}{L})(0)+(-U_o\frac{y}{L})(-\frac{U_o}{L}) = -\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\nabla^2 v = -\frac{1}{\rho}\frac{\partial p}{\partial y}+0$$

Solve for the two pressure gradients and cross-differentiate to see if they agree:

$$\frac{\partial p}{\partial x}=-\frac{\rho U_o^2}{L}(1+\frac{x}{L}) \qquad \frac{\partial p}{\partial y}=-\frac{\rho U_o^2}{L}\frac{y}{L} \qquad \text{Check } \frac{\partial^2 p}{\partial x\partial y}=0 \text{ for both}$$

Thus, before finding *p*(*x*,*y*), we know this is **an exact solution to Navier-Stokes**. *Ans.*(*b*)

(c) Integrate the two pressure gradients to find the pressure distribution:

$$p=\int\frac{\partial p}{\partial x}dx = -\frac{\rho U_o^2}{L}(x+\frac{x^2}{2L}) + f(y); \text{ Then } \frac{\partial p}{\partial y}=\frac{df}{dy},\quad f=-\frac{\rho U_o^2}{L}\frac{y^2}{2L}+const$$

$$p = -\frac{\rho U_o^2}{L}(x+\frac{x^2}{2L}+\frac{y^2}{2L}) + p_o \qquad \textit{Ans.}(c)$$

This is the same as **Bernoulli's equation**, but that is a bit hard to see.

P4.31 According to potential theory (Chap. 8) for the flow approaching a rounded two-dimensional body, as in Fig. P4.31, the velocity approaching the stagnation point is given by $u = U(1 - a^2/x^2)$, where *a* is the nose radius and *U* is the velocity far upstream.

Compute the value and position of the maximum viscous normal stress along this streamline. Is this also the position of maximum fluid deceleration? Evaluate the maximum viscous normal stress if the fluid is SAE 30 oil at 20°C, with $U = 2$ m/s and $a = 6$ cm.

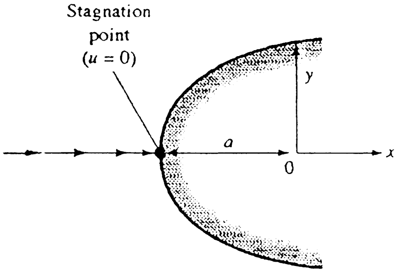


Fig. P4.31

Solution: (a) Along this line of symmetry the convective deceleration is one-dimensional:

$$a_x = u\frac{\partial u}{\partial x} = U\left(1 - \frac{a^2}{x^2}\right)U\left(\frac{2a^2}{x^3}\right) = 2U^2\left(\frac{a^2}{x^3} - \frac{a^4}{x^5}\right)$$

This has a maximum deceleration at $\frac{da_x}{dx} = 0$, or at $\mathbf{x = -\sqrt{(5/3)}\,a = -1.29a}$ *Ans.* (a)

The value of maximum deceleration at this point is $a_{x,max} = -0.372U^2/a$.

(b) The viscous normal stress along this line is given by

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} = 2\mu\left(\frac{2a^2U}{x^3}\right) \text{ with a maximum } \boldsymbol{\tau_{max} = \frac{4\mu U}{a}} \textbf{ at } \mathbf{x = -a} \quad \textit{Ans. (b)}$$

Thus maximum stress does not occur at the same position as maximum deceleration. For SAE 30 oil at 20°C, we obtain the numerical result

$$\text{SAE 30 oil,} \quad \rho = 917\ \frac{kg}{m^3}, \quad \mu = 0.29\ \frac{kg}{m\cdot s}, \quad \tau_{max} = \frac{4(0.29)(2.0)}{(0.06\ m)} \approx \mathbf{39\ Pa} \quad \textit{Ans. (b)}$$

P4.32 The answer to Prob. 4.14 is $\upsilon_\theta = f(r)$ only. Do not reveal this to your friends if they are still working on Prob. 4.14. Show that this flow field is an exact solution to the Navier-Stokes equations (4.38) for only two special cases of the function $f(r)$. Neglect gravity. Interpret these two cases physically.

Solution: Given $v_\theta = f(r)$ and $v_r = v_z = 0$, we need only satisfy the θ-momentum relation:

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}-\frac{v_\theta}{r^2}\right],$$

$$\text{or:}\quad \rho(0+0)=-0+\mu\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)+0-\frac{f}{r^2}\right],\quad \text{or:}\quad \mathbf{f''+\frac{1}{r}f'-\frac{1}{r^2}f=0}$$

This is the 'equidimensional' ODE and always has a solution in the form of a *power-law*, $f = Cr^n$. The two relevant solutions for these particular coefficients are $n = \pm 1$:

$\mathbf{f_1 = C_1 r}$ (solid-body rotation); $\mathbf{f_2 = C_2/r}$ (irrotational vortex) *Ans.*

P4.33 Consider incompressible flow at a volume rate Q toward a drain at the vertex of a 45° wedge of width b, as in Fig. P4.33. Neglect gravity and friction and assume purely radial inflow. (*a*) Find an expression for $v_r(r)$. (*b*) Show that the viscous term in the r-momentum equation is zero. (*c*) Find the pressure distribution $p(r)$ if $p = p_o$ at $r = R$.

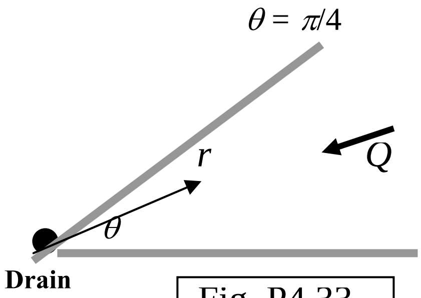


Fig. P4.33

Solution: (*a*) Assume one-dimensional, steady, radial inflow. Then, at any radius r,

$$v_r = \frac{-Q}{area} = \frac{-Q}{(\pi/4)rb} = \frac{-C}{r}, \quad where \quad C = \frac{4Q}{\pi b} \qquad Ans.(a)$$

The velocity is negative because the flow is inward. (*b*) The r-momentum equation is not written out in Chapter 4; it is Eq. (D.5) of Appendix D. The viscous term is

$$\nu\left(\nabla^2 v_r-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)=\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]=$$

$$=\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{-C}{r}\right)\right)-\frac{(-C/r)}{r^2}-0\right]=\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{C}{r}\right)+\frac{C}{r^3}\right]=\nu\left(-\frac{C}{r^3}+\frac{C}{r^3}\right)=\mathbf{0} \qquad Ans.(b)$$

(*c*) With the viscous term zero, the *r*-momentum equation reduces to

$$\rho(v_r \frac{\partial v_r}{\partial r}) = \rho(\frac{-C}{r})(\frac{C}{r^2}) = -\frac{\partial p}{\partial r}, \quad \text{or: } \frac{\partial p}{\partial r} = \frac{\rho C^2}{r^3}$$

$$\text{Integrate: } p = -\frac{\rho C^2}{2r^2} + C_1 \; ; \; at \; r = R, \; p = p_o = -\frac{\rho C^2}{2R^2} + C_1, \; C_1 = p_o + \frac{\rho C^2}{2R^2}$$

$$\text{Finally,} \quad p = p_o + \frac{\rho}{2}(\frac{C^2}{R^2} - \frac{C^2}{r^2}), \; where \; C = \frac{4Q}{\pi b} \qquad Ans.(c)$$

The two terms in parentheses are the velocities-squared at *r* = *R* and *r* = *r*, respectively. In other words, it integrates to *Bernoulli's equation* because the viscous term is zero (irrotational flow).

P4.34 A proposed three-dimensional incompressible flow field has the following vector form:

$$\mathbf{V} = Kx\mathbf{i} + Ky\mathbf{j} - 2Kz\mathbf{k}$$

(a) Determine if this field is a valid solution to continuity and Navier-Stokes. (b) If $\mathbf{g} = -g\mathbf{k}$, find the pressure field *p*(*x*, *y*, *z*). (c) Is the flow irrotational?

Solution: (a) Substitute this field into the three-dimensional incompressible continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial}{\partial x}(Kx) + \frac{\partial}{\partial y}(Ky) + \frac{\partial}{\partial z}(-2Kz)$$

$$= K + K - 2K = 0 \quad \textbf{Yes, satisfied}. \quad Ans. \text{ (a)}$$

(b) Substitute into the full incompressible Navier-Stokes equation (4.38). The laborious results are:

$$\text{x – momentum:} \quad \rho(K^2 x + 0 + 0) = -\frac{\partial p}{\partial x} + \mu(0 + 0 + 0)$$

$$\text{y – momentum:} \quad \rho(0 + K^2 y + 0) = -\frac{\partial p}{\partial y} + \mu(0 + 0 + 0)$$

$$\text{z – momentum:} \quad \rho\{0 + 0 + (-2Kz)(-2K)\} = -\frac{\partial p}{\partial z} + \rho(-g) + \mu(0 + 0 + 0)$$

Integrate each equation for the pressure and collect terms. The result is

$$\boldsymbol{p = p(0,0,0) - \rho g z - (\rho/2)K^2(x^2 + y^2 + 4z^2)} \quad \textit{Ans.}\ \text{(b)}$$

Note that the last term is identical to $(\rho/2)(u^2 + v^2 + w^2)$, in other words, Bernoulli's equation.

(c) For irrotational flow, the curl of the velocity field must be zero:

$$\nabla \times \mathbf{V} = \mathbf{i}(0-0) + \mathbf{j}(0-0) + \mathbf{k}(0-0) = \mathbf{0} \quad \textbf{Yes, irrotational}. \quad \textit{Ans.}\ \text{(c)}$$

P4.35 From the Navier-Stokes equations for incompressible flow in polar coordinates (App. E for cylindrical coordinates), find the most general case of purely circulating motion $\upsilon_\theta(r)$, $\upsilon_r = \upsilon_z = 0$, for flow with no slip between two fixed concentric cylinders, as in Fig. P4.35.

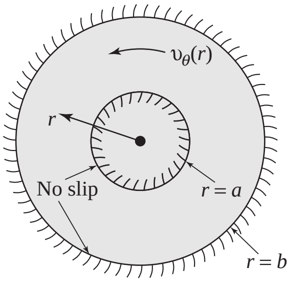


Fig. P4.35

Solution: The preliminary work for this problem is identical to Prob. 4.32 on an earlier page. That is, there are two possible solutions for purely circulating motion υ_θ(r), hence

$$\mathrm{v}_\theta = \mathrm{C}_1\mathrm{r} + \frac{\mathrm{C}_2}{\mathrm{r}}, \quad \text{subject to}\ \ \mathrm{v}_\theta(\mathrm{a}) = 0 = \mathrm{C}_1\mathrm{a} + \mathrm{C}_2/\mathrm{a} \quad \text{and} \quad \mathrm{v}_\theta(\mathrm{b}) = 0 = \mathrm{C}_1\mathrm{b} + \mathrm{C}_2/\mathrm{b}$$

This requires $\mathbf{C_1 = C_2 = 0}$, or $\mathbf{v_\theta = 0}$ (no steady motion possible between fixed walls) *Ans.*

P4.36 A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as in Fig. P4.36. The velocity profile is

$$\mathrm{u} = Cy(2h - y) \quad \mathrm{v} = \mathrm{w} = 0$$

Find the constant C in terms of the specific weight and viscosity and the angle θ. Find the volume flux Q per unit width in terms of these parameters.

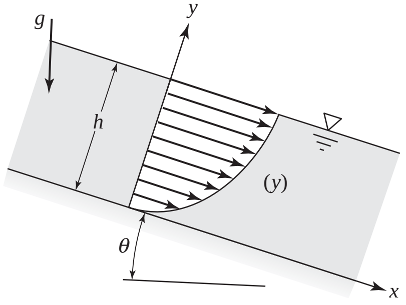


Fig. P4.36

Solution: There is atmospheric pressure all along the surface at y = h, hence $\partial p/\partial x = 0$. The x-momentum equation can easily be evaluated from the known velocity profile:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\nabla^2 u,\quad \text{or:}\quad 0=0+\rho g\sin\theta+\mu(-2C)$$

$$\text{Solve for}\quad \mathbf{C}=\frac{\boldsymbol{\rho}\mathbf{g}\,\mathbf{sin}\,\boldsymbol{\theta}}{\mathbf{2}\boldsymbol{\mu}}\quad \textit{Ans.}\text{ (a)}$$

The flow rate per unit width is found by integrating the velocity profile and using C:

$$Q=\int_0^h u\,dy=\int_0^h Cy(2h-y)\,dy=\frac{2}{3}Ch^3=\frac{\boldsymbol{\rho}\mathbf{gh^3 sin}\,\boldsymbol{\theta}}{\mathbf{3}\boldsymbol{\mu}}\ \text{per unit width}\quad \textit{Ans.}\text{ (b)}$$

P4.37 A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance 2h apart, as in the figure. The flow is fully developed, that is, w = w(x) only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile w(x).

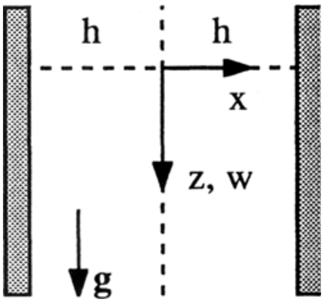


Fig. P4.37

Solution: Only the z-component of Navier-Stokes is relevant:

$$\rho\frac{dw}{dt}=0=\rho g+\mu\frac{d^2w}{dx^2},\quad \textit{or:}\quad w''=-\frac{\rho g}{\mu},\ \ w(-h)=w(+h)=0\ \ (\textit{no-slip})$$

The solution is very similar to Eqs. (4.142) to (4.143) of the text:

$$w=\frac{\boldsymbol{\rho}\mathbf{g}}{\mathbf{2}\boldsymbol{\mu}}(\mathbf{h^2-x^2})\quad \textit{Ans.}$$

P4.38 Show that the incompressible flow distribution, in cylindrical coordinates,

$$v_r = 0 \qquad v_\theta = Cr^n \qquad v_z = 0$$

where *C* is a constant, (*a*) satisfies the Navier-Stokes equation for only two values of *n*. Neglect gravity. (*b*) Knowing that $p = p(r)$ only, find the pressure distribution for each case, assuming that the pressure at $r = R$ is p_o. What might these two cases represent?

Solution: (a) The important direction here is the θ-momentum equation, Eq. (D.6):

$$\frac{\partial v_\theta}{\partial t} + (\mathbf{V}\bullet\nabla)v_\theta + \frac{1}{\mathrm{r}}v_r v_\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right),\ \text{or:}$$

$$0 + 0 + 0 = 0 + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + 0\right) = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(Cr^n)\right) - \frac{Cr^n}{r^2}\right],\ \text{or:}$$

$$\nu\left[\frac{1}{r}\frac{\partial}{\partial r}(rnCr^{n-1}) - Cr^{n-2}\right] = \nu(Cn^2 r^{n-2} - Cr^{n-2}) = 0$$

Cancel *C* and ν and r^{n-2}. These terms equal zero only if $n^2 = 1$, or $\boldsymbol{n = \pm 1}$. *Ans.*(*a*).

(*b*) Find the respective pressure distributions for $n = 1$ and $n = -1$. Use Eq. (D.5), which reduces simply to $\partial p/\partial r = \rho v_\theta^2/r$. Try this for each distribution, $n = \pm 1$:

$$Case\,1, n = 1: \int_{p_o}^{p} dp = \int_R^r \frac{\rho(C^2 r^2)}{r}dr\ ;\ \text{or:}\quad p = p_o + \frac{\rho C^2}{2}(r^2 - R^2) \qquad Ans.(b_1)$$

$$Case\,2, n = -1: \int_{p_o}^{p} dp = \int_R^r \frac{\rho(C^2/r^2)}{r}dr\ ;\ \text{or:}\quad p = p_o + \frac{\rho C^2}{2}\left(\frac{1}{R^2} - \frac{1}{r^2}\right) \qquad Ans.(b_2)$$

Case 1, $v_\theta = Cr$, is solid-body rotation. Case 2, $v_\theta = C/r$, is an irrotational potential vortex.

P4.39 Reconsider the angular-momentum balance of Fig. 4.5 by adding a concentrated *body couple Cz* about the *z* axis [6]. Determine a relation between the body couple and shear stress for equilibrium. What are the proper dimensions for *Cz*? (Body couples are important in continuous media with microstructure, such as granular materials.)

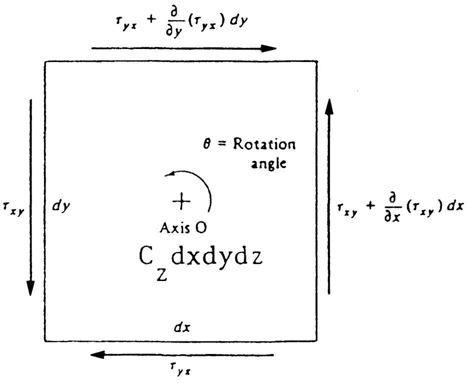

$\tau_{yx} + \frac{\partial}{\partial y}(\tau_{yx})\,dy$
θ = Rotation angle
Axis O
C_z dxdydz
τ_{xy}
dy
$\tau_{xy} + \frac{\partial}{\partial x}(\tau_{xy})\,dx$
dx
τ_{yx}

Fig. 4.5

Solution: The couple Cz has to be per unit volume to make physical sense in Eq. (4.39):

$$\left[\tau_{xy} - \tau_{yx} + \frac{1}{2}\frac{\partial \tau_{xy}}{\partial x}dx - \frac{1}{2}\frac{\partial \tau_{yx}}{\partial y}dy\right]dx\,dy\,dz + C_z dx\,dy\,dz = \frac{1}{12}\rho\,dx\,dy\,dz(dx^2 + dy^2)\frac{d^2\theta}{dt^2}$$

Reduce to third order terms and cancel (dx dy dz): $\boldsymbol{\tau_{yx} - \tau_{xy} = C_z}$ *Ans.*
The concentrated couple allows the stress tensor to have unsymmetrical shear stress terms.

P4.40 For pressure-driven laminar flow between parallel plates (see Fig. 4.12*b*), the velocity components are $u = U(1 - y^2/h^2)$, $v = 0$, and $w = 0$, where U is the centerline velocity. In the spirit of Ex. 4.6, find the temperature distribution $T(y)$ for a constant wall temperature T_w.

Solution: There are no variations with *x* or *z*, so the energy equation (4.53) reduces to

$$\rho c_p u\frac{\partial T}{\partial x} = 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 ,$$

$$or: \ \frac{d^2T}{dy^2} = -\frac{\mu}{k}\left(\frac{2Uy}{h^2}\right)^2 = -\left(\frac{4\mu U^2}{k h^4}\right)y^2; \quad \text{Integrate}: \ \frac{dT}{dy} = -\left(\frac{4\mu U^2}{k h^4}\right)\frac{y^3}{3} + C_1$$

The condition $T = T_w$ at ±h is equivalent to $dT/dy = 0$ at $y = 0$. Thus $C_1 = 0$. Integrate again:

$$T = -\left(\frac{4\mu U^2}{kh^4}\right)\frac{y^4}{12} + C_2\ ;\ \text{at } y = h: T = T_w = -\left(\frac{4\mu U^2}{kh^4}\right)\frac{h^4}{12} + C_2,\ \therefore C_2 = T_w + \frac{\mu U^2}{3k}$$

The final solution for $T(y)$ is, like Ex. 4.6, a quartic polynomial:

$$T(y) = T_w + \frac{\mu U^2}{3k}\left(1 - \frac{y^4}{h^4}\right) \qquad \textit{Ans.}$$

P4.41 As mentioned in Sec. 4.10, the velocity profile for laminar flow between two plates, as in Fig. P4.40, is

$$u = \frac{4u_{\max} y(h-y)}{h^2} \qquad \upsilon = w = 0$$

If the wall temperature is T_w at both walls, use

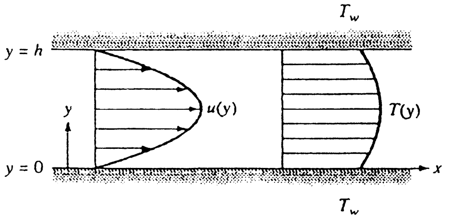


Fig. P4.41

the incompressible-flow energy equation (4.75) to solve for the temperature distribution $T(y)$ between the walls for steady flow.

Solution: Assume T = T(y) and use the energy equation with the known u(y):

$$\rho c_p \frac{dT}{dt} = k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2, \quad \text{or:} \quad \rho c_p(0) = k\frac{d^2T}{dy^2} + \mu\left[\frac{4u_{\max}}{h^2}(h-2y)\right]^2, \quad \text{or:}$$

$$\frac{d^2T}{dy^2} = -\frac{16\mu u_{\max}^2}{kh^4}(h^2 - 4hy + 4y^2), \quad \text{Integrate:} \ \frac{dT}{dy} = \frac{-16\mu u_{\max}^2}{kh^4}\left(h^2y - 2hy^2 + \frac{4y^3}{3} + C_1\right)$$

Before integrating again, note that dT/dy = 0 at y = h/2 (the symmetry condition), so $C_1 = -h^3/6$. Now integrate once more:

$$T = -\frac{16\mu u_{max}^2}{kh^4}\left(h^2\frac{y^2}{2} - 2h\frac{y^3}{3} + \frac{y^4}{3} + C_1 y\right) + C_2$$

If $T = T_W$ at y = 0 and at y = h, then $C_2 = T_W$. The final solution is:

$$\mathbf{T = T_w + \frac{8\mu u_{max}^2}{k}\left[\frac{y}{3h} - \frac{y^2}{h^2} + \frac{4y^3}{3h^3} - \frac{2y^4}{3h^4}\right]} \quad \textit{Ans.}$$

This is exactly the same solution as Problem P4.40 above, except that, here, the coordinate *y* is measured from the boo tom wall rather than the centerline.

P4.42 Suppose that we wish to analyze the rotating, partly-full cylinder of Fig. 2.23 as a *spin-up* problem, starting from rest and continuing until solid-body-rotation is achieved. What are the appropriate boundary and initial conditions for this problem?

Solution: Let *V* = *V*(*r*, *z*, *t*). The initial condition is: *V*(*r*, *z*, 0) = 0. The boundary conditions are

Along the side walls: $v_\theta(R, z, t) = R\Omega$, $v_r(R, z, t) = 0$, $v_z(R, z, t) = 0$.

At the bottom, z = 0: $v_\theta(r, 0, t) = r\Omega$, $v_r(r, 0, t) = 0$, $v_z(r, 0, t) = 0$.

At the free surface, $z = \eta$: $p = p_{atm}$, $\tau_{rz} = \tau_{\theta z} = 0$.

P4.43 For the draining liquid film of Fig. P4.36, what are the appropriate boundary conditions (a) at the bottom *y* = 0 and (b) at the surface *y* = *h*?

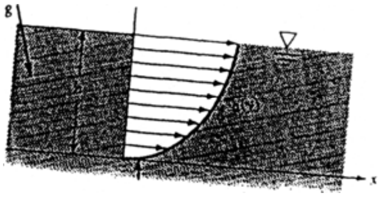


Fig. P4.36

Solution: The physically realistic conditions at the upper and lower surfaces are:

(a) at the bottom, y = 0, **no-slip:** $\mathbf{u(0) = 0}$ *Ans.* (a)

(b) At the surface, y = h, no shear stress, $\mu\frac{\partial u}{\partial y} = 0$, or $\mathbf{\frac{\partial u}{\partial y}(\mathit{h}) = 0}$ *Ans.* (b)

P4.44 Suppose that we wish to analyze the sudden pipe-expansion flow of Fig. P3.59, using the full continuity and Navier-Stokes equations. What are the proper boundary conditions to handle this problem?

Solution: First, at all walls, one would impose the no-slip condition: $\mathbf{u_r = u_z = 0}$ at all solid surfaces: at $r = r_1$ in the small pipe, at $r = r_2$ in the large pipe, and also on the flat-faced surface between the two.

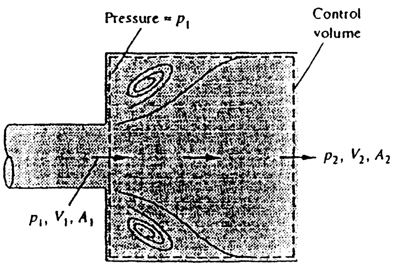


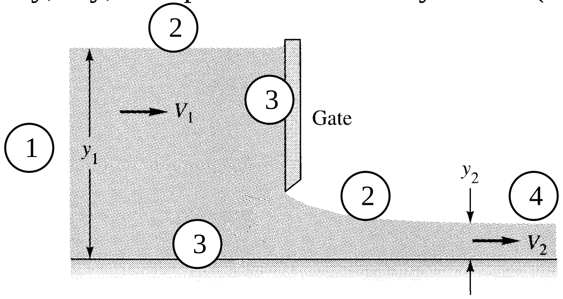
Fig. P3.59

Second, at some position upstream in the small pipe, the complete velocity distribution must be known: $\mathbf{u_1 = u_1(r)}$ at $z = z_1$. [Possibly the *paraboloid* of Prob. 4.34.]

Third, to be strictly correct, at some position downstream in the large pipe, the complete velocity distribution must be known: $\mathbf{u_2 = u_2(r)}$ at $z = z_2$. In numerical (computer) studies, this is often simplified by using a "free outflow" condition, $\partial u/\partial z = 0$.

Finally, the pressure must be specified at either the inlet or the outlet section of the flow, usually at the upstream section: $\mathbf{p = p_1(r)}$ at $z = z_1$.

P4.45 For the sluice gate problem of Example 3.10, list all the boundary conditions needed to solve this flow exactly by, say, Computational Fluid Dynamics (CFD).



Solution: There are four different kinds of boundary conditions needed, as labeled.

(1) Known velocity V_1 upstream, and of course the depth y_1 must be known.

(2) Known pressure p_{atm} at both the upstream and downstream free surfaces.

(3) No-slip ($V = 0$) all along the bottom and on the gate inner wall.

(4) The downstream flow is complicated because we don't know V_2 or y_2 and therefore cannot specify them. What CFD modelers do is to have an adjustable upper boundary and specify that the exit flow is "smooth", or "zero gradient", that is, $\partial V/\partial x = 0$.

P4.46 Fluid from a large reservoir at temperature T_o flows into a circular pipe of radius R. The pipe walls are wound with an electric-resistance coil which delivers heat to the fluid at a rate q_w (energy per unit wall area). If we wish to analyze this problem by using the full continuity, Navier-Stokes, and energy equations, what are the proper boundary conditions for the analysis?

Solution: Letting $z = 0$ be the pipe entrance, we can state inlet conditions: typically $u_z(r, 0) = U$ (a uniform inlet profile), $u_r(r, 0) = 0$, and $T(r, 0) = T_o$, also uniform.

At the wall, $r = R$, the no-slip and known-heat-flux conditions hold: $u_z(R, z) = u_r(R, z) = 0$ and $k(\partial T/\partial r) = q_w$ at (R, z) (assuming that q_w is positive for heat flow in).

At the exit, $z = L$, we would probably assume 'free outflow': $\partial u_z/\partial z = \partial T/\partial z = 0$. Finally, we would need to know the pressure at one point, probably the inlet, $z = 0$.

P4.47 Given the incompressible flow $\mathbf{V} = 3y\mathbf{i} + 2x\mathbf{j}$. Does this flow satisfy continuity? If so, find the stream function $\psi(x, y)$ and plot a few streamlines, with arrows.

Solution: With u = 3y and v = 2x, we may check $\partial u/\partial x + \partial v/\partial y = 0 + 0 = \mathbf{0}$, OK. Find the streamlines from $u = \partial\psi/\partial y = 3y$ and $v = -\partial\psi/\partial x = 2x$. Integrate to find

$$\boldsymbol{\psi = \frac{3}{2}y^2 - x^2} \quad \textit{Ans.}$$

Set $\psi = 0, \pm1, \pm2$, etc. and plot some streamlines at right: flow around corners of half-angles 39° and 51°.

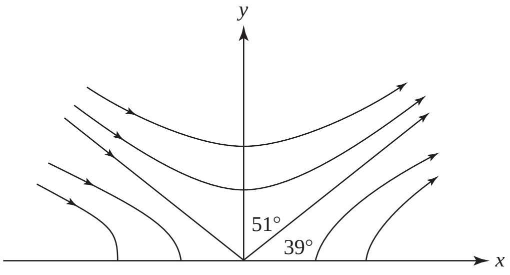


Fig. P4.47

P4.48 Consider the following two-dimensional incompressible flow, which clearly satisfies continuity:

$$u = U_o = \text{constant}, \quad v = V_o = \text{constant}$$

Find the stream function $\psi(r, \theta)$ of this flow, that is, using *polar coordinates*.

Solution: In cartesian coordinates the stream function is quite easy:

$$u = \partial\psi/\partial y = U_o \quad \text{and} \quad v = -\partial\psi/\partial x = V_o \quad \text{or:} \quad \psi = U_o y - V_o x + \text{constant}$$

But, in polar coordinates, $y = r\sin\theta$ and $x = r\cos\theta$. Therefore the desired result is

$$\boldsymbol{\psi(r, \theta) = U_o r\sin\theta - V_o r\cos\theta + \text{constant}} \quad \textit{Ans.}$$

P4.49 Investigate the stream function $\psi = K(x^2 - y^2)$, K = constant. Plot the streamlines in the full xy plane, find any stagnation points, and interpret what the flow could represent.

Solution: The velocities are given by

$$u = \frac{\partial\psi}{\partial y} = -2Ky; \quad v = -\frac{\partial\psi}{\partial x} = -2Kx$$

This is also stagnation flow, with the streamlines turned 45° from Prob. 4.48.

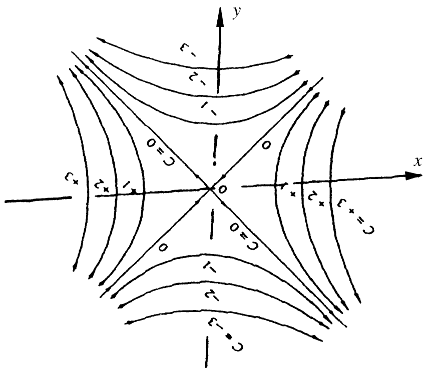


Fig. P4.49

P4.50 In 1851, George Stokes (of Navier-Stokes fame) solved the problem of steady incompressible low-Reynolds-number flow past a sphere, using *spherical polar* coordinates (r, θ) – [Ref. 5, page 168]. In these coordinates, the equation of continuity is

$$\frac{\partial}{\partial r}(r^2 v_r \sin\theta) + \frac{\partial}{\partial \theta}(r\, v_\theta \sin\theta) = 0$$

(*a*) Does a stream function exist for these coordinates? (*b*) If so, find its form.

Solution: Two velocity components and two continuity terms. **Yes, ψ exists!** *Ans.*(a)
(*b*) The stream function should be defined such that continuity takes the form

$$-\frac{\partial^2 \psi}{\partial r\,\partial \theta} + \frac{\partial^2 \psi}{\partial \theta\,\partial r} \equiv 0 \quad , \qquad \text{or:}$$

$$v_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r} \quad ; \quad v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta} \qquad \textit{Ans.}(b)$$

P4.51 The velocity profile for incompressible pressure-driven laminar flow between parallel plates (see Fig. 4.12*b*) has the form $u = C(h^2 - y^2)$, where C is a constant. (*a*) Determine if a stream function exists. (*b*) If so, determine a formula for the stream function,

Solution: (*a*) A stream function exists, for a single velocity component u, if $\partial u/\partial x = 0$, which it certainly is, since u is a function only of y. **Yes, ψ *exists*.** *Ans.*(*a*)
(*b*) Finding the stream function is just a matter of direct integration:

$$v = -\frac{\partial \psi}{\partial x} = 0 \quad , \quad \text{hence } \psi \text{ is a function only of } y$$

$$u = \frac{\partial \psi}{\partial y} = C(h^2 - y^2)\ ; \ \text{Integrate: } \psi = C(h^2 y - \frac{y^3}{3}) + \textit{constant} \qquad \textit{Ans.}(b)$$

P4.52 A two-dimensional, incompressible, frictionless fluid is guided by wedge-shaped walls into a small slot at the origin, as in Fig. P4.52. The width into the paper is *b*, and the volume flow rate is *Q*. At any given distance *r* from the slot, the flow is radial inward, with constant velocity. Find an expression for the polar-coordinate stream function of this flow.

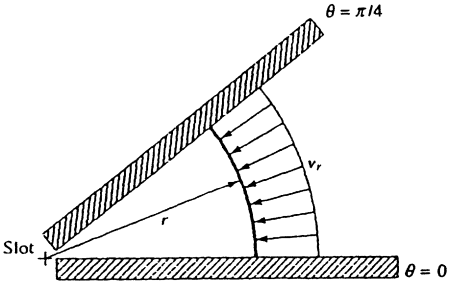


Fig. P4.52

Solution: We can find velocity from continuity:

$$v_r = -\frac{Q}{A} = -\frac{Q}{(\pi/4)rb} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad \text{from Eq. (4.101). Then}$$

$$\boldsymbol{\psi = -\frac{4Q}{\pi b}\theta + \text{constant}} \quad \textit{Ans.}$$

This is equivalent to the stream function for a line sink, Eq. (4.131).

P4.53 For the fully developed laminar-pipe-flow solution of Eq. (4.137), find the axisymmetric stream function $\psi(r, z)$. Use this result to determine the average velocity $V = Q/A$ in the pipe as a ratio of u_{max}.

Solution: The given velocity distribution, $v_z = u_{max}(1 - r^2/R^2)$, $v_r = 0$, satisfies continuity, so a stream function does exist and is found as follows:

$$v_z = u_{max}(1 - r^2/R^2) = \frac{1}{r}\frac{\partial\psi}{\partial r}, \quad \text{solve for } \psi = u_{max}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) + f(z), \text{ now use in}$$

$$v_r = 0 = -\frac{1}{r}\frac{\partial\psi}{\partial z} = 0 + \frac{df}{dz}, \quad \text{thus } f(z) = \text{const}, \quad \psi = \mathbf{u_{max}}\left(\frac{\mathbf{r^2}}{\mathbf{2}} - \frac{\mathbf{r^4}}{\mathbf{4R^2}}\right) \quad \textit{Ans.}$$

We can find the flow rate and average velocity from the text for polar coordinates:

$$Q_{1\text{-}2} = 2\pi(\psi_2 - \psi_1), \quad \text{or:} \quad Q_{0\text{-}R} = 2\pi\left[u_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) - u_{max}(0 - 0)\right] = \frac{\pi}{2}R^2 u_{max}$$

Then $V_{avg} = Q/A_{pipe} = [(\pi/2)R^2 u_{max}/(\pi R^2)] = \boldsymbol{\frac{1}{2}u_{max}}$ *Ans.*

P4.54 An incompressible stream function is defined by

$$\psi(x, y) = \frac{U}{L^2}(3x^2y - y^3)$$

where U and L are (positive) constants. Where in this chapter are the streamlines of this flow plotted? Use this stream function to find the volume flow Q passing through the rectangular surface whose corners are defined by $(x, y, z) = (2L, 0, 0)$, $(2L, 0, b)$, $(0, L, b)$, and $(0, L, 0)$. Show the direction of Q.

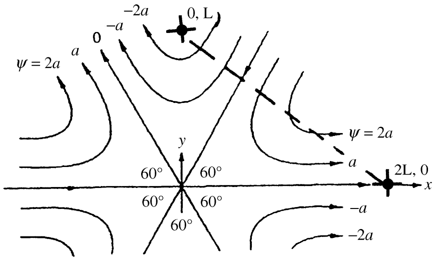


Fig. E4.7

Solution: This flow, with velocities $u = \partial\psi/\partial y = 3U/L^2(x^2 - y^2)$, and $v = -\partial\psi/\partial x = -6xyU/L^2$, is identical to Example 4.7 of the text, with "a" = $3U/L^2$. The streamlines are plotted in Fig. E4.7. The volume flow per unit width between the points (2L, 0) and (0, L) is

$$Q/b = \psi(2L, 0) - \psi(0, L) = \frac{U}{L^2}(0-0) - \frac{U}{L^2}[3(0)^2L - L^3] = UL, \quad \text{or:} \quad \mathbf{Q = ULb} \quad \textit{Ans.}$$

Since ψ at the *lower* point (2L, 0) is larger than at the *upper* point (0, L), the flow through this diagonal plane is to the left, as per Fig. 4.9 of the text.

P4.55 For the incompressible plane flow of Prob. P4.6, with velocity components $u = 2y$, $v = 8x$, $w = 0$, determine (*a*) if a stream function exists. (*b*) If so, determine the form of the stream function, and (*c*) plot a few representative streamlines.

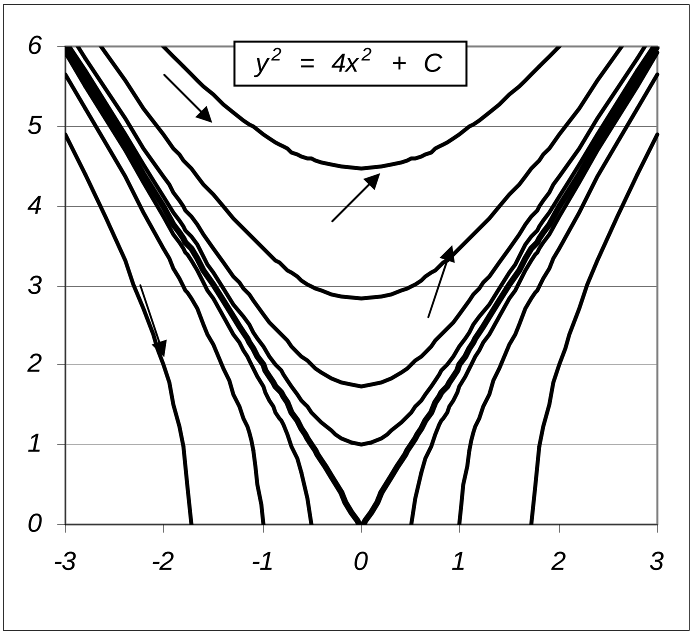
Solution: (*a*) Check to see is two-dimensional continuity is satisfied:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}(2y) + \frac{\partial}{\partial y}(8x) = 0 + 0 = 0 \quad \textbf{Yes, } \boldsymbol{\psi} \textbf{ exists.} \quad \textit{Ans.}(a)$$

(*b*) Find the stream function by relating velocities to derivatives of ψ:

$$u = 2y = \frac{\partial\psi}{\partial y}\ ;\ v = 8x = -\frac{\partial\psi}{\partial x}\ ;\ \text{Integrate: } \psi = y^2 - 4x^2 + \textit{constant} \quad \textit{Ans.}(b)$$

(*c*) Plot a few streamlines, that is, plot $y^2 = 4x^2 + C$ for various *C*. Here are the results:



We are showing only the upper half plane, which is the mirror image of the lower half.

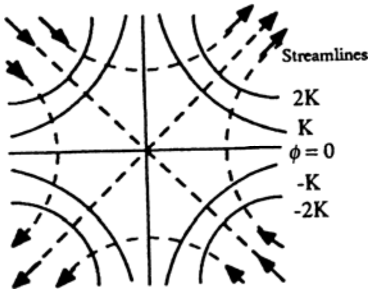


Fig. P4.56

P4.56 Investigate the velocity potential $\phi = Kxy$, K = constant. Sketch the potential lines in the full *xy* plane, find any stagnation points, and sketch in by eye the orthogonal streamlines. What could the flow represent?

Solution: The potential lines, ϕ = constant, are hyperbolas, as shown. The streamlines, sketched in as normal to the ϕ lines, are also hyperbolas. The pattern represents plane stagnation flow (Prob. 4.49) turned at 45°.

P4.57 A two-dimensional incompressible flow field is defined by the velocity components

$$u = 2V\left(\frac{x}{L} - \frac{y}{L}\right) \qquad v = -2V\frac{y}{L}$$

where V and L are constants. If they exist, find the stream function and velocity potential.

Solution: First check continuity and irrotationality:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{2V}{L}-\frac{2V}{L}=0 \quad \boldsymbol{\psi \text{ exists};}$$

$$\nabla x\mathbf{V}=\mathbf{k}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=\mathbf{k}\left(0+\frac{2V}{L}\right)\neq 0 \quad \boldsymbol{\phi \text{ does not exist}}$$

To find the stream function ψ, use the definitions of u and v and integrate:

$$u=\frac{\partial\psi}{\partial y}=2V\left(\frac{x}{L}-\frac{y}{L}\right), \quad \therefore \quad \psi=2V\left(\frac{xy}{L}-\frac{y^2}{2L}\right)+f(x)$$

$$\text{Evaluate} \quad \frac{\partial\psi}{\partial x}=\frac{2Vy}{L}+\frac{df}{dx}=-v=\frac{2Vy}{L}$$

$$\text{Thus} \quad \frac{df}{dx}=0 \quad \text{and} \quad \boldsymbol{\psi=V\left(\frac{2xy}{L}-\frac{y^2}{L}\right)+const} \quad \textit{Ans.}$$

P4.58 Show that the incompressible velocity potential in plane polar coordinates $\phi(r,\theta)$ is such that

$$\upsilon_r=\frac{\partial\phi}{\partial r} \qquad \upsilon_\theta=\frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

Finally show that ϕ as defined satisfies Laplace's equation in polar coordinates for incompressible flow.

Solution: Both of these things are quite true and easy to show from the definition of the gradient vector in polar coordinates. *Ans.*

P4.59 Consider the two-dimensional incompressible velocity potential $\phi = xy + x^2 - y^2$. (a) Is it true that $\nabla^2\phi = 0$, and, if so, what does this mean? (b) If it exists, find the stream function $\psi(x, y)$ of this flow. (c) Find the equation of the streamline which passes through $(x, y) = (2, 1)$.

Solution: (a) First check that $\nabla^2\phi = 0$, which means that **incompressible continuity is satisfied**.

$$\nabla^2\phi=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0+2-2=0 \qquad \textbf{Yes}$$

(b) Now use ϕ to find u and v and then integrate to find ψ.

$$u = \frac{\partial\phi}{\partial x} = y + 2x = \frac{\partial\psi}{\partial y}, \quad \text{hence} \quad \psi = \frac{y^2}{2} + 2xy + f(x)$$

$$v = \frac{\partial\phi}{\partial y} = x - 2y = -\frac{\partial\psi}{\partial x} = -2y - \frac{df}{dx}, \quad \text{hence} \quad f(x) = -\frac{x^2}{2} + const$$

The final stream function is thus $\boldsymbol{\psi = \frac{1}{2}(y^2 - x^2) + 2xy + const}$ *Ans.* (b)

(c) The streamline which passes through $(x, y) = (2, 1)$ is found by setting ψ = a constant:

$$At\ (x,y) = (2,\ 1), \quad \psi = \frac{1}{2}(1^2 - 2^2) + 2(2)(1) = -\frac{3}{2} + 4 = \frac{5}{2}$$

Thus the proper streamline is $\boldsymbol{\psi = \frac{1}{2}(y^2 - x^2) + 2xy = \frac{5}{2}}$ *Ans.* (c)

P4.60 Liquid drains from a small hole in a tank, as shown in Fig. P4.60, such that the velocity field set up is given by $\upsilon_r \approx 0$, $\upsilon_z \approx 0$, $\upsilon_\theta = \omega R^2/r$, where $z = H$ is the depth of the water far from the hole. Is this flow pattern rotational or irrotational? Find the depth z_C of the water at the radius $r = R$.

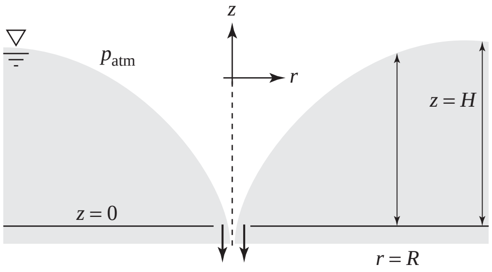


Fig. P4.60

Solution: From Appendix D, the angular velocity is

$$\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) - \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) = 0 \quad \text{(IRROTATIONAL)}$$

Incompressible continuity is valid for this flow, hence Bernoulli's equation holds at the surface, where $p = p_{atm}$, both at infinity and at $r = R$:

$$p_{atm} + \frac{1}{2}\rho V^2_{r=\infty} + \rho gH = p_{atm} + \frac{1}{2}\rho V^2_{r=R} + \rho g z_c$$

Introduce $V_{r=\infty} = 0$ and $V_{r=R} = \omega R$ to obtain $z_C = H - \dfrac{\omega^2 R^2}{2g}$ *Ans.*

P4.61 For the incompressible plane flow of Prob. P4.6, with velocity components $u = 2y$, $v = 8x$, $w = 0$, determine (*a*) if a velocity potential exists. (*b*) If so, determine the form of the velocity potential, and (*c*) plot a few representative potential lines.

Solution: (*a*) A velocity potential exists if the vorticity is zero. Here, for plane flow in (x, y) coordinates, we need only evaluate rotation around the z axis:

$$\varsigma_z = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 8 - 2 = +6 \neq 0 \qquad \text{Rotational, } \phi \text{ does not exist.} \quad \textit{Ans.}(a)$$

(*b, c*) There is **no velocity potential** – no plot, no formula. The flow has constant vorticity.

P4.62 Show that the linear Couette flow between plates in Fig. 1.6 has a stream function but no velocity potential. Why is this so?

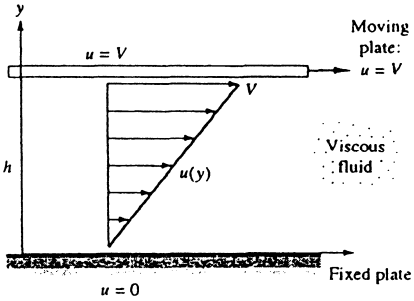


Fig. 1.6

Solution: Given u = Vy/h, v = 0, check continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \overset{?}{=} 0 = 0 + 0 \quad \textbf{(Satisfied therefore } \boldsymbol{\psi} \textbf{ exists).} \text{ Find } \psi \text{ from}$$

$$u = \frac{Vy}{h} = \frac{\partial \psi}{\partial y}, \quad v = 0 = -\frac{\partial \psi}{\partial x}, \quad \text{solve for } \psi = \mathbf{\frac{V}{2h} y^2 + const} \quad \textit{Ans.}$$

Now check irrotationality:

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \overset{?}{=} 0 = 0 - \frac{V}{h} \neq 0! \ \textbf{(Rotational, } \boldsymbol{\phi} \textbf{ does not exist.)} \quad \textit{Ans.}$$

P4.63 Find the two-dimensional velocity potential $\phi(r,\theta)$ for the polar-coordinate flow pattern $\upsilon_r = Q/r$, $\upsilon_\theta = K/r$, where Q and K are constants.

Solution: Relate these velocity components to the polar-coordinate definition of ϕ:

$$v_r = \frac{Q}{r} = \frac{\partial\phi}{\partial r},\quad v_\theta = \frac{K}{r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta};\quad \text{solve for } \boldsymbol{\phi = Q\ln(r) + K\theta + \text{const}} \quad \textit{Ans.}$$

P4.64 Show that the velocity potential $\phi(r, z)$ in axisymmetric cylindrical coordinates (see Fig. 4.2 of the text) is defined by the formulas:

$$\upsilon_r = \frac{\partial\phi}{\partial r} \qquad \upsilon_z = \frac{\partial\phi}{\partial z}$$

Further show that for incompressible flow this potential satisfies Laplace's equation in (r, z) coordinates.

Solution: Both of these things are quite true and are easy to show from their definitions. *Ans.*

P4.65 A two-dimensional incompressible flow is defined by

$$u = -\frac{Ky}{x^2 + y^2} \qquad \upsilon = \frac{Kx}{x^2 + y^2}$$

where K = constant. Is this flow irrotational? If so, find its velocity potential, sketch a few potential lines, and interpret the flow pattern.

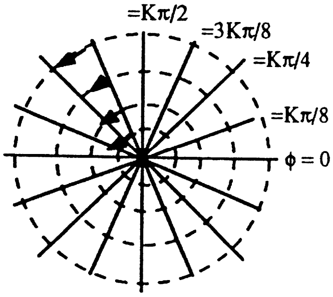


Fig. P4.65

Solution: Evaluate the angular velocity:

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{K}{x^2+y^2} - \frac{2Kx^2}{(x^2+y^2)^2} + \frac{K}{x^2+y^2} - \frac{2Ky^2}{(x^2+y^2)^2} = \mathbf{0\ (Irrotational)} \quad \textit{Ans.}$$

Introduce the definition of velocity potential and integrate to get $\phi(x, y)$:

$$u = \frac{\partial\phi}{\partial x} = -\frac{Ky}{x^2+y^2};\quad v = \frac{\partial\phi}{\partial y} = \frac{Kx}{x^2+y^2},\quad \text{solve for } \boldsymbol{\phi = K\tan^{-1}\left(\frac{y}{x}\right) = K\theta} \quad \textit{Ans.}$$

The ϕ lines are plotted above. They represent a counterclockwise line vortex.

P4.66 A plane polar-coordinate velocity potential is defined by

$$\phi = \frac{K\cos\theta}{r} \quad K = \text{const}$$

Find the stream function for this flow, sketch some streamlines and potential lines, and interpret the flow pattern.

Solution: Evaluate the velocities and thence find the stream function:

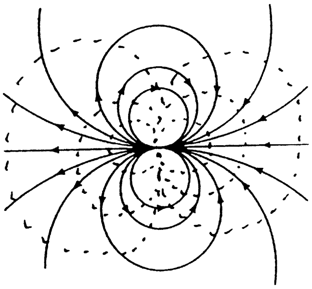


Fig. P4.66

$$v_r = \frac{\partial\phi}{\partial r} = -\frac{K\cos\theta}{r^2} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \quad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{K\sin\theta}{r^2} = -\frac{\partial\psi}{\partial r},$$

$$\text{solve} \quad \psi = -\frac{\mathbf{K\sin\theta}}{\mathbf{r}} \quad \textit{Ans.}$$

The streamlines and potential lines are shown above. This pattern is a line doublet.

P4.67 A stream function for a plane, irrotational, polar-coordinate flow is

$$\psi = C\theta - K\ln r \quad C \text{ and } K = \text{const}$$

Find the velocity potential for this flow. Sketch some streamlines and potential lines, and interpret the flow pattern.

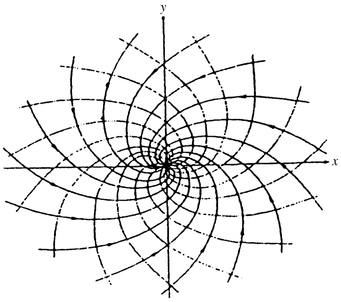


Fig. 4.14

Solution: If this problem is given *early* enough (before Section 4.10 of the text), the students will discover this pattern for themselves. It is a line source plus a line vortex, a tornado-like flow, Eq. (4.134) and Fig. 4.14 of the text. Find the velocity potential:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{C}{r} = \frac{\partial\phi}{\partial r}; \quad v_\theta = -\frac{\partial\psi}{\partial r} = \frac{K}{r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \quad \text{solve } \phi = \mathbf{C\,ln(r) + K\boldsymbol{\theta}} \quad \textit{Ans.}$$

The streamlines and potential lines are plotted above for *negative* C (a line sink).

P4.68 For the velocity distribution of Prob. P4.4, (*a*) determine if a velocity potential exists and, if it does, (*b*) find an expression for $\phi(x,y)$ and sketch the potential line which passes through the point $(x, y) = (L/2, L/2)$.

Solution: Recall the given flow, $u = U_o(1+x/L)$ and $v = -U_o(y/L)$. (*a*) Calculate if the flow is irrotational. For plane flow, only one term of the curl(**V**) is needed:

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 0 = 0 \; ; \quad \text{Yes, curl}(\mathbf{V}) = 0$$

Therefore **a velocity potential *does* exist**. *Ans.*(*a*) (*b*) To find ϕ, integrate from *u* and *v*:

$$\frac{\partial\phi}{\partial x} = u \quad ; \quad Thus \quad \phi = \int u\,dx = \int U_o\left(1+\frac{x}{L}\right)dx \;=\; U_o\left(x+\frac{x^2}{2L}\right) \;+\; f(y)$$

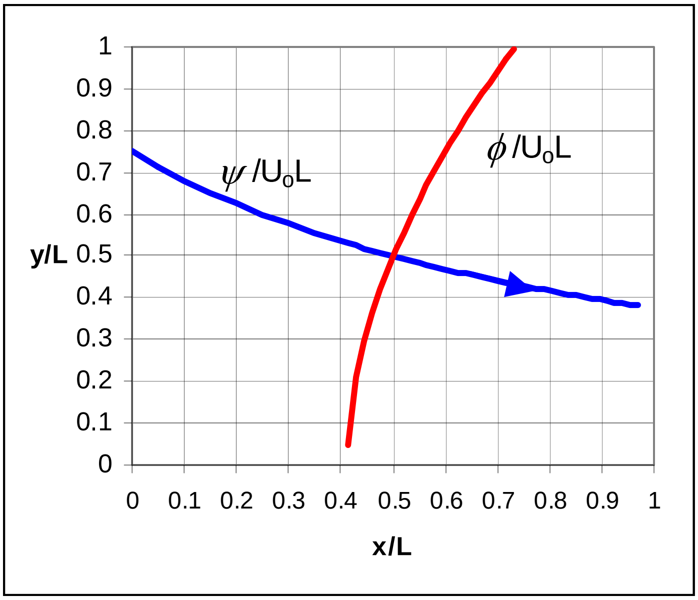
$$\frac{\partial\phi}{\partial y} = v \;=\; 0 + \frac{df}{dy} = -U_o\frac{y}{L}, \qquad \text{or}: \; f = -U_o\frac{y^2}{2L} + constant$$

$$\text{Thus} \qquad \phi \;=\; (U_oL)\left(\frac{x}{L} + \frac{x^2-y^2}{2L^2}\right) \;+\; const \qquad\qquad Ans.(b)$$

For a potential line to pass through (*L*/2, *L*/2), we must have $\phi/(U_o/L) = [1/2 + \{(1/2)^2 - (1/2)^2\}/2] = ½$. For convenience let the *const* = 0. Thus we are to plot this potential line:

$$\frac{\phi}{U_oL} = \frac{1}{2} = \frac{x}{L} + \frac{x^2-y^2}{2L^2}$$

The result is plotted (**red**) in the graph below, along with the (**blue**) ψ line, which has the analytic form $\psi = U_o(y + xy/L) = 3U_oL/4$.



P4.69 A steady, two-dimensional flow has the following polar-coordinate velocity potential:

$$\phi = C\,r\cos\theta + K\ln r$$

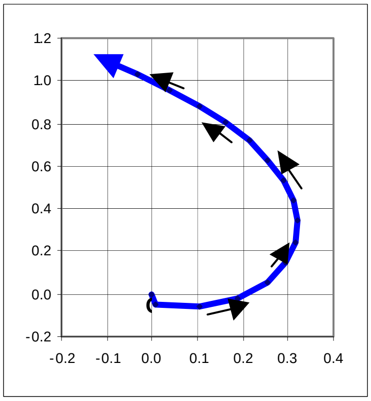
where C and K are constants. Determine the stream function $\psi(r, \theta)$ for this flow. For extra credit, let C be a velocity scale U, let $K = UL$, and sketch what the flow might represent.

Solution: Write out the ψ and ϕ expressions for polar-coordinate velocities:

$$v_r = \frac{\partial\phi}{\partial r} = C\cos\theta + \frac{K}{r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}\ , \quad \text{hence } \psi = C\,r\sin\theta + K\theta + f(r)$$

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -C\sin\theta + K(0) = -\frac{\partial\psi}{\partial r}\ , \quad \textit{hence } \boxed{\psi = C\,r\sin\theta + K\theta + \textit{constant}}\ \textit{Ans.}$$

Extra credit: Plot a typical streamline for $C = U$ and $K = UL$:



All the streamlines are logarithmic spirals coming out from the origin in every direction.

P4.70 A CFD model of steady two-dimensional incompressible flow has printed out the values of stream function $\psi(x, y)$, in m^2/s, at each of the four corners of a small 10cm-by-10cm cell, as shown in Fig. P4.70. Use these numbers to estimate the resultant velocity in the center of the cell and its angle α with respect to the x axis.

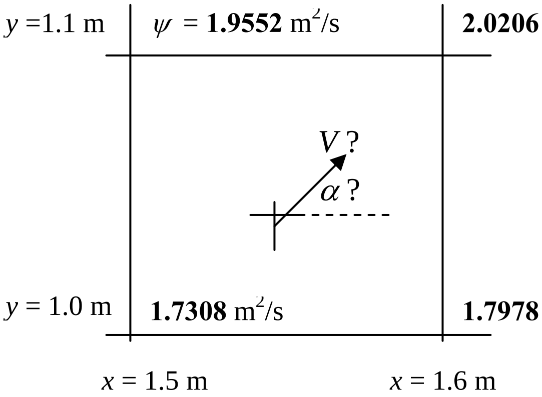
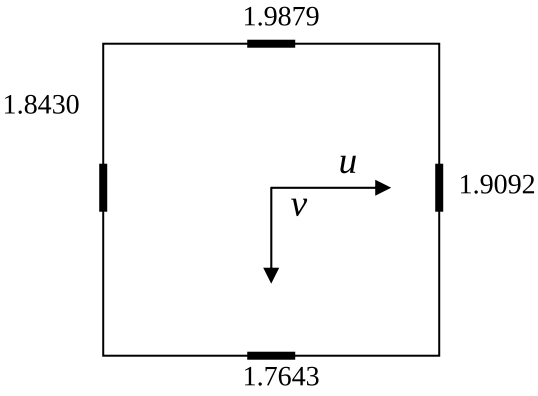


Fig. P4.70

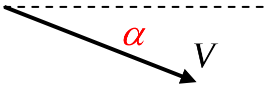
Solution: Quick analysis: the ψ values are higher on the top than the bottom, therefore u is to the *right*. The ψ values are higher on the right than the left, therefore v is *down*. There are several ways to estimate the center velocities. One simple way is to compute average values of ψ on the sides:



Then $u_{\text{center}} \approx \Delta\psi/\Delta y = (1.9879\text{-}1.7643\ \text{m}^2/\text{s})/(0.1\text{m}) = 2.236$ m/s **to the right**. And $v_{\text{center}} \approx \Delta\psi/\Delta x = (1.9092\text{-}1.8430\ \text{m}^2/\text{s})/(0.1\text{m}) = 0.662$ m/s **down**. The resultant and its angle are

$$V \approx \sqrt{(2.236)^2 + (0.662)^2} = \mathbf{2.332\ m/s}\ ;\ \alpha = \tan^{-1}\left(\frac{0.662}{2.236}\right) = \mathbf{16.5^o}\ \ down\ \ Ans.$$

The ψ values in this problem are in fact taken from an exact solution, $V = 2.3315$ m/s, $\alpha = 16.505°$.



P4.71 Consider the following two-dimensional function $f(x, y)$:

$$f = Ax^3 + Bxy^2 + Cx^2 + D\ , \quad \text{where}\ \ A > 0$$

(a) Under what conditions, if any, on (A,B,C,D) can this function f be a steady, plane-flow velocity potential? (b) If you find a $\phi(x, y)$ to satisfy part (a), also find the associated stream function $\psi(x, y)$, if any, for this flow.

Solution: (a) If f is to be a plane-flow velocity potential, it must satisfy Laplace's equation:

$$\nabla^2 f = 6Ax + Bx + 2C = 0 \quad if \quad B = -3A \quad \text{and} \quad C = 0$$

$$\text{The velocity potential is} \qquad \phi = Ax^3 - 3Axy^2 + D \qquad Ans.(a)$$

(b) To find ψ, use ϕ to get u and v and work backwards to get the stream function:

$$u = \frac{\partial\phi}{\partial x} = 3Ax^2 - 3Ay^2 = \frac{\partial\psi}{\partial y}\ ,\ \therefore\ \psi = 3Ax^2y - Ay^3 + f(x)$$

$$v = \frac{\partial\phi}{\partial y} = -6Axy = -\frac{\partial\psi}{\partial x} = -6Axy - \frac{df}{dx}\ ,\ \therefore\ f = const$$

$$\text{Finally,} \qquad \psi = 3Ax^2y - Ay^3 + const \qquad Ans.(b)$$

P4.72 Water flows through a two-dimensional narrowing wedge at 9.96 gal/min per meter of width into the paper. If this inward flow is purely radial, find an expression, in SI units, for (*a*) the stream function, and (*b*) the velocity potential of the flow. Assume one-dimensional flow. The included angle of the wedge is 45°.

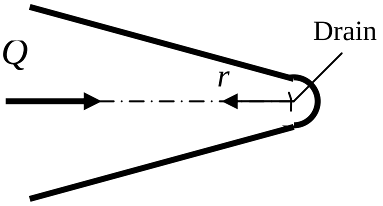


Fig. P4.72

Solution: The wedge angle equals π/4 radians. At any given position *r*, the inward flow equals

$$v_r = -\frac{Q}{A} = -\frac{Q}{(\pi/4)rb} = -\frac{4Q/(\pi b)}{r}$$

$$\text{where} \quad 4(Q/b)/\pi = \frac{4(9.96\,gal/\text{min}/m)(6.309E-5\,m^3/s/gpm)}{\pi} = 0.00080\frac{m^3}{s-m}$$

We have already been advised that $v_\theta = 0$. (*a*) Work from radial velocity to stream function:

$$v_r = -\frac{0.00080\,m^3/s-m}{r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}\;; \quad \text{Solve} \quad \psi = -0.00080\,\theta \qquad \textit{Ans.}(a)$$

Note that *r* must be in *meters*. (*b*) Work from radial velocity to obtain velocity potential:

$$v_r = -\frac{0.00080\,m^3 s-m}{r} = \frac{\partial\phi}{\partial r}\;; \quad \text{Solve} \quad \phi = -0.00080\,\ln(r) \qquad \textit{Ans.}(b)$$

P4.73 A CFD model of steady two-dimensional incompressible flow has printed out the values of velocity potential $\phi(x, y)$, in m^2/s at each of the four corners of a small 10cm-by10cm cell, as shown in Fig. P4.73. Use these numbers to estimate the resultant velocity in the center of the cell and its angle α with respect to the x axis.

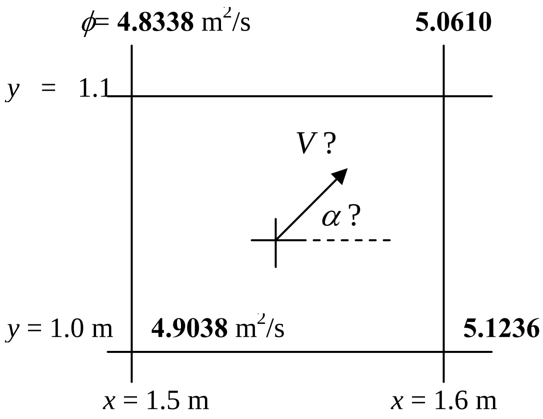
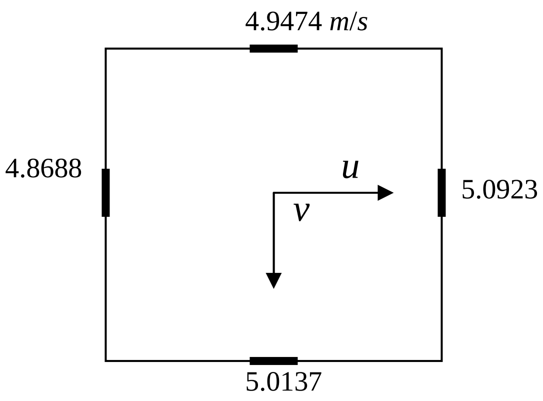


Fig. P4.73

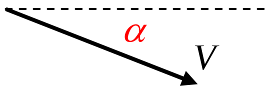
Solution: Quick analysis: the ϕ values are lower on the left than the right, therefore u is to the *right*. The ϕ values are lower on the top than the bottom, therefore v is *down*. There are several ways to estimate the center velocities. One simple way is to compute average values of ϕ on the sides:



Then $u_{center} \approx \Delta\phi/\Delta x$ = (5.0923-4.8688 m^2/s)/(0.1m) = 2.235 m/s **to the right**. And $v_{center} \approx |\Delta\phi/\Delta y|$ = (5.0137-4.9474 m^2/s)/(0.1m) = 0.663 m/s **down**. The resultant and its angle are

$$V \approx \sqrt{(2.235)^2 + (0.663)^2} = \mathbf{2.331\ m/s} \quad ; \quad \alpha = \tan^{-1}\left(\frac{0.663}{2.235}\right) = \mathbf{16.5^o} \quad \textit{Ans.}$$

The ϕ values in this problem are in fact taken from an exact solution, V = 2.3315 m/s, α = 16.505° down.



P4.74 Consider the two-dimensional incompressible polar-coordinate velocity potential

$$\phi = B r \cos\theta + B L \theta$$

where B is a constant and L is a constant length scale. (*a*) What are the dimensions of B? (*b*) Locate the only stagnation point in this flow field. (*c*) Prove that a stream function exists and then find the function $\psi(r, \theta)$.

Solution: (*a*) To give ϕ its correct dimensions of {L^2/T}, the constant *B* must have the dimensions of *velocity*, or {L/T}. *Ans.*(*a*)

(*b*) Calculate velocities in polar coordinates:

$$v_r = \frac{\partial\phi}{\partial r} = B\cos\theta \quad ; \quad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -B\sin\theta - \frac{BL}{r}$$

At first it doesn't look as if we can find a stagnation point, but indeed there is one:

$$r = L \ , \ \theta = 180^\circ : \ v_\theta = 0 \ , \ v_r = B - \frac{BL}{L} = 0 \qquad \textit{Ans.}(b)$$

As discussed later in Chap. 8, this is the velocity potential of a *Rankine half-body*.

(c) With the velocities known, check the continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} = 0 = \frac{B\cos\theta}{r} - \frac{B\cos\theta}{r} = 0 \qquad \text{Yes, satisfied}$$

Continuity is satisfied. Find the stream function from the definition of $\psi(r, \theta)$:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = B\cos\theta \quad ; \quad v_\theta = -\frac{\partial\psi}{\partial r} = -B\sin\theta - \frac{BL}{r}$$

$$\text{Integrate:} \quad \psi = B\,r\sin\theta + B\,L\ln r + const \qquad \textit{Ans.}(c)$$

P4.75 Given the following steady *axisymmetric* stream function:

$$\psi = \frac{B}{2}\left(r^2 - \frac{r^4}{2R^2}\right) , \quad \text{where } B \text{ and } R \text{ are constants}$$

valid in the region $0 \le r \le$ R and $0 \le z \le$ L. (*a*) What are the dimensions of the constant *B*?

(*b*) Show whether this flow possesses a velocity potential and, if so, find it. (*c*) What might this flow represent? [HINT: Examine the axial velocity v_z.]

Solution: (*a*) From the definition of $\psi(r, z)$ in Eqs. (4.105), the dimensions of ψ are {L^3/T}. Thus B has *velocity* dimensions, {B} = {L/T}. *Ans*.(*a*)

(*b*) To test for irrotationality, first find the velocity components from Eqs. (4.106):

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial z} = 0 \quad ; \quad v_z = \frac{1}{r}\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{B}{2}\left(2r - \frac{4r^3}{2R^2}\right) = B\left(1-\frac{r^2}{R^2}\right)$$

Now evaluate the curl of the velocity, which has only one possible non-zero component. From Appendix D, Eq. (D.11),

$$2\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} = 0 - \frac{2Br}{R^2} \neq 0 \qquad \textbf{Rotational}\ ,\ \phi \text{ does not exist. } \textit{Ans.}(b)$$

(*c*) The interpretation of the flow follows immediately from the velocity components. The velocity profile is a paraboloid of revolution and represents **Poiseuille pipe flow**, Eq. (4.137). *Ans*.(*c*)

***P4.76** A two-dimensional incompressible flow has the velocity potential

$$\phi = K(x^2 - y^2) + C\ln(x^2 + y^2)$$

where K and C are constants. In this discussion, avoid the origin, which is a singularity (infinite velocity). (*a*) Find the sole stagnation point of this flow, which is somewhere in the upper half plane. (*b*) Prove that a stream function exists and then find $\psi(x, y)$, using the hint that

$\int dx/(a^2+x^2) = (1/a)\tan^{-1}(x/a)$.

Solution: (*a*) Find the velocity components and see where they both equal zero:

$$u = \frac{\partial\phi}{\partial x} = 2Kx + \frac{2Cx}{x^2+y^2} \;;\quad v = \frac{\partial\phi}{\partial x} = -2Ky + \frac{2Cy}{x^2+y^2}$$

For positive *K* and *C*, *u* cannot be zero anywhere except at $x = 0$. Then $v = 0$ if

$$2Ky = \frac{2C}{y}\;,\quad or: \quad stagnation\ at \quad x=0 \;\; and \;\; y = \sqrt{\frac{C}{K}} \qquad Ans.(a)$$

(*b*) First check the velocities to see if continuity is satisfied:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = [2K + \frac{2C}{x^2+y^2} - \frac{4Cx^2}{(x^2+y^2)^2}] + [-2K + \frac{2C}{x^2+y^2} - \frac{4Cy^2}{(x^2+y^2)^2}] = 0$$

The algebra is messy but, indeed, continuity is satisfied, ψ **exists**. *Ans.*(*b*) – part 1. Now integrate the velocity components to find the stream function :

$$u = \frac{\partial\psi}{\partial y} = 2Kx + \frac{2Cx}{x^2+y^2} \quad and \quad v = -\frac{\partial\psi}{\partial x} = -2Ky + \frac{2Cy}{x^2+y^2}$$

$$\text{Integrate to obtain} \quad \psi = 2Kxy + 2C\tan^{-1}(\frac{y}{x}) + const \qquad Ans.(b)$$

P4.77 Outside an inner, intense-activity circle of radius *R*, a tropical storm can be simulated by a polar-coordinate velocity potential $\phi(r, \theta) = U_o R\, \theta$, where U_o is the wind velocity at radius *R*. (*a*) Determine the velocity components outside $r = R$. (*b*) If, at $R =$

25 mi, the velocity is 100 mi/h and the pressure 99 kPa, calculate the velocity and pressure at r = 100 mi.

Solution: (a) First, convert U_o = 100 mi/h = 44.7 m/s and R = 25 mi = 40,200 m. The velocities are calculated from ϕ, as requested in Prob. P4.58:

$$v_r = \frac{\partial}{\partial r}(U_o R\theta) = 0 \quad ; \quad v_\theta = \frac{1}{r}\frac{\partial}{\partial \theta}(U_o R\theta) = \frac{U_o R}{r} \qquad Ans.(a)$$

Outside the "intense" region, the wind is simulated as a circulating "potential vortex" whose velocity drops off inversely as the radius. (b) The flow is *irrotational*, otherwise ϕ would not exist. Thus Bernoulli's equation applies outside $r = R$, with no elevation change at the ocean surface. Take surface air density to be sea-level standard, ρ = 1.225 kg/m^3.

$$\text{At} \quad r = 4R = 100\,mi, \quad v_\theta = \frac{U_o R}{4R} = \frac{U_o}{4} = \frac{44.7}{4} = 11.2\,\frac{m}{s}$$

$$\text{Bernoulli}: \quad p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}V_2^2 \,, \quad \text{or}:$$

$$99{,}000 + (1.225/2)(44.7)^2 = p_2 + (1.225/2)(11.2)^2 \text{ , Solve } \; p_{100\,mi} = \mathbf{100{,}150}\; Pa \quad Ans.(b)$$

The pressure far from the storm is approximately sea-level standard pressure.

P4.78 An incompressible, irrotational, two-dimensional flow has the following stream function in polar coordinates:

$$\psi = Ar^n \sin(n\theta) \;, \qquad \text{where } A \text{ and } n \text{ are constants.}$$

Find an expression for the velocity potential of this flow.

Solution: Use ψ to find the velocity components, then integrate back to find ϕ.

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = nAr^{n-1}\cos(n\theta) = \frac{\partial\phi}{\partial r};\ \text{Integrate}: \phi = Ar^n\cos(n\theta) + f(\theta)$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -nAr^{n-1}\sin(n\theta) = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{1}{r}[-nAr^n\sin(n\theta)] + \frac{df}{d\theta}$$

$$\text{Compare}: \frac{df}{d\theta} = 0\ , \quad \text{and thus} \quad \phi = Ar^n\cos(n\theta) + \text{constant} \qquad Ans.$$

P4.79 Study the combined effect of the two viscous flows in Fig. 4.16. That is, find *u*(*y*) when the upper plate moves at speed *V* and there is also a constant pressure gradient (*dp/dx*). Is superposition possible? If so, explain why. Plot representative velocity profiles for (a) zero, (b) positive, and (c) negative pressure gradients for the same upper-wall speed *V*.

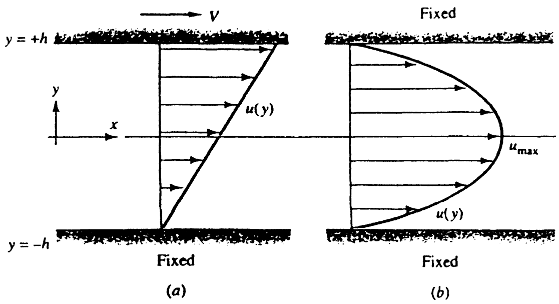


Fig. 4.16

Solution: The combined solution is

$$u = \frac{V}{2}\left(1+\frac{y}{h}\right)+\frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left(1-\frac{y^2}{h^2}\right)$$

The superposition is quite valid because the convective acceleration is zero, hence what remains is linear: $\nabla p = \mu\nabla^2\mathbf{V}$. Three representative velocity profiles are plotted at right for various (dp/dx).

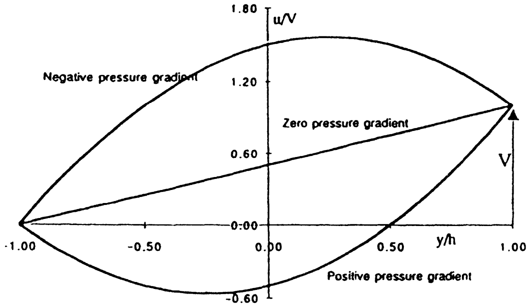
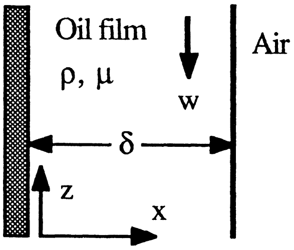


Fig. P4.79

P4.80 An oil film drains steadily down the side of a vertical wall, as shown. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness. Assume that w = w(x) only that the atmosphere offers no shear resistance to the film. (a) Solve Navier-Stokes for w(x). (b) Suppose that film thickness and $[\partial w/\partial x]$ at the wall are measured. Find an expression which relates μ to this slope $[\partial w/\partial x]$.



Solution: First, there is no pressure gradient $\partial p/\partial z$ because of the constant-pressure atmosphere. The Navier-Stokes z-component is $\mu(d^2w/dx^2) = \rho g$, and the solution requires w = 0 at x = 0 and (dw/dx) = 0 (no shear at the film edge) at x = δ. The solution is:

$$\boldsymbol{w = \frac{\rho g x}{2\mu}(x - 2\delta)} \quad \textit{Ans.} \text{ (a)} \quad \text{NOTE: } w \text{ is negative (down)}$$

The wall slope is $dw/dx\,|_{wall} = -\rho g\delta/\mu$, rearrange: $\mu = -\rho g\delta/[dw/dx\,|_{wall}]$ *Ans.* (b)

P4.81 Modify the analysis of Fig. 4.17 to find the velocity v_θ when the inner cylinder is fixed and the outer cylinder rotates at angular velocity Ω_o. May this solution be *added* to Eq. (4.146) to represent the flow caused when both inner and outer cylinders rotate? Explain your conclusion.

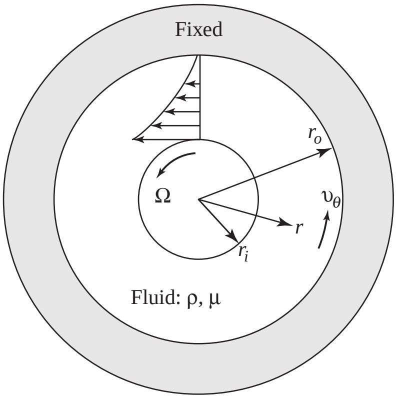


Fig. 4.17

Solution: We apply new boundary conditions to Eq. (4.145) of the text:

$$v_\theta = C_1 r + C_2/r;$$

$$\text{At } r = r_i, \quad v_\theta = 0 = C_1 r_i + C_2/r_i$$

$$\text{At } r = r_o, \quad v_\theta = \Omega_o r_o = C_1 r_o + C_2/r_o$$

Solve for C_1 and C_2. The final result: $\boldsymbol{v_\theta = \Omega_o r_o \left(\frac{r/r_i - r_i/r}{r_o/r_i - r_i/r_o}\right)}$ *Ans.*

This solution may indeed be added to the inner-rotation solution, Eq. (4.146), because the convective acceleration is zero and hence the Navier-Stokes equation is *linear*.

P4.82 A solid circular cylinder of radius *R* rotates at angular velocity Ω in a viscous incompressible fluid which is at rest far from the cylinder, as in Fig. P4.82. Make simplifying assumptions and derive the governing differential equation and boundary conditions for the velocity field v_θ in the fluid. Do not solve unless you are obsessed with this problem. What is the steady-state flow field for this problem?

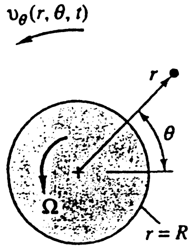


Fig. P4.82

Solution: We assume purely circulating motion: $v_z = v_r = 0$ and $\partial/\partial\theta = 0$. Thus the remaining variables are v_θ = fcn(r, t) and p = fcn(r, t). Continuity is satisfied identically, and the θ-momentum equation reduces to a partial differential equation for v_θ:

$$\frac{\partial v_\theta}{\partial t} = \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2}\right] \quad \text{subject to } v_\theta(R, t) = \Omega R \quad \text{and} \quad v_\theta(\infty, t) = 0 \quad \textit{Ans.}$$

I am not obsessed with this problem so will not attempt to find a solution. However, at large times, or t = ∞, the steady state solution is $\mathbf{v_\theta = \Omega R^2/r}$. *Ans.*

P4.83 The flow pattern in bearing lubrication can be illustrated by Fig. P4.83, where a viscous oil (ρ, μ) is forced into the gap $h(x)$ between a fixed slipper block and a wall moving at velocity *U*. If the gap is thin, $h \ll L$, it can be shown that the pressure and velocity distributions are of the form $p = p(x)$, $u = u(y)$, $\upsilon = w = 0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for $u(y)$. What are the proper boundary conditions? Integrate and show that

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh) + U\left(1 - \frac{y}{h}\right)$$

where $h = h(x)$ may be an arbitrary slowly varying gap width. (For further information on lubrication theory, see Ref. 16.)

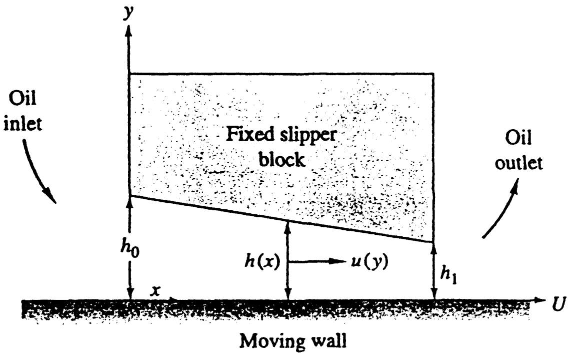


Fig. P4.83

Solution: With u = u(y) and p = p(x) only in the gap, the *x*-momentum equation becomes

$$\rho\frac{du}{dt}=0=-\frac{dp}{dx}+\mu\frac{\partial^2 u}{\partial y^2},\quad \text{or:}\quad \frac{d^2u}{dy^2}=\frac{1}{\mu}\frac{dp}{dx}=\text{constant}$$

$$\text{Integrate twice:}\quad u=\frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2}+C_1y+C_2,\quad \text{with } u(0)=U \quad \text{and} \quad u(h)=0$$

With C1 and C2 evaluated, the solution is exactly as listed in the problem statement:

$$\mathbf{u=\frac{1}{2\mu}\frac{dp}{dx}(y^2-yh)+U\left(1-\frac{y}{h}\right)}\quad \textit{Ans.}$$

P4.84 Consider a viscous film of liquid draining uniformly down the side of a vertical rod of radius *a*, as in Fig. P4.84. At some distance down the rod the film will approach a terminal or *fully developed* draining flow of constant outer radius *b*, with $\upsilon_z = \upsilon_z(r)$, $\upsilon_\theta = \upsilon_r = 0$. Assume that the atmosphere offers no shear resistance to the film motion. Derive a differential equation for υ_z, state the proper boundary conditions, and solve for the film velocity distribution. How does the film radius *b* relate to the total film volume flow rate *Q*?

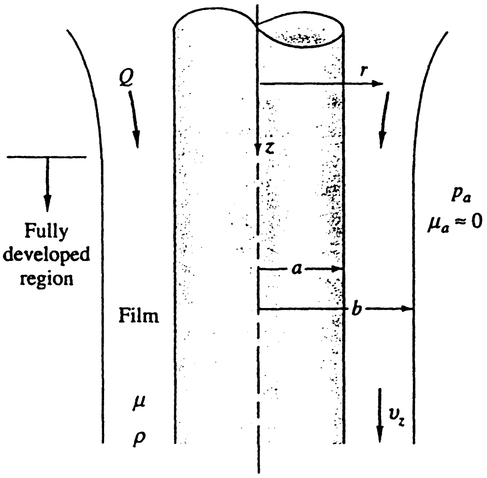


Fig. P4.84

Solution: With vz = fcn(r) only, the Navier-Stokes z-momentum relation is

$$\rho\frac{dv_z}{dt} = 0 = -\frac{\partial p}{\partial z} + \rho g + \mu\nabla^2 v_z,$$

$$\text{or:}\quad \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\frac{\rho g}{\mu},\quad \text{Integrate twice:}\quad v_z = -\frac{\rho g r^2}{4\mu} + C_1\ln(r) + C_2$$

The proper B.C. are: $u(a) = 0$ (no-slip) and $\mu\frac{\partial v_z}{\partial r}(b) = 0$ (no free-surface shear stress)

$$\text{The final solution is}\quad \mathbf{v_z = \frac{\rho g b^2}{2\mu}\ln\left(\frac{r}{a}\right) - \frac{\rho g}{4\mu}(r^2 - a^2)}\quad \textit{Ans.}$$

$$\text{The flow rate is}\quad Q = \int_a^b v_z 2\pi r\,dr = \frac{\pi\rho g a^4}{8\mu}(-3\sigma^4 - 1 + 4\sigma^2 + 4\sigma^4\ln\sigma),$$

$$\text{where}\ \sigma = \frac{b}{a}\quad \textit{Ans.}$$

P4.85 A flat plate of essentially infinite width and breadth oscillates sinusoidally in its own plane beneath a viscous fluid, as in Fig. P4.85. The fluid is at rest far above the plate. Making as many simplifying assumptions as you can, set up the governing differential equation and boundary conditions for finding the velocity field *u* in the fluid. Do not solve (if you *can* solve it immediately, you might be able to get exempted from the balance of this course with credit).

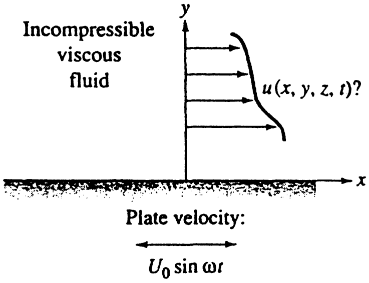


Fig. P4.85

Solution: Assume $u = u(y, t)$ and $\partial p/\partial x = 0$. The x-momentum relation is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$

$$\text{or:}\quad \rho\left(\frac{\partial u}{\partial t} + 0 + 0\right) = 0 + 0 + \mu\left(0 + \frac{\partial^2 u}{\partial y^2}\right),\quad \text{or, finally:}$$

$$\mathbf{\frac{\partial u}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}}\quad \text{subject to:}\ \mathbf{u(0,t) = U_o\sin(\omega t)}\quad \textbf{and}\quad \mathbf{u(\infty,t) = 0.}\quad \textit{Ans.}$$

P4.86 SAE 10 oil at 20°C flows between parallel plates 8 mm apart, as in Fig. P4.86. A mercury manometer, with wall pressure taps 1 m apart, registers a 6-cm height, as shown. Estimate the flow rate of oil for this condition.

Solution: Assuming laminar flow, this geometry fits Eqs. (4.143, 144) of the text:

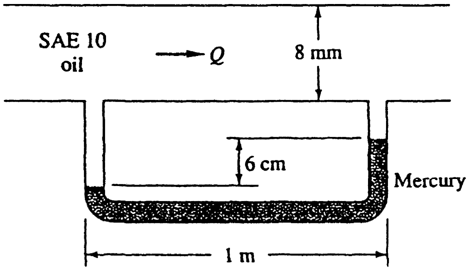


Fig. P4.86

$$V_{avg} = \frac{2}{3}u_{max} = \left(\frac{dp}{dx}\right)\frac{h^2}{3\mu}, \quad \text{where } h = \text{plate half-width} = 4\,\text{mm}$$

For SAE 10W oil, take $\rho = 870\ \text{kg/m}^3$ and $\mu = 0.104$ kg/m·s. The manometer reads

$$\Delta p = (\rho_{Hg} - \rho_{oil})g\Delta h = (13550 - 870)(9.81)(0.06) \approx 7463\ \text{Pa} \quad \text{for } \Delta x = L = 1\text{m}$$

$$\text{Then} \quad V = \frac{\Delta p}{\Delta x}\frac{h^2}{3\mu} = \left(\frac{7463\ \text{Pa}}{1\ \text{m}}\right)\frac{(0.004)^2}{3(0.104)} \approx 0.383\ \frac{\text{m}}{\text{s}}$$

The flow rate per unit width is $Q = VA = (0.383)(0.008) \approx \mathbf{0.00306\ \frac{m^3}{s\cdot m}}$ *Ans.*

NOTE: The Reynolds number, based upon plate half-width, is 16, *laminar*.

P4.87 SAE 30W oil at 20°C flows through the 9-cm-diameter pipe in Fig. P4.87 at an average velocity of 4.3 m/s. (a) Verify that the flow is laminar. (b) Determine the volume flow rate in m^3/h. (c) Calculate the expected reading *h* of the mercury manometer, in cm.

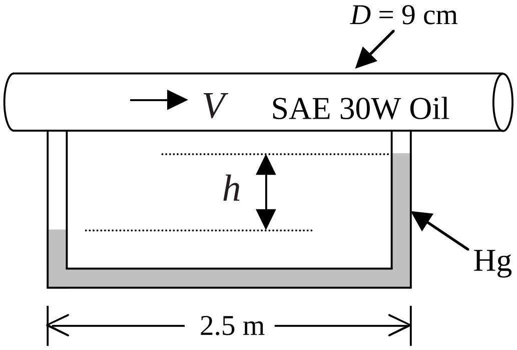


Fig. P4.87

Solution: (a) Check the Reynolds number. For SAE 30W oil, from Appendix A.3, $\rho = 891\ \text{kg/m}^3$ and $\mu = 0.29$ kg/(m·s). Then

$\text{Re}_d = \rho V d/\mu = (891\ \text{kg/m}^3)(4.3\ \text{m/s})(0.09\ \text{m})/[0.29\ \text{kg/(m·s)}] = 1190 < 2000$ **Laminar** *Ans.* (a)

(b) With average velocity known, the volume flow follows easily:

$$Q = AV = [(\pi/4)(0.09\ \text{m})^2](4.3\ \text{m/s})(3600\ \text{s/h}) = \mathbf{98.5\ m^3/h} \quad \textit{Ans. (b)}$$

(c) The manometer measures the pressure drop over a 2.5 m length of pipe. From Eq. (4.147),

$$V = 4.3\ \frac{m}{s} = \frac{\Delta p}{L}\frac{R^2}{8\mu} = \frac{\Delta p}{2.5\ m}\frac{(0.045\ m)^2}{8(0.29\ kg/m{\cdot}s)}, \quad solve\ for\ \ \Delta p = 12320\ Pa$$

$$\Delta p_{mano} = 12320 = (\rho_{merc} - \rho_{oil})gh = (13550 - 891)(9.81)h, \quad \text{Solve}\ \ \mathbf{h = 0.099\ m} \quad \textit{Ans. (c)}$$

P4.88 The viscous oil in Fig. P4.88 is set into steady motion by a concentric inner cylinder moving axially at velocity *U* inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve Eqs. (4.38) for the fluid velocity distribution $v_z(r)$. What are the proper boundary conditions?

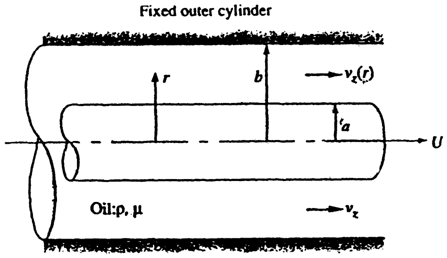


Fig. P4.88

Solution: If $v_z = \text{fcn}(r)$ only, the *z*-momentum equation (Appendix E) reduces to:

$$\rho\frac{dv_z}{dt} = -\frac{\partial p}{\partial z} + \rho g_z + \mu\nabla^2 v_z, \quad \text{or:} \quad 0 = 0 + 0 + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

The solution is $v_z = C_1 \ln(r) + C_2$, subject to $v_z(a) = U$ and $v_z(b) = 0$

Solve for $C_1 = U/\ln(a/b)$ and $C_2 = -C_1 \ln(b)$

The final solution is: $\mathbf{v_z = U\dfrac{\ln(r/b)}{\ln(a/b)}}$ *Ans.*

P4.89 Modify Prob. 4.88 so that the outer cylinder also moves to the *left* at constant speed *V*. Find the velocity distribution $\upsilon_z(r)$. For what ratio *V/U* will the wall shear stress be the same at both cylinder surfaces?

Solution: We merely modify the boundary conditions for the known solution in 4.88:

$v_z = C_1 \ln(r) + C_2$, subject to $v_z(a) = U$ and $v_z(b) = -V$

Solve for $C_1 = (U + V)/\ln(a/b)$ and $C_2 = U - (U + V)\ln(a)/\ln(a/b)$

The final solution is $\mathbf{v_z = U + (U + V)\dfrac{\ln(r/a)}{\ln(a/b)}}$ *Ans.*

The shear stress $\tau = \mu(U + V)/[r\ln(a/b)]$ and is never equal at both walls for any ratio of V/U unless the clearance is vanishingly small, that is, unless a ≈ b. *Ans.*

P4.90 It is desired to pump ethanol at 20°C through 25 meters of straight smooth tubing under laminar-flow conditions, $\text{Re}_d = \rho V d/\mu < 2300$. The available pressure drop is 10 kPa. (*a*) What is the maximum possible mass flow, in kg/h? (*b*) What is the appropriate diameter?

Solution: For ethanol at 20°C, $\rho = 789$ kg/m^3 and $\mu = 0.0012$ kg/m-s. From Eq. (4.138),

$$Q_{\text{laminar}} = \frac{\pi R^4}{8\mu}\frac{\Delta p}{L} = \frac{\pi d^4}{128\mu}\frac{\Delta p}{L}$$

Clearly, flow increases with diameter, so maximum mass flow requires the maximum diameter consistent with the maximum Reynolds number. The Reynolds number may be written out:

$$\text{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi d\mu} = \left(\frac{4\rho}{\pi d\mu}\right)\left(\frac{\pi d^4}{128\mu}\frac{\Delta p}{L}\right) = \frac{\rho d^3 \Delta p}{32\mu^2 L} < 2300$$

$$\text{Or}: \ d^3 = \frac{2300(32)\mu^2 L}{\rho\Delta p} = \frac{2300(32)(0.0012)^2(25)}{(789)(10{,}000)} = 3.36E-7\,m^3$$

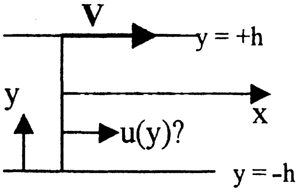
$$\textit{Solve for} \quad d_{\max} = 0.00695\,m \approx \mathbf{7}\ mm \qquad \textit{Ans.}(b)$$

The maximum mass flow is

$$\dot{m}_{\max} = \rho Q_{\max} = \left(789\frac{kg}{m^3}\right)\left[\frac{\pi(0.00695m)^4}{128(0.0012kg/m\bullet s)}\right]\left(\frac{10{,}000Pa}{25m}\right) = 0.0151\frac{kg}{s} = \mathbf{54}\frac{kg}{h} \quad \textit{Ans.}(a)$$

Light liquids like ethanol stay laminar only for tiny diameters. To work the same problem with, say, SAE 30W oil, $\mu = 0.29$ kg/m-s, would result in $d_{max} =$ 26 cm, or 37 times larger. The maximum oil mass flow would be nearly nine thousand times larger.

P4.91 Consider 2-D incompressible steady Couette flow between parallel plates with the upper plate moving at speed *V*, as in Fig. 4.16a. Let the fluid be *nonnewtonian*, with stress given by



$$\tau_{xx} = a\left(\frac{\partial u}{\partial x}\right)^c \quad \tau_{yy} = a\left(\frac{\partial v}{\partial y}\right)^c \quad \tau_{xy} = \tau_{yx} = \frac{a}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^c, \quad a \text{ and } c \text{ are constants}$$

Make all the same assumptions as in the derivation of Eq. (4.140). (a) Find the velocity profile *u*(*y*). (b) How does the velocity profile for this case compare to that of a newtonian fluid?

Solution: (a) Neglect gravity and pressure gradient. If u = u(y) and v = 0 at both walls, then continuity specifies that v = 0 everywhere. Start with the *x*-momentum equation:

$$\rho\left(u\cancel{\frac{\partial u}{\partial x}} + v\cancel{\frac{\partial u}{\partial y}}\right) = \rho\cancel{g_x} - \cancel{\frac{\partial p}{\partial x}} + \cancel{\frac{\partial \tau_{xx}}{\partial x}} + \frac{\partial \tau_{xy}}{\partial y}$$

Many terms drop out because v = 0 and τ_{xx} and $\partial u/\partial x = 0$ (because *u* does not vary with *x*). Thus we only have

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{d}{dy}\left[\frac{a}{2}\left(\frac{du}{dy}\right)^c\right] = 0, \quad or: \quad \frac{du}{dy} = constant, \quad u = C_1 y + C_2$$

The boundary conditions are no-slip at both walls:

$$u(y = -h) = 0 = C_1(-h) + C_2; \quad u(y = +h) = V = C_1(+h) + C_2, \quad solve\ C_1 = \frac{V}{2h},\ C_2 = \frac{V}{2}$$

The final solution for the velocity profile is:

$$\boldsymbol{u(y) = \frac{V}{2h}y + \frac{V}{2}} \quad \textit{Ans.}\ \text{(a)}$$

This is **exactly the same** as Eq. (4.140) for the newtonian fluid! *Ans.* (b)

P4.92 A tank of area *A*o is draining in laminar flow through a pipe of diameter *D* and length *L,* as shown in Fig. P4.92. Neglecting the exit-jet kinetic energy and assuming the pipe flow is driven by the hydrostatic pressure at its entrance, derive a formula for the tank level *h*(*t*) if its initial level is *h*o.

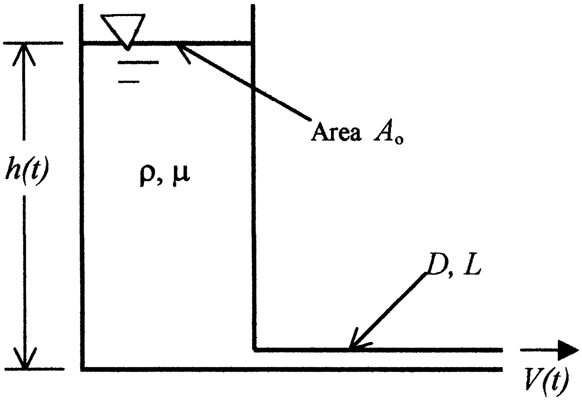


Fig. P4.92

Solution: For laminar flow, the flow rate out is given by Eq. (4.147). A control volume mass balance shows that this flow out is balanced by a tank level decrease:

$$Q_{out} = \frac{\pi D^4}{128\mu}\frac{\Delta p}{L} = -A_o\frac{dh}{dt} \quad \text{where } \Delta p \approx \rho g h(t)$$

Thus we can separate the variables and integrate to find the tank level change:

$$\int_{h_o}^{h}\frac{dh}{h} = -\int_0^t \frac{\pi D^4 \rho g}{128\mu L A_o}dt, \quad or: \quad \boldsymbol{h = h_o \exp\left[-\frac{\pi D^4 \rho g}{128\mu L A_o}t\right]} \quad \textit{Ans.}$$

P4.93 A number of straight 25-cm-long microtubes, of diameter *d*, are bundled together into a “honeycomb” whose total cross-sectional area is 0.0006 m^2. The pressure drop from entrance to exit is 1.5 kPa. It is desired that the total volume flow rate be 1 m^3/h of water at 20°C. (a) What is the appropriate microtube diameter? (b) How many microtubes are in the bundle? (c) What is the Reynolds number of each microtube?

Solution: For water at 20°C, $\rho = 998\ \mathrm{kg/m^3}$ and $\mu = 0.001$ kg/m·s. Each microtube of diameter D sees the same pressure drop. If there are N tubes,

$$Q = \frac{1}{3600}\ \frac{m^3}{s} = NQ_{tube} = N\frac{\pi D^4 \Delta p}{128\,\mu L} = N\frac{\pi D^4 (1500\,Pa)}{128(0.001\,kg/m\cdot s)(0.25\,m)} = 1.47E5\,N\ D^4$$

$$\text{At the same time,}\quad N = A_{bundle}/A_{tube} = \frac{0.0006\,m^2}{(\pi/4)D^2}$$

Combine to find $D^2 = 2.47E{-}6\ m^2$ or **D = 0.00157 m** *and* **N = 310** *Ans.*(a, b)

With D known, compute $V = Q/A_{\text{bundle}} = Q_{\text{tube}}/A_{\text{tube}} = 0.462$ m/s and

$\mathrm{Re}_D = \rho VD/\mu = (998)(0.462)(0.00157)/(0.001) =$ **724** (laminar) *Ans.* (c)

P4.94 A long solid cylinder rotates steadily in a very viscous fluid, as in Fig. P4.94. Assuming laminar flow, solve the Navier-Stokes equation in polar coordinates to determine the resulting velocity distribution. The fluid is at rest far from the cylinder. [HINT: the cylinder does not induce any radial motion.]

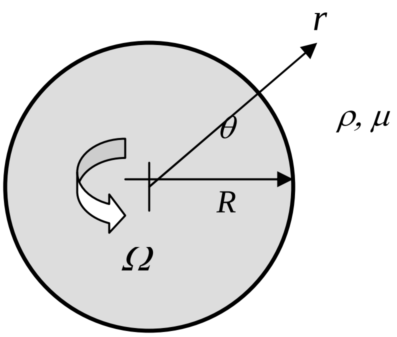


Fig. P4.94

Solution: We already have the useful hint that $v_r = 0$. Continuity then tells us that $(1/r)\partial v_\theta/\partial\theta = 0$, hence v_θ does not vary with θ. Navier-Stokes then yields the flow. From Eq. D.6, the tangential momentum relation, with $\partial p/\partial\theta = 0$ and $v_\theta = f(r)$, we obtain Eq. (4.139):

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) = \frac{v_\theta}{r^2}, \quad \text{Solution:} \quad v_\theta = C_1 r + \frac{C_2}{r}$$

$$\text{As } r\to\infty,\ v_\theta \to 0, \quad \text{hence } C_1 = 0$$

$$\text{At } r = R,\ v_\theta = \Omega R = \frac{C_2}{R};\ C_2 = \Omega R^2;\ \text{Finally,}\ v_\theta = \frac{\Omega R^2}{r} \quad \textit{Ans.}$$

Rotating a cylinder in a large expanse of fluid sets up (eventually) a *potential vortex flow*.

***P4.95** Two immiscible liquids of equal thickness h are being sheared between a fixed and a moving plate, as in Fig. P4.95. Gravity is neglected, and there is no variation with x. Find an expression for (a) the velocity at the interface; and (b) the shear stress in each fluid. Assume steady laminar flow.

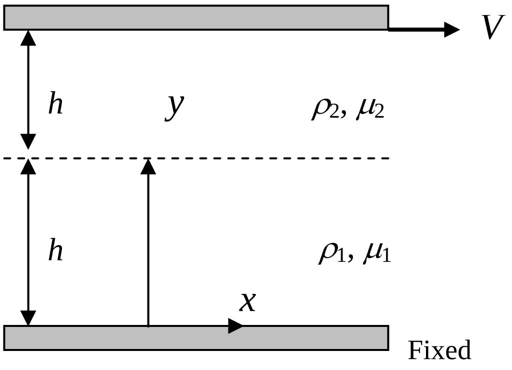


Fig. P4.95

Solution: Treat this as a *Ch. 4 problem* (not Ch. 1), use continuity and Navier-Stokes:

$$\text{Continuity:}\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + \frac{\partial v}{\partial y} = 0;\ \text{thus } v = const = 0 \text{ for no-slip at the walls}$$

This tells us that there is no velocity v, hence we need only consider $u(y)$ in Navier-Stokes:

$$\rho_{1,2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{1,2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \text{or: } 0 + 0 = 0 + \mu_{1,2}\left(0 + \frac{d^2 u}{dy^2}\right)$$

$$\text{Thus} \quad u = a + b\,y$$

The velocity profiles are linear in *y* but have a different slope in each layer. Let u_I be the velocity at the interface. (*a*) The shear stress is the same in each layer:

$$\tau = \mu_1 \frac{u_I}{h} = \mu_2 \frac{V - u_I}{h} \quad \text{Solve for} \quad u_I = \frac{\mu_2}{\mu_1 + \mu_2} V \qquad \textit{Ans.}(a)$$

(*b*) In terms of the upper plate velocity, *V*, the shear stress is

$$\tau = \left(\frac{\mu_1 \mu_2}{\mu_1 + \mu_2}\right) \frac{V}{h} \qquad \textit{Ans.}(b)$$

P4.96 Reconsider Prob. P1.44 and calculate (*a*) the inner shear stress and (*b*) the power required, if the exact laminar-flow formula, Eq. (4.140) is used. (*c*) Determine whether this flow pattern is stable. [HINT: The shear stress in (r, θ) coordinates is *not* like plane flow.]

Solution: The exact laminar-flow velocity is Eq. (4.140), and the shear stress is Eq. (D.9):

$$v_\theta = \Omega_i\, r_i \left[\frac{(r_o/r) - (r/r_o)}{(r_o/r_i) - (r_i/r_o)}\right]$$

$$\tau_{r\theta} = \mu\left(\frac{dv_\theta}{dr} - \frac{v_\theta}{r}\right) = \mu\left[\frac{\Omega_i\, r_i}{(r_o/r_i) - (r_i/r_o)}\right] \frac{2r_o}{r^2}$$

Recall the data from Prob. P1.44: $r_i = 5$ cm, $r_o = 6$ cm, $L = 120$ cm, $\mu = 0.86$ kg/m-s (SAE 50W oil), and Ωi = 900 rev/min = 94.25 rad/s. At the inner cylinder,

$$\tau_{inner} = \mu\left[\frac{\Omega_i\, r_i}{(r_o/r_i) - (r_i/r_o)}\right] \frac{2r_o}{r_i^2} = (0.86)\left[\frac{94.25(0.05)}{0.06/0.05 - 0.05/0.06}\right] \frac{2(0.06)}{(0.05)^2} = \mathbf{531}\, Pa \quad \textit{Ans.}(a)$$

The moment and power required are

$$M = \tau_i\, 2\pi r_i^2 L = (531)\, 2\pi (0.05)^2 (1.20) = \mathbf{10.0}\ N-m$$

$$Power = \Omega_i\, M = (94.25\, rad/s)(10.0\, N-m) = \mathbf{943}\ watts \qquad \textit{Ans.}(b)$$

The shear stress, moment, and power are all 31% larger than the approximate linear-profile analysis of Prob. 1.44.

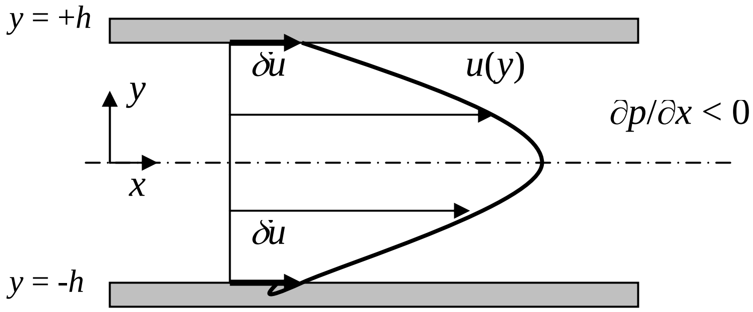
(*c*) The stability of this flow is determined by Taylor’s criterion, Eq. (4.141):

$$Ta = \frac{r_i(r_o - r_i)^3\Omega_i^2}{(\mu/\rho)^2} = \frac{(0.05)(0.06-0.05)^3(94.25)^2}{(0.86/902)^2} = 490 < 1700 \quad \textbf{STABLE} \quad \textit{Ans.}(c)$$

To finish this, we had to look up the density of SAE 50W oil, $\rho = 902\ \text{kg/m}^3$.

P4.97 For the pressure-gradient flow between two parallel plates of Fig. 4.12(*b*), reanalyze for the case of *slip flow* at both walls. Use the simple slip condition $\delta u_{\text{wall}} = \ell(du/dy)_{\text{wall}}$, where ℓ is the mean free path of the fluid. (*a*) Sketch the expected velocity profile. (*b*) Find an expression for the shear stress at each wall. (*c*) Find the volume flow between the plates.

Solution: The velocity profile will be similar to Fig. 4.12*b* except for wall slip. *Ans.*(*a*)



Fig, P4.97

The no-slip analysis of Eq. (4.133) is correct up to the point of integration:

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

The boundary conditions, however, give rise to new constants:

$$\text{At}\quad y = +h:\quad u = \delta u_{wall} = \frac{1}{\mu}\frac{dp}{dx}\frac{h^2}{2} + C_1 h + C_2$$

$$\text{At}\quad y = -h:\quad u = \delta u_{wall} = \frac{1}{\mu}\frac{dp}{dx}\frac{(-h)^2}{2} - C_1 h + C_2$$

Subtract these two and we find that $\boldsymbol{C_1 = 0}$, which is a symmetry condition. The slip velocity is

$$\delta u_{wall} = \ell\frac{du}{dy}|_{-h} = \ell\,\frac{1}{\mu}\frac{dp}{dx}(-h) = \frac{1}{\mu}\frac{dp}{dx}\frac{h^2}{2} + C_2 \;,$$

$$\text{whence} \quad C_2 = \frac{1}{2\mu}(-\frac{dp}{dx})(h^2 + 2h\ell)$$

Finally, the velocity profile with wall slip is given by

$$u = \frac{1}{2\mu}(-\frac{dp}{dx})(h^2 + 2h\ell - y^2) \qquad\qquad Ans.(b)$$

This is the shape that the writer attempted to sketch above in *Ans*.(*a*).

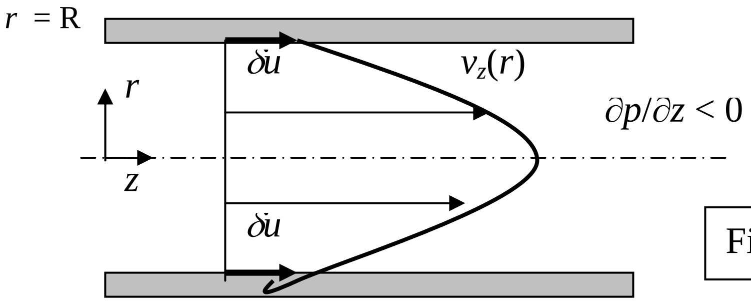
(*c*) The volume flow between the plates, per unit width into the paper, is

$$Q = \int u\,dA = \int_{-h}^{+h}[\frac{1}{2\mu}(-\frac{dp}{dx})(h^2 + 2h\ell - y^2)dy = \frac{2h^3}{3\mu}(-\frac{dp}{dx})(1 + \frac{2\ell}{h}) \qquad Ans.(c)$$

The added flow due to slip is proportional to twice the Knudsen number, 2Kn = 2 ℓ /h.

P4.98 For the pressure-gradient flow in a circular tube, in Sect. 4.10, reanalyze for the case of *slip flow* at the wall. Use the simple slip condition $\delta v_{z,\,wall} = \ell(dv_z/dr)_{wall}$, where ℓ is the mean free path of the fluid. (*a*) Sketch the expected velocity profile. (*b*) Find an expression for the shear stress at the wall. (*c*) Find the volume flow through the tube.

Solution: (*a*) The velocity profile has equal slip δu all around, as shown:



Fig, P4.98

(*b*) The analysis of the velocity is correct up to the first result after Eq. (4.136) of the text:

$$v_z = \frac{dp}{dz}\frac{r^2}{4\mu} + C_1\ln(r) + C_2$$

Once again, $\boldsymbol{C_1 = 0}$ to avoid a logarithmic singularity at the centerline. The constant C_2 is found from the slip boundary condition:

$$\text{At } r=R:\ v_z = \delta u = \frac{dp}{dz}\frac{R^2}{4\mu} + C_2 = \ell\,\frac{dv_z}{dr}\Big|_{r=R} = \ell\,\frac{dp}{dz}\left(-\frac{R}{2\mu}\right),$$

$$\text{whence}\quad C_2 = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)(R^2 + 2\ell R)$$

The velocity profile thus is given by the slip-flow formula

$$v_z = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)(R^2 + 2\ell R - r^2) \qquad \textit{Ans.}(b)$$

The volume flow, with slip, is given by

$$Q = \int v_z\,dA = \int_0^R \left[\frac{1}{4\mu}\left(-\frac{dp}{dz}\right)(R^2 + 2\ell R - r^2)\right]2\pi r\,dr = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right)\left(1+4\frac{\ell}{R}\right) \qquad \textit{Ans.}(c)$$

The slip-flow correction factor is 4 times the Knudsen number, 4Kn = 4ℓ/R.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

Chapter 4 is not a favorite of the people who prepare the FE Exam. Probably not a single problem from this chapter will appear on the exam, but if some did, they might be like these:

FE4.1 Given the steady, incompressible velocity distribution **V** = 3x**i** + Cy**j** + 0**k**, where C is a constant, if conservation of mass is satisfied, the value of C should be

(a) 3 (b) 3/2 (c) 0 (d) –3/2 **(e) –3**

FE4.2 Given the steady velocity distribution **V** = 3x**i** + 0**j** + Cy**k**, where C is a constant, if the flow is irrotational, the value of C should be

(a) 3 (b) 3/2 **(c) 0** (d) –3/2 (e) –3

FE4.3 Given the steady, incompressible velocity distribution **V** = 3x**i** + Cy**j** + 0**k**, where C is a constant, the shear stress τ_{xx} at the point (x, y, z) is given by

(a) 3μ (b) $(3x + Cy)\mu$ **(c) 0** (d) $C\mu$ (e) $(3 + C)\mu$

FE4.4 Given the steady incompressible velocity distribution $u = Ax$, $v = By$, and $w = Cxy$, where (A, B, C) are constants. This flow satisfies the equation of continuity if A equals

(*a*) B , (*b*) $B + C$, (*c*) $B - C$, **(*d*) $-B$** , (*e*) $-(B + C)$

FE4.5 For the velocity field in Prob. FE4.4, the convective acceleration in the *x* direction is

(*a*) Ax^2 , **(*b*) A^2x** , (*c*) B^2y , (*d*) By^2 , (*e*) Cx^2y

FE4.6 If, for laminar flow in a smooth straight tube, the tube diameter and length both double, while everything else remains the same, the volume flow rate will increase by a factor of

(*a*) 2 , (*b*) 4 , **(*c*) 8** , (*d*) 12 , (*e*) 16

COMPREHENSIVE PROBLEMS

C4.1 In a certain medical application, water at room temperature and pressure flows through a rectangular channel of length L = 10 cm, width s = 1 cm, and gap thickness b = 0.3 mm. The volume flow is sinusoidal, with amplitude Q_o = 0.5 ml/s and frequency f = 20 Hz, that is, $Q = Q_o\sin(2\pi f\ t)$.

(a) Calculate the maximum Reynolds number Re = Vb/ν, based on maximum average velocity and gap thickness. Channel flow remains *laminar* for Re < 2000, otherwise it will be *turbulent*. Is this flow laminar or turbulent?

(b) Assume quasi-steady flow, that is, solve as if the flow were steady at any given Q(t). Find an expression for streamwise velocity u as a function of y, μ, dp/dx, and b, where dp/dx is the pressure gradient required to drive the flow through the channel at flow rate Q. Also estimate the maximum magnitude of velocity component u.

(c) Find an analytic expression for flow rate Q(t) as a function of dp/dx.

(d) Estimate the wall shear stress τ_W as a function of Q, f, μ, b, s, and time t.

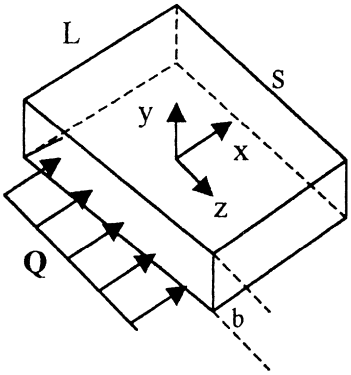
(e) Finally, use the given numbers to estimate the wall shear amplitude, τ_{WO}, in Pa.

Solution: (a) Maximum flow rate is the amplitude, Q_o = 0.5 ml/s, hence average velocity V = Q/A:

$$V = \frac{Q}{bs} = \frac{0.5E\text{–}6\ m^3/s}{(0.0003\,m)(0.01\,m)} = 0.167\,m/s$$

$$\mathrm{Re}_{\max} = \frac{Vb}{\nu} = \frac{(0.167)(0.0003)}{(0.001/998)}$$

$$= \mathbf{50\ (laminar)}\quad \textit{Ans.}\ \text{(a)}$$



(b, c) The quasi-steady analysis is just like Eqs. (4.142–144) of the text, with "h" = b/2:

$$\boldsymbol{u = \frac{-1}{2\mu}\frac{dp}{dx}\left(\frac{b^2}{4} - y^2\right),\quad u_{\max} = \frac{-1}{2\mu}\frac{dp}{dx}\frac{b^2}{4},\quad Q_{\max} = \frac{2}{3}u_{\max}bs = \frac{-sb^3}{12\mu}\frac{dp}{dx}}\quad \textit{Ans.}\ \text{(b, c)}$$

$$\text{(d) Wall shear:}\quad \tau_{wall} = \mu\left|\frac{du}{dy}\right|_{wall} = \boldsymbol{\frac{b}{2}\frac{dp}{dx} = \frac{6\mu Q}{sb^2} = \frac{6\mu Q_o}{sb^2}\sin(2\pi f\,t)}\quad \textit{Ans.}\ \text{(d)}$$

(e) For our given numerical values, the amplitude of wall shear stress is:

$$\tau_{wo} = \frac{6\mu Q_o}{sb^2} = \frac{6(0.001)(0.5E\text{–}6)}{(0.01)(0.0003)^2} = \mathbf{3.3}\ \boldsymbol{Pa}\quad \textit{Ans.}\ \text{(e)}$$

C4.2 A belt moves upward at velocity V, dragging a film of viscous liquid of thickness *h*, as in Fig. C4.2. Near the belt, the film moves upward due to no-slip. At its outer edge, the film moves downward due to gravity. Assuming that the only non-zero velocity is *v*(*x*), with zero shear stress at the outer film edge, derive a formula for (a) *v*(*x*); (b) the average velocity *V*avg in the film; and (c) the wall velocity *V*C for which there is no net flow either up or down.

(d) Sketch *v*(*x*) for case (c).

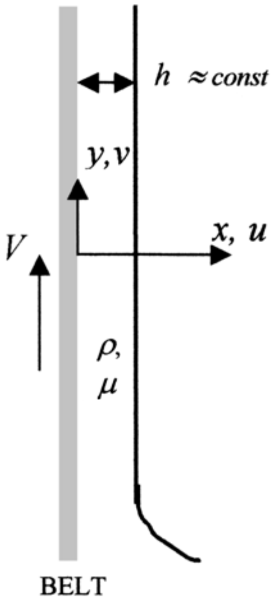


Fig. C4.2

Solution: (a) The assumption of parallel flow, *u* = *w* = 0 and *v* = *v*(*x*), satisfies continuity and makes the *x*- and *z*-momentum equations irrelevant. We are left with the *y*-momentum equation:

$$\rho\left(\cancel{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}}\right)=\cancel{-\frac{\partial p}{\partial y}}-\rho g+\mu\left(\frac{\partial^2 v}{\partial x^2}+\cancel{\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}}\right)$$

There is no convective acceleration, and the pressure gradient is negligible due to the free surface. We are left with a second-order linear differential equation for *v*(*x*):

$$\frac{d^2 v}{dx^2}=\frac{\rho g}{\mu}\quad \text{Integrate: }\frac{dv}{dx}=\frac{\rho g}{\mu}x+C_1\quad \text{Integrate again: }v=\frac{\rho g}{\mu}\frac{x^2}{2}+C_1 x+C_2$$

At the free surface, $x = h$, $\tau = \mu(dv/dx) = 0$, hence $C_1 = -\rho g h/\mu$. At the wall, $v = V = C_2$. The solution is

$$\boldsymbol{v = V - \frac{\rho g h}{\mu}x + \frac{\rho g}{2\mu}x^2}\quad \textit{Ans.}\text{ (a)}$$

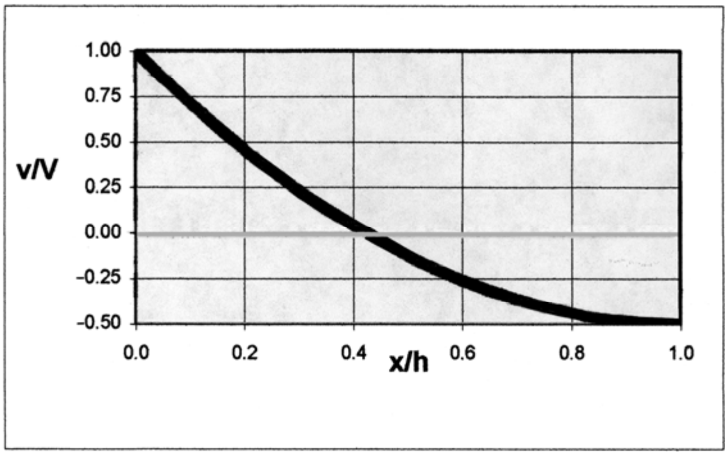
(b) The average velocity is found by integrating the distribution *v*(*x*) across the film:

$$v_{avg}=\frac{1}{h}\int_0^h v(x)\,dx=\frac{1}{h}\left[Vx-\frac{\rho g h x^2}{2\mu}+\frac{\rho g x^3}{6\mu}\right]_0^h=\boldsymbol{V-\frac{\rho g h^2}{3\mu}}\quad \textit{Ans.}\text{ (b)}$$

(c) Since $hv_{avg} \equiv Q$ per unit depth into the paper, there is no net up-or-down flow when

$$\boldsymbol{V = \rho g h^2/(3\mu)}\quad \textit{Ans.}\text{ (c)}$$

(d) A graph of case (c) is shown below. *Ans.* (d)



Chapter 5 • Dimensional Analysis and Similarity

5.1 For axial flow through a circular tube, the Reynolds number for transition to turbulence is approximately 2300 [see Eq. (6.2)], based upon the diameter and average velocity. If *d* = 5 cm and the fluid is kerosene at 20°C, find the volume flow rate in m^3/h which causes transition.

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 0.00192 kg/m·s. The only unknown in the transition Reynolds number is the fluid velocity:

$$\mathrm{Re}_{tr} \approx 2300 = \frac{\rho \mathrm{Vd}}{\mu} = \frac{(804)\mathrm{V}(0.05)}{0.00192}, \quad \text{solve for } \mathrm{V}_{tr} = 0.11\ \mathrm{m/s}$$

$$\text{Then} \quad \mathrm{Q} = \mathrm{VA} = (0.11)\frac{\pi}{4}(0.05)^2 = 2.16\mathrm{E{-}4}\ \frac{\mathrm{m}^3}{\mathrm{s}} \times 3600 \approx \mathbf{0.78}\ \frac{\mathbf{m^3}}{\mathbf{hr}} \quad \textit{Ans.}$$

P5.2 A prototype automobile is designed for cold weather in Denver, CO (-10°C, 83 kPa). Its drag force is to be tested in on a one-seventh-scale model in a wind tunnel at 150 mi/h and at 20°C and 1 atm. If model and prototype satisfy dynamic similarity, what prototype velocity, in mi/h, is matched? Comment on your result.

Solution: First assemble the necessary air density and viscosity data:

$$\text{Denver: } T = 263\,K \ ; \quad \rho_p = \frac{p}{RT} = \frac{83000}{287(263)} = 1.10\ \frac{kg}{m^3} \ ; \quad \mu_p = 1.75\,E-5\frac{kg}{m-s}$$

$$\text{Wind tunnel: } T = 293\,K \ ; \rho_m = \frac{p}{RT} = \frac{101350}{287(293)} = 1.205\ \frac{kg}{m^3} \ ; \quad \mu_m = 1.80\,E-5\frac{kg}{m-s}$$

Convert 150 mi/h = 67.1 m/s. For dynamic similarity, equate the Reynolds numbers:

$$\mathrm{Re}_p = \frac{\rho VL}{\mu}\Big|_p = \frac{(1.10)V_p(7L_m)}{1.75E-5} = \mathrm{Re}_m = \frac{\rho VL}{\mu}\Big|_m = \frac{(1.205)(67.1)(L_m)}{1.80E-5}$$

$$\text{Solve for} \quad V_{prototype} = 10.2\,m/s = 22.8\ mi/h \qquad \textit{Ans.}$$

This is too *slow*, hardly fast enough to turn into a driveway. Since the tunnel can go no faster, the model drag must be corrected for Reynolds number effects. Note that we did not need to know the actual *length* of the prototype auto, only that it is 7 times larger than the model length.

P5.3 The transfer of energy by viscous dissipation is dependent upon viscosity μ, thermal conductivity k, stream velocity U, and stream temperature T_o. Group these quantities, if possible, into the dimensionless *Brinkman number*, which is proportional to μ.

Solution: Here we have only a single dimensionless group. List the dimensions, from Table 5.1:

μ	k	U	T_o
$\{ML^{-1}T^{-1}\}$	$\{MLT^{-3}\Theta^{-1}\}$	$\{LT^{-1}\}$	$\{\Theta\}$

Four dimensions, four variables (MLTΘ) – perfect for making a pi group. Put μ in the numerator:

$$\text{Brinkman number} = k^a U^b T_o^c \mu^1 \quad \text{yields} \qquad \mathrm{Br} = \mu U^2/(kT_o) \qquad \textit{Ans.}$$

5.4 When tested in water at 20°C flowing at 2 m/s, an 8-cm-diameter sphere has a measured drag of 5 N. What will be the velocity and drag force on a 1.5-m-diameter weather balloon moored in sea-level standard air under dynamically similar conditions?

Solution: For water at 20°C take $\rho \approx 998$ kg/m^3 and $\mu \approx 0.001$ kg/m·s. For sea-level standard air take $\rho \approx 1.2255$ kg/m^3 and $\mu \approx$ 1.78E–5 kg/m·s. The balloon velocity follows from *dynamic similarity*, which requires identical Reynolds numbers:

$$\mathrm{Re}_{\text{model}} = \left.\frac{\rho VD}{\mu}\right|_{\text{model}} = \frac{998(2.0)(0.08)}{0.001} = 1.6\text{E}5 = \mathrm{Re}_{\text{proto}} = \frac{1.2255V_{\text{balloon}}(1.5)}{1.78\text{E}{-5}}$$

or $V_{\text{balloon}} \approx$ **1.55 m/s**. *Ans.* Then the two spheres will have identical drag coefficients:

$$C_{D,\text{model}} = \frac{F}{\rho V^2 D^2} = \frac{5\text{ N}}{998(2.0)^2(0.08)^2} = 0.196 = C_{D,\text{proto}} = \frac{F_{\text{balloon}}}{1.2255(1.55)^2(1.5)^2}$$

Solve for $\mathbf{F_{balloon} \approx 1.3\ N}$ *Ans.*

5.5 An automobile has a characteristic length and area of 8 ft and 60 ft^2, respectively. When tested in sea-level standard air, it has the following measured drag force versus speed:

V, mi/h:	20	40	60
Drag, lbf:	31	115	249

The same car travels in Colorado at 65 mi/h at an altitude of 3500 m. Using dimensional analysis, estimate (a) its drag force and (b) the horsepower required to overcome air drag.

Solution: For sea-level air in BG units, take $\rho \approx 0.00238$ slug/ft^3 and $\mu \approx$ 3.72E–7 slug/ft·s. Convert the raw drag and velocity data into dimensionless form:

V (mi/hr):	20	40	60
$C_D = F/(\rho V^2 L^2)$:	0.237	0.220	0.211
$\mathrm{Re}_L = \rho VL/\mu$:	1.50E6	3.00E6	4.50E6

Drag coefficient plots versus Reynolds number in a very smooth fashion and is well fit (to ±1%) by the Power-law formula $\mathbf{C_D \approx 1.07Re_L^{-0.106}}$.

(a) The new velocity is V = 65 mi/hr = 95.3 ft/s, and for air at 3500-m Standard Altitude (Table A-6) take $\rho = 0.001675$ slug/ft^3 and $\mu = 3.50E{-}7$ slug/ft·s. Then compute the new Reynolds number and use our Power-law above to estimate drag coefficient:

$$\mathbf{Re}_{Colorado} = \frac{\rho VL}{\mu} = \frac{(0.001675)(95.3)(8.0)}{3.50E{-}7} = 3.65E6, \quad hence$$

$$C_D \approx \frac{1.07}{(3.65E6)^{0.106}} = 0.2157, \quad \therefore\ \mathbf{F} = 0.2157(0.001675)(95.3)^2(8.0)^2 = \mathbf{210\ lbf} \quad Ans.\ \text{(a)}$$

(b) The horsepower required to overcome drag is

$$\text{Power} = FV = (210)(95.3) = 20030\ \text{ft·lbf/s} \div 550 = \mathbf{36.4\ hp} \quad Ans.\ \text{(b)}$$

P5.6 The full-scale parachute in the chapter-opener photo had a drag force of approximately 950 lbf when tested at a velocity of 12 mi/h in air at 20°C and 1 atm. Earlier, a model parachute of diameter 1.7 m was tested in the same tunnel. (*a*) For dynamic similarity, what should be the air velocity for the model? (*b*) What is the expected drag force of the model? (*c*) Is there anything surprising about your result to part (*b*)?

Solution: Convert the data to metric: 950 lbf = 4226 N, 12 mi/h = 5.36 m/s, and D_p = 55 ft = 16.8 m. (*a*) From Table A.2 for air at 20C, ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. (*a*) For similarity, equate the Reynolds numbers:

$$\text{Re}_p = \frac{\rho_p V_p D_p}{\mu_p} = \frac{(1.20)(5.36)(16.8)}{1.8E-5} = 6.0\text{E}6 = \text{Re}_m = \frac{\rho_m V_m D_m}{\mu_m} = \frac{(1.20)(V_m)(1.7)}{1.8E-5}$$

$$Solve\ \ for\ \ V_m = 52.9\frac{m}{s} = \mathbf{118}\ \frac{mi}{h} \qquad Ans.(a)$$

(*b*) For similarity, the force coefficients will be equal:

$$C_{Fp} = \frac{F_p}{\rho_p V_p^2 D_p^2} = \frac{4226N}{(1.20)(5.36)^2(16.8)^2} = 0.436 = \frac{F_m}{\rho_m V_m^2 D_m^2} = \frac{F_m}{(1.20)(52.9)^2(1.7)^2}$$

$$Solve\ \ for\ \ F_m = 4226\,N = \mathbf{950}\ lbf \qquad Ans.(b)$$

(*c*) It might be surprising that the drag forces are exactly the same for model and prototype! This is because, if ρ and μ are the same, the product *VD* is the same for both and the force is proportional to $(VD)^2$.

5.7 A body is dropped on the moon (g = 1.62 m/s^2) with an initial velocity of 12 m/s. By using option-2 variables, Eq. (5.11), the ground impact occurs at $t^{**} = 0.34$ and $S^{**} = 0.84$. Estimate (a) the initial displacement, (b) the final displacement, and (c) the time of impact.

Solution: (a) The initial displacement follows from the "option 2" formula, Eq. (5.12):

$$S^{**} = gS_o/V_o^2 + t^{**} + \frac{1}{2}t^{**2} = 0.84 = \frac{(1.62)S_o}{(12)^2} + 0.34 + \frac{1}{2}(0.34)^2$$

$$\text{Solve for} \quad S_o \approx \mathbf{39\ m} \quad \textit{Ans.}\ \text{(a)}$$

(b, c) The final time and displacement follow from the given dimensionless results:

$$S^{**} = gS/V_o^2 = 0.84 = (1.62)S/(12)^2, \quad \text{solve for } S_{\text{final}} \approx \mathbf{75\,m} \quad \textit{Ans.}\ \text{(b)}$$

$$t^{**} = gt/V_o = 0.34 = (1.62)t/(12), \quad \text{solve for } t_{\text{impact}} \approx \mathbf{2.52\ s} \quad \textit{Ans.}\ \text{(c)}$$

5.8 The *Morton number* Mo, used to correlate bubble-dynamics studies, is a dimensionless combination of acceleration of gravity *g*, viscosity μ, density ρ, and surface tension coefficient *Y*. If Mo is proportional to *g*, find its form.

Solution: The relevant dimensions are $\{g\} = \{LT^{-2}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{Y\} = \{MT^{-2}\}$. To have *g* in the numerator, we need the combination:

$$\{Mo\} = \{g\}\{\mu\}^a\{\rho\}^b\{Y\}^c = \left\{\frac{L}{T^2}\right\}\left\{\frac{M}{LT}\right\}^a\left\{\frac{M}{L^3}\right\}^b\left\{\frac{M}{T^2}\right\}^c = M^0L^0T^0$$

$$\text{Solve for} \quad a = 4,\ b = -1,\ c = -3, \quad \text{or:} \quad \boldsymbol{Mo = \frac{g\mu^4}{\rho Y^3}} \quad \textit{Ans.}$$

P5.9 The *Richardson number*, Ri, which correlates the production of turbulence by buoyancy, is a dimensionless combination of the acceleration of gravity *g*, the fluid temperature T_o, the local temperature gradient $\partial T/\partial z$, and the local velocity gradient $\partial u/\partial z$. Determine the form of the Richardson number if it is proportional to *g*.

Solution: In the {MLTΘ} system, these variables have the dimensions $\{g\} = \{L/T^2\}$, $\{T_o\} = \{\Theta\}$, $\{\partial T/\partial z\} = \{\Theta/L\}$, and $\{\partial u/\partial z\} = \{T^{-1}\}$. The ratio $g/(\partial u/\partial z)^2$ will cancel time, leaving {L} in the numerator, and the ratio $\{\partial T/\partial z\}/T_o$ will cancel {Θ}, leaving {L} in the denominator. Multiply them together and we have the standard form of the dimensionless Richardson number:

$$\mathrm{Ri} = \frac{g\,(\dfrac{\partial T}{\partial z})}{T_o\,(\dfrac{\partial u}{\partial z})^2} \qquad \textit{Ans.}$$

5.10 Determine the dimension $\{MLT\Theta\}$ of the following quantities:

$$\text{(a) } \rho u\frac{\partial u}{\partial x} \quad \text{(b) } \int_1^2 (p - p_0)\,dA \quad \text{(c) } \rho c_p \frac{\partial^2 T}{\partial x\,\partial y} \quad \text{(d) } \iiint \rho \frac{\partial u}{\partial t}\,dx\,dy\,dz$$

All quantities have their standard meanings; for example, ρ is density, etc.

Solution: Note that $\{\partial u/\partial x\} = \{U/L\}$, $\{\int p\,dA\} = \{pA\}$, etc. The results are:

$$\text{(a)}\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \text{(b)}\left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\};\quad \text{(c)}\left\{\frac{\mathbf{M}}{\mathbf{L^3T^2}}\right\};\quad \text{(d)}\left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\} \quad \textit{Ans.}$$

P5.11 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the wave speed of an atomic bomb explosion. He assumed that the blast wave radius R was a function of energy released E, air density ρ, and time t. Use dimensional analysis to show how wave radius must vary with time.

Solution: The proposed function is $R = f(E, \rho, t)$. There are four variables ($n = 4$) and three primary dimensions (MLT, or $j = 3$), thus we expect n-j = 4-3 = 1 pi group. List the

$$\{R\} = \{\mathrm{L}\} \;;\; \{E\} = \{\mathrm{ML}^2/\mathrm{T}^2\} \;;\quad \{\rho\} = \{\mathrm{M/L}^3\} \;;\quad \{t\} = \{\mathrm{T}\}$$

dimensions:

Assume arbitrary exponents and make the group dimensionless:

$$R^1 E^a \rho^b t^c \;=\; (\mathrm{L})^1 (\mathrm{ML}^2/\mathrm{T}^2)^a (\mathrm{M/L}^3)^b (\mathrm{T})^c \;=\; \mathrm{M}^0\,\mathrm{L}^0\,\mathrm{T}^0\,,$$

$$\text{whence } a+b=0 \;;\; 1+2a-3b=0 \;;\; -2a+c=0\,;\; \text{Solve } a=-\frac{1}{5}\,;\, b=+\frac{1}{5}\,;\, c=-\frac{2}{5}$$

The single pi group is

$$\Pi_1 \;=\; \frac{R\,\rho^{1/5}}{E^{1/5}\,t^{2/5}} \;=\; \text{constant}, \quad \text{thus} \quad R_{wave} \propto t^{2/5} \qquad \textit{Ans.}$$

5.12 The *Stokes number*, St, used in particle-dynamics studies, is a dimensionless combination of *five* variables: acceleration of gravity g, viscosity μ, density ρ, particle velocity U, and particle diameter D. (a) If St is proportional to μ and inversely proportional to g, find its form. (b) Show that St is actually the quotient of two more traditional dimensionless groups.

Solution: (a) The relevant dimensions are $\{g\} = \{\mathrm{LT}^{-2}\}$, $\{\mu\} = \{\mathrm{ML}^{-1}\mathrm{T}^{-1}\}$, $\{\rho\} = \{\mathrm{ML}^{-3}\}$, $\{U\} = \{\mathrm{LT}^{-1}\}$, and $\{D\} = \{\mathrm{L}\}$. To have μ in the numerator and g in the denominator, we need the combination:

$$\{St\} = \{\mu\}\{g\}^{-1}\{\rho\}^a\{U\}^b\{D\}^c = \left\{\frac{M}{LT}\right\}\left\{\frac{T^2}{L}\right\}\left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c = M^0L^0T^0$$

$$\text{Solve for} \quad a=-1,\ b=1,\ c=-2, \quad \text{or:} \quad St = \frac{\mu U}{\rho g D^2} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{This has the ratio form:} \quad \boldsymbol{St} = \frac{U^2/(gD)}{\rho UD/\mu} = \frac{\textit{Froude number}}{\textit{Reynolds number}} \quad \textit{Ans.}\ \text{(b)}$$

5.13 The speed of propagation *C* of a capillary (very small) wave in deep water is known to be a function only of density ρ, wavelength λ, and surface tension Y. Find the proper functional relationship, completing it with a dimensionless constant. For a given density and wavelength, how does the propagation speed change if the surface tension is doubled?

Solution: The "function" of ρ, λ, and Y must have velocity units. Thus

$$\{C\} = \{f(\rho,\lambda,Y)\}, \quad \text{or} \quad C = \text{const}\ \rho^a\lambda^b Y^c, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \left\{\frac{M}{L^3}\right\}^a \{L\}^b \left\{\frac{M}{T^2}\right\}^c$$

$$\text{Solve for} \quad a = b = -1/2 \text{ and } c = +1/2, \quad \text{or:} \quad \mathbf{C = const}\sqrt{\frac{\mathbf{Y}}{\rho\lambda}} \quad \textit{Ans.}$$

Thus, for constant ρ and λ, if Y is doubled, C increases as $\sqrt{2}$, or **+41%**. *Ans.*

P5.14 In forced convection, the heat transfer coefficient *h* is a function of thermal conductivity *k*, density ρ, viscosity μ, specific heat c_p, body length *L*, and velocity *V*. Heat transfer coefficient has units of W/(m^2-K) and dimensions $\{MT^{-3}\Theta^{-1}\}$. Rewrite this relation in dimensionless form, using (k, ρ, c_p, L) as repeating variables.

Solution: From Table 5.1, plus the given definition of *h*, list the dimensions:

h	k	ρ	μ	c_p	L	V
$\{MT^{-3}\Theta^{-1}\}$	$\{MLT^{-3}\Theta^{-1}\}$	$\{ML^{-3}\}$	$\{ML^{-1}T^{-1}\}$	$\{L^2T^{-2}\Theta^{-1}\}$	$\{L\}$	$\{LT^{-1}\}$

Four dimensions, 3 pi groups expected.

Add one variable successively to our repeating variables (k, ρ, c_p, L):

$$\Pi_1 = k^a \rho^b c_p^c L^d h^1 \quad \text{yields} \quad \Pi_1 = \frac{hL}{k}$$

$$\Pi_2 = k^a \rho^b c_p^c L^d \mu^1 \quad \text{yields} \quad \Pi_2 = \frac{\mu c_p}{k}$$

$$\Pi_3 = k^a \rho^b c_p^c L^d V^1 \quad \text{yields} \quad \Pi_3 = \frac{\rho L c_p V}{k}$$

The final desired dimensionless function is

$$\frac{hL}{k} = fcn\left(\frac{\mu c_p}{k}, \frac{\rho L c_p V}{k}\right) \qquad \textit{Ans.}$$

In words, the Nusselt number is a function of Prandtl number and Peclet number.

5.15 The wall shear stress τ_w in a boundary layer is assumed to be a function of stream velocity U, boundary layer thickness δ, local turbulence velocity u', density ρ, and local pressure gradient dp/dx. Using (ρ, U, δ) as repeating variables, rewrite this relationship as a dimensionless function.

Solution: The relevant dimensions are $\{\tau_w\} = \{ML^{-1}T^{-2}\}$, $\{U\} = \{LT^{-1}\}$, $\{\delta\} = \{L\}$, $\{u'\} = \{LT^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{dp/dx\} = \{ML^{-2}T^{-2}\}$. With $n = 6$ and $j = 3$, we expect $n - j = k = 3$ pi groups:

$$\Pi_1 = \rho^a U^b \delta^c \tau_w = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT^2}\right\} = M^0 L^0 T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 0$$

$$\Pi_2 = \rho^a U^b \delta^c u' = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{L}{T}\right\} = M^0 L^0 T^0, \quad solve\ \ a = 0,\ b = -1,\ c = 0$$

$$\Pi_3 = \rho^a U^b \delta^c \frac{dp}{dx} = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{L^2T^2}\right\} = M^0 L^0 T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 1$$

The final dimensionless function then is given by:

$$\Pi_1 = fcn(\Pi_2, \Pi_3), \quad \text{or:} \quad \boldsymbol{\frac{\tau_w}{\rho U^2} = fcn\left(\frac{u'}{U}, \frac{dp}{dx}\frac{\delta}{\rho U^2}\right)} \quad \textit{Ans.}$$

5.16 Convection heat-transfer data are often reported as a *heat-transfer coefficient h*, defined by

$$\dot{Q} = h A \Delta T$$

where $\dot{Q}$ = heat flow, J/s
A = surface area, m^2
ΔT = temperature difference, K

The dimensionless form of *h*, called the *Stanton number*, is a combination of *h*, fluid density ρ, specific heat c_p, and flow velocity V. Derive the Stanton number if it is proportional to *h*. What are the units of *h*?

Solution: If $\{\dot{Q}\} = \{hA\Delta T\}$, then $\left\{\dfrac{ML^2}{T^3}\right\} = \{h\}\{L^2\}\{\Theta\}$, or: $\{h\} = \left\{\dfrac{M}{\Theta T^3}\right\}$

$$\text{Then }\{\text{Stanton No.}\} = \{h^1\rho^b c_p{}^c V^d\} = \left\{\frac{M}{\Theta T^3}\right\}\left\{\frac{M}{L^3}\right\}^b \left\{\frac{L^2}{T^2\Theta}\right\}^c \left\{\frac{L}{T}\right\}^d = M^0L^0T^0\Theta^0$$

$$\text{Solve for } b = -1,\ c = -1, \text{ and } d = -1.$$

$$\text{Thus, finally, Stanton Number} = h\rho^{-1}c_p{}^{-1}V^{-1} = \boldsymbol{\frac{h}{\rho V c_p}} \quad \textit{Ans.}$$

P5.17 If you disturb a tank of length *L* and water depth *h*, the surface will oscillate back and forth at frequency Ω, assumed here to depend also upon water density ρ and the acceleration of gravity *g*. (*a*) Rewrite this as a dimensionless function. (*b*) If a tank of water sloshes at 2.0 Hz on earth, how fast would it oscillate on Mars ($g \approx 3.7$ m/s^2)?

Solution: Write out the dimensions of the five variables. We hardly even need Table 5.1:

Ω	h	L	ρ	g
$\{T^{-1}\}$	$\{L\}$	$\{L\}$	$\{ML^{-3}\}$	$\{LT^{-2}\}$

(*a*) There are five variables and three dimensions {MLT}, hence we expect two pi groups.

The writer thinks Ω should be correlated versus *h*, so he chooses (*L*, ρ, *g*) as repeating variables:

$$\Pi_1 = L^a \rho^b g^c \Omega^1 \quad \text{yields} \quad \Pi_1 = \Omega\sqrt{\frac{L}{g}}$$

$$\Pi_2 = L^a \rho^b g^c h^1 \quad \text{yields} \quad \Pi_2 = \frac{h}{L}$$

$$\text{Thus} \quad \Omega\sqrt{\frac{L}{g}} = fcn\left(\frac{h}{L}\right) \qquad Ans.(a)$$

Note that density ***drops out***, being the only variable containing mass {M}. If the tank sloshes on earth at 2.0 Hz, that sets the value of Π_1, which we use on Mars to get Ω_{Mars} at the same h/L.

$$\Omega_{earth}\sqrt{\frac{L}{g_{earth}}} = (2.0\,s^{-1})\sqrt{\frac{L}{9.81\,m/s^2}} = 0.639\,m^{-1/2}\sqrt{L} = \Omega_{Mars}\sqrt{\frac{L}{g_{Mars}}} = \Omega_{Mars}\sqrt{\frac{L}{3.7\,m/s^2}}$$

$$\text{Solve for} \quad \Omega_{Mars} \approx \mathbf{1.23}\ Hz \qquad Ans.(b)$$

5.18 Under laminar conditions, the volume flow Q through a small triangular-section pore of side length b and length L is a function of viscosity μ, pressure drop per unit length $\Delta p/L$, and b. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size b is doubled?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccc} Q & = \text{fcn}(\Delta p/L, & \mu, & b) \\ \{L^3/T\} & \{M/L^2T^2\} & \{M/LT\} & \{L\} \end{array}$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ Pi group, found as follows:

$$\Pi_1 = (\Delta p/L)^a(\mu)^b(b)^cQ^1 = \{M/L^2T^2\}^a\{M/LT\}^b\{L\}^c\{L^3/T\}^1 = M^0L^0T^0$$

$$M:\ a + b = 0; \quad L:\ -2a - b + c + 3 = 0; \quad T:\ -2a - b - 1 = 0,$$

$$\text{solve} \quad a = -1,\ b = +1,\ c = -4$$

$$\Pi_1 = \frac{\mathbf{Q\mu}}{\mathbf{(\Delta p/L)b^4}} = \textbf{constant} \qquad Ans.$$

Clearly, if b is doubled, the flow rate Q increases by a factor of $2^4 =$ **16**. *Ans.*

5.19 The period of oscillation T of a water surface wave is assumed to be a function of density ρ, wavelength λ, depth h, gravity g, and surface tension Y. Rewrite this relationship in dimensionless form. What results if Y is negligible?

Solution: Establish the variables and their dimensions:

$$\begin{array}{lccccc} T = \text{fcn}(& \rho & , \lambda & , h & , g & , Y \;) \\ \{T\} & \{M/L^3\} & \{L\} & \{L\} & \{L/T^2\} & \{M/T^2\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 =$ **3** Pi groups, capable of various arrangements and selected by the writer as follows:

$$\text{Typical final result:}\quad \mathbf{T(g/\lambda)^{1/2} = fcn\left(\frac{h}{\lambda}, \frac{Y}{\rho g \lambda^2}\right)} \quad \textit{Ans.}$$

$$\text{If Y is negligible, } \rho \text{ drops out also, leaving:}\quad \mathbf{T(g/\lambda)^{1/2} = fcn\left(\frac{h}{\lambda}\right)} \quad \textit{Ans.}$$

P5.20 A fixed cylinder of diameter D and length L, immersed in a stream flowing normal to its axis at velocity U, will experience zero average lift. However, if the cylinder is rotating at angular velocity Ω, a lift force F will arise. The fluid density ρ is important, but viscosity is secondary and can be neglected. Formulate this lift behavior as a dimensionless function.

Solution: No suggestion was given for the repeating variables, but for this type of problem (force coefficient, lift coefficient), we normally choose (ρ, U, D) for the task. List the dimensions:

D	L	U	Ω	F	ρ
$\{L\}$	$\{L\}$	$\{LT^{-1}\}$	$\{T^{-1}\}$	$\{MLT^{-2}\}$	$\{ML^{-3}\}$

There are three dimensions (MLT), which we knew when we chose (ρ, U, D). Combining these three, separately, with F, Ω, and L, we find this dimensionless function:

$$\frac{F}{\rho U^2 D^2} = fcn\left(\frac{\Omega D}{U}, \frac{L}{D}\right) \qquad \textit{Ans.}$$

This is a correct solution for Chapter 5, but in Chapter 8 we will use the "official" function, with extra factors of (1/2):

$$\frac{F}{(1/2)\rho U^2 LD} = fcn\left(\frac{\Omega D}{2U}, \frac{L}{D}\right)$$

5.21 In Example 5.1 we used the pi theorem to develop Eq. (5.2) from Eq. (5.1). Instead of merely listing the primary dimensions of each variable, some workers list the *powers* of each primary dimension for each variable in an array:

$$\begin{array}{c|ccccc|} & F & L & U & \rho & \mu \\ M & 1 & 0 & 0 & 1 & 1 \\ L & 1 & 1 & 1 & -3 & -1 \\ T & -2 & 0 & -1 & 0 & -1 \end{array}$$

This array of exponents is called the *dimensional matrix* for the given function. Show that the *rank* of this matrix (the size of the largest nonzero determinant) is equal to $j = n - k$, the desired reduction between original variables and the pi groups. This is a general property of dimensional matrices, as noted by Buckingham [1].

Solution: The **rank** of a matrix is the size of the largest submatrix within which has a *non-zero determinant*. This means that the constants in that submatrix, when considered as coefficients of algebraic equations, are *linearly independent*. Thus we establish the number of *independent* parameters—adding one more forms a dimensionless group. For the example shown, the rank is **three** (note the very first 3 × 3 determinant on the left has a non-zero determinant). Thus "j" = 3 for the drag force system of variables.

5.22 The angular velocity Ω of a windmill is a function of windmill diameter *D*, wind velocity *V*, air density ρ, windmill height H as compared to atmospheric boundary layer height *L*, and the number of blades *N*: that is, $\Omega = \text{fcn}(D, V, \rho, H/L, N)$. Viscosity effects are negligible. Rewrite this function in terms of dimensionless Pi groups.

$$\Omega = fcn(D, V, \rho, H/L, N)$$

Solution: We have $n = 6$ variables, $j = 3$ dimensions (M, L, T), thus expect $n - j = 3$ Pi groups. Since only ρ has *mass* dimensions, it drops out. After some thought, we realize that *H/L* and *N* are already dimensionless! The desired dimensionless function becomes:

$$\frac{\Omega D}{V} = fcn\left(\frac{H}{L}, N\right) \quad \textit{Ans.}$$

5.23 The period *T* of vibration of a beam is a function of its length *L*, area moment of inertia *I*, modulus of elasticity *E*, density ρ, and Poisson's ratio σ. Rewrite this relation in dimensionless form. What further reduction can we make if *E* and *I* can occur only in the product form *EI*?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} T = \text{fcn}(& L, & I, & E, & \rho, & \sigma\ ) \\ \{T\} & \{L\} & \{L^4\} & \{M/LT^2\} & \{M/L^3\} & \{\text{none}\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by myself as follows: [Note that σ must be a Pi group.]

$$\text{Typical final result: } \frac{T}{L}\sqrt{\frac{E}{\rho}} = \text{fcn}\left(\frac{L^4}{I}, \sigma\right) \quad \textit{Ans.}$$

$$\text{If E and I can only appear together as EI, then } \frac{T}{L^3}\sqrt{\frac{EI}{\rho}} = \text{fcn}(\sigma) \quad \textit{Ans.}$$

5.24 The lift force *F* on a missile is a function of its length *L*, velocity *V*, diameter *D*, angle of attack α, density ρ, viscosity μ, and speed of sound *a* of the air. Write out the dimensional matrix of this function and determine its rank. (See Prob. 5.21 for an explanation of this concept.) Rewrite the function in terms of pi groups.

Solution: Establish the variables and their dimensions:

$$\begin{array}{ccccccccc} F & = \text{fcn}(& L, & V, & D, & \alpha, & \rho, & \mu, & a\ ) \\ \{ML/T^2\} & & \{L\} & \{L/T\} & \{L\} & \{1\} & \{M/L^3\} & \{M/LT\} & \{L/T\} \end{array}$$

Then $n = 8$ and $j = 3$, hence we expect $n - j = 8 - 3 =$ **5** Pi groups. The matrix is

$$\begin{array}{c|cccccccc} & F & L & V & D & \alpha & \rho & \mu & a \\ M: & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ L: & 1 & 1 & 1 & 1 & 0 & -3 & -1 & 1 \\ T: & -2 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \end{array}$$

The rank of this matrix is indeed **three**, hence there are exactly 5 Pi groups. The writer chooses:

Typical final result: $$\frac{\mathbf{F}}{\rho \mathbf{V}^2\mathbf{L}^2} = \mathbf{fcn}\left(\alpha, \frac{\rho \mathbf{VL}}{\mu}, \frac{\mathbf{L}}{\mathbf{D}}, \frac{\mathbf{V}}{\mathbf{a}}\right)$$ *Ans.*

P5.25 The thrust *F* of a propeller is generally thought to be a function of its diameter *D* and angular velocity *Ω*, the forward speed *V*, and the density *ρ* and viscosity *μ* of the fluid. Rewrite this relationship as a dimensionless function.

Solution: Write out the function with the various dimensions underneath:

$$\begin{array}{ccccccc} F & = fcn(& D, & \Omega, & V, & \rho, & \mu\ ) \\ \{ML/T^2\} & & \{L\} & \{1/T\} & \{L/T\} & \{M/L^3\} & \{M/LT\} \end{array}$$

There are 6 variables and 3 primary dimensions (MLT), and we quickly see that $j = 3$, because

(ρ, V, D) cannot form a pi group among themselves. Use the pi theorem to find the three pi's:

$$\Pi_1 = \rho^a V^b D^c F \;;\; \text{Solve for } a=-1, b=-2, c=-2. \quad \text{Thus} \quad \Pi_1 = \frac{F}{\rho V^2 D^2}$$

$$\Pi_2 = \rho^a V^b D^c \Omega \;;\; \text{Solve for } a=0,\; b=-1,\; c=1. \quad \text{Thus} \quad \Pi_2 = \frac{\Omega D}{V}$$

$$\Pi_3 = \rho^a V^b D^c \mu \;;\; \text{Solve for } a=-1, b=-1, c=-1. \quad \text{Thus} \quad \Pi_3 = \frac{\mu}{\rho V D}$$

Thus one of many forms of the final desired dimensionless function is

$$\frac{F}{\rho V^2 D^2} = fcn\left(\frac{\Omega D}{V}, \frac{\mu}{\rho V D}\right) \qquad \textit{Ans.}$$

5.26 A pendulum has an oscillation period T which is assumed to depend upon its length L, bob mass m, angle of swing θ, and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200 g, is tested on earth and found to have a period of 2.04 s when swinging at 20°. (a) What is its period when it swings at 45°? A similarly constructed pendulum, with L = 30 cm and m = 100 g, is to swing on the moon (g = 1.62 m/s^2) at θ = 20°. (b) What will be its period?

Solution: First establish the variables and their dimensions so that we can do the numbers:

$$\begin{array}{cccccc} T & = fcn(L, & m, & g, & \theta) \\ \{T\} & \{L\} & \{M\} & \{L/T^2\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect n – j = 5 – 3 = **2** Pi groups. They are unique:

$$T\sqrt{\frac{g}{L}} = fcn(\theta) \quad \text{(mass drops out for dimensional reasons)}$$

(a) If we change the angle to 45°, this changes Π**2**, hence we lose dynamic similarity and **do not know the new period**. More testing is required. *Ans.* (a)

(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

$$T_1\left(\frac{g_1}{L_1}\right)^{1/2} = (2.04\text{ s})\left(\frac{9.81\text{ m/s}^2}{1.0\text{ m}}\right)^{1/2} = 6.39 = T_2\left(\frac{1.62\text{ m/s}^2}{0.3\text{ m}}\right)^{1/2},$$

or: $\mathbf{T_2 = 2.75\ s}$ *Ans.* (b)

5.27 In studying sand transport by ocean waves, A. Shields in 1936 postulated that the bottom shear stress τ required to move particles depends upon gravity g, particle size d and density ρ_p, and water density ρ and viscosity μ. Rewrite this in terms of dimensionless groups (which led to the *Shields Diagram* in 1936).

Solution: There are six variables (τ, g, d, ρ_p, ρ, μ) and three dimensions (M, L, T), hence we expect $n - j = 6 - 3 =$ **3** Pi groups. The author used (ρ, g, d) as repeating variables:

$$\boldsymbol{\frac{\tau}{\rho g d} = fcn\left(\frac{\rho g^{1/2} d^{3/2}}{\mu}, \frac{\rho_p}{\rho}\right)} \quad \textit{Ans.}$$

The shear parameter used by Shields himself was based on *net* weight: $\boldsymbol{\tau/[(\rho_p - \rho)gd]}$.

5.28 A simply supported beam of diameter D, length L, and modulus of elasticity E is subjected to a fluid crossflow of velocity V, density ρ, and viscosity μ. Its center deflection δ is assumed to be a function of all these variables. (a) Rewrite this proposed function in dimensionless form. (b) Suppose it is known that δ is independent of μ, inversely proportional to E, and dependent only upon ρV^2, not ρ and V separately. Simplify the dimensionless function accordingly.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccccc} \delta & = \text{fcn}(& \rho & , \text{D} , & \text{L} , & \text{E} , & \text{V} , & \mu \;) \\ \{L\} & & \{M/L^3\} & \{L\} & \{L\} & \{M/LT^2\} & \{L/T\} & \{M/LT\} \end{array}$$

Then $n = 7$ and $j = 3$, hence we expect $n - j = 7 - 3 =$ **4** Pi groups, capable of various arrangements and selected by the writer, as follows (a):

$$\text{Well-posed final result:} \quad \boldsymbol{\frac{\delta}{L} = fcn\left(\frac{L}{D}, \frac{\rho VD}{\mu}, \frac{E}{\rho V^2}\right)} \quad \textit{Ans. (a)}$$

(b) If μ is unimportant and δ proportional to E^{-1}, then the Reynolds number ($\rho VD/\mu$) drops out, and we have already cleverly combined E with ρV^2, which we can now slip out and turn upside down:

$$\text{If } \mu \text{ drops out and } \quad \delta \propto \frac{1}{E}, \quad \text{then } \frac{\delta}{L} = \frac{\rho V^2}{E}\text{fcn}\left(\frac{L}{D}\right),$$

$$\text{or:} \quad \boldsymbol{\frac{\delta E}{\rho V^2 L} = fcn\left(\frac{L}{D}\right)} \quad \textit{Ans. (b)}$$

5.29 When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to turbulence at a time t_{tr} which depends upon the pipe diameter D, fluid acceleration a, density ρ, and viscosity μ. Arrange this into a dimensionless relation between t_{tr} and D.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} t_{tr} & = \text{fcn}(& \rho, & D, & a, & \mu) \\ \{T\} & & \{M/L^3\} & \{L\} & \{L/T^2\} & \{M/LT\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 =$ **2** Pi groups, capable of various arrangements and selected by the writer, as required, to isolate t_{tr} versus D:

$$\mathbf{t_{tr}\left(\frac{\rho a^2}{\mu}\right)^{1/3} = fcn\left[D\left(\frac{\rho^2 a}{\mu^2}\right)^{1/3}\right]} \quad \textit{Ans.}$$

P5.30 When a large tank of high-pressure ideal gas discharges through a nozzle, the maximum exit mass flow $\dot{m}$ is a function of tank pressure p_o and temperature T_o, gas constant R, specific heat c_p, and nozzle diameter D. Rewrite this as a dimensionless function. Check to see if you can use (p_o, T_o, R, D) as repeating variables.

Solution: Using Table 5.1, write out the dimensions of the six variables:

$$\begin{array}{cccccc} \dot{m} & p_o & T_o & R & D & c_p \\ \{MT^{-1}\} & \{ML^{-1}T^{-2}\} & \{\Theta\} & \{L^2T^{-2}\Theta^{-1}\} & \{L\} & \{L^2T^{-2}\Theta^{-1}\} \end{array}$$

By inspection, we see that (p_o, T_o, R, D) are indeed good repeating variables. There are two pi groups:

$$\Pi_1 = p_o^a\, T_o^b\, R^c\, c_p^d\, \dot{m}^1 \quad \text{yields} \quad \Pi_1 = \frac{\dot{m}\sqrt{RT_o}}{p_o\, D^2}$$

$$\Pi_2 = p_o^a\, T_o^b\, R^c\, c_p^d\, c_p^1 \quad \text{yields} \quad \Pi_1 = \frac{c_p}{R}$$

$$\text{Thus} \quad \frac{\dot{m}\sqrt{RT_o}}{p_o\, D^2} = fcn(\frac{c_p}{R}) \qquad \textit{Ans.}$$

The group (c_p/R) = $k/(k$-1), where $k = c_p/c_v$. We usually write the right hand side as $fcn(k)$.

5.31 The heat-transfer rate per unit area q to a body from a fluid in natural or gravitational convection is a function of the temperature difference ΔT, gravity g, body length L, and three fluid properties: kinematic viscosity ν, conductivity k, and thermal expansion coefficient β. Rewrite in dimensionless form if it is known that g and β appear only as the product $g\beta$.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccccc} q & = \text{fcn}(& \Delta T, & g, & L, & \nu, & \beta, & k\ ) \\ \{M/T^3\} & & \{\Theta\} & \{L/T^2\} & \{L\} & \{L^2/T\} & \{1/\Theta\} & \{ML/\Theta T^3\} \end{array}$$

Then $n = 7$ and $j = 4$, hence we expect $n - j = 7 - 4 =$ **3** Pi groups, capable of various arrangements and selected by myself, as follows:

$$\text{If } \beta \text{ and } \Delta T \text{ kept separate, then} \quad \frac{qL}{k\Delta T} = \text{fcn}\left(\beta\Delta T, \frac{gL^3}{\nu^2}\right)$$

If, in fact, β and g must appear together, then $\Pi 2$ and $\Pi 3$ above combine and we get

$$\underbrace{\mathbf{\frac{qL}{k\Delta T}}}_{\text{Nusselt No.}} = \mathbf{fcn}\underbrace{\left(\mathbf{\frac{\beta\Delta T g L^3}{\nu^2}}\right)}_{\text{Grashof Number}} \quad \textit{Ans.}$$

5.32 A *weir* is an obstruction in a channel flow which can be calibrated to measure the flow rate, as in Fig. P5.32. The volume flow Q varies with gravity g, weir width b into the paper, and upstream water height H above the weir crest. If it is known that Q is proportional to b, use the pi theorem to find a unique functional relationship $Q(g, b, H)$.

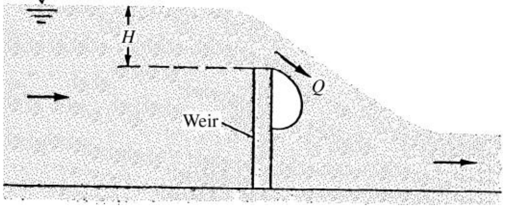


Fig. P5.32

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccc} Q & = \text{fcn}(\; g \;, & b \;, & H\;) \\ \{L^3/T\} & \{L/T^2\} & \{L\} & \{L\} \end{array}$$

Then $n = 4$ and $j = 2$, hence we expect $n - j = 4 - 2 = \mathbf{2}$ Pi groups, capable of various arrangements and selected by myself, as follows:

$$\frac{Q}{g^{1/2}H^{5/2}} = \text{fcn}\left(\frac{b}{H}\right); \quad \text{but if } Q \propto b, \text{ then we reduce to } \quad \mathbf{\frac{Q}{bg^{1/2}H^{3/2}} = constant} \quad \textit{Ans.}$$

5.33 A spar buoy (see Prob. 2.113) has a period *T* of vertical (heave) oscillation which depends upon the waterline cross-sectional area *A*, buoy mass *m*, and fluid specific weight γ. How does the period change due to doubling of (a) the mass and (b) the area? Instrument buoys should have long periods to avoid wave resonance. Sketch a possible long-period buoy design.

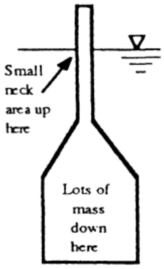


Fig. P5.33

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccc} T & = \text{fcn}(\; A \;, & m \;, & \gamma \;) \\ \{T\} & \{L^2\} & \{M\} & \{M/L^2T^2\} \end{array}$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ single Pi group, as follows:

$$\mathbf{T\sqrt{\frac{A\gamma}{m}} = dimensionless\ constant} \quad \textit{Ans.}$$

Since we can't do anything about γ, the specific weight of water, we *can* increase period T by increasing buoy mass m and decreasing waterline area A. See the illustrative long-period buoy in Figure P5.33 above.

5.34 To good approximation, the thermal conductivity k of a gas (see Ref. 21 of Chap. 1) depends only on the density ρ, mean free path ℓ, gas constant R, and absolute temperature T. For air at 20°C and 1 atm, $k \approx 0.026$ W/m·K and $\ell \approx 6.5E{-}8$ m. Use this information to determine k for hydrogen at 20°C and 1 atm if $\ell \approx 1.2E{-}7$ m.

Solution: First establish the variables and their dimensions and then form a pi group:

$$\begin{array}{cccccc} k & = \text{fcn}(& \rho\ , & \ell\ , & R\ , & T\ ) \\ \{ML/\Theta T^3\} & & \{M/L^3\} & \{L\} & \{L^2/T^2\Theta\} & \{\Theta\} \end{array}$$

Thus $n = 5$ and $j = 4$, and we expect $n - j = 5 - 4 = \mathbf{1}$ single pi group, and the result is

$$k/(\rho R^{3/2} T^{1/2} \ell) = \text{a dimensionless constant} = \Pi_1$$

The value of Π_1 is found from the air data, where $\rho = 1.205$ kg/m^3 and R = 287 m^2/s^2·K:

$$\Pi_{1,air} = \frac{0.026}{(1.205)(287)^{3/2}(293)^{1/2}(6.5E{-}8)} = 3.99 = \Pi_{1,hydrogen}$$

For hydrogen at 20°C and 1 atm, calculate $\rho = 0.0839$ kg/m^3 with $R = 4124$ m^2/s^2·K. Then

$$\Pi_1 = 3.99 = \frac{k_{hydrogen}}{(0.0839)(4124)^{3/2}(293)^{1/2}(1.2E{-}7)}, \quad \textit{solve for}\ k_{hydrogen} = \mathbf{0.182\ \frac{W}{m{\cdot}K}} \quad \textit{Ans.}$$

This is slightly larger than the accepted value for hydrogen of $k \approx 0.178$ W/m·K.

5.35 The torque *M* required to turn the cone-plate viscometer in Fig. P5.35 depends upon the radius *R*, rotation rate Ω, fluid viscosity *μ*, and cone angle *θ*. Rewrite this relation in dimensionless form. How does the relation simplify if it is known that *M* is proportional to *θ*?

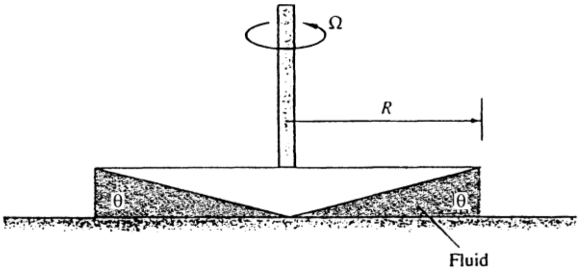


Fig. P5.35

Solution: Establish the variables and their dimensions:

$$\underset{\{ML^2/T^2\}}{M} = \text{fcn}(\underset{\{L\}}{R}, \underset{\{1/T\}}{\Omega}, \underset{\{M/LT\}}{\mu}, \underset{\{1\}}{\theta})$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups, capable of only one reasonable arrangement, as follows:

$$\frac{\mathbf{M}}{\mu\mathbf{\Omega R}^3} = \mathbf{fcn}(\theta); \quad \text{if } M \propto \theta, \quad \text{then } \frac{\mathbf{M}}{\mu\mathbf{\Omega}\theta\mathbf{R}^3} = \mathbf{constant} \quad \textit{Ans.}$$

See Prob. 1.56 of this Manual, for an analytical solution.

5.36 The rate of heat loss, Q_{loss} through a window is a function of the temperature difference ΔT, the surface area *A*, and the *R* resistance value of the window (in units of $ft^2\cdot hr\cdot °F/Btu$): $Q_{loss} = \text{fcn}(\Delta T, A, R)$. (a) Rewrite in dimensionless form. (b) If the temperature difference doubles, how does the heat loss change?

Solution: First figure out the dimensions of *R*: $\{R\} = \{T^3\Theta/M\}$. Then note that $n = 4$ variables and $j = 3$ dimensions, hence we expect only 4 – 3 = *one* Pi group, and it is:

$$\Pi_1 = \frac{Q_{loss}R}{A\,\Delta T} = Const, \quad or: \quad \boldsymbol{Q_{loss} = Const\frac{A\,\Delta T}{R}} \quad \textit{Ans.}\ \text{(a)}$$

(b) Clearly (to me), $Q \propto \Delta T$: **if ΔT doubles, Q_{loss} also doubles**. *Ans.* (b)

P5.37 The volume flow *Q* through an orifice plate is a function of pipe diameter *D*, pressure drop Δ*p* across the orifice, fluid density *ρ* and viscosity *μ*, and orifice diameter *d*. Using *D*, *ρ*, and Δ*p* as repeating variables, express this relationship in dimensionless form.

Solution: There are 6 variables and 3 primary dimensions (MLT), and we already know that

j = 3, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$$\Pi_1 = D^a \rho^b \Delta p^c Q \;;\; \text{Solve for } a = -2,\, b = 1/2,\, c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{Q\rho^{1/2}}{D^2\,\Delta p^{1/2}}$$

$$\Pi_2 = D^a \rho^b \Delta p^c d \;;\; \text{Solve for } a = -1 \;\; b = 0 \;\; c = 0. \quad \text{Thus} \quad \Pi_1 = \frac{d}{D}$$

$$\Pi_3 = D^a \rho^b \Delta p^c \mu \;;\; \text{Solve for } a = -1,\, b = -1/2,\, c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{\mu}{D\rho^{1/2}\Delta p^{1/2}}$$

The final requested orifice-flow function (see Sec. 6.12 later for a different form) is:

$$\frac{Q\rho^{1/2}}{D^2\,\Delta p^{1/2}} = fcn\left(\frac{d}{D},\ \frac{\mu}{D\rho^{1/2}\Delta p^{1/2}}\right) \qquad \textit{Ans.}$$

5.38 The size d of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter D, jet velocity U, and the properties of the liquid ρ, μ, and Y. Rewrite this relation in dimensionless form. *Hint*: Take D, ρ, and U as repeating variables.

Solution: Establish the variables and their dimensions:

$$\begin{array}{ccccccc} d & = \text{fcn}(& D\,, & U\,, & \rho\,, & \mu\,, & Y\;) \\ \{L\} & & \{L\} & \{L/T\} & \{M/L^3\} & \{M/LT\} & \{M/T^2\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect n – j = 6 – 3 = **3** Pi groups, capable of various arrangements and selected by the writer, as follows:

$$\text{Typical final result:}\quad \mathbf{\frac{d}{D} = fcn\left(\frac{\rho UD}{\mu},\ \frac{\rho U^2 D}{Y}\right)}\quad \textit{Ans.}$$

P5.39 Consider natural convection in a rotating, fluid-filled enclosure. The average wall shear stress τ in the enclosure is assumed to be a function of rotation rate Ω, enclosure height H, density ρ, temperature difference ΔT, viscosity μ, and thermal expansion coefficient β. (*a*) Rewrite this relationship as a dimensionless function. (*b*) Do you see a severe *flaw* in the analysis?

Solution: (*a*) Using Table 5.1, write out the dimensions of the seven variables:

τ	ρ	H	Ω	μ	β	ΔT
$\{ML^{-1}T^{-2}\}$	$\{ML^{-3}\}$	$\{L\}$	$\{T^{-1}\}$	$\{ML^{-1}T^{-1}\}$	$\{\Theta^{-1}\}$	$\{\Theta\}$

There are four primary dimensions (MLTQ), and we can easily find four variables (ρ, H, Ω, β) that do not form a pi group. Therefore we expect 7-4 = 3 dimensionless groups. Adding each remaining variable in turn, we find three nice pi groups:

$$\Pi_1 = \rho^a H^b \Omega^c \beta^d \tau \quad \text{leads to} \quad \Pi_1 = \frac{\tau}{\rho H^2 \Omega^2}$$

$$\Pi_2 = \rho^a H^b \Omega^c \beta^d \mu \quad \text{leads to} \quad \Pi_2 = \frac{\mu}{\rho H^2 \Omega}$$

$$\Pi_3 = \rho^a H^b \Omega^c \beta^d \Delta T \quad \text{leads to} \quad \Pi_3 = \beta \Delta T$$

Thus one very nice arrangement of the desired dimensionless function is

$$\frac{\tau}{\rho H^2 \Omega^2} = fcn\left(\frac{\mu}{\rho H^2 \Omega},\ \beta \Delta T\right) \qquad \textit{Ans.}(a)$$

(*b*) This is a good dimensional analysis exercise, ***but in real life it would fail miserably***, because natural convection is highly dependent upon the acceleration of gravity, *g*, which we left out by mistake.

P5.40 The time t_d to drain a liquid from a hole in the bottom of a tank is a function of the hole diameter *d*, the initial fluid volume υ_o, the initial liquid depth *h*o, and the density ρ and viscosity μ of the fluid. Rewrite this relation as a dimensionless function, using Ipsen's method.

Solution: As asked, use *Ipsen's method*. Write out the function with the dimensions beneath:

$$\begin{array}{cccccccccccc} t_d & = & fcn(& d & , & \upsilon_o & , & h_o & , & \rho & , & \mu\ ) \\ \{T\} & & & \{L\} & & \{L^3\} & & \{L\} & & \{M/L^3\} & & \{M/LT\} \end{array}$$

Eliminate the dimensions by multiplication or division. Divide by μ to eliminate {M}:

$$\begin{array}{cccccccccccc} t_d & = & fcn(& d & , & \upsilon_o & , & h_o & , & \dfrac{\rho}{\mu} & , & \cancel{\mu}\ ) \\ \{T\} & & & \{L\} & & \{L^3\} & & \{L\} & & \{T/L^2\} & & \end{array}$$

Recall Ipsen's rules: Only divide into variables containing mass, in this case only ρ. Now eliminate {T}. Again only one division is necessary:

$$\begin{array}{cccccccccc} \dfrac{t_d\,\mu}{\rho} & = & fcn(& d & , & \upsilon_o & , & h_o & , & \cancel{\dfrac{\rho}{\mu}}\ ) \\ \{L^2\} & & & \{L\} & & \{L^3\} & & \{L\} & & \end{array}$$

Finally, eliminate {L} by dividing by appropriate powers of *d*. This completes our task when we discard *d*: itself:

$$\underset{\{1\}}{\frac{t_d \mu}{\rho d^2}} = fcn\left(\underset{\{1\}}{\frac{\upsilon_o}{d^3}},\ \underset{\{1\}}{\frac{h_o}{d}} \right) \qquad \textit{Ans.}$$

Just divide out the dimensions, don't worry about *j* or selecting repeating variables. Of course, the Pi Theorem would give the same, or comparable, results.

5.41 A certain axial-flow turbine has an output torque *M* which is proportional to the volume flow rate *Q* and also depends upon the density ρ, rotor diameter *D*, and rotation rate Ω. How does the torque change due to a doubling of (a) *D* and (b) Ω?

Solution: List the variables and their dimensions, one of which can be **M/Q**, since M is stated to be proportional to Q:

$$\underset{\{M/LT\}}{M/Q} = \text{fcn}(\underset{\{L\}}{D},\ \underset{\{M/L^3\}}{\rho},\ \underset{\{1/T\}}{\Omega})$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ single Pi group:

$$\frac{M/Q}{\rho\Omega D^2} = \text{dimensionless constant}$$

(a) If turbine diameter D is doubled, the torque M increases by a factor of **4**. *Ans.* (a)
(b) If turbine speed Ω is doubled, the torque M increases by a factor of **2**. *Ans.* (b)

P5.42 When disturbed, a floating buoy will bob up and down at frequency *f*. Assume that this frequency varies with buoy mass *m* and waterline diameter *d* and with the specific weight γ of the liquid. (*a*) Express this as a dimensionless function. (*b*) If d and γ are constant and the buoy mass is halved, how will the frequency change?

Solution: The proposed function is $f = fcn(m, d, \gamma)$. Write out their dimensions:

$$\{f\} = \{T^{-1}\} \quad ; \quad \{m\} = \{M\} \quad ; \quad \{d\} = \{L\} \quad ; \quad \{\gamma\} = \{ML^{-2}T^{-2}\}$$

There are four variables and $j = 3$. Hence we expect only *one* Pi group. We find that

$$\Pi_1 = \frac{f}{d}\sqrt{\frac{m}{\gamma}} = \text{constant} \qquad Ans.(a)$$

Hence, for these simplifying assumptions, f is proportional to $m^{-1/2}$. If m halves, f rises by a factor $(0.5)^{-1/2} = 1.414$. In other words, halving m increases f by about 41%. *Ans.*(*b*)

5.43 Non-dimensionalize the thermal energy partial differential equation (4.75) and its boundary conditions (4.62), (4.63), and (4.70) by defining dimensionless temperature $T^* = T/T_o$, where To is the fluid inlet temperature, assumed constant. Use other dimensionless variables as needed from Eqs. (5.23). Isolate all dimensionless parameters which you find, and relate them to the list given in Table 5.2.

Solution: Recall the previously defined variables in addition to T^*:

$$u^* = \frac{u}{U}; \quad x^* = \frac{x}{L}; \quad t^* = \frac{Ut}{L}; \quad \text{similarly,} \quad v^* \text{ or } w^* = \frac{v \text{ or } w}{U}; \quad y^* \text{ or } z^* = \frac{y \text{ or } z}{L}$$

Then the dimensionless versions of Eqs. (4.75, 62, 63, 70) result as follows:

$$\textbf{(4.75):} \quad \frac{dT^*}{dt^*} = \underbrace{\left(\frac{k}{\rho c_p UL}\right)}_{\text{1/Peclet Number}} \nabla^{*2}T^* + \underbrace{\left(\frac{\mu U}{\rho c_p T_o L}\right)}_{\text{Eckert Number divided by Reynolds Number}} \Phi^*$$

5.44 The differential energy equation for incompressible two-dimensional flow through a "Darcy-type" porous medium is approximately

$$\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} + \rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial y}\frac{\partial T}{\partial y} + k\frac{\partial^2 T}{\partial y^2} = 0$$

where σ is the *permeability* of the porous medium. All other symbols have their usual meanings. (a) What are the appropriate dimensions for σ? (b) Nondimensionalize this equation, using (L, U, ρ, T_o) as scaling constants, and discuss any dimensionless parameters which arise.

Solution: (a) The only way to establish $\{\sigma\}$ is by comparing two terms in the PDE:

$$\left\{\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}\right\} = \left\{k\frac{\partial^2 T}{\partial x^2}\right\}, \quad \text{or:} \quad \left\{\frac{M}{L^3T^3}\right\}\{\sigma\} \overset{?}{=} \left\{\frac{M}{LT^3}\right\},$$

$$\text{Thus} \quad \{\sigma\} = \{\mathbf{L^2}\} \quad \textit{Ans.}\text{ (a)}$$

(b) Define dimensionless variables using the stated list of (L, U, ρ, T_o) for scaling:

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad p^* = \frac{p}{\rho U^2}; \quad T^* = \frac{T}{T_o}$$

Substitution into the basic PDE above yields only a *single* dimensionless parameter:

$$\zeta\left(\frac{\partial p^*}{\partial x^*}\frac{\partial T^*}{\partial x^*} + \frac{\partial p^*}{\partial y^*}\frac{\partial T^*}{\partial y^*}\right) + \frac{\partial^2 T^*}{\partial y^{*2}} = 0, \quad \text{where } \boldsymbol{\zeta = \frac{\rho^2 c_p U^2 \sigma}{\mu k}} \quad \textit{Ans.}\text{ (b)}$$

I don't know the name of this parameter. It is related to the "Darcy-Rayleigh" number.

5.45 A model differential equation, for chemical reaction dynamics in a plug reactor, is as follows:

$$u\frac{\partial C}{\partial x} = \mathrm{D}\frac{\partial^2 C}{\partial x^2} - kC - \frac{\partial C}{\partial t}$$

where u is the velocity, Δ is a diffusion coefficient, k is a reaction rate, x is distance along the reactor, and C is the (dimensionless) concentration of a given chemical in the reactor. (a) Determine the appropriate dimensions of Δ and k. (b) Using a characteristic length scale L and average velocity V as parameters, rewrite this equation in dimensionless form and comment on any Pi groups appearing.

Solution: (a) Since all terms in the equation contain C, we establish the dimensions of k and Δ by comparing $\{k\}$ and $\{\Delta\partial^2/\partial x^2\}$ to $\{u\partial/\partial x\}$:

$$\{k\} = \{D\}\left\{\frac{\partial^2}{\partial x^2}\right\} = \{D\}\left\{\frac{1}{L^2}\right\} = \{u\}\left\{\frac{\partial}{\partial x}\right\} = \left\{\frac{L}{T}\right\}\left\{\frac{1}{L}\right\},$$

$$\text{hence}\quad \boldsymbol{\{k\} = \left\{\frac{1}{T}\right\}} \quad\text{and}\quad \boldsymbol{\{D\} = \left\{\frac{L^2}{T}\right\}} \quad \textit{Ans.}\text{ (a)}$$

(b) To non-dimensionalize the equation, define $u^* = u/V$, $t^* = Vt/L$, and $x^* = x/L$ and substitute into the basic partial differential equation. The dimensionless result is

$$\boldsymbol{u^*\frac{\partial C}{\partial x^*} = \left(\frac{D}{VL}\right)\frac{\partial^2 C}{\partial x^{*2}} - \left(\frac{kL}{V}\right)C - \frac{\partial C}{\partial t^*}},\ \text{where } \frac{VL}{D} = \text{mass-transfer Peclet number} \quad \textit{Ans.}\text{ (b)}$$

P5.46 If a vertical wall at temperature T_w is surrounded by a fluid at temperature T_o, a natural convection boundary layer flow will form. For laminar flow, the momentum equation is

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho\beta(T - T_o)g + \mu\frac{\partial^2 u}{\partial y^2}$$

to be solved, along with continuity and energy, for (u, v, T) with appropriate boundary conditions. The quantity β is the thermal expansion coefficient of the fluid. Use ρ, g, L, and $(T_w - T_o)$ to nondimensionalize this equation. Note that there is no "stream" velocity in this type of flow.

Solution: For the given constants used to define dimensionless variables, there is only one pairing which will give a velocity unit: $(gL)^{1/2}$. Here are the writer's dimensionless variables:

$$u^* = \frac{u}{\sqrt{gL}}\ ;\ v^* = \frac{v}{\sqrt{gL}}\ ;\ x^* = \frac{x}{L}\ ;\ y^* = \frac{y}{L}\ ;\ T^* = \frac{T - T_o}{T_w - T_o}$$

Substitute into the momentum equation above and clean up so all terms are dimensionless:

$$\rho(u^*\frac{\partial u^*}{\partial x^*}\frac{gL}{L}) + \rho(v^*\frac{\partial u^*}{\partial y^*}\frac{gL}{L}) = \rho\beta g(T_w - T_o)T^* + \mu\frac{\partial^2 u^*}{\partial y^{*2}}\frac{\sqrt{gL}}{L^2}$$

$$\text{or:}\qquad u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = [\beta(T_w - T_o)]T^* + [\frac{\mu}{\rho L\sqrt{gL}}]\frac{\partial^2 u^*}{\partial y^{*2}}\qquad \textit{Ans.}$$

There are two dimensionless parameters: $\beta(T_w - T_o)$ and $\mu/[\rho L\sqrt{gL}]$. Neither has a name, to the writer's knowledge, because a much cleverer analysis would result in only a single dimensionless parameter, the *Grashof number*, $g\beta(T_w - T_o)L^3/\nu^2$. (See, for example, White, *Viscous Fluid Flow*, 3rd edition, Section 4-14.3, page 323.)

5.47 The differential equation for small-amplitude vibrations *y*(*x*, *t*) of a simple beam is given by

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus

Use only the quantities ρ, E, and A to nondimensionalize y, x, and t, and rewrite the differential equation in dimensionless form. Do any parameters remain? Could they be removed by further manipulation of the variables?

Solution: The appropriate dimensionless variables are

$$y^* = \frac{y}{\sqrt{A}};\quad t^* = t\sqrt{\frac{E}{\rho A}};\quad x^* = \frac{x}{\sqrt{A}}$$

Substitution into the PDE above yields a dimensionless equation with *one* parameter:

$$\boldsymbol{\frac{\partial^2 y^*}{\partial t^{*2}} + \left(\frac{I}{A^2}\right)\frac{\partial^4 y^*}{\partial x^{*4}} = 0;}\quad \text{One geometric parameter: } \boldsymbol{\frac{I}{A^2}}\quad \textit{Ans.}$$

We could *remove* (I/A^2) completely by redefining $\mathbf{x^* = x/I^{1/4}}$. *Ans.*

5.48 A smooth steel (SG = 7.86) sphere is immersed in a stream of ethanol at 20°C moving at 1.5 m/s. Estimate its drag in N from Fig. 5.3*a*. What stream velocity would quadruple its drag? Take *D* = 2.5 cm.

Solution: For ethanol at 20°C, take $\rho \approx 789\ \text{kg/m}^3$ and $\mu \approx 0.0012$ kg/m·s. Then

$$\text{Re}_D = \frac{\rho UD}{\mu} = \frac{789(1.5)(0.025)}{0.0012} \approx 24700; \quad \text{Read Fig. 5.3(a):} \quad C_{D,\text{sphere}} \approx 0.4$$

$$\text{Compute drag} \quad F = C_D\left(\frac{1}{2}\right)\rho U^2 \frac{\pi}{4} D^2 = (0.4)\left(\frac{1}{2}\right)(789)(1.5)^2\left(\frac{\pi}{4}\right)(0.025)^2$$

$$\approx \mathbf{0.17\ N} \quad \textit{Ans.}$$

Since $C_D \approx$ constant in this range of Re_D, **doubling U quadruples** the drag. *Ans.*

5.49 The sphere in Prob. 5.48 is dropped in gasoline at 20°C. Ignoring its acceleration phase, what will be its terminal (constant) fall velocity, from Fig. 5.3*a*?

Solution: For gasoline at 20°C, take $\rho \approx 680\ \text{kg/m}^3$ and $\mu \approx 2.92\text{E}{-}4$ kg/m·s. For steel take $\rho \approx 7800\ \text{kg/m}^3$. Then, in "terminal" velocity, the net weight equals the drag force:

$$\text{Net weight} = (\rho_{\text{steel}} - \rho_{\text{gasoline}})g\frac{\pi}{6}D^3 = \text{Drag force} = C_D\frac{\rho}{2}V^2\frac{\pi}{4}D^2,$$

$$\text{or:} \quad (7800-680)(9.81)\frac{\pi}{6}(0.025)^3 = 0.571\ \text{N} = C_D\left(\frac{1}{2}\right)(680)U^2\frac{\pi}{4}(0.025)^2$$

$$\text{Guess} \quad C_D \approx 0.4 \quad \text{and compute} \quad \mathbf{U \approx 2.9\ \frac{m}{s}} \quad \textit{Ans.}$$

Now check $\text{Re}_D = \rho UD/\mu = 680(2.9)(0.025)/(2.92\text{E}{-}4) \approx 170000$. **Yes**, $C_D \approx 0.4$, OK.

P5.50 The parachute in the chapter-opener photo is, of course, meant to decelerate the payload on Mars. The wind tunnel test gave a drag coefficient of about 1.1, based upon the projected area of the parachute. Suppose it was falling on *earth* and, at an altitude of 1000 m, showed a steady descent rate of about 18 mi/h. Estimate the weight of the payload.

Solution: Let's convert everything to metric. The diameter is 55 ft = 16.8 m. Standard air density at 1000 m is 1.112 kg/m^3. Descent velocity is 18 mi/h = 8.05 m/s. Then

$$C_D = 1.1 = \frac{F}{(1/2)\rho V^2(\pi/4)D^2} = \frac{F}{(1/2)(1.112 kg/m^3)(8.05 m/s)^2(\pi/4)(16.8 m)^2}$$

$$\text{Solve for} \quad F = 8740\,N \approx 1970\,lbf \quad \text{(on earth)} \qquad \textit{Ans.}$$

5.51 A ship is towing a sonar array which approximates a submerged cylinder 1 ft in diameter and 30 ft long with its axis normal to the direction of tow. If the tow speed is 12 kn (1 kn = 1.69 ft/s), estimate the horsepower required to tow this cylinder. What will be the frequency of vortices shed from the cylinder? Use Figs. 5.2 and 5.3.

Solution: For seawater at 20°C, take $\rho \approx 1.99$ slug/ft^3 and $\mu \approx 2.23E{-}5$ slug/ft·s. Convert $V = 12$ knots ≈ 20.3 ft/s. Then the Reynolds number and drag of the towed cylinder is

$$\mathrm{Re_D} = \frac{\rho \mathrm{UD}}{\mu} = \frac{1.99(20.3)(1.0)}{2.23\mathrm{E}{-}5} \approx 1.8\mathrm{E}6. \quad \text{Fig. 5.3(a) cylinder:} \quad \text{Read } \mathrm{C_D} \approx 0.3$$

$$\text{Then} \quad \mathrm{F} = \mathrm{C_D}\left(\frac{1}{2}\right)\rho \mathrm{U}^2\mathrm{DL} = (0.3)\left(\frac{1}{2}\right)(1.99)(20.3)^2(1)(30) \approx 3700 \text{ lbf}$$

$$\text{Power} \quad \mathrm{P} = \mathrm{FU} = (3700)(20.3) \div 550 \approx \mathbf{140\ hp} \quad \textit{Ans.}\text{ (a)}$$

Data for cylinder vortex shedding is found from Fig. 5.2*b*. At a Reynolds number ReD ≈ 1.8E6, read fD/U ≈ 0.24. Then

$$f_{shedding} = \frac{StU}{D} = \frac{(0.24)(20.3\, ft/s)}{1.0\, ft} \approx \mathbf{5\ Hz} \quad \textit{Ans.}\text{ (b)}$$

5.52 When fluid in a long pipe starts up from rest at a uniform acceleration *a*, the initial flow is laminar. The flow undergoes transition to turbulence at a time *t** which depends, to first approximation, only upon *a*, ρ, and μ. Experiments by P. J. Lefebvre, on water at 20°C starting from rest with 1-g acceleration in a 3-cm-diameter pipe, showed transition at *t** = 1.02 s. Use this data to estimate (*a*) the transition time, and (*b*) the transition Reynolds number Re_D for water flow accelerating at 35 m/s^2 in a 5-cm-diameter pipe.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and *m* = 0.001 kg/m-s. There are four variables. Write out their dimensions:

$$\begin{array}{cccc} t^* & a & \rho & \mu \\ \{T\} & \{LT^{-2}\} & \{ML^{-3}\} & \{ML^{-1}T^{-1}\} \end{array}$$

There are three primary dimensions, (MLT), hence we expect 4 – 3 = one pi group:

$$\Pi_1 = \rho^a\,\mu^b\,a^c\,t^{*1} \quad \text{yields} \quad \Pi_1 = t^*\left(\frac{\rho a^2}{\mu}\right)^{1/3}, \ or \ t^* = (const)\left(\frac{\mu}{\rho a^2}\right)^{1/3}$$

Use LeFebvre's data point to establish the constant value of Π_1:

$$t^* = 1.02 = (const)\left[\frac{0.001\,kg/m-s}{(998\,kg/m^3)(9.81\,m/s^2)^2}\right]^{1/3} = (const)(0.00218)$$

Thus the constant, or Π_1, equals 1.02/0.00218 = 467 (dimensionless). Use this value to establish the new transition time for *a* = 35 m/s^2 in a 5-cm-diameter pipe:

$$t^* = (467)\left(\frac{\mu}{\rho a^2}\right)^{1/3} = (467)\left[\frac{0.001}{998(35)^2}\right]^{1/3} = \mathbf{0.44}\,s \qquad Ans.(a)$$

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{\rho(aT^*)D}{\mu} = \frac{998[35(0.44)](0.05)}{(0.001)} = \mathbf{768,000} \qquad Ans.(b)$$

This transition Reynolds number is more than 300 times the value for which *steady* laminar pipe flow undergoes transition. The reason is that this is a thin-boundary-layer flow, and the laminar velocity profile never even approaches the Poiseuille parabola.

5.53 Vortex shedding can be used to design a *vortex flowmeter* (Fig. 6.34). A blunt rod stretched across the pipe sheds vortices whose frequency is read by the sensor downstream. Suppose the pipe diameter is 5 cm and the rod is a cylinder of diameter 8 mm. If the sensor reads 5400 counts per minute, estimate the volume flow rate of water in m^3/h. How might the meter react to other liquids?

Solution: 5400 counts/min = 90 Hz = f.

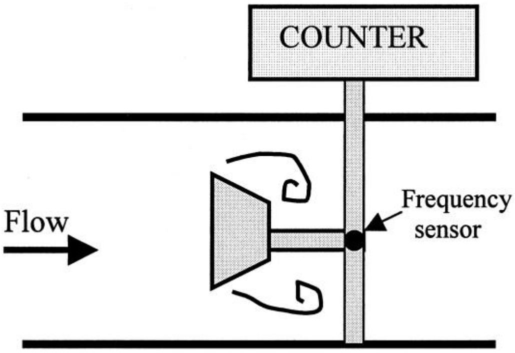


Fig. 6.34

$$\text{Guess}\quad \frac{fD}{U} \approx 0.2 = \frac{90(0.008)}{U}, \quad \text{or} \quad U \approx 3.6\ \frac{m}{s}$$

$$\text{Check}\quad Re_{D,water} = \frac{998(3.6)(0.008)}{0.001} \approx 29000; \quad \text{Fig. 5.2: Read St} \approx 0.2, \quad \text{OK.}$$

If the centerline velocity is 3.6 m/s and the flow is turbulent, then $V_{avg} \approx 0.82V_{center}$ (see Ex. 3.4 of the text). Then the pipe volume flow is approximately:

$$Q = V_{avg}A_{pipe} = (0.82\times 3.6)\frac{\pi}{4}(0.05\ m)^2 \approx 0.0058\ \frac{m^3}{s} \approx \mathbf{21}\ \frac{\mathbf{m^3}}{\mathbf{hr}} \quad \textit{Ans.}$$

5.54 A fishnet is made of 1-mm-diameter strings knotted into 2 × 2 cm squares. Estimate the horsepower required to tow 300 ft^2 of this netting at 3 kn in seawater at 20°C. The net plane is normal to the flow direction.

Solution: For seawater at 20°C, take $\rho \approx 1025\ kg/m^3$ and $\mu \approx 0.00107$ kg/m·s. Convert V = 3 knots = 1.54 m/s. Then, considering the strings as "cylinders in crossflow," the Reynolds number is Re

$$Re_D = \frac{\rho VD}{\mu} = \frac{(1025)(1.54)(0.001)}{0.00107} \approx 1500; \quad \text{Fig. 5.3(a): } C_{D,cyl} \approx 1.0$$

Drag of one 2-cm strand:

$$F = C_D \frac{\rho}{2} V^2 DL = (1.0)\left(\frac{1025}{2}\right)(1.54)^2(0.001)(0.02) \approx 0.0243\ \text{N}$$

Now 1 m^2 of net contains *5000* of these 2-cm strands, and 300 ft^2 = 27.9 m^2 of net contains (5000)(27.9) = *139400* strands total, for a total net force F = 139400(0.0243) ≈ 3390 N ÷ 4.4482 = **762 lbf** on the net. Then the horsepower required to tow the net is

$$\text{Power} = FV = (3390\ \text{N})(1.54\ \text{m/s}) = 5220\ \text{W} \div 746\ W/hp \approx \mathbf{7.0\,hp} \quad \textit{Ans.}$$

5.55 The radio antenna on a car begins to vibrate wildly at **8 Hz** when the car is driven at 45 mi/h over a rutted road which approximates a sine wave of amplitude 2 cm and wavelength λ = 2.5 m. The antenna diameter is 4 mm. Is the vibration due to the road or to vortex shedding?

Solution: Convert U = 45 mi/h = 20.1 m/s. Assume sea level air, ρ = 1.2 kg/m^3, μ = 1.8E−5 kg/m·s. Check the Reynolds number based on antenna diameter: Re_d = (1.2)(20.1)(0.004)/(1.8E−5) = 5400. From Fig. 5.2b, read St ≈ 0.21 = $(\omega/2\pi)d/U$ = (f_{shed})(0.004 m)/(20.1 m/s), or f_{shed} ≈ 1060 Hz ≠ 8 Hz, so **rule out** vortex shedding. Meanwhile, the rutted road introduces a forcing frequency $f_{road} = U/\lambda$ = (20.1 m/s)/(2.5 m) ≈ **8.05 Hz**. We conclude that this resonance is due to ***road roughness***.

5.56 Flow past a long cylinder of square cross-section results in more drag than the comparable round cylinder. Here are data taken in a water tunnel for a square cylinder of side length b = 2 cm:

V, m/s:	1.0	2.0	3.0	4.0
Drag, N/(m of depth):	21	85	191	335

(a) Use this data to predict the drag force per unit depth of wind blowing at 6 m/s, in air at 20°C, over a tall square chimney of side length b = 55 cm. (b) Is there any uncertainty in your estimate?

Solution: Convert the data to the dimensionless form $F/(\rho V^2 bL)$ = fcn($\rho Vb/\mu$), like Eq. (5.2). For air, take ρ = 1.2 kg/m^3 and μ = 1.8E−5 kg/m·s. For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Make a new table using the water data, with L = 1 m:

$F/(\rho V^2 bL)$:	1.05	1.06	1.06	1.05
$\rho Vb/\mu$:	19960	39920	59880	79840

In this Reynolds number range, the force coefficient is approximately constant at about 1.055. Use this value to estimate the air drag on the large chimney:

$$F_{air} = C_F \rho_{air} V_{air}^2 (bL)_{chimney} = (1.055)\left(1.2\ \frac{kg}{m^3}\right)\left(6\ \frac{m}{s}\right)^2 (0.55\ m)(1\ m) \approx \mathbf{25\ N/m} \quad \textit{Ans.}\ \text{(a)}$$

(b) Yes, there **is** uncertainty, because $\mathrm{Re}_{chimney} = 220{,}000 > \mathrm{Re}_{model} = 80{,}000$ or less.

5.57 The simply supported 1040 carbon-steel rod of Fig. P5.57 is subjected to a crossflow stream of ai at 20°C and 1 atm. For what stream velocity *U* will the rod center deflection be approximately 1 cm?

Solution: For air at 20°C, take $\rho \approx 1.2\ \mathrm{kg/m^3}$ and $\mu \approx 1.8\mathrm{E}{-5}\ \mathrm{kg/m\cdot s}$. For carbon steel take Young's modulus E ≈ 29E6 psi ≈ 2.0E11 Pa.

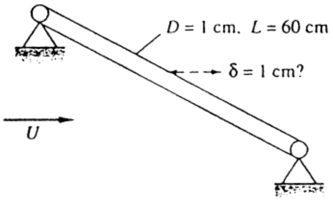


Fig. P5.57

This is not an elasticity course, so just use the formula for center deflection of a simply-supported beam:

$$\delta_{\text{center}} = \frac{FL^3}{48EI} = 0.01\ \mathrm{m} = \frac{F(0.6)^3}{48(2.0\mathrm{E}11)[(\pi/4)(0.005)^4]}, \quad \text{solve for } F \approx 218\ \mathrm{N}$$

$$\text{Guess} \quad C_D \approx 1.2, \quad \text{then } F = 218\ \mathrm{N} = C_D \frac{\rho}{2} V^2 DL = (1.2)\left(\frac{1.2}{2}\right) V^2 (0.01)(0.6)$$

Solve for V ≈ 225 m/s, check $\mathrm{Re}_D = \rho VD/\mu \approx 150{,}000$: OK, $C_D \approx 1.2$ from Fig. 5.3a.
Then **V ≈ 225 m/s**, which is quite high subsonic speed, Mach number ≈ 0.66. *Ans.*

5.58 For the steel rod of Prob. 5.57, at what airstream velocity *U* will the rod begin to vibrate laterally in resonance in its first mode (a half sine wave)? (*Hint:* Consult a vibration text [Ref. 34 or 35] under "lateral beam vibration.")

Solution: From a vibrations book, the first mode frequency for a simply-supported slender beam is given by

$$\omega_n = \pi^2\sqrt{\frac{EI}{mL^4}} \quad \text{where } m = \rho_{steel}\pi R^2 = \text{beam mass per unit length}$$

$$\text{Thus} \quad f_n = \frac{\omega_n}{2\pi} = \frac{\pi}{2}\left[\frac{2.0E11(\pi/4)(0.005)^4}{(7840)\pi(0.005)^2(0.6)^4}\right]^{1/2} \approx 55.1 \text{ Hz}$$

The beam will resonate if its vortex shedding frequency is the same. Guess fD/U ≈ 0.2:

$$St = \frac{fD}{U} \approx 0.2 = \frac{55.1(0.01)}{U}, \quad \text{or} \quad U \approx 2.8 \ \frac{m}{s}$$

Check $Re_D = \rho VD/\mu \approx 1800$. Fig. 5.2, OK, St ≈ 0.2. Then $\mathbf{V \approx 2.8\ \frac{m}{s}}$ *Ans.*

P5.59 A long, slender, 3-cm-diameter smooth flagpole bends alarmingly in 20 mi/h sea-level winds, causing patriotic citizens to gasp. An engineer claims that the pole will bend less if its surface is deliberately roughened. Is she correct, at least qualitatively?

Solution: For sea-level air, take ρ = 1.2255 kg/m^3 and μ = 1.78E-5 kg/m-s. Convert 20 mi/h = 8.94 m/s. Calculate the Reynolds number of the pole as a "cylinder in crossflow":

$$Re_D = \frac{\rho VD}{\mu} = \frac{(1.2255 kg/m^3)(8.94 m/s)(0.03 m)}{1.78E-5 kg/m-s} = 18{,}500$$

From Fig. 5.3*b*, we see that this Reynolds number is *below* the region where roughness is effective in reducing cylinder drag. Therefore we think the engineer is ***incorrect***. *Ans*.

[It is more likely that the drag of the *flag* is causing the problem.]

***P5.60** The thrust F of a free propeller, either aircraft or marine, depends upon density ρ, the rotation rate n in r/s, the diameter D, and the forward velocity V. Viscous effects are slight and neglected here. Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, r/min	4800	6000	8000
Measured thrust, N	6.1	19	47

(*a*) Use this data to make a crude but effective dimensionless plot. (b) Use the dimensionless data to predict the thrust, in newtons, of a similar 1.6-m-diameter prototype propeller when rotating at 3800 r/min and flying at 225 mi/h at 4000 m standard altitude.

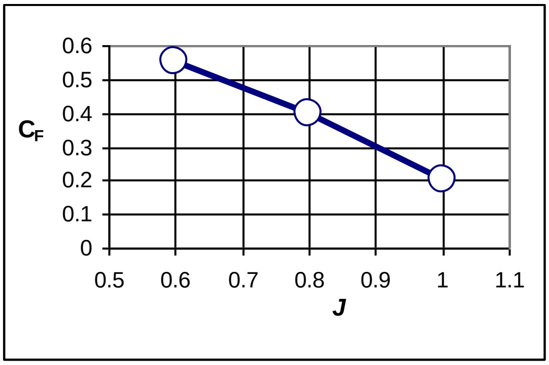
Solution: The given function is $F = \text{fcn}(\rho, n, D, V)$, and we note that $j = 3$. Hence we expect 2 pi groups. The writer chose (ρ, n, D) as repeating variables and found this:

$$C_F = fcn(J)\,, \quad where \quad C_F = \frac{F}{\rho n^2 D^4} \quad and \quad J = \frac{V}{nD}$$

The quantity C_F is called the *thrust coefficient*, while J is called the *advance ratio*. Now use the data (at $\rho = 1.2255$ kg/m^3) to fill out a new table showing the two pi groups:

n, r/s	133.3	100.0	80.0
C_F	0.55	0.40	0.20
J	0.60	0.80	1.00

A crude but effective plot of this data is as follows. *Ans.*(*a*)



(*b*) At 4000 m altitude, from Table A.6, ρ = 0.8191 kg/m^3. Convert 225 mi/h = 101.6 m/s. Convert 3800 r/min = 63.3 r/s. Then find the prototype advance ratio:

$$J = (101.6 \text{ m/s})/[(63.3 \text{ r/s})(1.6 \text{ m})] = 1.00$$

Well, lucky us, that's our third data point! Therefore $C_{F,\text{prototype}} \approx 0.20$. And the thrust is

$$F_{prototype} = C_F \rho n^2 D^4 = (0.20)(0.8191\frac{kg}{m^3})(63.3\frac{r}{s})^2(1.6m)^4 \approx \mathbf{4300\ N} \quad Ans.(b)$$

5.61 If viscosity is neglected, typical pump-flow results are shown in Fig. P5.61 for a model pump tested in water. The pressure rise decreases and the power required increases with the dimensionless flow coef-ficient. Curve-fit expressions are given for the data. Suppose a similar pump of 12-cm diameter is built to move gasoline at 20°C and a flow rate of 25 m^3/h. If the pump rotation speed is 30 r/s, find (a) the pressure rise and (b) the power required.

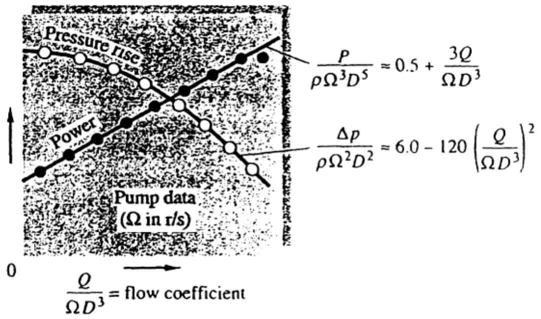


Fig. P5.61

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx 2.92\text{E}{-4}$ kg/m·s. Convert Q = 25 m^3/hr = 0.00694 m^3/s. Then we can evaluate the "flow coefficient":

$$\frac{Q}{\Omega D^3} = \frac{0.00694}{(30)(0.12)^3} \approx 0.134, \quad \text{whence} \quad \frac{\Delta p}{\rho\Omega^2 D^2} \approx 6 - 120(0.134)^2 \approx 3.85$$

$$\text{and} \quad \frac{P}{\rho\Omega^3 D^5} \approx 0.5 + 3(0.134) \approx 0.902$$

With the dimensionless pressure rise and dimensionless power known, we thus find

$$\Delta p = (3.85)(680)(30)^2(0.12)^2 \approx \mathbf{34000\ Pa} \quad \textit{Ans.}\ \text{(a)}$$

$$P = (0.902)(680)(30)^3(0.12)^5 \approx \mathbf{410\ W} \quad \textit{Ans.}\ \text{(b)}$$

P5.62 Extend Prob. P5.30 as follows. Let the maximum mass flow $\dot{m}$ again be a function of tank pressure p_o and temperature T_o, gas constant R, and nozzle diameter D, but replace c_p by the specific heat ratio, k. For an air tank at 190 kPa and 330 K, with a 2-cm nozzle diameter, experiments show a mass flow of 0.133 kg/s. (*a*) Can this data be used to correlate an *oxygen* tank? (*b*) If so, estimate the oxygen mass flow if the tank conditions are 300 kPa and 450 K, with a nozzle diameter of 3 cm.

Solution: Problem P5.30, with c_p replaced by the specific heat ratio, k, led to the function

$$\frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = fcn(k)$$

For air, R = 287 m^2/s^2-K and k = 1.40. The given data point reduces to

$$\frac{\dot{m}\sqrt{RT_o}}{p_o\, D^2} = \frac{(0.133)\sqrt{(287)(330)}}{(190{,}000)(0.02)^2} = 0.539 \quad \text{for} \quad k = 1.40$$

(a) Can we use this air data for oxygen? **Yes**, because k_{oxygen} also equals 1.40. So let's do it. For oxygen, from Table A.4, R = 260 m^2/s^2-K and k = 1.40. The correlation yields

$$\frac{\dot{m}\sqrt{RT_o}}{p_o\, D^2} = \frac{\dot{m}\sqrt{(260)(450)}}{(300{,}000)(0.03)^2} = 0.539, \text{ solve for } \dot{m}_{oxygen} = \mathbf{0.425}\ kg/s \qquad Ans.(b)$$

5.63 The pressure drop per unit length $\Delta p/L$ in smooth pipe flow is known to be a function only of the average velocity V, diameter D, and fluid properties ρ and μ. The following data were obtained for flow of water at 20°C in an 8-cm-diameter pipe 50 m long:

Q, m^3/s	0.005	0.01	0.015	0.020
Δp, Pa	5800	20,300	42,100	70,800

Verify that these data are slightly outside the range of Fig. 5.10. What is a suitable power-law curve fit for the present data? Use these data to estimate the pressure drop for flow of kerosene at 20°C in a smooth pipe of diameter 5 cm and length 200 m if the flow rate is 50 m^3/h.

Solution: For water at 20°C, take $\rho \approx$ 998 kg/m^3 and $\mu \approx$ 0.001 kg/m·s. In the spirit of Fig. 5.10 and Example 5.7 in the text, we generate dimensionless Δp and V:

Q, m^3/s:	0.005	0.010	0.015	0.020
V = Q/A, m/s:	0.995	1.99	2.98	3.98
Re = ρVD/μ:	79400	158900	238300	317700
$\rho D^3\Delta p/(L\mu^2)$:	5.93E7	2.07E8	4.30E8	7.24E8

These data, except for the first point, exceed Re = 1E5 and are thus off to the right of the plot in Fig. 5.10. They could fit a "1.75" Power-law, as in *Ans.* (c) as in Ex. 5.7 of the text, but only to ±4%. They fit a "1.80" power-law much more accurately:

$$\frac{\rho \Delta p D^3}{L\mu^2} \approx 0.0901\left(\frac{\rho V D}{\mu}\right)^{1.80} \pm 1\%$$

For kerosene at 20°C, take $\rho \approx 804$ kg/m^3 and $\mu \approx 1.92\text{E}{-}3$ kg/m·s. The new length is 200 m, the new diameter is 5 cm, and the new flow rate is 50 m^3/hr. Then evaluate Re:

$$V = \frac{50/3600}{(\pi/4)(0.05)^2} \approx 7.07\ \frac{\text{m}}{\text{s}}, \quad \text{and} \quad \text{Re}_D = \frac{\rho V D}{\mu} = \frac{804(7.07)(0.05)}{1.92\text{E–}3} \approx 148100$$

$$\text{Then} \quad \rho \Delta p D^3/(L\mu^2) \approx 0.0901(148100)^{1.80} \approx 1.83\text{E}8 = \frac{(804)\Delta p(0.05)^3}{(200)(1.92\text{E–}3)^2}$$

Solve for **Δp ≈ 1.34E6 Pa** *Ans.*

5.64 The natural frequency *ω* of vibra-tion of a mass *M* attached to a rod, as in Fig. P5.64, depends only upon *M* and the stiffness *EI* and length *L* of the rod. Tests with a 2-kg mass attached to a 1040 carbon-steel rod of diameter 12 mm and length 40 cm reveal a natural frequency of 0.9 Hz. Use these data to predict the natural frequency of a 1-kg mass attached to a 2024 aluminum-alloy rod of the same size.

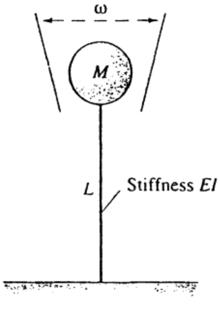


Fig. P5.64

Solution: For steel, E ≈ 29E6 psi ≈ 2.03E11 Pa. If $\omega = f(M, EI, L)$, then $n = 4$ and $j = 3$ (MLT), hence we get only **1** pi group, which we can evaluate from the steel data:

$$\frac{\omega(ML^3)^{1/2}}{(EI)^{1/2}} = \text{constant} = \frac{0.9[(2.0)(0.4)^3]^{1/2}}{[(2.03\text{E}11)(\pi/4)(0.006)^4]^{1/2}} \approx 0.0224$$

For 2024 aluminum, E ≈ 10.6E6 psi ≈ 7.4E10 Pa. Then re-evaluate the same pi group:

$$\text{New}\quad \frac{\omega(\mathrm{ML}^3)^{1/2}}{(\mathrm{EI})^{1/2}} = 0.0224 = \frac{\omega\,[(1.0)(0.4)^3]^{1/2}}{[(7.4\mathrm{E}10)(\pi/4)(0.006)^4]^{1/2}},\quad \text{or}\quad \boldsymbol{\omega_{\mathrm{alum}} \approx 0.77\ \mathrm{Hz}}\quad \textit{Ans.}$$

5.65 In turbulent flow near a flat wall, the local velocity *u* varies only with distance *y* from the wall, wall shear stress τw, and fluid properties ρ and μ. The following data were taken in the University of Rhode Island wind tunnel for airflow, $\rho = 0.0023$ slug/ft^3, $\mu = 3.81\text{E}{-}7$ slug/(ft·s), and $\tau w = 0.029$ lbf/ft^2:

y, in	0.021	0.035	0.055	0.080	0.12	0.16
u, ft/s	50.6	54.2	57.6	59.7	63.5	65.9

(a) Plot these data in the form of dimensionless *u* versus dimensionless *y*, and suggest a suitable power-law curve fit. (b) Suppose that the tunnel speed is increased until *u* = 90 ft/s at *y* = 0.11 in. Estimate the new wall shear stress, in lbf/ft^2.

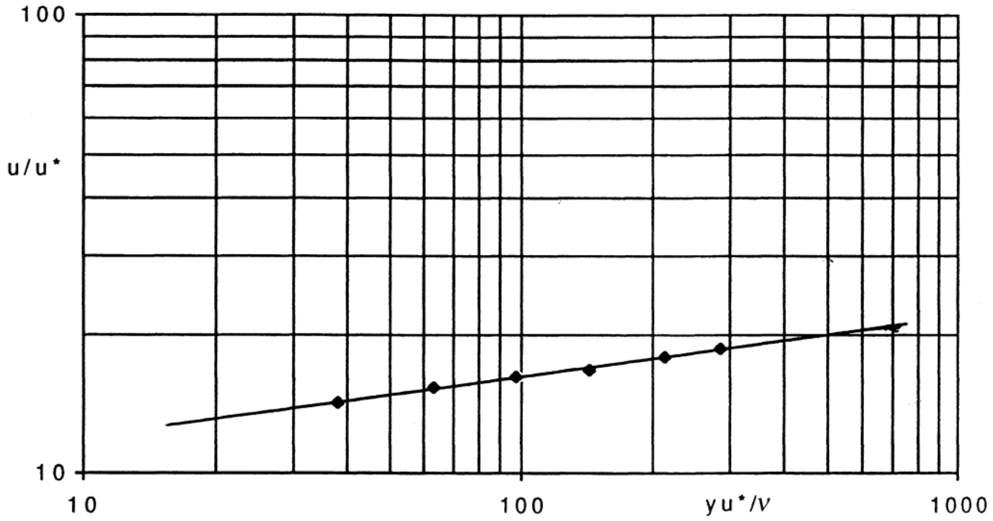
Solution: Given that u = fcn(y, τw, ρ, μ), then *n* = 5 and *j* = 3 (MLT), so we expect n – j = 5 – 3 = **2** pi groups, and they are traditionally chosen as follows (Chap. 6, Section 6.5):

$$\frac{u}{u^*} = \mathrm{fcn}\left(\frac{\rho u^* y}{\mu}\right),\quad \text{where}\ u^* = (\tau_w/\rho)^{1/2} = \text{the ‘friction velocity’}$$

We may compute $u^* = (\tau w/\rho)^{1/2} = (0.029/0.0023)^{1/2} = 3.55$ ft/s and then modify the given data into dimensionless parameters:

y, in:	0.021	0.035	0.055	0.080	0.12	0.16
$\rho u^* y/\mu$:	38	63	98	143	214	286
u/u*:	14.3	15.3	16.2	16.8	17.9	18.6

When plotted on log-log paper as follows, they form nearly a straight line:



The slope of the line is 0.13 and its intercept (at $yu^*/\nu = 1$) is 8.9. Hence the formula:

$$u/u^* \approx 8.9(yu^*/\nu)^{0.13} \pm 1\% \quad \textit{Ans.}\ \text{(a)}$$

Now if the tunnel speed is increased until u = 90 ft/s at y = 0.11 in, we may substitute in:

$$\frac{90}{u^*} \approx 8.9\left[\frac{0.0023(0.11/12)u^*}{3.87\text{E}{-7}}\right]^{0.13} = 8.9(54.5u^*)^{0.13}, \quad \text{solve for } u^* \approx 4.89 \text{ ft/s}$$

$$\text{Solve for} \quad \tau_w = \rho u^{*2} = (0.0023)(4.89)^2 \approx \mathbf{0.055\ lbf/ft^2} \quad \textit{Ans.}\ \text{(b)}$$

5.66 A torpedo 8 m below the surface in 20°C seawater cavitates at a speed of 21 m/s when atmospheric pressure is 101 kPa. If Reynolds-number and Froude-number effects are negligible, at what speed will it cavitate when running at a depth of 20 m? At what depth should it be to avoid cavitation at 30 m/s?

Solution: For seawater at 20°C, take ρ = 1025 kg/m^3 and p_v = 2337 Pa. With Reynolds and Froude numbers neglected, the cavitation numbers must simply be the same:

$$\text{Ca} = \frac{p_a + \rho gz - p_v}{\rho V^2} \quad \text{for Flow } 1 = \frac{101000 + (1025)(9.81)(8) - 2337}{(1025)(21)^2} \approx 0.396$$

$$\text{(a) At z} = 20\text{ m:}\quad \text{Ca} = 0.396 = \frac{101000 + 1025(9.81)(20) - 2337}{1025\text{V}_\text{a}^2},$$

$$\text{or}\quad \text{V}_\text{a} \approx \mathbf{27.2\ \frac{m}{s}}\quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) At } \text{V}_\text{b} = 30\ \frac{\text{m}}{\text{s}}\text{:}\quad \text{Ca} = 0.396 = \frac{101000 + 1025(9.81)\text{z}_\text{b} - 2337}{1025(30)^2},$$

$$\text{or}\quad \text{z}_\text{b} \approx \mathbf{26.5\ m}\quad \textit{Ans.}\text{ (b)}$$

5.67 A student needs to measure the drag on a prototype of characteristic length *d*p moving at velocity *U*p in air at sea-level conditions. He constructs a model of characteristic length *d*m, such that the ratio *d*p/*d*m = a factor *f*. He then measures the model drag under dynamically similar conditions, in sea-level air. The student claims that the drag force on the prototype will be identical to that of the model. Is this claim correct? Explain.

Solution: Assuming no compressibility effects, dynamic similarity requires that

$$Re_m = Re_p,\quad or:\quad \frac{\rho_m U_m d_m}{\mu_m} = \frac{\rho_p U_p d_p}{\mu_p},\quad whence\ \frac{U_m}{U_p} = \frac{d_p}{d_m} = f$$

Run the tunnel at "*f*" times the prototype speed, then drag coefficients match:

$$\frac{F_m}{\rho_m U_m^2 d_m^2} = \frac{F_p}{\rho_p U_p^2 d_p^2},\quad or:\quad \frac{F_m}{F_p} = \left(\frac{U_m d_m}{U_p d_p}\right)^2 = \left(\frac{f}{f}\right)^2 = \mathbf{1}\quad \mathbf{Yes,}\ \textit{drags are the same}!$$

P5.68 For the rotating-cylinder function of Prob. P5.20, if $L >> D$, the problem can be reduced to only two groups, $F/(\rho U^2 LD)$ versus $(\Omega D/U)$. Here are experimental data for a cylinder 30 cm in diameter and 2 m long, rotating in sea-level air, with U = 25 m/s.

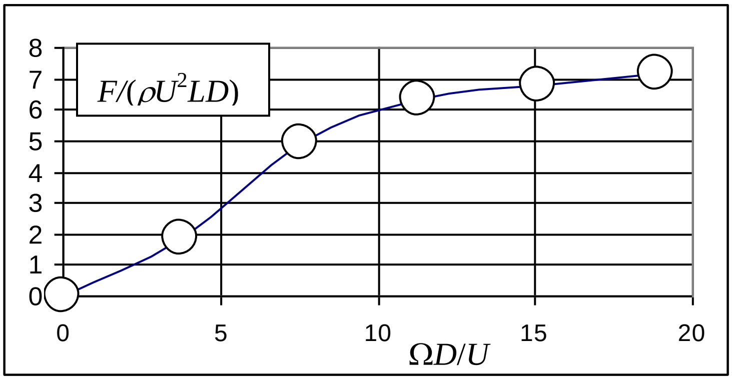
Ω, rev/min	0	3000	6000	9000	12000	15000
F, N	0	850	2260	2900	3120	3300

(*a*) Reduce this data to the two dimensionless groups and make a plot. (*b*) Use this plot to predict the lift of a cylinder with *D* = 5 cm, *L* = 80 cm, rotating at 3800 rev/min in water at *U* = 4 m/s.

Solution: (*a*) In converting the data, the writer suggests using Ω in rad/s, not rev/min. For sea-level air, $\rho = 1.2255\ \text{kg/m}^3$. Take, for example, the first data point, Ω = 3000 rpm x (2π/60) = 314 rad/s, and *F* = 850 N.

$$\Pi_1 = \frac{F}{\rho U^2 LD} = \frac{850}{(1.2255)(25)^2(2.0m)(0.3m)} = 1.85\ ;\ \Pi_2 = \frac{\Omega D}{U} = \frac{(314)(0.3)}{25} = 3.77$$

Do this for the other four data points, and plot as follows. *Ans*.(*a*)



(*b*) For water, take $\rho = 998\ \text{kg/m}^3$. The new data are *D* = 5 cm, *L* = 80 cm, 3800 rev/min in water at *U* = 4 m/s. Convert 3800 rev/min = 398 rad/s. Compute the rotation Pi group:

$$\Pi_2 = \frac{\Omega D}{U} = \frac{(398 rad/s)(0.05m)}{4 m/s} = 4.97$$

Read the chart for Π_1. The writer reads $\Pi_1 \approx 2.8$. Thus we estimate the water lift force:

$$F = \Pi_1 \rho U^2 LD = (2.8)(998)(4)^2(0.8m)(0.05m) \approx 1788 N \approx \mathbf{1800} N \quad Ans.(b)$$

5.69 A simple flow-measurement device for streams and channels is a notch, of angle α, cut into the side of a dam, as shown in Fig. P5.69. The volume flow *Q* depends only on α, the acceleration of gravity g, and the height δ of the upstream water surface above the notch vertex. Tests of a model notch, of angle $\alpha = 55°$, yield the following flow rate data:

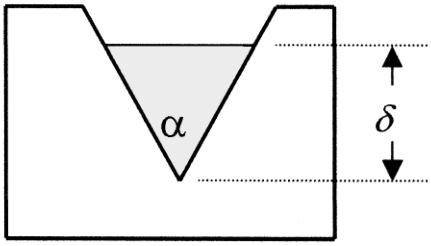


Fig. P5.69

δ, cm:	10	20	30	40
Q, m^3/h:	8	47	126	263

(a) Find a dimensionless correlation for the data. (b) Use the model data to predict the flow rate of a prototype notch, also of angle $\alpha = 55°$, when the upstream height δ is 3.2 m.

Solution: (a) The appropriate functional relation is $Q = \text{fcn}(\alpha, g, \delta)$ and its dimensionless form is $Q/(g^{1/2}\delta^{5/2}) = \text{fcn}(\alpha)$. Recalculate the data in this dimensionless form, with α constant:

$$Q/(g^{1/2}\delta^{5/2}) = \mathbf{0.224} \quad \mathbf{0.233} \quad \mathbf{0.227} \quad \mathbf{0.230} \quad \text{respectively} \quad \textit{Ans.}\ \text{(a)}$$

(b) The average coefficient in the data is about 0.23. Since the notch angle is still 55°, we may use the formula to predict the larger flow rate:

$$Q_{prototype} = 0.23 g^{1/2}\delta^{5/2} = 0.23\left(9.81\ \frac{m}{s^2}\right)^{1/2}(3.2\ m)^{5/2} \approx \mathbf{13.2\ m^3/s} \quad \textit{Ans.}\ \text{(b)}$$

5.70 A diamond-shaped body, of characteristic length 9 in, has the following measured drag forces when placed in a wind tunnel at sea-level standard conditions:

V, ft/s:	30	38	48	56	61
F, lbf	1.25	1.95	3.02	4.05	4.81

Use these data to predict the drag force of a similar 15-in diamond placed at similar orientation in 20°C water flowing at 2.2 m/s.

Solution: For sea-level air, take $\rho = 0.00237$ slug/ft^3, $\mu = 3.72\text{E}{-7}$ slug/ft·s. For water at 20°C, take ρ = 1.94 kg/m^3, μ = 2.09E–5 slug/ft·s. Convert the model data into drag coefficient and Reynolds number, taking $L_m = 9$ in = 0.75 ft:

V_m, ft/s:	30	38	48	56	61
$F/(\rho V^2L^2)$:	0.667	0.649	0.630	0.621	0.621
$\rho VL/\mu$:	143000	182000	229000	268000	291000

An excellent curve-fit to this data is the power-law

$$C_F \approx 2.5\text{Re}_L^{-0.111} \pm 1\%$$

Now introduce the new case, V_{proto} = 2.2 m/s = 7.22 ft/s, L_{proto} = 15 in = 1.25 ft. Then $\text{Re}_{L,proto} = \dfrac{1.94(7.22)(1.25)}{2.09\text{E}{-5}} \approx 837000$, which is outside the range of the model data. Strictly speaking, we **cannot** use the model data to predict this new case. *Ans.*

If we wish to *extrapolate* to get an estimate, we obtain

$$C_{F,proto} \approx \frac{2.5}{(837000)^{0.111}} \approx 0.550 \approx \frac{F_{proto}}{1.94(7.22)^2(1.25)^2},$$

$$\text{or:}\quad F_{proto} \approx \mathbf{87\ lbf} \quad \textit{Approximately}$$

5.71 The pressure drop in a venturi meter (Fig. P3.128) varies only with the fluid density, pipe approach velocity, and diameter ratio of the meter. A model venturi meter tested in water at 20°C shows a 5-kPa drop when the approach velocity is 4 m/s. A geometrically similar prototype meter is used to measure gasoline at 20°C and a flow rate of 9 m^3/min. If the prototype pressure gage is most accurate at 15 kPa, what should the upstream pipe diameter be?

Solution: Given Δp = fcn(ρ, V, d/D), then by dimensional analysis $\Delta p/(\rho V^2)$ = fcn(d/D). For water at 20°C, take ρ = 998 kg/m^3. For gasoline at 20°C, take ρ = 680 kg/m^3. Then, using the water 'model' data to obtain the function "fcn(d/D)", we calculate

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{5000}{(998)(4.0)^2} = 0.313 = \frac{\Delta p_p}{\rho_p V_p^2} = \frac{15000}{(680)V_p^2},\quad \text{solve for } V_p \approx 8.39\ \frac{m}{s}$$

$$\text{Given}\quad Q = \frac{9}{60}\ \frac{m^3}{s} = V_pA_p = (8.39)\frac{\pi}{4}D_p^2,\quad \text{solve for best } \mathbf{D_p \approx 0.151\ m} \quad \textit{Ans.}$$

P5.72 A one-twelfth-scale model of a large commercial aircraft is tested in a wind tunnel at 20°C and 1 atm. The model chord length is 27 cm, and its wing area is 0.63 m^2. Test results for the drag of the model are as follows:

V, mi/h	50	75	100	125
Drag, N	15	32	53	80

In the spirit of Fig. 5.8, use this data to estimate the drag of the full-scale aircraft when flying at 550 mi/h, for the same angle of attack, at 32,800 ft standard altitude. Neglect Mach number differences between model and prototype.

Solution: Compute the model drag coefficients and Reynolds numbers, plot them, and extrapolate in the spirit of Fig. 5.8 of the text. For the first point, 50 mi/h = 22.35 m/s. At 20°C and 1 atm, ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. Compute the first dimensionless data point:

$$C_D = \frac{F}{(1/2)\rho V^2 A} = \frac{15\,N}{(1/2)(1.20)(22.35)^2(0.63)} = 0.0794$$

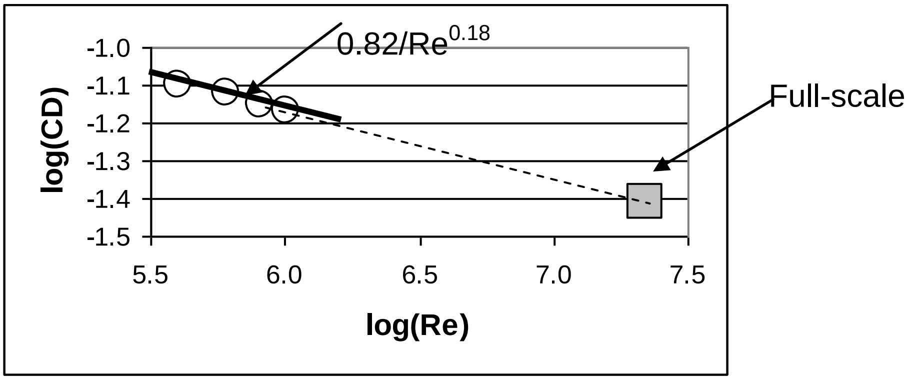
$$\mathrm{Re}_{chord} = \frac{\rho V c}{\mu} = \frac{(1.20)(22.35)(0.27)}{1.8\mathrm{E}-5} = 402{,}000$$

Do this for all four model data points:

Re_c	402,000	604,000	805,000	1,006,000
C_D	0.0794	0.00753	0.00702	0.00678

Now plot them and extrapolate to the prototype Reynolds number. At 32,800 ft = 10,000 m, ρ = 0.4125 kg/m^3 and $\mu \approx$ 1.5E-5 kg/m-s (at that altitude, T = 223 K). Full-scale c = 12(0.27) = 3.24 m, and the full-scale wing area is A = (12)^2(0.63) = 90.7 m^2. The full-scale velocity is 550 mi/h = 246 m/s. Full-scale Reynolds number is Re_p =

(0.4125)(246)(3.24)/(1.5E-5) = 21,920,000, or $\log(\mathrm{Re}_p) = 7.34$. The log-log plot and extrapolation would look like this:



You can see that it is a long way out from those four closely packed model points to a Reynolds number of 21,920,000. Uncertainty is high. The model curve-fit $C_D \approx 0.82/\mathrm{Re}^{0.18}$ can be used to estimate C_D(prototype) = $0.82/(21{,}920{,}000)^{0.18} \approx 0.039$. Our rather uncertain estimate for the drag of the full-scale aircraft is thus

$$\text{Full-scale drag} \approx C_D(\rho_p/2)V_p^2 A_p = (0.039)(0.4125/2)(246)^2(90.7) \approx$$

$$\approx \mathbf{44{,}000}\,N \approx 10{,}000\,lbf \quad \mathit{Ans.}$$

5.73 The power *P* generated by a certain windmill design depends upon its diameter *D*, the air density ρ, the wind velocity *V*, the rotation rate Ω, and the number of blades *n*. (a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when *V* = 40 m/s and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype?

Solution: (a) For the function $P = \text{fcn}(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\boldsymbol{\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right)} \quad \textit{Ans.}\text{ (a)}$$

(c) "Geometrically similar" requires that *n* is the same for both windmills. For "dynamic similarity," the advance ratio ($\Omega D/V$) must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

$$\text{or:}\quad \Omega_{proto} = \mathbf{144\ \frac{rev}{min}} \quad \textit{Ans.}\text{ (c)}$$

(b) At 2000 m altitude, $\rho = 1.0067$ kg/m^3. At sea level, $\rho = 1.2255$ kg/m^3. Since $\Omega D/V$ and *n* are the same, it follows that the power coefficients equal for model and prototype:

$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = 0.138 = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$\textit{solve}\quad \mathbf{P_{proto} = 5990\ W \approx 6\ kW} \quad \textit{Ans.}\text{ (b)}$$

5.74 A one-tenth-scale model of a supersonic wing tested at 700 m/s in air at 20°C and 1 atm shows a pitching moment of 0.25 kN·m. If Reynolds-number effects are negligible, what will the pitching moment of the prototype wing be flying at the same Mach number at 8-km standard altitude?

Solution: If Reynolds number is unimportant, then the dimensionless moment coefficient $M/(\rho V^2 L^3)$ must be a function only of the Mach number, Ma = V/a. For sea-level air, take $\rho = 1.225$ kg/m^3 and sound speed *a* = 340 m/s. For air at 8000-m standard altitude (Table A-6), take $\rho = 0.525$ kg/m^3 and sound speed *a* = 308 m/s. Then

$$\mathrm{Ma_m} = \frac{\mathrm{V_m}}{\mathrm{a_m}} = \frac{700}{340} = 2.06 = \mathrm{Ma_p} = \frac{\mathrm{V_p}}{308},\quad \text{solve for }\ \mathrm{V_p} \approx 634\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\text{Then}\quad \mathrm{M_p} = \mathrm{M_m}\left(\frac{\rho_p V_p^2 L_p^3}{\rho_m V_m^2 L_m^3}\right) = 0.25\left(\frac{0.525}{1.225}\right)\left(\frac{634}{700}\right)^2\left(\frac{10}{1}\right)^3 \approx \mathbf{88\ kN{\cdot}m} \quad \textit{Ans.}$$

P5.75 According to the web site *USGS Daily Water Data for the Nation*, the mean flow rate in the New River near Hinton, WV is 10,100 ft^3/s. If the hydraulic model in Fig. 5.9 is to match this condition with Froude number scaling, what is the proper model flow rate?

Solution: For Froude scaling, the volume flow rate is a blend of velocity and length terms:

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \sqrt{\frac{L_m}{L_p}}\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \quad \text{or} \quad \alpha^{5/2}$$

$$Fig.5/9: \alpha = 1:65\ ; \quad \therefore\ Q_{model} = (10100\,\frac{ft^3}{s})(\frac{1}{65})^{5/2} = 0.30\,\frac{ft^3}{s} \quad Ans.$$

5.76 A 2-ft-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into "friction" drag (Reynolds scaling) and "wave" drag (Froude scaling). The model data are as follows:

Tow speed, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
Friction drag, lbf:	0.016	0.057	0.122	0.208	0.315	0.441
Wave drag, lbf:	0.002	0.021	0.083	0.253	0.509	0.697

The prototype ship is 150 ft long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

Solution: For fresh water at 20°C, take ρ = 1.94 slug/ft^3, μ = 2.09E−5 slug/ft·s. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

V_m, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
$Re_m = V_mL_m/\nu$:	143000	297000	446000	594000	743000	892000
$C_{F,friction}$:	0.00322	0.00287	0.00273	0.00261	0.00254	0.00247
$Fr_m = V_m/\sqrt{(gL_m)}$:	0.099	0.199	0.299	0.399	0.498	0.598
$C_{F,\ wave}$:	0.00040	0.00106	0.00186	0.00318	0.00410	0.00390

For seawater, take ρ = 1.99 slug/ft^3, μ = 2.23E–5 slug/ft·s. With L_p = 150 ft and V_p = 15 knots = 25.3 ft/s, evaluate

$$\mathrm{Re}_{\mathrm{proto}} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23\mathrm{E}{-5}} \approx 3.39\mathrm{E}8; \quad \mathrm{Fr}_p = \frac{25.3}{[32.2(150)]^{1/2}} \approx 0.364$$

$$\text{For} \quad \mathrm{Fr} \approx 0.364, \quad \text{interpolate to } C_{F,\mathrm{wave}} \approx 0.0027$$

Thus we can immediately estimate $F_{\mathrm{wave}} \approx 0.0027(1.99)(25.3)^2(150)^2 \approx \underline{77000}$ lbf. However, as mentioned in Fig. 5.8 of the text, **Re_p is far outside the range of the friction force data**, therefore we must *extrapolate* as best we can. A power-law curve-fit is

$$C_{F,\mathrm{friction}} \approx \frac{0.0178}{\mathrm{Re}^{0.144}}, \quad \text{hence } C_{F,\mathrm{proto}} \approx \frac{0.0178}{(3.39\mathrm{E}8)^{0.144}} \approx 0.00105$$

$$\text{Thus} \quad F_{\mathrm{friction}} \approx 0.00105(1.99)(25.3)^2(150)^2 \approx \underline{30000}\ \text{lbf.}\ \mathbf{F_{total} \approx 107000\ lbf.} \quad \textit{Ans.}$$

5.77 A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6 m/s and a volume flow of 0.05 m^3/s. What will the velocity and flow of the prototype be? If the measured force on a certain part of the model is 1.5 N, what will the corresponding force on the prototype be?

Solution: Given $\alpha = L_m/L_p = 1/30$, Froude scaling requires that

$$V_p = \frac{V_m}{\sqrt{\alpha}} = \frac{0.6}{(1/30)^{1/2}} \approx \mathbf{3.3\ \frac{m}{s}}; \quad Q_p = \frac{Q_m}{\alpha^{5/2}} = \frac{0.05}{(1/30)^{5/2}} \approx \mathbf{246\ \frac{m^3}{s}} \quad \textit{Ans.}\ \text{(a)}$$

The force scales in similar manner, assuming that the density remains constant (water):

$$F_p = F_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 = F_m(1)\left(\frac{1}{\sqrt{\alpha}}\right)^2\left(\frac{1}{\alpha}\right)^2 = (1.5\,\mathrm{N})(30)^3 \approx 40500\,\mathbf{N} \quad \textit{Ans.}\ \text{(b)}$$

5.78 A prototype spillway has a characteristic velocity of 3 m/s and a characteristic length of 10 m. A small model is constructed by using Froude scaling. What is the minimum scale ratio of the model which will ensure that its minimum Weber number is 100? Both flows use water at 20°C.

Solution: For water at 20°C, ρ = 998 kg/m^3 and Y = 0.073 N/m, for both model and prototype. Evaluate the Weber number of the prototype:

$$\mathrm{We_p} = \frac{\rho_p V_p^2 L_p}{Y_p} = \frac{998(3.0)^2(10.0)}{0.073} \approx 1.23\mathrm{E6}; \quad \text{for Froude scaling,}$$

$$\frac{\mathrm{We_m}}{\mathrm{We_p}} = \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2\left(\frac{L_m}{L_p}\right)\left(\frac{Y_p}{Y_m}\right) = (1)(\sqrt{\alpha})^2(\alpha)(1) = \alpha^2 = \frac{100}{1.23\mathrm{E6}} \quad \text{if } \alpha = 0.0090$$

Thus the model Weber number will be ≥100 if $\alpha = \mathrm{Lm/Lp} \geq 0.0090 =$ **1/111**. *Ans.*

5.79 An East Coast estuary has a tidal period of 12.42 h (the semidiurnal lunar tide) and tidal currents of approximately 80 cm/s. If a one-five-hundredth-scale model is constructed with tides driven by a pump and storage apparatus, what should the period of the model tides be and what model current speeds are expected?

Solution: Given Tp = 12.42 hr, Vp = 80 cm/s, and $\alpha = \mathrm{Lm/Lp} = 1/500$. Then:

$$\text{Froude scaling:} \quad T_m = T_p\sqrt{\alpha} = \frac{12.42}{\sqrt{500}} = 0.555\,\text{hr} \approx \mathbf{33\ min} \quad \textit{Ans.}\text{ (a)}$$

$$V_m = V_p\sqrt{\alpha} = 80\big/\sqrt{(500)} \approx \mathbf{3.6\ cm/s} \quad \textit{Ans.}\text{ (b)}$$

5.80 A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1-m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

Solution: Given $\alpha = 1/35$, then Froude scaling determines everything:

$$V_{tow} = V_m = V_p\sqrt{\alpha} = 11/\sqrt{(35)} \approx \mathbf{1.86\ m/s}$$

$$F_m/F_p = (V_m/V_p)^2(L_m/L_p)^2 = (\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (1/35)^3 \approx \frac{\mathbf{1}}{\mathbf{42900}} \quad \textit{Ans.}$$

$$P_m/P_p = (F_m/F_p)(V_m/V_p) = \alpha^3(\sqrt{\alpha}) = \alpha^{3.5} = 1/35^{3.5} \approx \frac{\mathbf{1}}{\mathbf{254000}}$$

5.81 An airplane, of overall length 55 ft, is designed to fly at 680 m/s at 8000-m standard altitude. A one-thirtieth-scale model is to be tested in a pressurized helium wind tunnel at 20°C. What is the appropriate tunnel pressure in atm? Even at this (high) pressure, exact dynamic similarity is not achieved. Why?

Solution: For air at 8000-m standard altitude (Table A-6), take $\rho = 0.525$ kg/m^3, $\mu =$ 1.53E−5 kg/m·s, and sound speed $a = 308$ m/s. For helium at 20°C (Table A-4), take gas constant R = 2077 J/(kg·°K), $\mu = 1.97$E−5 kg/m·s, and $a = 1005$ m/s. For similarity at this supersonic speed, we must match both the Mach and Reynolds numbers. First convert $L_p =$ 55 ft = 16.8 m. Then

$$\mathrm{Ma_p} = \frac{680}{308} = 2.21 = \mathrm{Ma_m} = \frac{V_m}{1005}, \quad \text{solve for } V_{\text{model}} \approx 2219\ \frac{m}{s}$$

$$\mathrm{Re_p} = \frac{\rho VL}{\mu}\Big|_p = \frac{0.525(680)(16.8)}{1.53\text{E–}5} = 3.91\text{E}8 = \mathrm{Re_m} = \frac{\rho_{He}(2219)(16.8/30)}{1.97\text{E–}5}$$

$$\text{Solve for} \quad \rho_{He} \approx 6.21\ \text{kg/m}^3 = \frac{p}{RT} = \frac{p_{He}}{(2077)(293)},$$

$$\text{or} \quad \mathbf{p_{He} \approx 3.78\ MPa = 37.3\ atm} \quad \textit{Ans.}$$

Even with Ma and Re matched, true dynamic similarity is not achieved, because the specific heat ratio of helium, $k \approx 1.66$, is not equal to $k_{air} \approx 1.40$.

P5.82 A one-fiftieth scale model of a military airplane is tested at 1020 m/s in a wind tunnel at sea-level conditions. The model wing area is 180 cm^2. The angle of attack is 3 degrees. If the measured model lift is 860 N, what is the prototype lift, using Mach number scaling, when it flies at 10,000 m standard altitude under dynamically similar conditions? [NOTE: Be careful with the area scaling.]

Solution: At sea-level, $\rho = 1.2255$ kg/m^3 and $T = 288$ K. Compute the speed of sound and Mach number for the model:

$$a_m = \sqrt{kRT} = \sqrt{1.4(287)(288)} = 340\frac{m}{s}; \quad Ma_m = \frac{V_m}{a_m} = \frac{1020\,m/s}{340\,m/s} = 3.0$$

Now compute the lift-force coefficient of the model:

$$C_{L,m} = \frac{F_m}{(1/2)\rho_m V_m^2 A_m} = \frac{860\,N}{(1/2)(1.2255kg/m^3)(1020m/s)^2(0.0180m^2)} = 0.0749$$

For dynamically similar conditions, the prototype must have the **same lift coefficient**. At 10,000 m standard altitude, from Table A.6, read $\rho = 0.4125$ kg/m^3 and $T = 223.16$ K.

The prototype wing area is $(0.0180\text{m}^2)(50)^2 =$ **45** m^2. (The writer cautioned about this scaling.) Then compute

$$a_p = \sqrt{kRT} = \sqrt{1.4(287)(223)} = 299\, m/s, \quad then\ V_p = Ma_p a_p = (3.0)(299) = 898\, m/s$$

$$F_{proto} = C_{L,p}\frac{\rho_p}{2}V_p^2 A_p = (0.0749)(\frac{0.4125}{2})(898)^2(45) = \mathbf{561{,}000}\,N = 126{,}000\,lbf \quad Ans.$$

5.83 A one-fortieth-scale model of a ship's propeller is tested in a tow tank at 1200 r/min and exhibits a power output of 1.4 ft·lbf/s. According to Froude scaling laws, what should the revolutions per minute and horsepower output of the prototype propeller be under dynamically similar conditions?

Solution: Given $\alpha = 1/40$, use Froude scaling laws:

$$\Omega_p/\Omega_m = T_m/T_p = \sqrt{\alpha}, \quad \text{thus } \Omega_p = \frac{1200}{(40)^{1/2}} \approx \mathbf{190}\ \frac{\textbf{rev}}{\textbf{min}} \quad \textit{Ans.}\text{ (a)}$$

$$P_p = P_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\Omega_p}{\Omega_m}\right)^3\left(\frac{D_p}{D_m}\right)^5 = (1.4)(1)\left(\frac{1}{\sqrt{40}}\right)^3(40)^5$$

$$= 567000 \div 550 = \mathbf{1030\ hp} \quad \textit{Ans.}\text{ (b)}$$

5.84 A prototype ocean-platform piling is expected to encounter currents of 150 cm/s and waves of 12-s period and 3-m height. If a one-fifteenth-scale model is tested in a wave channel, what current speed, wave period, and wave height should be encountered by the model?

Solution: Given $\alpha = 1/15$, apply straight Froude scaling (Fig. 5.6*b*) to these results:

$$\text{Velocity:} \quad V_m = V_p\sqrt{\alpha} = \frac{150}{\sqrt{15}} = \mathbf{39}\ \frac{\textbf{cm}}{\textbf{s}}$$

$$\text{Period:} \quad T_m = T_p\sqrt{\alpha} = \frac{12}{\sqrt{15}} = \mathbf{3.1\ s}; \quad \text{Height:} \quad H_m = \alpha H_p = \frac{3}{15} = \mathbf{0.20\ m} \quad \textit{Ans.}$$

***P5.85** As shown in Ex. 5.3, pump performance data can be non-dimensionalized. Problem P5.61 gave typical dimensionless data for centrifugal pump "head", $H = \Delta p/\rho g$, as follows:

$$\frac{gH}{n^2D^2} \approx 6.0 - 120\left(\frac{Q}{nD^3}\right)^2$$

where Q is the volume flow rate, n the rotation rate in r/s, and D the impeller diameter. This type of correlation allows one to compute H when (ρ, Q, D) are known. (*a*) Show how to rearrange these Pi groups so that one can *size* the pump, that is, compute D directly when (Q, H, n) are known. (*b*) Make a crude but effective plot of your new function. (*c*) Apply part (*b*) to the following example: When H = 37 m, Q = 0.14 m^3/s, and n = 35 r/s, find the pump diameter for this condition.

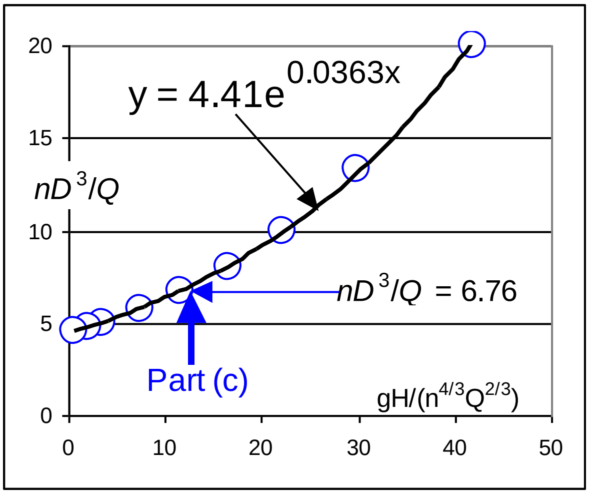
Solution: (*a*) We have to eliminate D from one or the other of the two parameters. The writer chose to remove D from the left side. The new parameter will be

$$\Pi_3 = \frac{gH}{n^2D^2}\left(\frac{nD^3}{Q}\right)^{2/3} = \frac{gH}{n^{4/3}Q^{2/3}}$$

$$\frac{nD^3}{Q} = fcn\left(\frac{gH}{n^{4/3}Q^{2/3}}\right)$$

For convenience, we inverted the right-hand parameter to feature D. Thus the function

will enable one to input (Q, H, n) and immediately solve for the impeller diameter. *Ans.*(*a*) (*b*) The new variable hopelessly complicates the algebra of the original parabolic formula. However, with a little (well, maybe a *lot of*) work, one can compute and plot a few values:



It fits a least-squared exponential curve quite well, as you see. *Ans.*(*b*)

$$\frac{gH}{n^{4/3}Q^{2/3}} = \frac{(9.81\,m/s^2)(37\,m)}{(35r/s)^{4/3}(0.14m^3/s)^{2/3}} = 11.76 \qquad \textit{Hence}$$

$$\frac{nD^3}{Q} \approx 4.41\exp[0.0363(11.76)] = 6.76 = \frac{35\,D^3}{0.14}, \quad \textit{Solve}\quad D \approx 0.30\,m \quad \textit{Ans.}(c)$$

(*c*) For the given data, $H = 37$ m, $Q = 0.14$ m^3/s, and $n = 35$ r/s, calculate Π_3:

A 30-cm pump fits these conditions. These Π value solutions are shown on the crude plot above. [NOTE: This problem was set up from the original parabolic function by using $D = 30$ cm, so the curve-fit is quite accurate.]

5.86 Solve Prob. 5.49 for glycerin, using the modified sphere-drag plot of Fig. 5.11.

Solution: Recall this problem is identical to Prob. 5.85 above except that the fluid is **glycerin**, with $\rho = 1260$ kg/m^3 and $\mu = 1.49$ kg/m·s. Evaluate the net weight:

$$W = (7800 - 1260)(9.81)\frac{\pi}{6}(0.025)^3 \approx 0.525\text{ N},\quad \text{whence}\quad \frac{\rho F}{\mu^2} = \frac{1260(0.525)}{(1.49)^2} \approx 298$$

From Fig. 5.11 read Re ≈ 15, or V = 15(1.49)/[1260(0.025)] ≈ **0.7 m/s**. *Ans.*

5.87 In Prob. 5.61 it would be difficult to solve for Ω because it appears in all three dimensionless coefficients. Rescale the problem, using the data of Fig. P5.61, to make a plot of dimensionless power versus dimensionless rotation speed. Enter this plot directly to solve for Ω for D = 12 cm, Q = 25 m^3/hr, and a maximum power P = 300 W, in gasoline at 20°C.

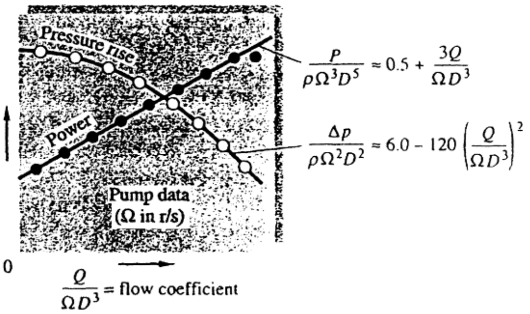


Fig. P5.61

Solution: For gasoline, $\rho = 680$ kg/m^3 and $\mu = 2.92\text{E}{-4}$ kg/m·s. We can eliminate Ω from the power coefficient for a new type of coefficient:

$$\Pi_3 = \frac{P}{\rho\Omega^3D^5}\cdot\frac{\Omega^3D^9}{Q^3} = \boldsymbol{\frac{PD^4}{\rho Q^3}},\quad \text{to be plotted versus}\quad \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.61.

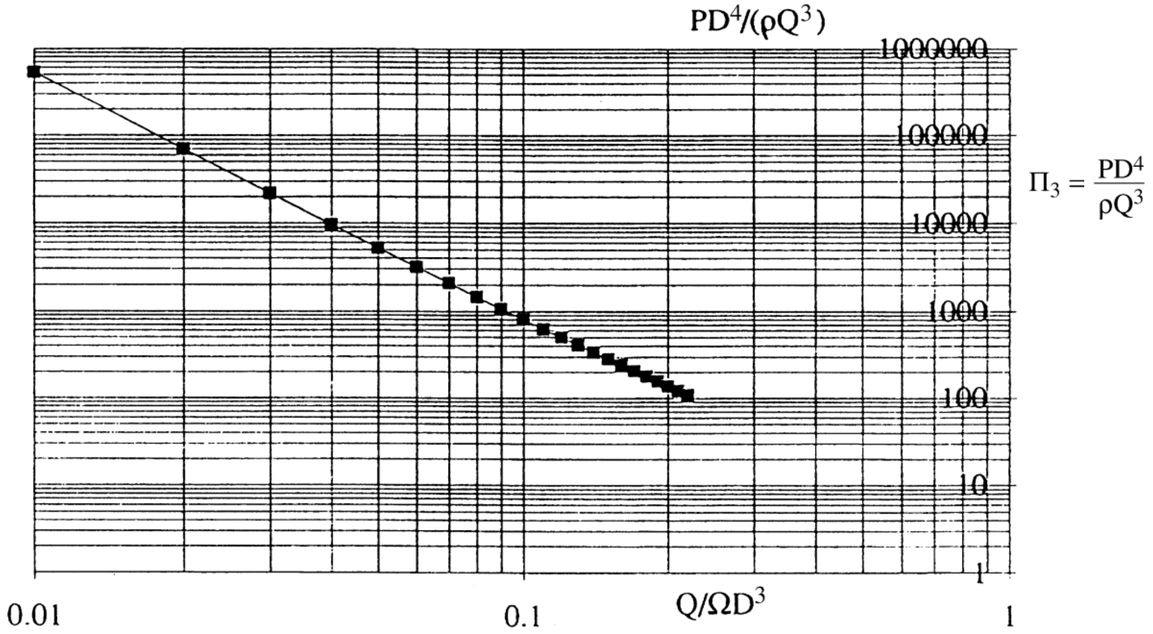


Fig. P5.87

Below $\Pi_3 < 10,000$, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 1.43/\Pi_3^{0.4} \pm 1\%$. We use the given data to evaluate Π_3 and hence compute $Q/\Omega D^3$:

$$\Pi_3 = \frac{(300)(0.12)^4}{(680)(25/3600)^3} = 273, \quad \text{whence} \quad \frac{Q}{\Omega D^3} \approx \frac{1.43}{(273)^{0.4}} \approx 0.152 = \frac{25/3600}{\Omega(0.12)^3}$$

Solve for $\Omega \approx$ **26.5 rev/s** *Ans.*

5.88 Modify Prob. 5.61 as follows: Let Ω = 32 r/s and Q = 24 m^3/h for a geometrically similar pump. What is the maximum diameter if the power is not to exceed 340 W? Solve this problem by rescaling the data of Fig. P5.61 to make a plot of dimensionless power versus dimensionless diameter. Enter this plot directly to find the desired diameter.

Solution: We can eliminate D from the power coefficient for an alternate coefficient:

$$\Pi_4 = \frac{P}{\rho\Omega^3 D^5}\cdot\left(\frac{\Omega D^3}{Q}\right)^{5/3} = \frac{\mathbf{P}}{\boldsymbol{\rho\Omega^{4/3}Q^{5/3}}},\quad \text{to be plotted versus } \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.61.

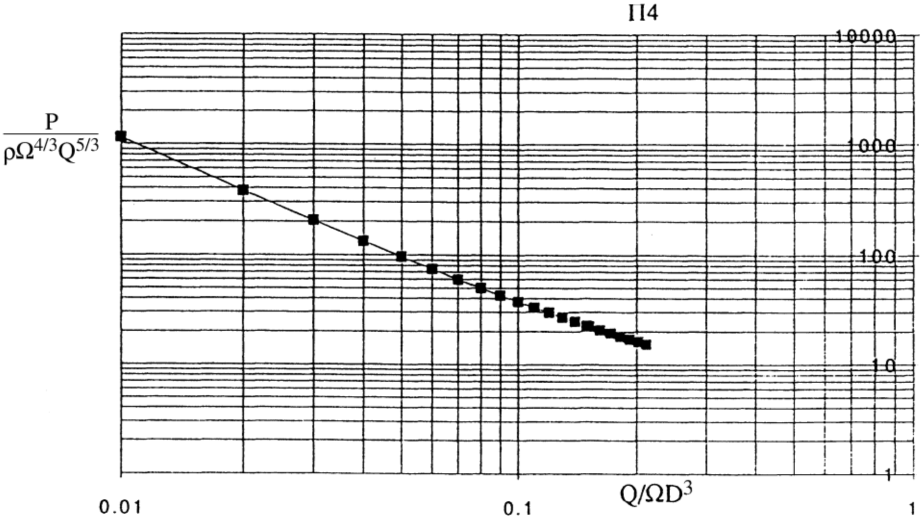


Fig. P5.88

Below Π4 < 1,000, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 2.12/\Pi_4^{0.85} \pm 1\%$.
We use the given data to evaluate Π4 and hence compute $Q/\Omega D^3$:

$$\Pi_4 = \frac{340}{680(32)^{4/3}(24/3600)^{5/3}} = 20.8,\quad \text{whence } \frac{Q}{\Omega D^3} = \frac{2.12}{(20.8)^{0.85}} \approx 0.161 = \frac{24/3600}{32D^3}$$

Solve for $D \approx \mathbf{0.11\ m}$ *Ans.*

P5.89 Wall friction τ_w, for turbulent flow at velocity U in a pipe of diameter D, was correlated, in 1911, with a dimensionless correlation by Ludwig Prandtl's student H. Blasius:

$$\frac{\tau_w}{\rho U^2} \approx \frac{0.632}{(\rho UD/\mu)^{1/4}}$$

Suppose that (ρ, U, μ, τ_w) were all known and it was desired to find the unknown velocity U. Rearrange and rewrite the formula so that U can be immediately calculated.

Solution: The easiest path the writer can see is to get rid of U^2 on the left hand side by multiplying both sides by the Reynolds number squared:

$$\frac{\tau_w}{\rho U^2}\left(\frac{\rho UD}{\mu}\right)^2 = \frac{\tau_w \rho D^2}{\mu^2} \approx \frac{0.632}{(\rho UD/\mu)^{1/4}}\left(\frac{\rho UD}{\mu}\right)^2 = 0.632\left(\frac{\rho UD}{\mu}\right)^{7/4}$$

$$\textit{Solve for } \left(\frac{\rho UD}{\mu}\right) \textit{ and clean up}: \quad \frac{\rho UD}{\mu} \approx 1.30\left(\frac{\tau_w \rho D^2}{\mu^2}\right)^{4/7} \qquad \textit{Ans.}$$

5.90 Knowing that Δp is proportional to L, rescale the data of Example 5.7 to plot dimensionless Δp versus dimensionless *viscosity*. Use this plot to find the viscosity required in the first row of data in Example 5.7 if the pressure drop is increased to 10 kPa for the same flow rate, length, and density.

Solution: Recall that Example 5.7, where $\Delta p/L = \text{fcn}(\rho, V, \mu, D)$, led to the correlation

$$\frac{\rho D^3 \Delta p}{L\mu^2} \approx 0.155\left(\frac{\rho VD}{\mu}\right)^{1.75},$$ which is awkward because μ occurs on both sides.

We can form a "μ-free" parameter by dividing the left side by Reynolds-number-squared:

$$\Pi_4 = \frac{\rho D^3 \Delta p/L\mu^2}{(\rho VD/\mu)^2} = \frac{\Delta p D}{\rho V^2 L} \approx \frac{0.155}{(\rho VD/\mu)^{0.25}} \qquad (3)$$

Correlation “3” can now be used to solve for an unknown viscosity. The data are the first row of Example 5.7, with viscosity unknown and a new pressure drop listed:

$$\mathrm{L = 5\ m;\quad D = 1\ cm;\quad Q = 0.3\ m^3/hr;\quad \Delta p = 10{,}000\ Pa;\quad \rho = 680\ \frac{kg}{m^3};\quad V = 1.06\ \frac{m}{s}}$$

$$\text{Evaluate}\quad \Pi_4 = \frac{(10000)(0.01)}{(680)(1.06)^2(5.0)} = 0.0262 \overset{?}{=} \frac{0.155}{\mathrm{Re}^{0.25}},\quad \text{or}\quad \mathrm{Re} \approx 1230\ \ ???$$

This is a trap for the unwary: Re = 1230 is **far below the range of the data** in Ex. 5.7, for which 15000 < Re < 95000. The solution cannot be trusted and in fact is quite incorrect, for the flow would be laminar and follow an entirely different correlation. *Ans.*

***P5.91** The traditional “Moody-type” pipe friction correlation in Chap. 6 is of the form

$$f = \frac{2\Delta p\, D}{\rho V^2 L} = fcn\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

where D is the pipe diameter, L the pipe length, and ε the wall roughness. Note that fluid average velocity V is used on both sides. This form is meant to find Δp when V is known.

(*a*) Suppose that Δp is known and we wish to find V. Rearrange the above function so that V is isolated on the left-hand side. Use the following data, for $\varepsilon/D = 0.005$, to make a plot of your new function, with your velocity parameter as the ordinate of the plot.

f	0.0356	0.0316	0.0308	0.0305	0.0304
$\rho VD/\mu$	15,000	75,000	250,000	900,000	3,330,000

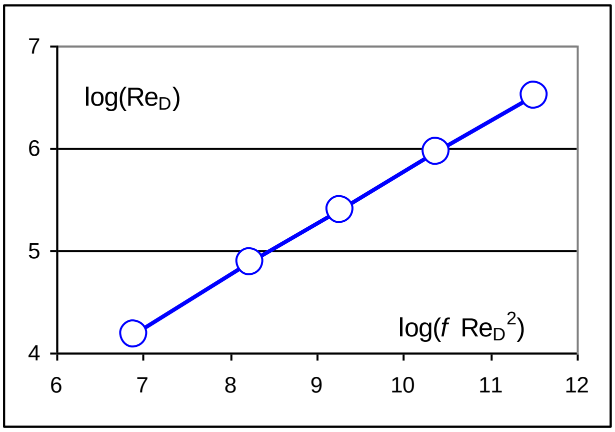
(*b*) Use your plot to determine *V*, in m/s, for the following pipe flow: D = 5 cm, ε = 0.025 cm, L = 10 m, for water flow at 20°C and 1 atm. The pressure drop Δp is 110 kPa.

Solution: We can eliminate *V* from the left side by multiplying by Re^2. Then rearrange:

$$\mathrm{Re}_D = fcn(f\,\mathrm{Re}_D^2, \frac{\varepsilon}{D}), \quad or: \quad \frac{\rho VD}{\mu} = fcn(\frac{2\rho D^3\,\Delta p}{L\,\mu^2}, \frac{\varepsilon}{D})$$

We can add a third row to the data above and make a log-log plot:

$f\,\mathrm{Re}_D^2$	8.01E6	1.78E8	1.92E9	2.47E10	3.31E11



It is a pretty good straight line on a log-log plot, which means a power-law. A good fit is

$$\frac{\rho VD}{\mu} \approx 4.85\left(\frac{2\rho D^3 \Delta p}{L\mu^2}\right)^{0.507} \quad for \quad \frac{\varepsilon}{D} = 0.005$$

Different power-law constants would be needed for other roughness ratios.

(*b*) Given pipe pressure drop data. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Calculate the value of ($f\,\mathrm{Re}_D^2$) for this data:

$$f\,\mathrm{Re}_D^2 = \frac{2\rho D^3 \Delta p}{L\mu^2} = \frac{2(998\,kg/m^3)(0.05m)^3(110000\,Pa)}{(10\,m)(0.001\,kg/m-s)^2} = 2.75E9$$

$$\text{Power-law}: \ \frac{\rho VD}{\mu} = 4.85(2.75E9)^{0.507} \approx 296{,}000 = \frac{(998)V(0.05)}{0.001}$$

$$Solve\ for \quad V \approx 5.93\ m/s \qquad Ans.(b)$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE5.1 Given the parameters (U, L, g, ρ, μ) which affect a certain liquid flow problem. The ratio $V^2/(Lg)$ is usually known as the

(a) velocity head (b) Bernoulli head **(c) Froude No.** (d) kinetic energy (e) impact energy

FE5.2 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. The appropriate tow velocity is

(a) 0.19 m/s (b) 0.35 m/s **(c) 1.31 m/s** (d) 2.55 m/s (e) 8.35 m/s

FE5.3 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. If the model wave drag is 2.2 N, the estimated full-size ship wave drag is

(a) 5500 N (b) 8700 N (c) 38900 N (d) 61800 N **(e) 275000 N**

FE5.4 A tidal estuary is dominated by the semi-diurnal lunar tide, with a period of 12.42 hours. If a 1:500 model of the estuary is tested, what should be the model tidal period?

(a) 4.0 s (b) 1.5 min (c) 17 min **(d) 33 min** (e) 64 min

FE5.5 A football, meant to be thrown at 60 mi/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E–5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the proper model water velocity?

(a) 7.5 mi/hr (b) 15.0 mi/hr (c) 15.6 mi/hr **(d) 16.5 mi/hr** (e) 30 mi/hr

FE5.6 A football, meant to be thrown at 60 mi/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E–5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the ratio of model force to prototype force?

(a) 3.86:1 (b) 16:1 (c) 32:1 (d) 56.2:1 (e) 64:1

FE5.7 Consider liquid flow of density ρ, viscosity μ, and velocity U over a very small model spillway of length scale L, such that the liquid surface tension coefficient Y is important. The quantity $\rho U^2 L/Y$ in this case is important and is called the

(a) capillary rise (b) Froude No. (c) Prandtl No. **(d) Weber No.** (e) Bond No.

FE5.8 If a stream flowing at velocity U past a body of length L causes a force F on the body which depends only upon U, L and fluid viscosity μ, then F must be proportional to

(a) $\rho UL/\mu$ (b) ρU^2L^2 (c) $\mu U/L$ **(d) μUL** (e) UL/μ

FE5.9 In supersonic wind tunnel testing, if different gases are used, dynamic similarity requires that the model and prototype have the same Mach number and the same

(a) Euler number (b) speed of sound (c) stagnation enthalpy

(d) Froude number **(e) specific heat ratio**

FE5.10 The Reynolds number for a 1-ft-diameter sphere moving at 2.3 mi/hr through seawater (specific gravity 1.027, viscosity 1.07E–3 N·s/m^2) is approximately

(a) 300 (b) 3000 (c) 30,000 **(d) 300,000** (e) 3,000,000

FE5.11 The Ekman number, important in physical oceanography, is a dimensionless combination of μ, L, ρ, and the earth's rotation rate Ω. If the Ekman number is proportional to Ω, it should take the form

$(a)\ \rho\Omega^2L^2/\mu$ $(b)\ \mu\Omega L/\rho$ $(c)\ \rho\Omega L/\mu$ $(d)\ \boldsymbol{\rho\Omega L^2/\mu}$ $(e)\ \rho\Omega/L\mu$

FE5.12 A valid, but probably useless, dimensionless group is given by $(\mu T_o g)/(\Upsilon L\alpha)$, where everything has its usual meaning, except α. What are the dimensions of α?

(a) **$\Theta L^{-1}T^{-1}$** (b) $\Theta L^{-1}T^{-2}$ (c) ΘML^{-1} (d) $\Theta^{-1}LT^{-1}$ (e) ΘLT^{-1}

COMPREHENSIVE PROBLEMS

C5.1 Estimating pipe wall friction is one of the most common tasks in fluids engineering. For long circular, rough pipes in turbulent flow, wall shear τ_w is a function of density ρ, viscosity μ, average velocity V, pipe diameter d, and wall roughness height ε. Thus, functionally, we can write $\tau_w = \text{fcn}(\rho, \mu, V, d, \varepsilon)$. (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has $d = 5$ cm and $\varepsilon = 0.25$ mm. For flow of water at 20°C, measurements show the following values of wall shear stress:

Q (in gal/min)	~	1.5	3.0	6.0	9.0	12.0	14.0
τ_w (in Pa)	~	0.05	0.18	0.37	0.64	0.86	1.25

Plot this data in the dimensionless form suggested by your part (a) and suggest a curve-fit formula. Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are 6 variables and 3 primary dimensions, therefore we expect 3 Pi groups. The traditional choices are:

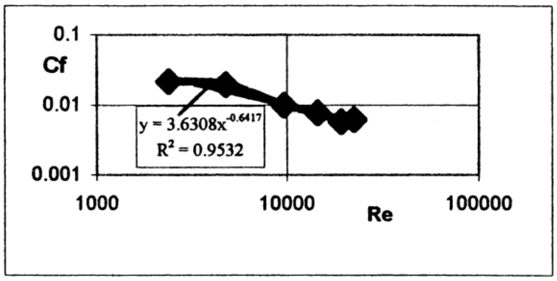
$$\frac{\tau_w}{\rho V^2} = fcn\left(\frac{\rho V d}{\mu}, \frac{\varepsilon}{d}\right) \quad or: \quad \boldsymbol{C_f = fcn\left(\mathbf{Re}, \frac{\varepsilon}{d}\right)} \quad Ans.\text{ (a)}$$

(b) In nondimensionalizing and plotting the above data, we find that $\varepsilon/d = 0.25$ mm/50 mm = 0.005 *for all the data*. Therefore we only plot dimensionless shear versus Reynolds number, using $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s for water. The results are tabulated as follows:

V, m/s	Re	Cf
0.0481972	2405	0.021567
0.0963944	4810	0.019411
0.1927888	9620	0.009975
0.2891832	14430	0.007668
0.3855776	19240	0.005796
0.4498406	22447	0.00619

When plotted on log-log paper, Cf versus Re makes a slightly curved line.

A reasonable power-law curve-fit is shown o the chart: **Cf ≈ $\mathbf{3.63Re^{-0.642}}$** with 95% correlatio *Ans*. (b)
This curve is *only* for the narrow Reynolds numbe range 2000–22000 and a *single* ε/d.



C5.2 When the fluid exiting a nozzle, as in Fig. P3.49, is a *gas*, instead of water, compressibility may be important, especially if upstream pressure p1 is large and exit diameter d2 is small. In this case, the difference (p1 – p2) is no longer controlling, and the gas mass flow, $\dot{m}$, reaches a maximum value which depends upon p1 and d2 and also upon the absolute upstream temperature, T1, and the gas constant, R. Thus, functionally, $\dot{m} = \text{fcn}(p_1, d_2, T_1, R)$. (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has d2 = 1 cm. For flow of air, measurements show the following values of mass flow through the nozzle:

T1 (in °K)	300	300	300	500	800
p1 (in kPa)	200	250	300	300	300
$\dot{m}$ (in kg/s)	0.037	0.046	0.055	0.043	0.034

Plot this data in the dimensionless form suggested by your part (a). Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are *n* = 5 variables and *j* = 4 dimensions (M, L, T, Θ), hence we expect only *n* – *j* = 5 – 4 = **1** Pi group, which turns out to be

$$\Pi_1 = \frac{\dot{\mathbf{m}}\sqrt{\mathbf{RT_1}}}{\mathbf{p_1 d_2^2}} = \textbf{Constant} \quad \textit{Ans. } \text{(a)}$$

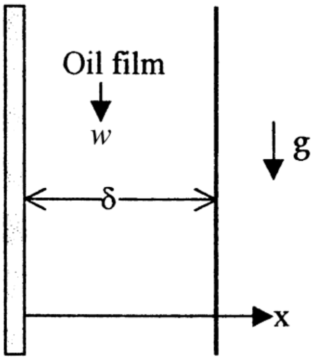
(b) The data should yield a *single* measured value of Π1 for all five points:

T1 (in °K)	~	300	300	300	500	800
$\dot{m}\sqrt{(RT_1)}/(p_1 d_2{}^2)$:		*0.543*	*0.540*	*0.538*	*0.543*	*0.543*

Thus the measured value of Π1 is about **0.543 ± 0.005** (dimensionless), which is very close to the theoretical value of 0.538 developed in Chap. 9 for air, *k* = 1.40. The problem asks you to *plot* this function, but since it is a *constant*, we shall not bother. *Ans.* (a, b)

C5.3 Reconsider the fully-developed drain-ing vertical oil-film problem (see Fig. P4.80) as an exercise in dimensional analysis. Let the vertical velocity be a function only of distance from the plate, fluid properties, gravity, and film thickness. That is, $w = \text{fcn}(x, \rho, \mu, g, \delta)$.
(a) Use the Pi theorem to rewrite this function in terms of dimensionless parameters.

b) Verify that the exact solution from Prob. 4.80 is consistent with your result in part (a).



Solution: There are $n = 6$ variables and $j = 3$ dimensions (M, L, T), hence we expect only $n - j = 6 - 3 = \mathbf{3}$ Pi groups. The author selects (ρ, g, δ) as repeating variables, whence

$$\Pi_1 = \frac{w}{\sqrt{g\delta}}; \quad \Pi_2 = \frac{\mu}{\rho\sqrt{g\delta^3}}; \quad \Pi_3 = \frac{x}{\delta}$$

Thus the expected function is

$$\boldsymbol{\frac{w}{\sqrt{g\delta}} = fcn\left(\frac{\mu}{\rho\sqrt{g\delta^3}}, \frac{x}{\delta}\right)} \quad \textit{Ans. (a)}$$

(b) The exact solution from Problem 4.80 can be written in just this form:

$$w = \frac{\rho g x}{2\mu}(x - 2\delta), \quad or: \quad \underbrace{\frac{w}{\sqrt{g\delta}}}_{\Pi_1}\ \underbrace{\frac{\mu}{\rho\sqrt{g\delta^3}}}_{\Pi_2} = \frac{1}{2}\underbrace{\frac{x}{\delta}}_{\Pi_3}\left(\frac{x}{\delta} - 2\right)$$

Yes, the two forms of dimensionless function are the same. *Ans.* (b)

C5.4 The Taco Inc. Model 4013 centrifugal pump has an impeller of diameter D = 12.95 in. When pumping 20°C water at Ω = 1160 rev/min, the measured flow rate Q and pressure rise Δp are given by the manufacturer as follows:

Q (gal/min)	~	200	300	400	500	600	700
Δp (psi)	~	36	35	34	32	29	23

(a) Assuming that Δp = fcn(ρ, Q, D, Ω), use the Pi theorem to rewrite this function in terms of dimensionless parameters and then plot the given data in dimensionless form. (b) It is desired to use the same pump, running at 900 rev/min, to pump 20°C gasoline at 400 gal/min. According to your dimensionless correlation, what pressure rise Δp is expected, in lbf/in^2?

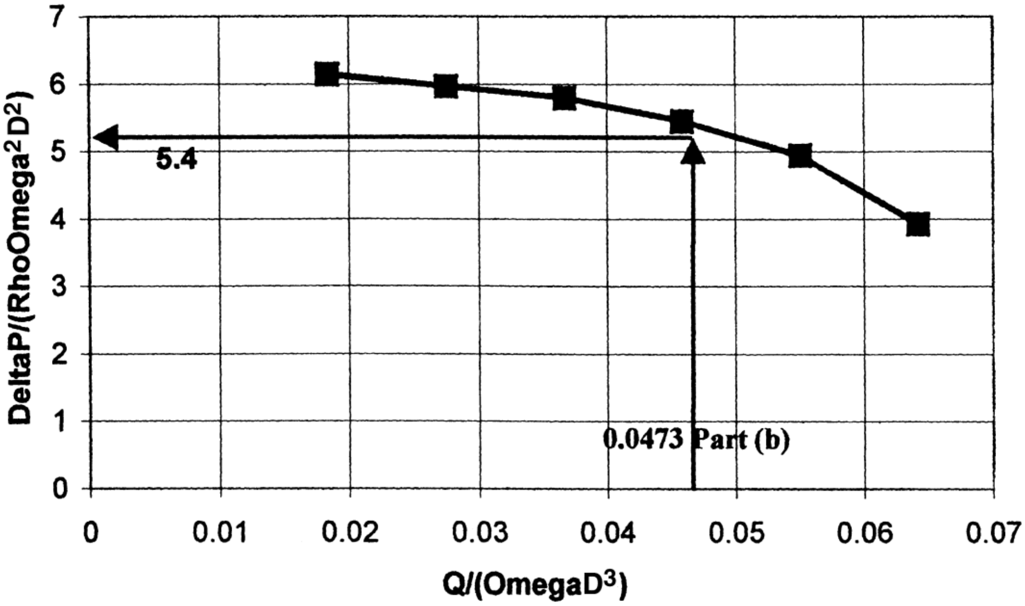
Solution: There are n = 5 variables and j = 3 dimensions (M, L, T), hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups. The author selects (ρ, D, Ω) as repeating variables, whence

$$\Pi_1 = \frac{\Delta p}{\rho\Omega^2 D^2}; \quad \Pi_2 = \frac{Q}{\Omega D^3}, \quad or: \quad \boldsymbol{\frac{\Delta p}{\rho\Omega^2 D^2} = fcn\left(\frac{Q}{\Omega D^3}\right)} \quad \textit{Ans.}\ \text{(a)}$$

Convert the data to this form, using Ω = 19.33 rev/s, D = 1.079 ft, ρ = 1.94 slug/ft^3, and use Δp in lbf/ft^2, not psi, and Q in ft^3/s, not gal/min:

Q (gal/min)	~	200	300	400	500	600	700
$\Delta p/(\rho\Omega^2 D^2)$:		*6.14*	*5.97*	*5.80*	*5.46*	*4.95*	*3.92*
$Q/(\Omega D^3)$:		*0.0183*	*0.0275*	*0.0367*	*0.0458*	*0.0550*	*0.0642*

The dimensionless plot of Π1 versus Π2 is shown below.



(b) The dimensionless chart above is valid for the new conditions, also. Convert 400 gal/min to 0.891 ft^3/s and Ω = 900 rev/min to 15 rev/s. Then evaluate Π2:

$$\Pi_2 = \frac{Q}{\Omega D^3} = \frac{0.891}{15(1.079)^3} = \mathbf{0.0473}$$

This value is entered in the chart above, from which we see that the corresponding value of Π_1 is about **5.4**. For gasoline (Table A-3), ρ = 1.32 slug/ft^3. Then this new running condition with gasoline corresponds to

$$\Pi_2 = 5.4 = \frac{\Delta p}{\rho\Omega^2 D^2} = \frac{\Delta p}{1.32(15)^2(1.079)^2}, \quad \textit{solve for } \Delta p = 1870\ \frac{lbf}{ft^2} = \mathbf{13}\ \frac{\mathbf{lbf}}{\mathbf{in^2}} \quad \textit{Ans.}\text{ (b)}$$

C5.5 Does an automobile radio antenna vibrate in resonance due to vortex shedding? Consider an antenna of length L and diameter D. According to beam-vibration theory [e.g. Kelly [34], p. 401], the first mode natural frequency of a solid circular cantilever beam is ω_n = 3.516$[EI/(\rho AL^4)]^{1/2}$, where E is the modulus of elasticity, I is the area moment of inertia, ρ is the beam material density, and A is the beam cross-section area. (a) Show that ω_n is proportional to the antenna radius R. (b) If the antenna is steel, with L = 60 cm and D = 4 mm, estimate the natural vibration frequency, in Hz. (c) Compare with the shedding frequency if the car moves at 65 mi/h.

Solution: (a) From Fig. 2.13 for a circular cross-section, $A = \pi R^2$ and $I = \pi R^4/4$. Then the natural frequency is predicted to be:

$$\omega_n = 3.516\sqrt{\frac{E\pi R^4/4}{\rho\pi R^2 L^4}} = \mathbf{1.758}\sqrt{\frac{\boldsymbol{E}}{\boldsymbol{\rho}}}\frac{\boldsymbol{R}}{\boldsymbol{L^2}} = Const \times RP \quad Ans.\text{ (a)}$$

(b) For steel, E = 2.1E11 Pa and ρ = 7840 kg/m^3. If L = 60 cm and D = 4 mm, then

$$\omega_n = 1.758\sqrt{\frac{2.1E11}{7840}}\frac{0.002}{0.6^2} \approx 51\ \frac{rad}{s} \approx \mathbf{8\ Hz} \quad Ans.\text{ (b)}$$

(c) For U = 65 mi/h = 29.1 m/s and sea-level air, check Re$_D$ = ρUD/μ = 1.2(29.1)(0.004)/(0.000018) ≈ 7800. From Fig. 5.2*b*, read Strouhal number St ≈ 0.21. Then,

$$\frac{\omega_{shed}D}{2\pi U} = \frac{\omega_{shed}(0.004)}{2\pi(29.1)} \approx 0.21, \quad or: \quad \omega_{shed} \approx 9600\ \frac{rad}{s} \approx \mathbf{1500\ Hz} \quad Ans.\text{ (c)}$$

Thus, for a typical antenna, the shedding frequency is far higher than the natural vibration frequency.

Chapter 6 • Viscous Flow in Ducts

P6.1 An engineer claims that flow of SAE 30W oil, at 20°C, through a 5-cm-diameter smooth pipe at 1 million N/h, is laminar. Do you agree? A million newtons is a lot, so this sounds like an awfully high flow rate.

Solution: For SAE 30W oil at 20°C (Table A.3), take ρ = 891 kg/m^3 and μ = 0.29 kg/m-s. Convert the weight flow rate to volume flow rate in SI units:

$$Q = \frac{\dot{w}}{\rho g} = \frac{(1E6\,N/h)(1/3600\,h/s)}{(891kg/m^3)(9.81m/s^2)} = 0.0318\frac{m^3}{s} = \frac{\pi}{4}(0.05m)^2 V\ ,\ solve\ V = 16.2\frac{m}{s}$$

$$Calculate\ \ \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(891kg/m^3)(16.2m/s)(0.05m)}{0.29\,kg/m-s} \approx \mathbf{2500}\ \ (transitional)$$

This is not high, but **not laminar**. *Ans.* With careful inlet design, low disturbances, and a very smooth wall, it might still be laminar, but **No**, this is *transitional*, not definitely laminar.

P6.2 The present pumping rate of North Slope crude oil through the Alaska Pipeline (see the chapter-opener photo) is about 600,000 barrels per day (1 barrel = 42 U.S. gallons). What would be the maximum rate if the flow were constrained to be *laminar*? Assume that Alaskan crude oil fits Fig. A.1 of the Appendix at 60°C.

Solution: From Fig. A.1 for crude at 60°C, ρ = 0.86(1000) = 860 kg/m^3 and μ = 0.0040 kg/m-s. From Eq. (6.2), the maximum laminar Reynolds number is about 2300. Convert the pipe diameter from 48 inches to 1.22 m. Solve for velocity:

$$\mathrm{Re}_D = 2300 = \frac{\rho VD}{\mu} = \frac{(860kg/m^3)V(1.22m)}{0.0040\,kg/m-s}\ ;\ \ \text{Solve for}\ \ V = 0.00877\ \frac{m}{s}$$

$$Q = VA = (0.00877\,m/s)(\pi/4)(1.22m)^2 = 0.01025m^3/s \times 3600 \times 24 =$$

$$= 886m^3/day \approx \mathbf{5600}\ \text{barrels/day}\ \ \textit{Ans.}$$

P6.3 Following up Prob. P6.2, suppose the Alaska Pipeline were carrying 30 million U.S. gallons per day of SAE 30W oil at 20°C? Estimate the Reynolds number. Is the flow laminar?

Solution: For SAE 30W oil at 20°C, Table A.3, $\rho = 891$ kg/m^3, and $\mu = 0.29$ kg/m-s. Convert the flow rate into cubic meters per second and then find the Reynolds number:

$$Q = 30\text{E}6\,\frac{gal}{day} \div 24 \div 3600 = 347\,\frac{gal}{s} \times 0.0037854 = 1.314\,\frac{m^3}{s} \text{ (or } V = 1.13\,\frac{m}{s}\text{)}$$

$$\text{Re}_D = \frac{4\rho Q}{\pi\mu D} = \frac{4(891)(1.314)}{\pi(0.29)(1.22m)} = \mathbf{4200} \quad \textit{Ans.} \quad (\textit{Transitional}\text{, not laminar})$$

6.4 For flow of SAE 30W oil through a 5-cm-diameter pipe, from Fig. A.1, for what flow rate in m^3/h would we expect transition to turbulence at (a) 20°C and (b) 100°C?

Solution: For SAE 30W oil take $\rho = 891$ kg/m^3 and take $\mu = 0.29$ kg/m·s at 20°C (Table A.3) and 0.01 kg/m-s at 100°C (Fig A.1). Write the critical Reynolds number in terms of flow rate *Q*:

$$\text{(a) } \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.29\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve} \quad Q = 0.0293\ \frac{m^3}{s} = \mathbf{106\ \frac{m^3}{h}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.010\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve} \quad Q = 0.00101\ \frac{m^3}{s} = \mathbf{3.6\ \frac{m^3}{h}} \quad \textit{Ans.}\text{ (b)}$$

6.5 In flow past a body or wall, early transition to turbulence can be induced by placing a trip wire on the wall across the flow, as in Fig. P6.5. If the trip wire in Fig. P6.5 is placed where the local velocity is *U*, it will trigger turbulence if $Ud/\nu = 850$, where *d* is the wire diameter [Ref. 3 of Ch. 6]. If the sphere diameter is 20 cm and transition is observed at $Re_D = 90{,}000$, what is the diameter of the trip wire in mm?

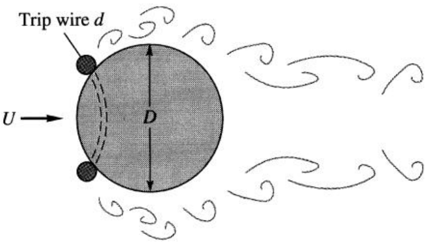


Fig. P6.5

Solution: For the same U and ν,

$$Re_d = \frac{Ud}{\nu} = 850; \quad Re_D = \frac{UD}{\nu} = 90000,$$

$$\text{or} \quad d = D\frac{Re_d}{Re_D} = (200\text{ mm})\left(\frac{850}{90000}\right) \approx \mathbf{1.9\ mm}$$

P6.6 For flow of a uniform stream parallel to a sharp flat plate, transition to a turbulent boundary layer on the plate may occur at $Re_x = \rho Ux/\mu \approx 1E6$, where *U* is the approach velocity and *x* is distance along the plate. If $U = 2.5$ m/s, determine the distance *x* for the following fluids at 20°C and 1 atm: (*a*) hydrogen; (*b*) air; (*c*) gasoline; (*d*) water; (*e*) mercury; and (*f*) glycerin.

Solution: We are to calculate $x = (Re_x)(\mu)/(\rho U) = (1E6)(\mu)/[\rho\,(2.5\text{m/s})]$. Make a table:

FLUID	ρ – kg/m^3	μ - kg/m-s	x - meters
Hydrogen	0.00839	9.05E-5	43.
Air	1.205	1.80E-5	6.0
Gasoline	680	2.92E-4	0.17
Water	998	0.0010	0.40

Mercury	13,550	1.56E-3	0.046
Glycerin	1260	1.49	470.

Clearly there are vast differences between fluid properties and their effects on flows.

6.7 Cola, approximated as pure water at 20°C, is to fill an 8-oz container (1 U.S. gal = 128 fl oz) through a 5-mm-diameter tube. Estimate the minimum filling time if the tube flow is to remain laminar. For what cola (water) temperature would this minimum time be 1 min?

Solution: For cola "water", take $\rho = 998\ \text{kg/m}^3$ and $\mu = 0.001$ kg/m·s. Convert 8 fluid ounces = $(8/128)(231\ \text{in}^3) \approx 2.37\text{E}{-4}\ \text{m}^3$. Then, if we assume transition at Re = 2300,

$$\text{Re}_{\text{crit}} = 2300 = \frac{\rho V D}{\mu} = \frac{4\rho Q}{\pi\mu D}, \quad \text{or:} \quad Q_{\text{crit}} = \frac{2300\pi(0.001)(0.005)}{4(998)} \approx 9.05\text{E}{-6}\ \frac{\text{m}^3}{\text{s}}$$

$$\text{Then} \quad \Delta t_{\text{fill}} = \upsilon/Q = 2.37\text{E}{-4}/9.05\text{E}{-6} \approx \mathbf{26\ s} \quad \textit{Ans.}\ \text{(a)}$$

(b) We fill in exactly one minute if $Q_{\text{crit}} = 2.37\text{E}{-4}/60 = 3.94\text{E}{-6}\ \text{m}^3/\text{s}$. Then

$$Q_{\text{crit}} = 3.94\text{E}{-6}\ \frac{\text{m}^3}{\text{s}} = \frac{2300\pi\nu D}{4} \quad \text{if } \nu_{\text{water}} \approx 4.36\text{E}{-7}\ \text{m}^2/\text{s}$$

From Table A-1, this kinematic viscosity occurs at **T ≈ 66°C** *Ans.* (b)

6.8 When water at 20°C ($\rho = 998\ \text{kg/m}^3$, $\mu = 0.001$ kg/m·s) flows through an 8-cm-diameter pipe, the wall shear stress is 72 Pa. What is the axial pressure gradient ($\partial p/\partial x$) if the pipe is (a) horizontal; and (b) vertical with the flow *up*?

Solution: Equation (6.9b) applies in both cases, noting that τ_w is negative:

$$\text{(a)} \quad \textit{Horizontal:} \quad \frac{dp}{dx} = \frac{2\tau_w}{R} = \frac{2(-72\ Pa)}{0.04\ m} = \mathbf{-3600}\ \frac{\mathbf{Pa}}{\mathbf{m}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)} \quad \textit{Vertical, up:} \quad \frac{dp}{dx} = \frac{2\tau_w}{R} - \rho g \cancelto{1}{\frac{dz}{dx}} = -3600 - 998(9.81) = \mathbf{-13{,}400}\ \frac{\mathbf{Pa}}{\mathbf{m}} \quad \textit{Ans.}\ \text{(b)}$$

6.9 A light liquid (ρ = 950 kg/m^3) flows at an average velocity of 10 m/s through a horizontal smooth tube of diameter 5 cm. The fluid pressure is measured at 1-m intervals along the pipe, as follows:

x, m:	0	1	2	3	4	5	6
p, kPa:	304	273	255	240	226	213	200

Estimate (a) the total head loss, in meters; (b) the wall shear stress in the fully developed section of the pipe; and (c) the overall friction factor.

Solution: As sketched in Fig. 6.6 of the text, the pressure drops fast in the entrance region (31 kPa in the first meter) and levels off to a linear decrease in the "fully developed" region (13 kPa/m for this data).

(a) The overall head loss, for $\Delta z = 0$, is defined by Eq. (6.8) of the text:

$$h_f = \frac{\Delta p}{\rho g} = \frac{304{,}000 - 200{,}000\ Pa}{(950\ kg/m^3)(9.81\ m/s^2)} = \mathbf{11.2\ m} \quad \textit{Ans.}\text{ (a)}$$

(b) The wall shear stress in the fully-developed region is defined by Eq. (6.9*b*):

$$\left.\frac{\Delta p}{\Delta L}\right|_{fully\ developed} = \frac{13000\ Pa}{1\ m} = \frac{4\tau_w}{d} = \frac{4\tau_w}{0.05\ m}, \quad \text{solve for } \tau_w = \mathbf{163\ Pa} \quad \textit{Ans.}\text{ (b)}$$

(c) The overall friction factor is defined by Eq. (6.10) of the text:

$$f_{overall} = h_{f,overall}\frac{d}{L}\frac{2g}{V^2} = (11.2\ m)\left(\frac{0.05\ m}{6\ m}\right)\frac{2(9.81\ m/s^2)}{(10\ m/s)^2} = \mathbf{0.0182} \quad \textit{Ans.}\text{ (c)}$$

NOTE: The fully-developed friction factor is only 0.0137.

6.10 Water at 20°C (ρ = 998 kg/m^3) flows through an inclined 8-cm-diameter pipe. At sections A and B, p_A = 186 kPa, V_A = 3.2 m/s, z_A = 24.5 m, while p_B = 260 kPa, V_B = 3.2 m/s, and z_B = 9.1 m. Which way is the flow going? What is the head loss?

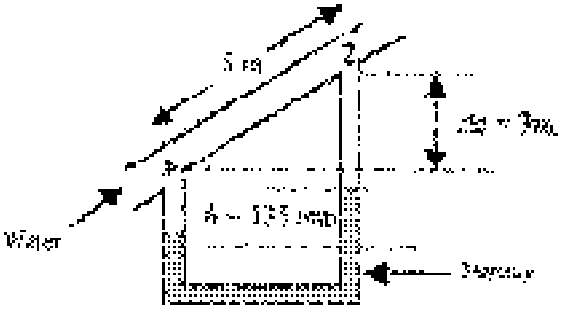
Solution: Guess that the flow is from A to B and write the steady flow energy equation:

$$\frac{p_A}{\rho g}+\cancel{\frac{V_A^2}{2g}}+z_A=\frac{p_B}{\rho g}+\cancel{\frac{V_B^2}{2g}}+z_B+h_f,\quad \text{or:}\quad \frac{186000}{9790}+24.5=\frac{260000}{9790}+9.1+h_f,$$

or: $43.50 = 35.66 + h_f$, solve: $\mathbf{h_f = +7.84\ m}$ *Yes, flow is from A to B.* *Ans.* (a, b)

6.11 Water at 20°C flows upward at 4 m/s in a 6-cm-diameter pipe. The pipe length between points 1 and 2 is 5 m, and point 2 is 3 m higher. A mercury manometer, connected between 1 and 2, has a reading h = 135 mm, with $p1$ higher. (a) What is the pressure change ($p1 - p2$)? (b) What is the head loss, in meters? (c) Is the manometer reading proportional to head loss? Explain. (d) What is the friction factor of the flow?

Solution: A sketch of this situation is shown at right. By moving through the manometer, we obtain the pressure change between points 1 and 2, which we compare with Eq. (6.9b):



$$p_1+\gamma_w h-\gamma_m h-\gamma_w\Delta z=p_2,$$

$$\text{or:}\quad p_1-p_2=\left(133100-9790\ \frac{N}{m^3}\right)(0.135\ m)+\left(9790\ \frac{N}{m^3}\right)(3\ m)$$

$$=16650+29370=\mathbf{46{,}000\ Pa}\quad \textit{Ans.}\,\text{(a)}$$

$$\textit{From Eq. (6.9b),}\quad h_f=\frac{\Delta p}{\gamma_w}-\Delta z=\frac{46000\ Pa}{9790\ N/m^3}-3\ m=4.7-3.0=\mathbf{1.7\ m}\quad \textit{Ans.}\ \text{(b)}$$

$$\textit{The friction factor is}\quad f=h_f\frac{d}{L}\frac{2g}{V^2}=(1.7\ m)\left(\frac{0.06\ m}{5\ m}\right)\frac{2(9.81\ m/s^2)}{(4\ m/s)^2}=\mathbf{0.025}\quad \textit{Ans.}\,\text{(d)}$$

By comparing the manometer relation to the head-loss relation above, we find that:

$$h_f=\frac{(\gamma_m-\gamma_w)}{\gamma_w}h$$ ***and thus head loss*** **is** ***proportional to manometer reading.*** *Ans.* (c)

NOTE: IN PROBLEMS 6.12 TO 6.99, MINOR LOSSES ARE NEGLECTED.

6.12 A 5-mm-diameter capillary tube is used as a viscometer for oils. When the flow rate is 0.071 m^3/h, the measured pressure drop per unit length is 375 kPa/m. Estimate the viscosity of the fluid. Is the flow laminar? Can you also estimate the density of the fluid?

Solution: Assume laminar flow and use the pressure drop formula (6.12):

$$\frac{\Delta p}{L} \overset{?}{=} \frac{8Q\mu}{\pi R^4}, \quad \text{or:} \quad 375000\ \frac{Pa}{m} \overset{?}{=} \frac{8(0.071/3600)\mu}{\pi(0.0025)^4}, \quad \text{solve } \mu \approx \mathbf{0.292\ \frac{kg}{m{\cdot}s}} \quad \textit{Ans.}$$

$$\text{Guessing} \quad \rho_{oil} \approx 900\ \frac{kg}{m^3},$$

$$\text{check } \mathrm{Re} = \frac{4\rho Q}{\pi\mu d} = \frac{4(900)(0.071/3600)}{\pi(0.292)(0.005)} \approx \mathbf{16} \quad \textbf{OK, laminar} \quad \textit{Ans.}$$

It is not possible to find density from this data, laminar pipe flow is independent of density.

6.13 A soda straw is 20 cm long and 2 mm in diameter. It delivers cold cola, approximated as water at 10°C, at a rate of 3 cm^3/s. (a) What is the head loss through the straw? What is the axial pressure gradient $\partial p/\partial x$ if the flow is (b) vertically up or (c) horizontal? Can the human lung deliver this much flow?

Solution: For water at 10°C, take $\rho = 1000\ kg/m^3$ and $\mu = 1.307E{-}3$ kg/m·s. Check Re:

$$\mathrm{Re} = \frac{4\rho Q}{\pi\mu d} = \frac{4(1000)(3E{-}6\ m^3/s)}{\pi(1.307E{-}3)(0.002)} = 1460 \quad \text{(OK, laminar flow)}$$

Then, from Eq. (6.12), $h_f = \dfrac{128\mu LQ}{\pi\rho g d^4} = \dfrac{128(1.307E{-}3)(0.2)(3E{-}6)}{\pi(1000)(9.81)(0.002)^4} \approx \mathbf{0.204\ m}$ *Ans.* (a)

If the straw is *horizontal*, then the pressure gradient is simply due to the head loss:

$$\left.\frac{\Delta p}{L}\right|_{horiz} = \frac{\rho g h_f}{L} = \frac{1000(9.81)(0.204\ m)}{0.2\ m} \approx \mathbf{9980\ \frac{Pa}{m}} \quad \textit{Ans. (c)}$$

If the straw is *vertical*, with flow *up*, the head loss and elevation change add together:

$$\left.\frac{\Delta p}{L}\right|_{vertical} = \frac{\rho g(h_f + \Delta z)}{L} = \frac{1000(9.81)(0.204 + 0.2)}{0.2} \approx \mathbf{19800\ \frac{Pa}{m}} \quad \textit{Ans. (b)}$$

The human lung can certainly deliver case (c) and strong lungs can develop case (b) also.

6.14 Water at 20°C is to be siphoned through a tube 1 m long and 2 mm in diameter, as in Fig. P6.14. Is there any height *H* for which the flow might not be laminar? What is the flow rate if *H* = 50 cm? Neglect the tube curvature.

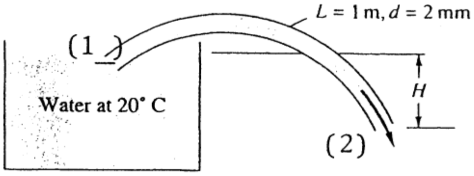


Fig. P6.14

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Write the steady flow energy equation between points 1 and 2 above:

$$\frac{p_{atm}}{\rho g}+\frac{0^2}{2g}+z_1=\frac{p_{atm}}{\rho g}+\frac{V_{tube}^2}{2g}+z_2+h_f,\quad \text{or:}\quad H-\frac{V^2}{2g}=h_f=\frac{32\mu L}{\rho g d^2}V \qquad (1)$$

$$\text{Enter data in Eq. (1):}\quad 0.5-\frac{V^2}{2(9.81)}=\frac{32(0.001)(1.0)V}{(998)(9.81)(0.002)^2},\quad \text{solve}\ V\approx 0.590\ \frac{m}{s}$$

Equation (1) is quadratic in V and has only one positive root. The siphon flow rate is

$$Q_{H=50\text{ cm}}=\frac{\pi}{4}(0.002)^2(0.590)=1.85\text{E–}6\ \frac{m^3}{s}\approx \mathbf{0.0067\ \frac{m^3}{h}}\quad \textbf{if } \boldsymbol{H}\textbf{ = 50 cm}\quad \textit{Ans.}$$

$$\text{Check}\quad Re=(998)(0.590)(0.002)/(0.001)\approx 1180\ \text{(OK, laminar flow)}$$

It is possible to approach Re ≈ 2000 (possible transition to turbulent flow) for H < 1 m, for the case of the siphon bent over nearly vertical. We obtain **Re = 2000 at H ≈ 0.87 m**.

6.15 Professor Gordon Holloway and his students at the University of New Brunswick went to a fast-food emporium and tried to drink chocolate shakes (ρ ≈ 1200 kg/m^3, μ ≈ 6 kg/m·s) through fat straws 8 mm in diameter and 30 cm long. (a) Verify that their human lungs, which can develop approximately 3000 Pa of vacuum pressure, would be unable to drink the milkshake through the vertical straw. (b) A student cut 15 cm from his straw and proceeded to drink happily. What rate of milkshake flow was produced by this strategy?

Solution: (a) Assume the straw is barely inserted into the milkshake. Then the energy equation predicts

$$\frac{p_1}{\rho g}=\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}=\frac{V_2^2}{2g}+z_2+h_f$$

$$=0+0+0=\frac{(-3000\ Pa)}{(1200\ kg/m^3)(9.81\ m/s^2)}+\frac{V_{tube}^2}{2g}+0.3\ m+h_f$$

$$\textit{Solve for}\ \ \boldsymbol{h_f=0.255\ m-0.3\ m-\frac{V_{tube}^2}{2g}<0}\quad \textbf{\textit{which is impossible}}\quad \textit{Ans.}\ \text{(a)}$$

(b) By cutting off 15 cm of vertical length and assuming laminar flow, we obtain a new energy equation

$$h_f = 0.255 - 0.15 - \frac{V^2}{2g} = \frac{32\mu LV}{\rho g d^2} = 0.105\ m - \frac{V^2}{2(9.81)} = \frac{32(6.0)(0.15)V}{(1200)(9.81)(0.008)^2} = 38.23V$$

$$\text{Solve for}\quad V = 0.00275\ m/s,\quad Q = AV = (\pi/4)(0.008)^2(0.00275)$$

$$Q = 1.4E{-}7\ \frac{m^3}{s} = \mathbf{0.14\ \frac{cm^3}{s}}\quad \textit{Ans.}\ \text{(b)}$$

Check the Reynolds number: $\text{Re}_d = \rho Vd/\mu = (1200)(0.00275)(0.008)/(6) = 0.0044$ (Laminar).

P6.16 Fluid flows steadily, at volume rate Q, through a large horizontal pipe and then divides into two small pipes, the larger of which has an inside diameter of 25 mm and carries three times the flow of the smaller pipe. Both small pipes have the same length and pressure drop. If all flows are laminar, estimate the diameter of the smaller pipe.

Solution: For laminar flow in a horizontal pipe, the volume flow is a simple formula, Eq. (6.12):

$$Q_{\text{laminar}} = \frac{\pi d^4}{128\mu}\left(\frac{\Delta p}{L}\right)$$

Since Δp, L, and μ are the same in the two small pipes, it follows that the flows simply vary as the 4th power of their diameters. Let pipe 1 have the 25-mm diameter. Then we compute

$$Q_1 = (const)(d_1^4) = 3Q_2 = 3(const)(d_2^4)$$

$$\textit{Thus}\quad d_2 = \frac{d_1}{3^{1/4}} = \frac{25\,mm}{1.316} = \mathbf{19.0}\,mm\quad \textit{Ans.}$$

6.17 A *capillary viscometer* measures the time required for a specified volume υ of liquid to flow through a small-bore glass tube, as in Fig. P6.17. This transit time is then correlated with fluid viscosity. For the system shown, (a) derive an approximate formula for the time required, assuming laminar flow with no entrance and exit losses. (b) If L = 12 cm, l = 2 cm, υ = 8 cm^3, and the fluid is water at 20°C, what capillary diameter D will result in a transit time t of 6 seconds?

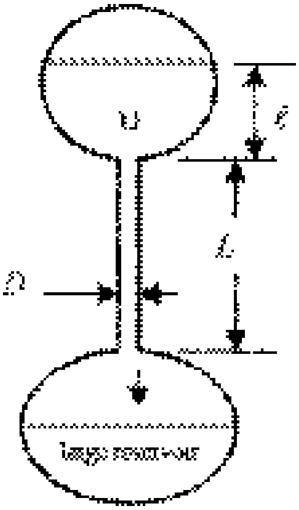


Fig. P6.17

Solution: (a) Assume no pressure drop and neglect velocity heads. The energy equation reduces to:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=0+0+(L+l)=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f=0+0+0+h_f,\quad \textbf{or}\text{:}\quad h_f\approx L+l$$

$$\text{For laminar flow,}\quad h_f=\frac{128\mu LQ}{\pi\rho g d^4}\quad \text{and, for uniform draining,}\quad Q=\frac{\upsilon}{\Delta t}$$

$$\text{Solve for}\quad \boldsymbol{\Delta t=\frac{128\mu L\upsilon}{\pi\rho g d^4(L+l)}}\quad \textit{Ans.}\text{ (a)}$$

(b) Apply to Δt = 6 s. For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Formula (a) predicts:

$$\Delta t=6\ s=\frac{128(0.001\ kg/m\cdot s)(0.12\ m)(8E\text{–}6\ m^3)}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4(0.12+0.02\ m)},$$

Solve for **d ≈ 0.0015 m** *Ans.* (b)

6.18 To determine the viscosity of a liquid of specific gravity 0.95, you fill, to a depth of 12 cm, a large container which drains through a 30-cm-long vertical tube attached to the bottom. The tube diameter is 2 mm, and the rate of draining is found to be 1.9 cm^3/s. What is your estimate of the fluid viscosity? Is the tube flow laminar?

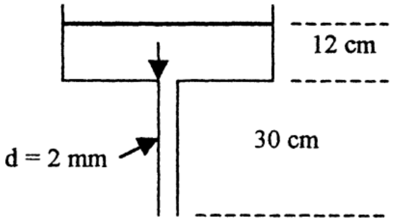


Fig. P6.18

Solution: The known flow rate and diameter enable us to find the velocity in the tube:

$$V = \frac{Q}{A} = \frac{1.9E\text{–}6\ m^3/s}{(\pi/4)(0.002\ m)^2} = 0.605\ \frac{m}{s}$$

Evaluate $\rho_{\text{liquid}} = 0.95(998) = 948\ \mathrm{kg/m^3}$. Write the energy equation between the top surface and the tube exit:

$$\cancel{\frac{p_a}{\rho g}} = \cancel{\frac{V_{top}^2}{2g}} + z_{top} = \cancel{\frac{p_a}{\rho g}} + \frac{V^2}{2g} + 0 + h_f,$$

$$or\colon\quad 0.42 = \frac{V^2}{2g} + \frac{32\mu LV}{\rho g d^2} = \frac{(0.605)^2}{2(9.81)} + \frac{32\mu(0.3)(0.605)}{948(9.81)(0.002)^2}$$

Note that “L” in this expression is the tube length only (L = 30 cm).

$$\text{Solve for } \mu = \mathbf{0.00257}\ \frac{\mathbf{kg}}{\mathbf{m\cdot s}}\quad (laminar\ flow)\quad Ans.$$

$$Re_d = \frac{\rho V d}{\mu} = \frac{948(0.605)(0.002)}{0.00257} = 446\ (laminar)$$

6.19 An oil (SG = 0.9) issues from the pipe in Fig. P6.19 at $Q = 35\ \mathrm{ft^3/h}$. What is the kinematic viscosity of the oil in $\mathrm{ft^3/s}$? Is the flow laminar?

Solution: Apply steady-flow energy:

$$\frac{p_{atm}}{\rho g}+\frac{0^2}{2g}+z_1=\frac{p_{atm}}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f,$$

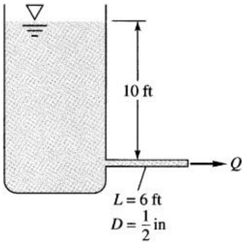


Fig. P6.19

where $V_2=\frac{Q}{A}=\frac{35/3600}{\pi(0.25/12)^2}\approx 7.13\ \frac{ft}{s}$

$$\text{Solve}\quad h_f=z_1-z_2-\frac{V_2^2}{2g}=10-\frac{(7.13)^2}{2(32.2)}=9.21\text{ ft}$$

Assuming laminar pipe flow, use Eq. (6.12) to relate head loss to viscosity:

$$h_f=9.21\text{ ft}=\frac{128\nu LQ}{\pi g d^4}=\frac{128(6)(35/3600)\nu}{\pi(32.2)(0.5/12)^4},\quad\text{solve}\quad \nu=\frac{\mu}{\rho}\approx \mathbf{3.76E\text{–}4}\ \frac{\mathbf{ft^2}}{\mathbf{s}}\qquad \textit{Ans.}$$

$$\text{Check Re}=4Q/(\pi\nu d)=4(35/3600)/[\pi(3.76E\text{–}4)(0.5/12)]\approx 790\ \text{(OK, laminar)}$$

P6.20 The oil tanks in Tinyland are only 160 cm high, and they discharge to the Tinyland oil truck through a smooth tube 4 mm in diameter and 55 cm long. The tube exit is open to the atmosphere and 145 cm below the tank surface. The fluid is medium fuel oil, $\rho = 850$ kg/m^3 and $\mu = 0.11$ kg/m-s. Estimate the oil flow rate in cm^3/h.

Solution: The steady flow energy equation, with 1 at the tank surface and 2 the exit, gives

$$z_1=z_2+\frac{\alpha V^2}{2g}+f\frac{L}{d}\frac{V^2}{2g},\ or:\ \Delta z=1.45m=\frac{V^2}{2g}(2.0+\frac{64}{\text{Re}_d}\frac{0.55m}{0.004m}),\ \text{Re}_d=\frac{850V(0.004)}{0.11}$$

We have taken the energy correction factor $\alpha = 2.0$ for laminar pipe flow.

Solve for $V = 0.10$ m/s, $\mathrm{Re}_d = 3.1$ (laminar), $Q = 1.26\mathrm{E}\text{-}6\ \mathrm{m^3/s} \approx$ **4500 cm³/h**. *Ans.*

The exit jet energy $\alpha V^2/2g$ is properly included but is very small (0.001 m).

6.21 In Tinyland, houses are less than a foot high! The rainfall is laminar! The drainpipe in Fig. P6.21 is only 2 mm in diameter. (a) When the gutter is full, what is the rate of draining? (b) The gutter is designed for a sudden rainstorm of up to 5 mm per hour. For this condition, what is the maximum roof area that can be drained successfully? (c) What is Red?

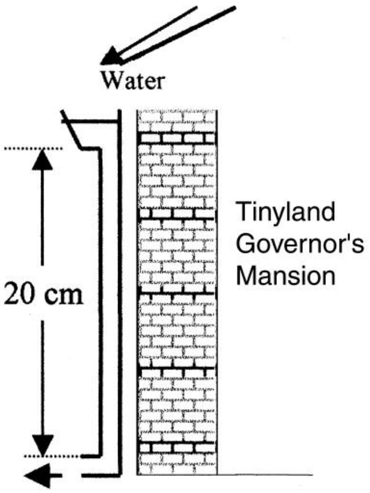


Fig. P6.21

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad \text{where } h_{f,\text{laminar}} = \frac{32\mu L V}{\rho g d^2}$$

For water, take $\rho = 998\ \mathrm{kg/m^3}$ and $\mu = 0.001$ kg/m·s. (a) With Δz known, this is a quadratic equation for the pipe velocity V:

$$0.2\ m = \frac{V^2}{2(9.81\ m/s^2)} + \frac{32(0.001\ kg/m{\cdot}s)(0.2\ m)V}{(998\ kg/m^3)(9.81\ m/s^2)(0.002\ m)^2},$$

$$\text{or:}\quad 0.051V^2 + 0.1634V - 0.2 = 0, \quad \textit{Solve for}\ V = 0.945\ \frac{m}{s},$$

$$Q = \frac{\pi}{4}(0.002\ m)^2\left(0.945\ \frac{m}{s}\right) = 2.97E{-}6\ \frac{m^3}{s} = \mathbf{0.0107\ \frac{m^3}{h}} \quad \textit{Ans.}\ \text{(a)}$$

(b) The roof area needed for maximum rainfall is 0.0107 m³/h ÷ 0.005 m/h = **2.14 m²**. *Ans.* (b)
(c) The Reynolds number of the gutter is Red = (998)(0.945)(0.002)/(0.001) = **1890** *laminar. Ans.* (c)

6.22 A steady push on the piston in Fig. P6.22 causes a flow rate $Q = 0.15\ \text{cm}^3/\text{s}$ through the needle. The fluid has $\rho = 900\ \text{kg/m}^3$ and $\mu = 0.002\ \text{kg/(m·s)}$. What force F is required to maintain the flow?

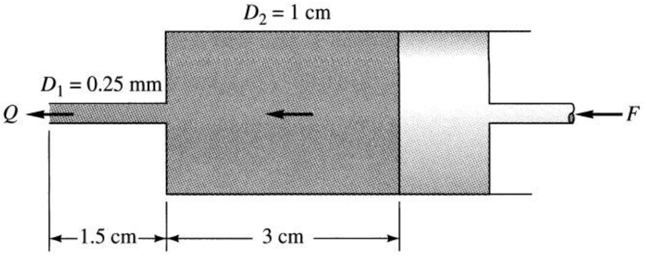


Fig. P6.22

Solution: Determine the velocity of exit from the needle and then apply the steady-flow energy equation:

$$V_1 = \frac{Q}{A} = \frac{0.15}{(\pi/4)(0.025)^2} = 306\ \text{cm/s}$$

$$\text{Energy:}\quad \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{f1} + h_{f2}, \quad \text{with } z_1 = z_2,\ V_2 \approx 0,\ h_{f2} \approx 0$$

Assume laminar flow for the head loss and compute the pressure difference on the piston:

$$\frac{p_2 - p_1}{\rho g} = h_{f1} + \frac{V_1^2}{2g} = \frac{32(0.002)(0.015)(3.06)}{(900)(9.81)(0.00025)^2} + \frac{(3.06)^2}{2(9.81)} \approx 5.79\ \text{m}$$

$$\text{Then}\quad F = \Delta p A_{\text{piston}} = (900)(9.81)(5.79)\frac{\pi}{4}(0.01)^2 \approx \mathbf{4.0\ N} \quad \textit{Ans.}$$

6.23 SAE 10 oil at 20°C flows in a vertical pipe of diameter 2.5 cm. It is found that the pressure is constant throughout the fluid. What is the oil flow rate in m^3/h? Is the flow up or down?

Solution: For SAE 10 oil, take $\rho = 870\ \text{kg/m}^3$ and $\mu = 0.104$ kg/m·s. Write the energy equation between point 1 upstream and point 2 downstream:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f, \quad \text{with } p_1 = p_2 \text{ and } V_1 = V_2$$

Thus $h_f = z_1 - z_2 > 0$ by definition. Therefore, **flow is down**. *Ans.*

While flowing down, the pressure drop due to friction exactly balances the pressure rise due to gravity.

Assuming laminar flow and noting that $\Delta z = L$, the pipe length, we get

$$h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \Delta z = L,$$

$$\text{or:}\quad Q = \frac{\pi(8.70)(9.81)(0.025)^4}{128(0.104)} = 7.87E\text{–}4\ \frac{m^3}{s} = \mathbf{2.83\ \frac{m^3}{h}} \quad \textit{Ans.}$$

6.24 Two tanks of water at 20°C are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. (a) Estimate the flow rate in m^3/h. Is the flow laminar? (b) For what tube diameter will Re_d be 500?

Solution: For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. (a) Both tank surfaces are at atmospheric pressure and have negligible velocity. The energy equation, when neglecting minor losses, reduces to:

$$\Delta z = 0.3\ m = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)(0.004\ m)^4}$$

$$\text{Solve for}\quad Q = 5.3E\text{–}6\ \frac{m^3}{s} = \mathbf{0.019\ \frac{m^3}{h}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Check}\quad \mathrm{Re}_d = 4\rho Q/(\pi\mu d) = 4(998)(5.3E\text{–}6)/[\pi(0.001)(0.004)]$$

$$\mathbf{Re_d = 1675}\ \ \textbf{\textit{laminar.}}\quad \textit{Ans.}\ \text{(a)}$$

(b) If $\mathrm{Re}_d = 500 = 4\rho Q/(\pi\mu d)$ and $\Delta z = h_f$, we can solve for both Q and d:

$$\mathrm{Re}_d = 500 = \frac{4(998\ kg/m^3)Q}{\pi(0.001\ kg/m{\cdot}s)d}, \quad \textbf{\textit{or}} \quad Q = 0.000394d$$

$$h_f = 0.3\ m = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4}, \quad or \quad Q = 20600d^4$$

$$\textit{Combine these two to solve for}\quad Q = 1.05E\text{–}6\ m^3/s \quad \textit{and} \quad \mathbf{d = 2.67\ mm} \quad \textit{Ans.}\ \text{(b)}$$

6.25 For the configuration shown in Fig. P6.25, the fluid is ethyl alcohol at 20°C, and the tanks are very wide. Find the flow rate that occurs, in m^3/h. Is the flow laminar?

Solution: For ethanol, take $\rho = 789\ kg/m^3$ and $\mu = 0.0012\ kg/m{\cdot}s$. Write the energy equation from upper free surface (1) to lower free surface (2):

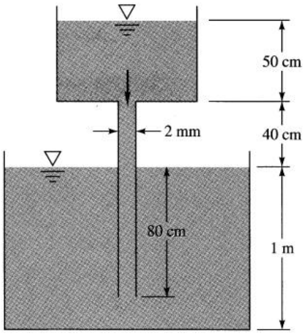


Fig. P6.25

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f,\quad \text{with } p_1=p_2 \text{ and } V_1\approx V_2\approx 0$$

$$\text{Then}\quad h_f = z_1 - z_2 = 0.9\text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.0012)(1.2\text{ m})Q}{\pi(789)(9.81)(0.002)^4}$$

$$\text{Solve for}\quad Q \approx 1.90\text{E–}6\ m^3/s = \mathbf{0.00684\ m^3/h.}\quad \textit{Ans.}$$

Check the Reynolds number $Re = 4\rho Q/(\pi\mu d) \approx 795$ – **OK, laminar flow.**

P6.26 Two oil tanks are connected by two 9-m-long pipes, as in Fig. P6.26. Pipe 1 is 5 cm in diameter and is 6 m higher than pipe 2. It is found that the flow rate in pipe 2 is twice as large as the flow in pipe 1. (*a*) What is the diameter

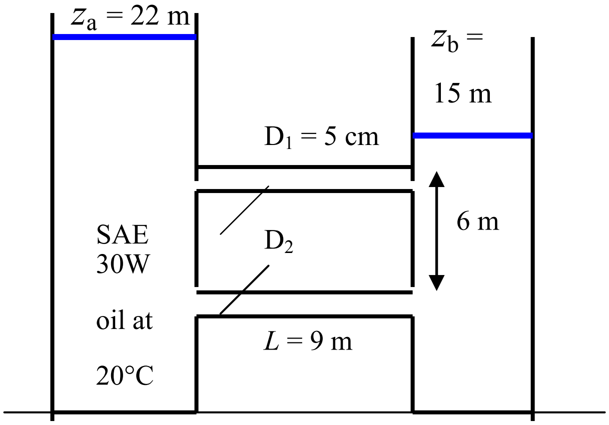


Fig. P6.26

of pipe 2? (*b*) Are both pipe flows laminar?

(*c*) What is the flow rate in pipe 2 (m^3/s)?

Neglect minor losses.

Solution: (*a*) If we *know* the flows are laminar, and (L, ρ, μ) are constant, then $Q \propto D^4$:

$$\text{From Eq.(6.12), } \frac{Q_2}{Q_1} = 2.0 = \left(\frac{D_2}{D_1}\right)^4, \text{ hence } D_2 = (5\,cm)(2.0)^{1/4} = \mathbf{5.95cm} \quad \textit{Ans.(a)}$$

We will check later in part (*b*) to be sure the flows are laminar. [Placing pipe 1 six meters higher was meant to be a confusing trick, since both pipes have exactly the same head loss and Δz.] (*c*) Find the flow rate first and then backtrack to the Reynolds numbers. For SAE 30W oil at 20°C (Table A.3), take $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/m-s. From the energy equation, with $V_1 = V_2 = 0$, and Eq. (6.12) for the laminar head loss,

$$\Delta z = 22 - 15 = 7\,m = h_f = \frac{128\mu L Q}{\pi\rho g D_2^4} = \frac{128(0.29\,kg/m-s)(9\,m)\,Q_2}{\pi(891\,kg/m^3)(9.81\,m/s^2)(0.0595\,m)^4}$$

$$\text{Solve for} \quad Q_2 = \mathbf{0.0072\,m^3/s} \qquad \textit{Ans.(c)}$$

In a similar manner, insert $D_1 = 0.05$m and compute $Q_1 = \mathbf{0.0036}$ m^3/s = $(1/2)Q_1$.

(*b*) Now go back and compute the Reynolds numbers:

$$\mathrm{Re}_1 = \frac{4\rho Q_1}{\pi\mu D_1} = \frac{4(891)(0.0036)}{\pi(0.29)(0.050)} = \mathbf{281} \;;\; \mathrm{Re}_2 = \frac{4\rho Q_2}{\pi\mu D_2} = \frac{4(891)(0.0072)}{\pi(0.29)(0.0595)} = \mathbf{473} \quad \textit{Ans.(b)}$$

Both flows are **laminar**, which verifies our flashy calculation in part (*a*).

6.27 Let us attack Prob. 6.25 in symbolic fashion, using Fig. P6.27. All parameters are constant except the upper tank depth *Z*(*t*). Find an expression for the flow rate *Q*(*t*) as a function of *Z*(*t*). Set up a differential equation, and solve for the time *t*0 to drain the upper tank completely. Assume quasi-steady laminar flow.

Solution: The energy equation of Prob. 6.25, using symbols only, is combined with a control-volume mass balance for the tank to give the basic differential equation for Z(t):

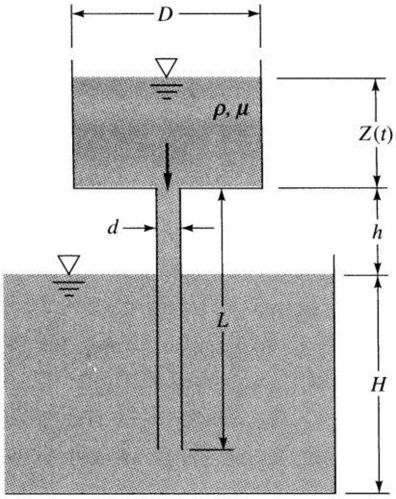


Fig. P6.27

$$\text{energy:}\quad h_f = \frac{32\mu LV}{\rho g d^2} = h + Z; \quad \text{mass balance:}\quad \frac{d}{dt}\left[\frac{\pi}{4}D^2 Z + \frac{\pi}{4}d^2 L\right] = -Q = -\frac{\pi}{4}d^2 V,$$

$$\text{or:}\quad \frac{\pi}{4}D^2\frac{dZ}{dt} = -\frac{\pi}{4}d^2 V, \quad \text{where } V = \frac{\rho g d^2}{32\mu L}(h+Z)$$

Separate the variables and integrate, combining all the constants into a single "C":

$$\int_{Z_o}^{Z}\frac{dZ}{h+Z} = -C\int_0^t dt, \quad \text{or:}\quad \mathbf{Z = (h+Z_o)e^{-Ct} - h}, \quad \text{where } \mathbf{C = \frac{\rho g d^4}{32\mu L D^2}} \quad \textit{Ans.}$$

$$\text{Tank drains completely when } Z = 0, \text{ at } \mathbf{t_0 = \frac{1}{C}\ln\left(1+\frac{Z_o}{h}\right)} \quad \textit{Ans.}$$

6.28 For straightening and smoothing an airflow in a 50-cm-diameter duct, the duct is packed with a “honeycomb” of thin straws of length 30 cm and diameter 4 mm, as in Fig. P6.28. The inlet flow is air at 110 kPa and 20°C, moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb.

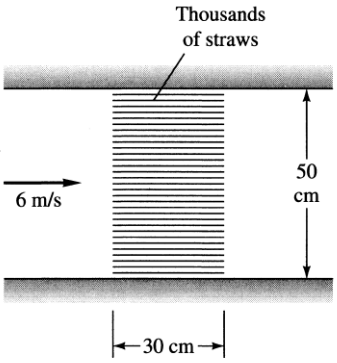


Fig. P6.28

Solution: For air at 20°C, take $\mu \approx$ 1.8E−5 kg/m·s and ρ = 1.31 kg/m^3. There would be approximately 12000 straws, but each one would see the average velocity of 6 m/s. Thus

$$\Delta p_{\text{laminar}} = \frac{32\mu LV}{d^2} = \frac{32(1.8E\text{–}5)(0.3)(6.0)}{(0.004)^2} \approx \mathbf{65\ Pa} \quad \textit{Ans.}$$

Check Re = ρVd/μ = (1.31)(6.0)(0.004)/(1.8E−5) ≈ 1750   OK, laminar flow.

P6.29 SAE 30W oil at 20°C flows through a straight pipe 25 m long, with diameter 4 cm. The average velocity is 2 m/s. (*a*) Is the flow laminar? Calculate (*b*) the pressure drop; and (*c*) the power required. (*d*) If the pipe diameter is doubled, for the same average velocity, by what percent does the required power increase?

Solution: For SAE 30W oil at 20°C, Table A.3, ρ = 891 kg/m^3, and μ = 0.29 kg/m-s. (a) We have enough information to calculate the Reynolds number:

$$Re_D = \frac{\rho VD}{\mu} = \frac{(891)(2.0)(0.04)}{0.29} = \mathbf{246} < 2300 \qquad \textbf{Yes, laminar flow} \quad \textit{Ans.}(a)$$

(*b*, c) The pressure drop and power follow from the laminar formulas of Eq. (6.12):

$$\Delta p = \frac{32\mu LV}{D^2} = \frac{32(0.29)(25)(2.0)}{(0.04)^2} = \mathbf{290,000}\ Pa \qquad Ans.(b)$$

$$Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}(0.04)^2(2.0) = 0.00251\,\frac{m^3}{s}$$

$$Power = Q\Delta p = (0.00251\frac{m^3}{2})(290,000\,Pa) = \mathbf{729}\ W \quad Ans.(c)$$

(*d*) If *D* doubles to 8 cm and *V* remains the same at 2.0 m/s, the new pressure drop will be 72,500 Pa, and the new flow rate will be $Q = 0.01005$ m^3/s, hence the new power will be

$$P = Q\,\Delta p = (0.01005)(72{,}500) = \mathbf{729}\ \text{W} \qquad \textbf{\textit{Zero percent change}}!$$

This is because D^2 cancels in the product $P = Q\,\Delta p = 8\,\pi\,\mu\,L\,V^2$. *Ans.*(*d*)
NOTE: The flow is still laminar, $\text{Re}_D = 492$.

6.30 SAE 10 oil at 20°C flows through the 4-cm-diameter vertical pipe of Fig. P6.30. For the mercury manometer reading *h* = 42 cm shown, (a) calculate the volume flow rate in m^3/h, and (b) state the direction of flow.

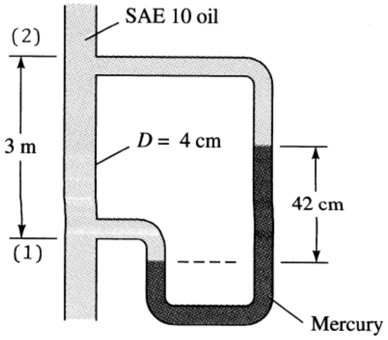


Fig. P6.30

Solution: For SAE 10 oil, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The pressure at the lower point (1) is considerably higher than p2 according to the manometer reading:

$$p_1 - p_2 = (\rho_{Hg} - \rho_{oil})g\Delta h = (13550 - 870)(9.81)(0.42) \approx 52200\ \text{Pa}$$
$$\Delta p/(\rho_{oil}g) = 52200/[870(9.81)] \approx 6.12\ \text{m}$$

This is more than 3 m of oil, therefore it must include a friction loss: **flow is *up***. *Ans.* (b)
The energy equation between (1) and (2), with $V_1 = V_2$, gives

$$\frac{p_1 - p_2}{\rho g} = z_2 - z_1 + h_f, \quad \text{or} \quad 6.12\ \text{m} = 3\ \text{m} + h_f, \quad \text{or:} \quad h_f \approx 3.12\ \text{m} = \frac{128\mu LQ}{\pi\rho g d^4}$$

$$\text{Compute} \quad Q = \frac{(6.12 - 3)\pi(870)(9.81)(0.04)^4}{128(0.104)(3.0)} = 0.00536\ \frac{\text{m}^3}{\text{s}} \approx \mathbf{19.3}\ \frac{\mathbf{m^3}}{\mathbf{h}} \quad Ans.\ \text{(a)}$$

Check $\text{Re} = 4\rho Q/(\pi\mu d) = 4(870)(0.00536)/[\pi(0.104)(0.04)] \approx 1430$ (OK, laminar flow).

P6.31 A *laminar flow element* or LFE (Meriam Instrument Co.) measures low gas-flow rates with a bundle of capillary tubes packed inside a large outer tube. Consider oxygen at 20°C and 1 atm flowing at 84 $\mathrm{ft^3/min}$ in a 4-in-diameter pipe. (*a*) Is the flow approaching the element turbulent? (*b*) If there are 1000 capillary tubes, L = 4 in, select a tube diameter to keep Re_d below 1500 and also to keep the tube pressure drop no greater than 0.5 $\mathrm{lbf/in^2}$. (*c*) Do the tubes selected in part (*b*) fit nicely within the approach pipe?

Solution: For oxygen at 20°C and 1 atm (Table A.4), take $R = 260\ \mathrm{m^2/(s^2K)}$, hence $\rho = p/RT$ = (101350Pa)/[260(293K)] = 1.33 $\mathrm{kg/m^3}$ = 0.00258 $\mathrm{slug/ft^3}$. Also read μ = 2.0E-5 kg/m-s = 4.18E-7 slug/ft-s. Convert Q = 84 $\mathrm{ft^3/min}$ = 1.4 $\mathrm{ft^3/s}$. Then the entry pipe Reynolds number is

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(0.00258\,slug/ft^3)(1.4\,ft^3/s)}{\pi(4.18E-7\,slug/ft-s)(4/12\,ft)} = 33{,}000 \quad (\text{turbulent}) \quad Ans.(a)$$

(*b*) To keep Re_d below 1500 and keep the (laminar) pressure drop no more than 72 psf (0.5 psi),

$$\mathrm{Re}_d = \frac{\rho Vd}{\mu} \le 1500 \quad \text{and} \quad \Delta p = \frac{32\mu LV}{d^2} \le 72\frac{lbf}{ft^2}, \quad \text{where } V = \frac{Q/1000}{(\pi/4)d^2}$$

Select values of d and iterate, or use EES. The upper limit on Reynolds number gives

$$\mathrm{Re}_d = 1500 \quad \text{if} \quad d = 0.00734\ ft = \mathbf{0.088in}\ ; \quad \Delta p = 2.74\ lbf/ft^2 \quad Ans.(b)$$

This is a satisfactory answer, since the pressure drop is no problem, quite small. One thousand of these tubes would have an area about one-half of the pipe area, so would fit nicely. *Ans.*(*c*)

Increasing the tube diameter would lower Re_d and have even smaller pressure drop. Example: d = 0.01 ft, Re_d = 1100, Δp = 0.8 psf. These 0.01-ft-diameter tubes would just barely fit into the larger pipe. One disadvantage, however, is that these tubes are *short*:

the entrance length is *longer* than the tube length, and thus Δp will be larger than calculated by "fully-developed" formulas.

6.32 SAE 30 oil at 20°C flows in the 3-cm-diameter pipe in Fig. P6.32, which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m^3/h.

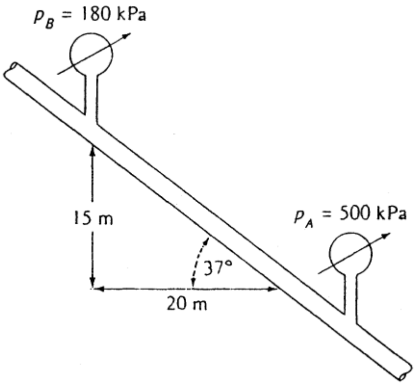


Fig. P6.32

Solution: For SAE 30 oil, take ρ = 891 kg/m^3 and μ = 0.29 kg/m·s. Evaluate the hydraulic grade lines:

$$\mathrm{HGL_B} = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6 \text{ m}; \quad \mathrm{HGL_A} = \frac{500000}{891(9.81)} + 0 = 57.2 \text{ m}$$

$$\text{Since} \quad \mathrm{HGL_A} > \mathrm{HGL_B} \text{ the } \textbf{flow is } \boldsymbol{up} \quad \textit{Ans.} \text{(a)}$$

The head loss is the difference between hydraulic grade levels:

$$h_f = 57.2 - 35.6 = 21.6 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

$$\text{Solve for} \quad Q = 0.000518 \text{ m}^3/\text{s} \approx \mathbf{1.86\ m^3/h} \quad \textit{Ans.} \text{ (b)}$$

Finally, check Re = $4\rho Q/(\pi\mu d) \approx 68$ (OK, laminar flow).

P6.33 Water at 20°C is pumped from a reservoir through a vertical tube 10 ft long and 1/16th inch in diameter. The pump provides a pressure rise of 11 lbf/in^2 to the flow. Neglect entrance losses. (*a*) Calculate the exit velocity. (*b*) Approximately how high will the exit water jet rise? (*c*) Verify that the flow is laminar.

Solution: For water at 20°C, Table A.3, $\rho = 998\ \text{kg/m}^3 = 1.94\ \text{slug/ft}^3$, and $\mu = 0.001$ kg/m-s = 2.09E-5 slug/ft-s. The energy equation, with 1 at the bottom and 2 at the top of the tube, is:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{11(144)}{1.94(32.2)} + 0 + 0 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f = 0 + \frac{V_2^2}{2g} + 10 + \frac{32\mu L V_2}{\rho g D^2}$$

$$\text{or}: \quad 25.4 = \frac{V_{exit}^2}{2(32.2)} + 10 + \frac{32(0.0000209)(10)V}{(1.94)(32.2)(0.00521)^2}; \text{ or}: \ 15.4\,ft = \frac{V_{exit}^2}{64.4} + 3.94V_{exit}$$

(*a*, *c*) The velocity head is very small (<1 ft), so the dominant term is 3.94 V_{exit}. One can easily iterate, or simply use EES to find the result:

$$V_{exit} = 3.84\ \frac{ft}{s} \quad Ans.(a) \quad ; \quad \text{Re}_D = \frac{\rho V D}{\mu} = \frac{(1.94)(3.84)(0.00521)}{0.0000209} = \mathbf{1860\ \ laminar}\ \ Ans.(c)$$

(*b*) Assuming frictionless flow outside the tube, the jet would rise due to the velocity head:

$$H_{rise} = \frac{V_{exit}^2}{2g} = \frac{(3.84\ ft/s)^2}{2(32.2\ ft/s^2)} = 0.229\ ft \approx \mathbf{2.75}\ inches \qquad Ans.(b)$$

6.34 Derive the time-averaged *x*-momentum equation (6.21) by direct substitution of Eqs. (6.19) into the momentum equation (6.14). It is convenient to write the convective acceleration as

$$\frac{d\text{u}}{d\text{t}} = \frac{\partial}{\partial \text{x}}(\text{u}^2) + \frac{\partial}{\partial \text{y}}(\text{uv}) + \frac{\partial}{\partial \text{z}}(\text{uw})$$

which is valid because of the continuity relation, Eq. (6.14).

Solution: Into the *x*-momentum eqn. substitute u = u + u’, v = v + v’, etc., to obtain

$$\rho\left[\frac{\partial}{\partial x}(\bar{u}^2+2\bar{u}u'+u'^2)+\frac{\partial}{\partial y}(\bar{v}\,\bar{u}+\bar{v}u'+v'\bar{u}+v'u')+\frac{\partial}{\partial z}(\bar{w}\bar{u}+\bar{w}u'+w'\bar{u}+w'u')\right]$$

$$=-\frac{\partial}{\partial x}(\bar{p}+p')+\rho g_x+\mu[\nabla^2(\bar{u}+u')]$$

Now take the time-average of the entire equation to obtain Eq. (6.21) of the text:

$$\rho\left[\frac{\mathbf{d\bar{u}}}{\mathbf{dt}}+\frac{\partial}{\partial \mathbf{x}}(\overline{\mathbf{u'^2}})+\frac{\partial}{\partial \mathbf{y}}(\overline{\mathbf{u'v'}})+\frac{\partial}{\partial \mathbf{z}}(\overline{\mathbf{u'w'}})\right]=-\frac{\partial\mathbf{\bar{p}}}{\partial \mathbf{x}}+\rho\mathbf{g_x}+\mu\nabla^2(\mathbf{\bar{u}}) \quad \textit{Ans.}$$

P6.35 In the overlap layer of Fig. 6.9*a*, turbulent shear is large. If we neglect viscosity, we can replace Eq. (6.24) by the approximate velocity-gradient function

$$\frac{du}{dy} = fcn(\,y,\tau_w,\rho)$$

Show that, by dimensional analysis, this leads to the logarithmic overlap relation (6.28).

Solution: There are four variables, and we may list their dimensions in the (MLT) system:

$\frac{du}{dy}$	y	τ_w	ρ
$\{T^{-1}\}$	$\{L\}$	$\{ML^{-1}T^{-2}\}$	$\{ML^{-3}\}$

These can be formed into a single pi group that is therefore equal to a dimensionless constant:

$$\Pi_1 = y^a\,\tau_w^b\,\rho^c\,\frac{du}{dy} \quad \text{yields} \quad \Pi_1 = \frac{du}{dy}\left(\frac{y\sqrt{\rho}}{\sqrt{\tau_w}}\right) = \text{constant} = C_1$$

Rearrange this into a differential equation and then integrate:

$$\frac{du}{dy} = C_1\sqrt{\frac{\tau_w}{\rho}}\,\frac{1}{y} = C_1\,\frac{v^*}{y} \;;\quad \text{Integrate:}\;\; u = C_1 v^*\ln(y) + C_2 \qquad \textit{Ans.}$$

We recognize the square-root term as the friction velocity *v** from Eq. (6.25). If the constants are rearranged so that the logarithm has a dimensionless argument, we would obtain Eq. (6.28):

$$\frac{u}{v^*} = \frac{1}{\kappa}\ln\left(\frac{y v^*}{\nu}\right) + B$$

6.36 The following turbulent-flow velocity data *u*(*y*), for air at 75°F and 1 atm near a smooth flat wall, were taken in the University of Rhode Island wind tunnel:

y, in:	0.025	0.035	0.047	0.055	0.065
u, ft/s:	51.2	54.2	56.8	57.6	59.1

Estimate (a) the wall shear stress and (b) the velocity *u* at $y = 0.22$ in.

Solution: For air at 75°F and 1 atm, take $\rho = 0.00230$ slug/ft^3 and $\mu = 3.80\text{E}{-7}$ slug/ft·s. We fit each data point to the logarithmic-overlap law, Eq. (6.28):

$$\frac{u}{u^*} \approx \frac{1}{\kappa}\ln\frac{\rho u^* y}{\mu} + B \approx \frac{1}{0.41}\ln\left[\frac{0.0023u^*y}{3.80\text{E}{-7}}\right] + 5.0, \quad u^* = \sqrt{\tau_w/\rho}$$

Enter each value of u and y from the data and estimate the friction velocity u*:

y, in:	0.025	0.035	0.047	0.055	0.065
u*, ft/s:	**3.58**	**3.58**	**3.59**	**3.56**	**3.56**
yu^*/ν (approx):	45	63	85	99	117

Each point gives a good estimate of u*, because each point is within the logarithmic layer in Fig. 6.10 of the text. The overall average friction velocity is

$$u^*_{avg} \approx 3.57\ \frac{\text{ft}}{\text{s}} \pm 1\%, \quad \tau_{w,avg} = \rho u^{*2} = (0.0023)(3.57)^2 \approx \mathbf{0.0293\ \frac{lbf}{ft^2}} \quad \textit{Ans.}\text{ (a)}$$

(*b*) Out at y = 0.22 inches, we may estimate that the log-law still holds:

$$\frac{\rho u^* y}{\mu} = \frac{0.0023(3.57)(0.22/12)}{3.80\text{E}-7} \approx 396, \quad u \approx u^*\left[\frac{1}{0.41}\ln(396) + 5.0\right]$$

$$\text{or:} \quad u \approx (3.57)(19.59) \approx \mathbf{70\ \frac{ft}{s}} \quad \textit{Ans.}\text{ (b)}$$

Figure 6.10 shows that this point ($y^+ \approx 396$) seems also to be within the logarithmic layer.

6.37 Two infinite plates a distance *h* apart are parallel to the *xz* plane with the upper plate moving at speed *V*, as in Fig. P6.37. There is a fluid of viscosity μ and constant pressure between the plates. Neglecting gravity and assuming incompressible turbulent flow *u*(*y*) between the plates, use the logarithmic law and appropriate boundary conditions to derive a formula for dimensionless wall shear stress versus dimensionless plate velocity. Sketch a typical shape of the profile *u*(*y*).

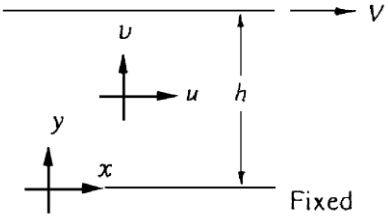
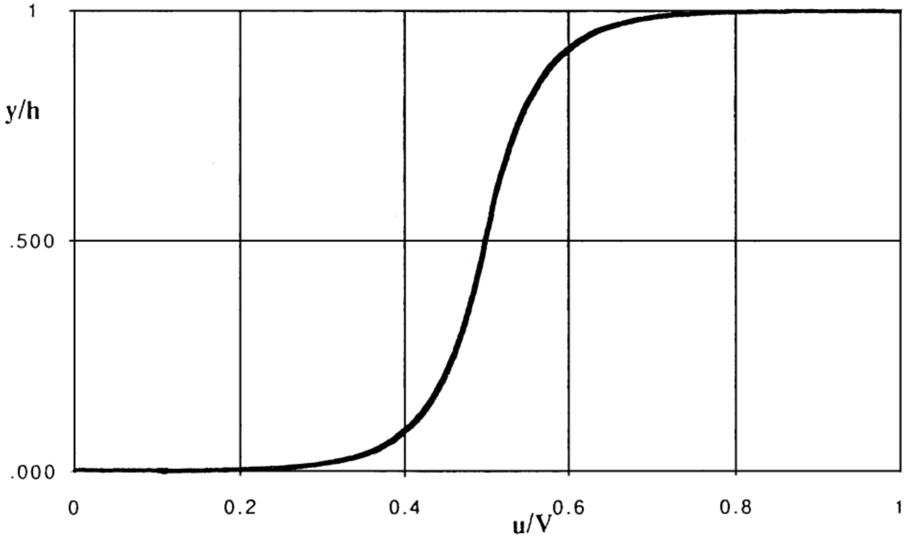


Fig. P6.37

Solution: The shear stress between parallel plates is *constant*, so the centerline velocity must be exactly $u = V/2$ at $y = h/2$. Anti-symmetric log-laws form, one with increasing velocity for $0 < y < h/2$, and a reverse mirror image for $h/2 < y < h$, as shown below:



The match-point at the center gives us a log-law estimate of the shear stress:

$$\frac{V}{2u^*} \approx \frac{1}{\kappa}\ln\left(\frac{hu^*}{2\nu}\right) + \mathrm{B}, \quad \kappa \approx 0.41,\ \mathrm{B} \approx 5.0,\ \boldsymbol{u^* = (\tau_w/\rho)^{1/2}} \quad \textit{Ans.}$$

This is one form of "dimensionless shear stress." The more normal form is friction coefficient versus Reynolds number. Calculations from the log-law fit a Power-law curve-fit expression in the range $2000 < \mathrm{Re_h} < 1\mathrm{E}5$:

$$\mathbf{C_f} = \frac{\tau_w}{(1/2)\rho V^2} \approx \frac{0.018}{(\rho V h/\nu)^{1/4}} = \frac{\mathbf{0.018}}{\mathbf{Re_h^{1/4}}} \quad \textit{Ans.}$$

6.38 Suppose in Fig. P6.37 that $h = 3$ cm, the fluid is water at 20°C ($\rho = 998$ kg/m^3, $\mu = 0.001$ kg/m·s), and the flow is turbulent, so that the logarithmic law is valid. If the shear stress in the fluid is 15 Pa, estimate V in m/s.

Solution: Just as in Prob. 6.37, apply the log-law at the center between the wall, that is, $y = h/2$, $u = V/2$. With τ_w known, we can evaluate u^* immediately:

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{15}{998}} = 0.123\ \frac{m}{s},\quad \frac{V/2}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{u^* h/2}{\nu}\right) + B,$$

$$\text{or:}\quad \frac{V/2}{0.123} = \frac{1}{0.41}\ln\left[\frac{0.123(0.03/2)}{0.001/998}\right] + 5.0 = 23.3,\quad \textit{Solve for}\ \mathbf{V \approx 5.72\ \frac{m}{s}}\quad \textit{Ans.}$$

6.39 By analogy with laminar shear, $\tau = \mu\, du/dy$. T. V. Boussinesq in 1877 postulated that turbulent shear could also be related to the mean-velocity gradient $\tau_{turb} = \varepsilon\, du/dy$, where ε is called the *eddy viscosity* and is much larger than μ. If the logarithmic-overlap law, Eq. (6.28), is valid with $\tau \approx \tau_w$, show that $\varepsilon \approx \kappa\rho u^* y$.

Solution: Differentiate the log-law, Eq. (6.28), to find du/dy, then introduce the eddy viscosity into the turbulent stress relation:

$$\textit{If}\quad \frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B,\quad \text{then}\quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Then, if}\quad \tau \approx \tau_w \equiv \rho u^{*2} = \varepsilon\frac{du}{dy} = \varepsilon\frac{u^*}{\kappa y},\quad \text{solve for}\ \boldsymbol{\varepsilon = \kappa\rho u^* y}\quad \textit{Ans.}$$

Note that $\varepsilon/\mu = \kappa y^+$, which is much larger than unity in the overlap region.

6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{turb} = \varepsilon\, du/dy$ where $\varepsilon = \rho\kappa^2 y^2 |du/dy|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán's dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{turb} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{turb} \approx \tau_w = \rho u^{*2} = \varepsilon\frac{du}{dy} = \left[\rho\kappa^2 y^2\left|\frac{du}{dy}\right|\right]\frac{du}{dy},\quad \text{solve for}\quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate:}\quad \int du = \frac{u^*}{\kappa}\int\frac{dy}{y},\quad \text{or:}\quad \mathbf{u = \frac{u^*}{\boldsymbol{\kappa}}\ln(y) + constant}\quad \textit{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data.

P6.41 Two reservoirs, which differ in surface elevation by 40 m, are connected by 350 m of new pipe of diameter 8 cm. If the desired flow rate is at least 130 N/s of water at 20°C, may the pipe material be (a) galvanized iron, (b) commercial steel, or (c) cast iron? Neglect minor losses.

Solution: Applying the extended Bernoulli equation between reservoir surfaces yields

$$\Delta z = 40\,m = f\frac{L}{D}\frac{V^2}{2g} = f\left(\frac{350\,m}{0.08\,m}\right)\frac{V^2}{2(9.81\,m/s^2)}$$

where f and V are related by the friction factor relation:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right) \quad \text{where} \quad \mathrm{Re}_D = \frac{\rho V D}{\mu}$$

When V is found, the weight flow rate is given by $w = \rho g Q$ where $Q = AV = (\pi D^2/4)V$. For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Given the desired $w = 130$ N/s, solve this system of equations by EES to yield the allowed wall roughness. The results are:

$f = 0.0257$; $V = 2.64$ m/s ; $\mathrm{Re}_D = 211{,}000$; $\varepsilon_{max} = 0.000203$ m = **0.203 mm**

Any less roughness is OK. From Table 6-1, the three pipe materials have
(a) galvanized: $\varepsilon = 0.15$ mm ; (b) commercial steel: $\varepsilon = 0.046$ mm ; cast iron: $\varepsilon = 0.26$ mm

Galvanized and steel are fine, but cast iron is too rough.. *Ans.* Actual flow rates are

(a) galvanized: 135 N/s; (b) steel: 152 N/s; (c) cast iron: 126 N/s (*not enough*)

P6.42 Fluid flows steadily, at volume rate Q, through a large horizontal pipe and then divides into two small smooth pipes, the larger of which has an inside diameter of 25 mm and carries three times the flow of the smaller pipe. Both small pipes have the same length and pressure drop. If all flows are turbulent, at Re_d near 10^4, estimate the diameter of the smaller pipe.

Solution: For *turbulent* flow, the formulas are algebraically complicated, such as Eq. (6.38). However, in the low Reynolds number region, the Blasius power-law approximation, Eq. (6.39), applies, leading to a simple approximate formula for pressure drop, Eq. (6.41):

$$\Delta p \approx 0.241 L \rho^{3/4} \mu^{1/4} d^{-4.75} Q^{1.75}$$

Since Δp, L, ρ, and μ are the same for both pipes, it follows that

$$Q^{1.75} \propto d^{4.75}, \quad \text{or:} \quad Q_{low\ turbulent} \propto d^{4.75/1.75} = d^{19/7}$$

$$\text{Thus, } Q_1 = const(d_1)^{19/7} = 3Q_2 = 3(const)(d_2)^{19/7},$$

$$\text{or: } d_2 = \frac{d_1}{3^{7/19}} = \frac{25\,mm}{1.499} = \mathbf{16.7}\ mm \quad Ans.$$

This is slightly smaller than the laminar-flow estimate of Prob. P6.29, where $d_2 \approx 19$ mm.

P6.43 A reservoir supplies water through 100 m of 30-cm-diameter cast iron pipe to a turbine that extracts 80 hp from the flow. The water then exhausts to the atmosphere. Neglect minor losses. (*a*) Assuming that $f \approx 0.019$, find the flow rate (there is a cubic polynomial). Explain why there are two solutions.

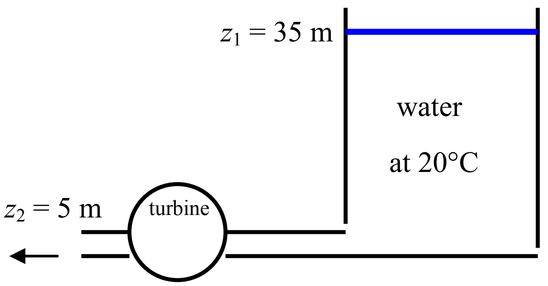


Fig. P6.43

(*b*) For extra credit, solve for the flow rate using the actual friction factors.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. The energy equation yields a relation between elevation, friction, and turbine power:

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{\rho g}} + \frac{V_2^2}{2g} + z_2 + h_{turb} + h_f$$

$$z_1 - z_2 = 35 - 5m = 30m = h_{turb} + h_f = \frac{Power}{\rho g Q} + (1 + f\frac{L}{D})\frac{V_2^2}{2g}\ , \quad Q = \frac{\pi}{4}D^2 V_2$$

$$30\,m = \frac{(80\,hp)(745.7W/hp)}{(9790\,N/m^3)(\pi/4)(0.3m)^2 V_2} + [1 + (0.019)\frac{100m}{0.3m}]\frac{V_2^2}{2(9.81m/s^2)}]$$

Clean this up into a cubic polynomial:

$$30 = \frac{86.2}{V} + 0.373V^2\ , \quad \text{or}: \quad V^3 - 80.3V + 231 = 0$$

$$\textit{Three roots}: \quad V = 3.34\,m/s \ ; \quad 6.81\,m/s \ ; \quad -10.15\,m/s$$

The third (negative) root is meaningless. The other two are correct. Either

$$\mathbf{Q = 0.481\ m^3/s}\ , \quad h_{\text{turbine}} = 12.7\text{ m} \ , \quad h_{\text{f}} = 17.3\text{ m}$$

$$\mathbf{Q = 0.236\ m^3/s}\ , \quad h_{\text{turbine}} = 25.8\text{ m} \ , \quad h_{\text{f}} = 4.2\text{ m} \qquad \textit{Ans.}(a)$$

Both solutions are valid. The higher flow rate wastes a lot of water and creates 17 meters of friction loss. The lower rate uses 51% less water and has proportionately much less friction.

(*b*) The *actual* friction factors are very close to the problem's "Guess". Thus we obtain

Re = 2.04E6, f = 0.0191; $\mathbf{Q = 0.479\ m^3/s}$, $h_{\text{turbine}} = 12.7$ m , $h_{\text{f}} = 17.3$ m

Re = 1.01E6, f = 0.0193 ; $\mathbf{Q = 0.237\ m^3/s}$, $h_{\text{turbine}} = 25.7$ m , $h_{\text{f}} = 4.3$ m

Ans.(b)

The same remarks apply: The lower flow rate is better, less friction, less water used.

6.44 Mercury at 20°C flows through 4 meters of 7-mm-diameter glass tubing at an average velocity of 5 m/s. Estimate the head loss in meters and the pressure drop in kPa.

Solution: For mercury at 20°C, take ρ = 13550 kg/m^3 and μ = 0.00156 kg/m·s. Glass tubing is considered hydraulically "smooth," ε/d = 0. Compute the Reynolds number:

$$Re_d = \frac{\rho V d}{\mu} = \frac{13550(5)(0.007)}{0.00156} = 304{,}000; \quad \text{Moody chart smooth: } f \approx 0.0143$$

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0143\left(\frac{4.0}{0.007}\right)\frac{5^2}{2(9.81)} = \mathbf{10.4\ m} \quad \textit{Ans.}\ \text{(a)}$$

$$\Delta p = \rho g h_f = (13550)(9.81)(10.4) = 1{,}380{,}000\ Pa = \mathbf{1380\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

6.45 Oil, SG = 0.88 and ν = 4E–5 m^2/s, flows at 400 gal/min through a 6-inch asphalted cast-iron pipe. The pipe is 0.5 miles long (2640 ft) and slopes upward at 8° in the flow direction. Compute the head loss in feet and the pressure change.

Solution: First convert 400 gal/min = 0.891 ft^3/s and ν = 0.000431 ft^2/s. For asphalted cast-iron, ε = 0.0004 ft, hence ε/d = 0.0004/0.5 = 0.0008. Compute V, Red, and f:

$$V = \frac{0.891}{\pi(0.25)^2} = 4.54\ \frac{ft}{s}; \quad Re_d = \frac{4.54(0.5)}{0.000431} = 5271; \quad \text{calculate} \quad f_{Moody} = 0.0377$$

$$\text{then} \quad h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0377\left(\frac{2640}{0.5}\right)\frac{(4.54)^2}{2(32.2)} = \mathbf{63.8\ ft} \quad \textit{Ans.}\ \text{(a)}$$

If the pipe slopes upward at 8°, the pressure drop must balance both friction and gravity:

$$\Delta p = \rho g(h_f + \Delta z) = 0.88(62.4)[63.8 + 2640\sin 8°] = \mathbf{23700\ \frac{lbf}{ft^2}} \quad \textit{Ans.}\ \text{(b)}$$

P6.46 Repeat Prob. P3.5, for the same data, by using the more exact turbulent flow formulas to compute the volume flow rate in gallons per minute. Recall the problem: Water at 20°C flows through a 5-inch-diameter smooth pipe at a centerline velocity of 25 ft/s. Estimate the volume flow rate in gallons per minute.

Solution: For water at 20°C, Table A.3, $\rho = 998\ \text{kg/m}^3 = 1.94\ \text{slug/ft}^3$, and $\mu = 0.001$ kg/m-s = 2.09E-5 slug/ft-s. A bit of iteration, or EES, is needed to get the proper Reynolds number and friction factor. Our estimate in Prob. P3.5 was $V_{\text{av}} \approx 21$ ft/s, whence the Reynolds number is

$$\text{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.94\,slug/ft^3)(21\,ft/s)(5/12\,ft)}{2.09E-5\,slug/ft\cdot s} \approx 812{,}000$$

$$\text{Then estimate}\quad f_{smooth} \approx 0.0121\ \text{ from Eq.(6.48)}$$

$$\text{Eq.(6.43)}:\ V_{av} = u_{\max}(1+1.3\sqrt{f})^{-1} = (25\ ft/s)(0.875) \approx 21.9\ \ ft/s$$

Iterate once to obtain $f_{\text{smooth}} = 0.0120$, $V_{\text{av}} = \mathbf{21.89}$ ft/s, hardly any change at all.

$$Q = V_{av}A_{pipe} = (21.89\,ft/s)\pi(\frac{2.5}{12}\ ft)^2 = 2.98\frac{ft^3}{s}\times 7.48\frac{gal}{ft^3}\times 60\frac{s}{\text{min}} = \mathbf{1340}\ \frac{gal}{min}\qquad Ans.$$

This is about 4.6% greater than our simple power-law estimate in Prob. P3.5.

6.47 The gutter and smooth drainpipe in Fig. P6.47 remove rainwater from the roof of a building. The smooth drainpipe is 7 cm in diameter. (a) When the gutter is full, estimate the rate of draining. (b) The gutter is designed for a sudden rainstorm of up to 5 inches per hour. For this condition, what is the maximum roof area that can be drained successfully?

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

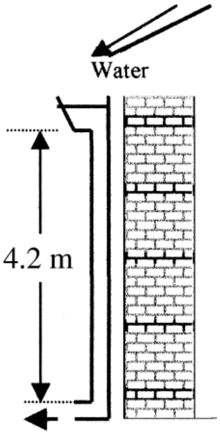


Fig. P6.47

$$\Delta z = \frac{V^2}{2g} + h_f, \quad h_f = f\frac{L}{d}\frac{V^2}{2g}, \quad \text{solve } V^2 = \frac{2g\Delta z}{1 + fL/d} = \frac{2(9.81)(4.2)}{1 + f(4.2/0.07)}$$

For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Guess $f \approx 0.02$ to obtain the velocity estimate V ≈ 6 m/s above. Then Red ≈ $\rho Vd/\mu \approx (998)(6)(0.07)/(0.001) \approx 428{,}000$ (turbulent). Then, for a smooth pipe, $f \approx 0.0135$, and V is changed slightly to 6.74 m/s. After convergence, we obtain

$$V = 6.77 \text{ m/s}, \quad Q = V(\pi/4)(0.07)^2 = \mathbf{0.026\ m^3/s} \quad \textit{Ans.}\text{ (a)}$$

A rainfall of 5 in/h = (5/12 ft/h)(0.3048 m/ft)/(3600 s/h) = 0.0000353 m/s. The required roof area is

$$A_{\text{roof}} = Q_{\text{drain}}/V_{\text{rain}} = (0.026 \text{ m}^3\text{/s})/0.0000353 \text{ m/s} \approx \mathbf{740\ m^2} \quad \textit{Ans.}\text{ (b)}$$

6.48 Show that if Eq. (6.33) is accurate, the position in a turbulent pipe flow where local velocity *u* equals average velocity *V* occurs exactly at *r* = 0.777*R*, independent of the Reynolds number.

Solution: Simply find the log-law position y^+ where u^+ exactly equals V/u*:

$$V = u^*\left[\frac{1}{\kappa}\ln\frac{Ru^*}{\nu} + B - \frac{3}{2\kappa}\right] \overset{?}{=} u^*\left[\frac{1}{\kappa}\ln\frac{yu^*}{\nu} + B\right] \quad \text{if} \quad \frac{1}{\kappa}\ln\frac{y}{R} = -\frac{3}{2\kappa}$$

$$\text{Since } y = R - r, \text{ this is equivalent to } \frac{r}{R} = 1 - e^{-3/2} = 1 - 0.223 \approx \mathbf{0.777} \quad \textit{Ans.}$$

6.49 The tank-pipe system of Fig. P6.49 is to deliver at least 11 m^3/h of water at 20°C to the reservoir. What is the maximum roughness height ε allowable for the pipe?

Solution: For water at 20°C, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. Evaluate V and Re for the expected flow rate:

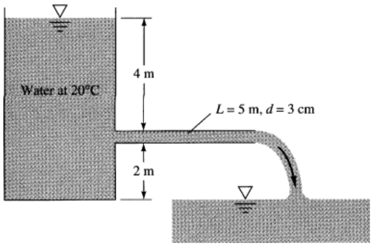


Fig. P6.49

$$V = \frac{Q}{A} = \frac{11/3600}{(\pi/4)(0.03)^2} = 4.32\ \frac{m}{s};\quad Re = \frac{\rho V d}{\mu} = \frac{998(4.32)(0.03)}{0.001} = 129000$$

The energy equation yields the value of the head loss:

$$\frac{p_{atm}}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \quad \text{or} \quad h_f = 4 - \frac{(4.32)^2}{2(9.81)} = 3.05\ m$$

$$\text{But also}\quad h_f = f\frac{L}{d}\frac{V^2}{2g},\quad \text{or:}\quad 3.05 = f\left(\frac{5.0}{0.03}\right)\frac{(4.32)^2}{2(9.81)},\quad \text{solve for } f \approx 0.0192$$

With f and Re known, we can find ε/d from the Moody chart or from Eq. (6.48):

$$\frac{1}{(0.0192)^{1/2}} = -2.0\log_{10}\left[\frac{\varepsilon/d}{3.7} + \frac{2.51}{129000(0.0192)^{1/2}}\right],\quad \text{solve for } \frac{\varepsilon}{d} \approx 0.000394$$

Then $\varepsilon = 0.000394(0.03) \approx 1.2E\text{–}5\ m \approx 0.012$ **mm** (pretty smooth) *Ans.*

6.50 Ethanol at 20°C flows at 125 U.S. gal/min through a horizontal cast-iron pipe with L = 12 m and d = 5 cm. Neglecting entrance effects, estimate (a) the pressure gradient, dp/dx; (b) the wall shear stress, τ_w; and (c) the percent reduction in friction factor if the pipe walls are polished to a smooth surface.

Solution: For ethanol (Table A-3) take $\rho = 789$ kg/m^3 and $\mu = 0.0012$ kg/m·s. Convert 125 gal/min to 0.00789 m^3/s. Evaluate V = Q/A = 0.00789/[$\pi(0.05)^2/4$] = 4.02 m/s.

$$Re_d = \frac{\rho V d}{\mu} = \frac{789(4.02)(0.05)}{0.0012} = 132{,}000, \quad \frac{\varepsilon}{d} = \frac{0.26\ mm}{50\ mm} = 0.0052 \ \text{ Then } f_{Moody} \approx 0.0314$$

$$\text{(b)} \quad \tau_w = \frac{f}{8}\rho V^2 = \frac{0.0314}{8}(789)(4.02)^2 = \mathbf{50\ Pa} \quad \textit{Ans.}\text{ (b)}$$

$$\text{(a)} \quad \frac{dp}{dx} = -\frac{4\tau_w}{d} = \frac{-4(50)}{0.05} = \mathbf{-4000\ \frac{Pa}{m}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(c)} \quad Re = 132000, \quad f_{smooth} = 0.0170, \text{ hence the reduction in f is}$$

$$\left(1 - \frac{0.0170}{0.0314}\right) = \mathbf{46\%} \quad \textit{Ans.}\text{ (c)}$$

6.51 The viscous sublayer (Fig. 6.10) is normally less than 1 percent of the pipe diameter and therefore very difficult to probe with a finite-sized instrument. In an effort to generate a thick sublayer for probing, Pennsylvania State University in 1964 built a pipe with a flow of glycerin. Assume a smooth 12-in-diameter pipe with $V = 60$ ft/s and glycerin at 20°C. Compute the sublayer thickness in inches and the pumping horsepower required at 75 percent efficiency if $L = 40$ ft.

Solution: For glycerin at 20°C, take $\rho = 2.44$ slug/ft^3 and $\mu = 0.0311$ slug/ft·s. Then

$$\mathrm{Re} = \frac{\rho \mathrm{V d}}{\mu} = \frac{2.44(60)(1\ \mathrm{ft})}{0.0311} = 4710 \ \text{ (barely turbulent!)} \quad \text{Smooth: } \mathrm{f_{Moody}} \approx 0.0380$$

$$\text{Then} \quad \mathrm{u^*} = \mathrm{V(f/8)^{1/2}} = 60\left(\frac{0.0380}{8}\right)^{1/2} \approx 4.13\ \frac{\mathrm{ft}}{\mathrm{s}}$$

The sublayer thickness is defined by $y^+ \approx 5.0 = \rho y u^*/\mu$. Thus

$$\mathrm{y_{sublayer}} \approx \frac{5\mu}{\rho \mathrm{u^*}} = \frac{5(0.0311)}{(2.44)(4.13)} = 0.0154\ \mathrm{ft} \approx \mathbf{0.185\ inches} \quad \textit{Ans.}$$

With *f* known, the head loss and the power required can be computed:

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0380)\left(\frac{40}{1}\right)\frac{(60)^2}{2(32.2)} \approx 85\text{ ft}$$

$$P = \frac{\rho g Q h_f}{\eta} = \frac{1}{0.75}(2.44)(32.2)\left[\frac{\pi}{4}(1)^2(60)\right](85) = 419000 \div 550 \approx \mathbf{760\ hp} \quad \textit{Ans.}$$

6.52 The pipe flow in Fig. P6.52 is driven by pressurized air in the tank. What gage pressure p_1 is needed to provide a 20°C water flow rate $Q = 60\ \text{m}^3/\text{h}$?

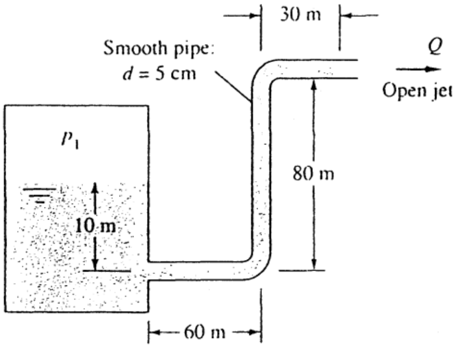


Fig. P6.52

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Get V, Re, f:

$$V = \frac{60/3600}{(\pi/4)(0.05)^2} = 8.49\ \frac{m}{s};$$

$$Re = \frac{998(8.49)(0.05)}{0.001} \approx 424000; \quad f_{smooth} \approx \mathbf{0.0136}$$

Write the energy equation between points (1) (the tank) and (2) (the open jet):

$$\frac{p_1}{\rho g} + \frac{0^2}{2g} + 10 = \frac{0}{\rho g} + \frac{V_{pipe}^2}{2g} + 80 + h_f, \quad \text{where } h_f = f\frac{L}{d}\frac{V^2}{2g} \text{ and } V_{pipe} = 8.49\ \frac{m}{s}$$

$$\text{Solve} \quad p_1 = (998)(9.81)\left[80 - 10 + \frac{(8.49)^2}{2(9.81)}\left\{1 + 0.0136\left(\frac{170}{0.05}\right)\right\}\right]$$

$$\approx \mathbf{2.38E6\ Pa} \quad \textit{Ans.}$$

[This is a *gage* pressure (relative to the pressure surrounding the open jet.)]

P6.53 Water at 20°C flows by gravity through a smooth pipe from one reservoir to a lower one. The elevation difference is 60 m. The pipe is 360 m long, with a diameter of 12 cm. Calculate the expected flow rate in m^3/h. Neglect minor losses.

Solution: For water at 20°C, Table A.3, ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. With no minor losses, the gravity head matches the Moody friction loss in the pipe:

$$\Delta z = 60\,m = h_f = f\frac{L}{D}\frac{V^2}{2g} = f\,(\frac{360\,m}{0.12\,m})\frac{V^2}{2(9.81\,m/s^2)}\;,\quad Q = \frac{\pi}{4}D^2 V$$

$$\text{where}\quad \frac{1}{\sqrt{f}} \approx 2.0\log_{10}(\frac{\mathrm{Re}_D\sqrt{f}}{2.51})\;,\quad \mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(998)V(0.12)}{0.001}$$

The unknown is V, since f can be found as soon as the Reynolds number is known. You could iterate your way to the answer by, say, guessing $f = 0.02$, getting V, repeating. Or you could put the above four equations into EES, which will promptly return the correct answer:

$$V = 5.61\frac{m}{s}\;;\;\mathrm{Re}_D = 672{,}000\;;\; f = 0.0125\,;\; Q = 0.0634\frac{m^3}{s} = \mathbf{228}\frac{m^3}{h}\qquad \textit{Ans.}$$

6.54* A swimming pool W by Y by h deep is to be emptied by gravity through the long pipe shown in Fig. P6.54. Assuming an average pipe friction factor f_{av} and neglecting minor losses, derive a formula for the time to empty the tank from an initial level h_{o}.

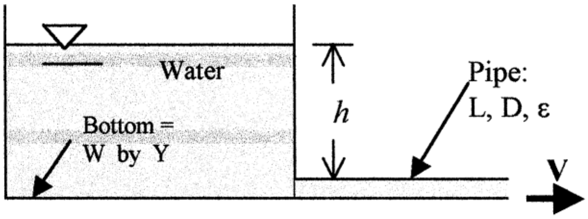


Fig. P6.54

Solution: With no driving pressure and negligible tank surface velocity, the energy equation can be combined with a control-volume mass conservation:

$$h(t) = \frac{V^2}{2g} + f_{av}\frac{L}{D}\frac{V^2}{2g},\quad \textit{or:}\quad Q_{out} = A_{pipe}V = \frac{\pi}{4}D^2\sqrt{\frac{2gh}{1+f_{av}L/D}} = -WY\frac{dh}{dt}$$

We can separate the variables and integrate for time to drain:

$$\frac{\pi}{4}D^2\sqrt{\frac{2g}{1+f_{av}L/D}}\int_0^t dt = -WY\int_{h_o}^0 \frac{dh}{\sqrt{h}} = -WY\left(0-2\sqrt{h_o}\right)$$

$$\textit{Clean this up to obtain:}\quad \boldsymbol{t_{drain} \approx \frac{4WY}{\pi D^2}\left[\frac{2h_o(1+f_{av}L/D)}{g}\right]^{1/2}}\qquad \textit{Ans.}$$

6.55 The reservoirs in Fig. P6.55 contain water at 20°C. If the pipe is smooth with L = 4500 m and d = 4 cm, what will the flow rate in m^3/h be for Δz = 100 m?

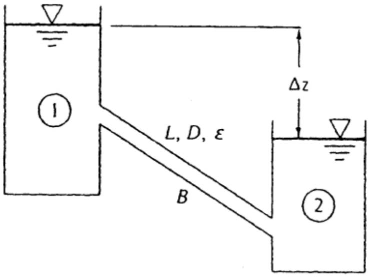


Fig. P6.55

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The energy equation from surface 1 to surface 2 gives

$$p_1 = p_2 \quad \text{and} \quad V_1 = V_2,$$
$$\text{thus} \quad h_f = z_1 - z_2 = 100 \text{ m}$$

$$\text{Then } 100 \text{ m} = f\left(\frac{4500}{0.04}\right)\frac{V^2}{2(9.81)}, \quad \text{or} \quad fV^2 \approx 0.01744$$

Iterate with an initial guess of f ≈ 0.02, calculating V and Re and improving the guess:

$$V \approx \left(\frac{0.01744}{0.02}\right)^{1/2} \approx 0.934 \ \frac{m}{s}, \quad Re \approx \frac{998(0.934)(0.04)}{0.001} \approx 37300, \quad f_{smooth} \approx 0.0224$$

$$V_{better} \approx \left(\frac{0.01744}{0.0224}\right)^{1/2} \approx 0.883 \ \frac{m}{s}, \quad Re_{better} \approx 35300, \quad f_{better} \approx 0.0226, \text{ etc......}$$

Alternately, one could, of course, use EES. The above process converges to

$$f = 0.0227, \ Re = 35000, \ V = 0.877 \text{ m/s}, \ Q \approx 0.0011 \text{ m}^3/\text{s} \approx \mathbf{4.0\ m^3/h}. \quad \textit{Ans.}$$

P6.56 The Alaska Pipeline in the chapter-opener photo has a design flow rate of 4.4E7 gallons per day of crude oil at 60°C (see Fig. A.1). (*a*) Assuming a galvanized-iron wall, estimate the total pressure drop required for the 800-mile trip. (*b*) If there are nine equally spaced pumps, estimate the horsepower each pump must deliver.

Solution: From Fig. A.1 for crude oil at 60°C, ρ = 860 kg/m^3 and μ = 0.004 kg/m-s. The pipe diameter is 4 ft. For galvanized iron, ε = 0.0005 ft, hence ε/D = 0.0005/4 = 0.000125. Convert the data to metric: 4.4E7 gal/day = 1.93 m^3/s, and D = 4 ft = 1.22 m. Calculate Reynolds number:

$$V = \frac{Q}{A} = \frac{1.93 m^3/s}{(\pi/4)(1.22m)^2} = 1.65\frac{m}{s} \ ; \ Re_D = \frac{\rho VD}{\mu} = \frac{(860)(1.65)(1.22)}{0.004} = 433{,}000$$

Calculate the friction factor from Eq. (6.48):

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{0.000125}{3.7}+\frac{2.51}{433000\sqrt{f}}\right) \quad \text{Calculate} \quad f \approx 0.0149$$

(*a*) The total 800-mile pressure drop is given by the usual Darcy-Moody expression, Eq. (6.10):

$$\Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.0149)\left[\frac{(800)(5280)}{4\ ft}\right]\left(\frac{860}{2}\right)(1.65)^2 = 1.85\text{E}7\,Pa = \mathbf{2680}\ psi \quad Ans.(a)$$

(*b*) The power delivered by each of the 9 pumps is

$$Power = Q\frac{\Delta p_{total}}{9} = \left(1.93\frac{m^3}{s}\right)\left(\frac{1.85\text{E}7\,Pa}{9}\right) = 396,000\,W \div 746 = \mathbf{5300}\ hp \quad Ans.(b)$$

6.57 Apply the analysis of Prob. 6.54 to the following data. Let $W = 5$ m, $Y = 8$ m, $h_o = 2$ m, $L = 15$ m, $D = 5$ cm, and $\varepsilon = 0$. (a) By letting $h = 1.5$ m and 0.5 m as representative depths, estimate the average friction factor. Then (b) estimate the time to drain the pool.

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The velocity in Prob. 6.54 is calculated from the energy equation:

$$V = \sqrt{\frac{2gh}{1+fL/D}} \quad \text{with} \quad f = fcn(\text{Re}_D)_{smooth\ pipe} \quad \text{and} \quad \text{Re}_D = \frac{\rho VD}{\mu},\ L/D = 300$$

(a) With a bit of iteration for the Moody chart, we obtain $\text{Re}_D = 108,000$ and $f \approx 0.0177$ at $h = 1.5$ m, and $\text{Re}_D = 59,000$ and $f \approx .0202$ at $h = 0.5$ m; thus the average value $\boldsymbol{f_{av} \approx 0.019}$. *Ans.* (a)

The drain formula from Prob. 6.54 then predicts:

$$t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1+f_{av}L/D)}{g}} \approx \frac{4(5)(8)}{\pi(0.05)^2}\sqrt{\frac{2(2)[1+0.019(300)]}{9.81}}$$

$$= 33700\ s = \mathbf{9.4\ h} \quad Ans.\ (b)$$

P6.58 For the system in Prob. P6.53, a pump is used at night to drive water back to the upper reservoir. If the pump delivers 15,000 W to the water, estimate the flow rate.

Solution: For water at 20°C, Table A.3, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Since the pressures and velocities cancel, the energy equation becomes

$$z_{lower} = z_{upper} + h_f - h_{pump}\ ,\ or:\ h_p = \Delta z + h_f = 60m + f\frac{360m}{0.12m}\frac{V^2}{2(9.81)}$$

$$\text{where}\ \frac{1}{\sqrt{f}} \approx 2.0\log_{10}\left(\frac{\text{Re}_D\sqrt{f}}{2.51}\right)\ ,\ \text{Re}_D = \frac{\rho VD}{\mu} = \frac{(998)V(0.12)}{0.001},\ Q = V\frac{\pi}{4}D^2$$

$$Power = \rho g Q h_p = (998)(9.81)Q h_p = 15{,}000\ W$$

The unknown is *V*, since *f* can be found as soon as the Reynolds number is known. You could iterate your way to the answer by, say, guessing *f* = 0.02, getting *V* (from a *cubic* equation!), and repeating. Or put the above five equations into EES, which returns the only positive answer:

$$\text{Re}_D = 235{,}000\ ;\ f = 0.0152\ ;\ V = 1.97\ m/s\ ;\ Q = 0.0222\,m^3/s = \mathbf{80}\ \frac{m^3}{h}\quad Ans.$$

6.59 The following data were obtained for flow of 20°C water at 20 m^3/hr through a badly corroded 5-cm-diameter pipe which slopes downward at an angle of 8°: p_1 = 420 kPa, z_1 = 12 m, p_2 = 250 kPa, z_2 = 3 m. Estimate (a) the roughness ratio of the pipe; and (b) the percent change in head loss if the pipe were smooth and the flow rate the same.

Solution: The pipe length is given indirectly as $L = \Delta z/\sin\theta = (9\text{ m})/\sin 8° = 64.7$ m. The steady flow energy equation then gives the head loss:

$$\frac{p_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{p_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f,\quad or:\quad \frac{420000}{9790} + 12 = \frac{250000}{9790} + 3 + h_f,$$

$$\text{Solve}\quad h_f = 26.4\ m$$

Now relate the head loss to the Moody friction factor:

$$h_f = 26.4 = f\frac{L}{d}\frac{V^2}{2g} = f\frac{64.7}{0.05}\frac{(2.83)^2}{2(9.81)},\quad \text{Solve}\ f = 0.050,\ Re = 141000,\ \text{Read}\ \frac{\varepsilon}{d} \approx 0.0211$$

The estimated (and uncertain) pipe roughness is thus $\varepsilon = 0.0211d \approx$ **1.06 mm** *Ans.* (a)
(b) At the same $\text{Re}_d = 141000$, $f_{\text{smooth}} = 0.0168$, or **66% less head loss**. *Ans.* (b)

P6.60 In the spirit of Haaland's explicit pipe friction factor approximation, Eq. (6.49), Jeppson [20] proposed the following explicit formula:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{5.74}{\mathrm{Re}_d^{0.9}}\right)$$

(*a*) Is this identical to Haaland's formula and just a simple rearrangement? Explain.

(*b*) Compare Jeppson to Haaland for a few representative values of (turbulent) $\mathrm{Re_d}$ and ε/d and their deviations compared to the Colebrook formula (6.48).

Solution: (*a*) No, it *looks* like a rearrangement of Haaland's formula, but it is not. Haaland started with Colebrook's smooth-wall formula and added just enough ε/d effect for accuracy. Jeppson started with the rough-wall formula and added just enough Re_d effect for accuracy. Both are excellent approximations over the full (turbulent) range of Re_d and ε/d. Their predicted values of f are nearly the same and very close to the implicit Colebrook formula. Here is a table of their standard deviations of their values when subtracted from Colebrook:

1E4 < Re*d* < 1e8	$\varepsilon/d = 0.03$	0.01	0.001	0.0001	0.00001
Jeppson rms error	0.000398	0.000328	0.000195	0.000067	0.000088
Haaland rms error	0.000034	0.000043	0.000129	0.000113	0.000083

As expected, Jeppson is slightly better for smooth walls, Haaland for rough walls. Both are within ±2% of the Colebrook formula over the entire range of Re_d and ε/d.

6.61 What level *h* must be maintained in Fig. P6.61 to deliver a flow rate of 0.015 ft^3/s through the $\frac{1}{2}$-in commercial-steel pipe?

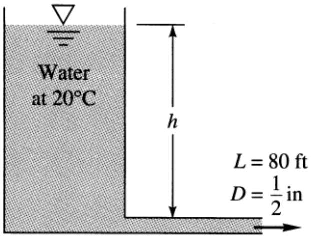


Fig. P6.61

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d = 0.00015/(0.5/12) \approx 0.0036$. Compute

$$V = \frac{Q}{A} = \frac{0.015}{(\pi/4)(0.5/12)^2} = 11.0\ \frac{\text{ft}}{\text{s}};$$

$$\text{Re} = \frac{\rho V d}{\mu} = \frac{1.94(11.0)(0.5/12)}{2.09\text{E}{-5}} \approx 42500 \quad \varepsilon/d = 0.0036, \quad f_{\text{Moody}} \approx 0.0301$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, yields an expression for surface elevation:

$$h = h_f + \frac{V^2}{2g} = \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = \frac{(11.0)^2}{2(32.2)}\left[1 + 0.0301\left(\frac{80}{0.5/12}\right)\right] \approx \mathbf{111\ ft} \quad \textit{Ans.}$$

6.62 Water at 20°C is to be pumped through 2000 ft of pipe from reservoir 1 to 2 at a rate of 3 ft^3/s, as shown in Fig. P6.62. If the pipe is cast iron of diameter 6 in and the pump is 75 percent efficient, what horsepower pump is needed?

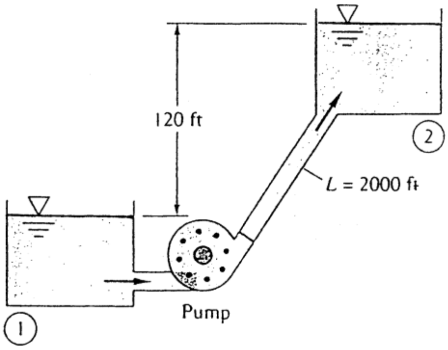


Fig. P6.62

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft, or $\varepsilon/d = 0.00085/(6/12) \approx 0.0017$. Compute V, Re, and *f*:

$$V = \frac{Q}{A} = \frac{3}{(\pi/4)(6/12)^2} = 15.3\ \frac{\text{ft}}{\text{s}};$$

$$\mathrm{Re} = \frac{\rho V d}{\mu} = \frac{1.94(15.3)(6/12)}{2.09\mathrm{E}{-}5} \approx 709000 \quad \varepsilon/d = 0.0017, \quad f_{\mathrm{Moody}} \approx 0.0227$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx V_2 \approx 0$, yields an expression for pump head:

$$h_{\mathrm{pump}} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\ \mathrm{ft} + 0.0227\left(\frac{2000}{6/12}\right)\frac{(15.3)^2}{2(32.2)} = 120 + 330 \approx 450\ \mathrm{ft}$$

$$\text{Power:}\quad P = \frac{\rho g Q h_p}{\eta} = \frac{1.94(32.2)(3.0)(450)}{0.75} = 112200 \div 550 \approx \mathbf{204\ hp} \quad \textit{Ans.}$$

6.63 A tank contains 1 m^3 of water at 20°C and has a drawn-capillary outlet tube at the bottom, as in Fig. P6.63. Find the outlet volume flux Q in m^3/h at this instant.

Solution: For water at 20°C, take $\rho = 998\ \mathrm{kg/m^3}$ and $\mu = 0.001$ kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or $\varepsilon/d = 0.0015/40 \approx 0.0000375$. The steady-flow energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, gives

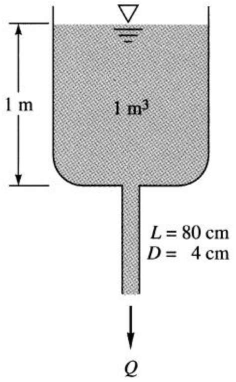


Fig. P6.63

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \Delta z - \frac{V^2}{2g}, \quad \text{or:}\quad \frac{V^2}{2g}\left(1 + \frac{0.8}{0.04}f\right) \approx 1.8\ \mathrm{m},\ V^2 \approx \frac{35.32}{1+20f}$$

$$\text{Guess}\quad f \approx 0.015, \quad V = \left[\frac{35.32}{1+20(0.015)}\right]^{1/2} \approx 5.21\ \frac{\mathrm{m}}{\mathrm{s}}, \quad \mathrm{Re} = \frac{998(5.21)(0.04)}{0.001} \approx 208000$$

$$f_{\mathrm{better}} \approx 0.0158, \quad V_{\mathrm{better}} \approx 5.18\ \mathrm{m/s}, \quad \mathrm{Re} \approx 207000\ \text{(converged)}$$

$$\text{Thus}\quad V \approx 5.18\ \mathrm{m/s}, \quad Q = (\pi/4)(0.04)^2(5.18) = 0.00651\ \mathrm{m^3/s} \approx \mathbf{23.4\ m^3/h}. \quad \textit{Ans.}$$

6.64 Repeat Prob. 6.63 to find the flow rate if the fluid is SAE 10 oil. Is the flow laminar or turbulent?

Solution: For SAE 10 oil at 20°C, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or $\varepsilon/d = 0.0015/40 \approx 0.0000375$. Guess laminar flow:

$$h_f = 1.8\ \text{m} - \frac{V^2}{2g} \overset{?}{=} \frac{32\mu LV}{\rho g d^2}, \quad \text{or:} \quad 1.8 - \frac{V^2}{2(9.81)} = \frac{32(0.104)(0.8)V}{870(9.81)(0.04)^2} = 0.195V$$

$$\text{Quadratic equation:} \quad V^2 + 3.83V - 35.32 = 0, \quad \text{solve } V = 4.33\ \text{m/s}$$

$$\text{Check} \quad \text{Re} = (870)(4.33)(0.04)/(0.104) \approx \mathbf{1450\ \ (OK, laminar)}$$

So it is laminar flow, and Q = $(\pi/4)(0.04)^2(4.33)$ = 0.00544 m^3/s = **19.6 m^3/h**. *Ans.*

6.65 In Prob. 6.63 the initial flow is turbulent. As the water drains out of the tank, will the flow revert to laminar motion as the tank becomes nearly empty? If so, at what tank depth? Estimate the time, in h, to drain the tank completely.

Solution: Recall that $\rho = 998$ kg/m^3, $\mu = 0.001$ kg/m·s, and $\varepsilon/d \approx 0.0000375$. Let Z be the depth of water in the tank (Z = 1 m in Fig. P6.63). When Z = 0, find the flow rate:

$$Z = 0,\ h_f = 0.8\ \text{m}, \quad V^2 \approx \frac{2(9.81)(0.8)}{1 + 20f} \quad \text{converges to } f = 0.0171,\ \text{Re} = 136000$$

$$V \approx 3.42\ \text{m/s}, \quad Q \approx 12.2\ \text{m}^3/\text{h}\ (Z = 0)$$

So even when the tank is empty, the flow is still turbulent. *Ans.*

$$\text{The time to drain the tank is} \quad \frac{d}{dt}(\upsilon_{\text{tank}}) = -Q = \frac{d}{dt}(A_{\text{tank}}Z) = (1\ \text{m}^2)\frac{dZ}{dt} = -Q,$$

$$\text{or} \quad t_{\text{drain}} = -\int_{1\text{m}}^{0\text{m}} \frac{dZ}{Q} = \left(\frac{1}{Q}\right)_{\text{avg}} (1\ \text{m})$$

So all we need is the average value of (1/Q) during the draining period. We know Q at Z = 0 and Z = 1 m, let's check it also at Z = 0.5 m: Calculate $Q_{\text{midway}} \approx 19.8$ m^3/h. Then estimate, by Simpson's Rule,

$$\left.\frac{1}{Q}\right|_{\text{avg}} \approx \frac{1}{6}\left[\frac{1}{23.4} + \frac{4}{19.8} + \frac{1}{12.2}\right] \approx 0.0544\ \frac{\text{h}}{\text{m}^3}, \quad t_{\text{drain}} = \mathbf{0.0544\ h \approx 3.3\ min} \quad \textit{Ans.}$$

6.66 Ethyl alcohol at 20°C flows through a 10-cm horizontal drawn tube 100 m long. The fully developed wall shear stress is 14 Pa. Estimate (a) the pressure drop, (b) the volume flow rate, and (c) the velocity u at $r = 1$ cm.

Solution: For ethyl alcohol at 20°C, $\rho = 789\ \text{kg/m}^3$, $\mu = 0.0012$ kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or $\varepsilon/d = 0.0015/100 \approx 0.000015$. From Eq. (6.12),

$$\Delta p = 4\tau_w \frac{L}{d} = 4(14)\left(\frac{100}{0.1}\right) \approx \mathbf{56000\ Pa} \quad \textit{Ans.}\ \text{(a)}$$

The wall shear is directly related to *f*, and we may iterate to find V and Q:

$$\tau_w = \frac{f}{8}\rho V^2, \quad \text{or:} \quad fV^2 = \frac{8(14)}{789} = 0.142 \quad \text{with} \quad \frac{\varepsilon}{d} = 0.000015$$

$$\text{Guess } f \approx 0.015, \quad V = \left[\frac{0.142}{0.015}\right]^{1/2} \approx 3.08\ \frac{m}{s}, \quad Re = \frac{789(3.08)(0.1)}{0.0012} \approx 202000$$

$$f_{better} \approx 0.0158, \quad V_{better} \approx 3.00\ \text{m/s}, \quad Re_{better} \approx 197000\ \text{(converged)}$$

Then V ≈ **3.00 m/s**, and $Q = (\pi/4)(0.1)^2(3.00) = 0.0236\ \text{m}^3/\text{s} = \mathbf{85\ m^3/h}$. *Ans.* (b)
Finally, the log-law Eq. (6.28) can estimate the velocity at r = 1 cm, "y" = R − r = 4 cm:

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} = \left(\frac{14}{789}\right)^{1/2} = 0.133\ \frac{m}{s};$$

$$\frac{u}{u^*} \approx \frac{1}{\kappa}\ln\left[\frac{\rho u^* y}{\mu}\right] + B = \frac{1}{0.41}\ln\left[\frac{789(0.133)(0.04)}{0.0012}\right] + 5.0 = 24.9$$

$$\text{Then} \quad u \approx 24.9(0.133) \approx \mathbf{3.3\ m/s} \quad \text{at } r = 1\ \text{cm.} \quad \textit{Ans.}\ \text{(c)}$$

6.67 A straight 10-cm commercial-steel pipe is 1 km long and is laid on a constant slope of 5°. Water at 20°C flows downward, due to gravity only. Estimate the flow rate in m^3/h. What happens if the pipe length is 2 km?

Solution: For water at 20°C, take $\rho = 998\ \text{kg/m}^3$ and $\mu = 0.001$ kg/m·s. If the flow is due to gravity only, then the head loss exactly balances the elevation change:

$$h_f = \Delta z = L\sin\theta = f\frac{L}{d}\frac{V^2}{2g}, \quad \text{or} \quad fV^2 = 2gd\sin\theta = 2(9.81)(0.1)\sin 5° \approx 0.171$$

Thus the flow rate is independent of the pipe length L if laid on a constant slope. *Ans.*
For commercial steel, take $\varepsilon \approx 0.046$ mm, or $\varepsilon/d \approx 0.00046$.

Begin by guessing fully-rough flow for the friction factor, and iterate V and Re and f:

$$f \approx 0.0164, \quad V \approx \left(\frac{0.171}{0.0164}\right)^{1/2} \approx 3.23\ \frac{m}{s}, \quad Re = \frac{998(3.23)(0.1)}{0.001} \approx 322000$$

$$f_{better} \approx 0.0179, \quad V_{better} \approx 3.09\ m/s, \quad Re \approx 308000\ \text{(converged)}$$

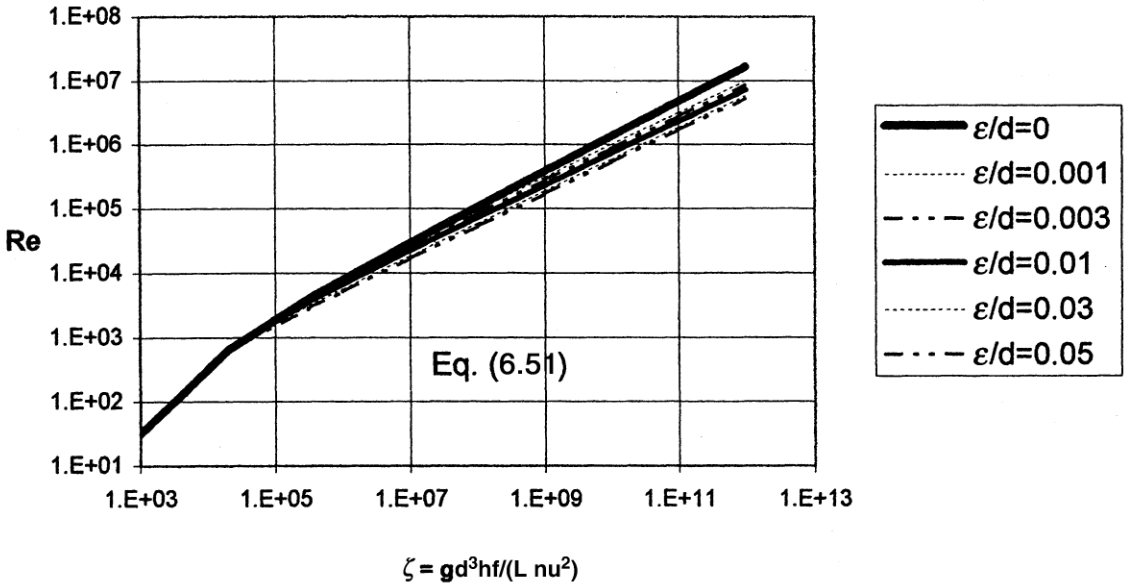
$$\text{Then} \quad Q \approx (\pi/4)(0.1)^2(3.09) \approx 0.0243\ m^3/s \approx \mathbf{87\ m^3/h}. \quad \textit{Ans.}$$

***6.68** The Moody chart, Fig. 6.13, is best for finding head loss (or Δp) when Q, V, d, and L are known. It is awkward for the “2nd” type of problem, finding Q when hf or Δp are known (see Ex. 6.9). Prepare a modified Moody chart whose abscissa is independent of Q and V, using ε/d as a parameter, from which one can immediately read the ordinate to find (dimensionless) Q or V. Use your chart to solve Example 6.9.

Solution: This problem was mentioned *analytically* in the text as Eq. (6.51). The proper parameter which contains head loss only, and *not* flow rate, is ζ:

$$\zeta = \frac{gd^3 h_f}{L\nu^2} \qquad Re_d = -(8\zeta)^{1/2}\log\left(\frac{\varepsilon/d}{3.7} + \frac{1.775}{\sqrt{\zeta}}\right) \qquad \text{Eq. (6.51)}$$

We simply plot Reynolds number versus ζ for various ε/d, as shown below:



To solve Example 6.9, a 100-m-long, 30-cm-diameter pipe with a head loss of 8 m and ε/d = 0.0002, we use that data to compute ζ = **5.3E7**. The oil properties are ρ = 950 kg/m^3 and ν =

2E–5 m^2/s. Enter the chart above: let's face it, the scale is very hard to read, but we estimate, at $\zeta = 5.3E7$, that $6E4 < Re_d < 9E4$, which translates to a flow rate of

$$\mathbf{0.28 < Q < 0.42\ m^3/s}. \quad \textit{Ans.} \quad (\text{Example 6.9 gave } Q = 0.342\ m^3/s.)$$

6.69 For Prob. 6.62 suppose the only pump available can deliver only 80 hp to the fluid. What is the proper pipe size in inches to maintain the 3 ft^3/s flow rate?

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. We can't specify ε/d because we don't know d. The energy analysis above is correct and should be modified to replace V by Q:

$$h_p = 120 + f\frac{L}{d}\frac{(4Q/\pi d^2)^2}{2g} = 120 + f\frac{2000}{d}\frac{[4(3.0)/\pi d^2]^2}{2(32.2)} = 120 + 453\frac{f}{d^5}$$

$$\text{But also}\quad h_p = \frac{\text{Power}}{\rho g Q} = \frac{80(550)}{62.4(3.0)} = 235 = 120 + \frac{453\ f}{d^5},\quad \text{or:}\quad \mathbf{d^5 \approx 3.94f}$$

Guess $f \approx 0.02$, calculate d, ε/d and Re and get a better f and iterate:

$$f \approx 0.020,\quad d \approx [3.94(0.02)]^{1/5} \approx 0.602\ \text{ft},\quad Re = \frac{4\rho Q}{\pi\mu d} = \frac{4(1.94)(3.0)}{\pi(2.09E{-}5)(0.602)},$$

$$\text{or}\quad Re \approx 589000,\quad \frac{\varepsilon}{d} = \frac{0.00085}{0.602} \approx 0.00141,\quad \text{Moody chart:}\quad f_{better} \approx 0.0218\ (\text{repeat})$$

We are nearly converged. The final solution is $f \approx 0.0217$, $d \approx 0.612$ ft ≈ **7.3 in** *Ans.*

P6.70 Ethylene glycol at 20°C flows through 80 meters of cast iron pipe of diameter 6 cm. The measured pressure drop is 250 kPa. Neglect minor losses. Using a non-iterative formulation, estimate the flow rate in m^3/h.

Solution: For ethylene glycol at 20°C, Table A.3, $\rho = 1117$ kg/m^3 and $\mu = 0.0214$ kg/m-s. The head loss is given: $\Delta p/\rho g = 250{,}000/[1117(9.81)] = 22.8$ m. For cast iron, $\varepsilon = 0.26$ mm, hence the roughness ratio is $\varepsilon/d = 0.26/60 = 0.00433$. We can use the direct approach of Eq. (6.51):

$$\mathrm{Re}_d = -\sqrt{8\zeta}\ \log\left(\frac{\varepsilon/d}{3.7} + \frac{1.775}{\sqrt{\zeta}}\right),\quad \zeta = \frac{g\,d^3\,h_f}{L\nu^2} = \frac{9.81(0.06)^3(22.8)}{80(0.0214/1117)^2} = 1.65\mathrm{E}6$$

$$\text{Evaluate: } \mathrm{Re}_d = -\sqrt{8(1.65\mathrm{E}6)}\ \log\left(\frac{0.00433}{3.7} + \frac{1.775}{\sqrt{1.65\mathrm{E}6}}\right) = 9410 \quad \text{(yes, turbulent)}$$

With the Reynolds number known, we can find the velocity and flow rate:

$$V = \frac{\mu \mathrm{Re}_d}{\rho d} = \frac{(0.0214)(9410)}{1117(0.06)} = 3.00\frac{m}{s}\ ;\ Q = \frac{\pi}{4}(0.06)^2(3.00) = 0.0085\frac{m^3}{2} = \mathbf{30.6}\frac{m^3}{h}\ \textit{Ans.}$$

6.71 It is desired to solve Prob. 6.62 for the most economical pump and cast-iron pipe system. If the pump costs \$125 per horsepower delivered to the fluid and the pipe costs \$7000 per inch of diameter, what are the minimum cost and the pipe and pump size to maintain the 3 ft^3/s flow rate? Make some simplifying assumptions.

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\mathrm{E}{-5}$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. Write the energy equation (from Prob. 6.62) in terms of Q and d:

$$P_{\text{in hp}} = \frac{\rho g Q}{550}(\Delta z + h_f) = \frac{62.4(3.0)}{550}\left\{120 + f\left(\frac{2000}{d}\right)\frac{[4(3.0)/\pi d^2]^2}{2(32.2)}\right\} = 40.84 + \frac{154.2f}{d^5}$$

$$\text{Cost} = \$125P_{hp} + \$7000d_{inches} = 125(40.84 + 154.2f/d^5) + 7000(12d), \quad \text{with d in ft.}$$

$$\text{Clean up:}\quad \text{Cost} \approx \$5105 + 19278f/d^5 + 84000d$$

Regardless of the (unknown) value of f, this Cost relation does show a minimum. If we assume for simplicity that *f* is constant, we may use the differential calculus:

$$\frac{d(\text{Cost})}{d(d)}\Big|_{f\approx\text{const}} = \frac{-5(19278)f}{d^6} + 84000, \quad \text{or} \quad d_{best} \approx (1.148\ f)^{1/6}$$

$$\text{Guess } f \approx 0.02,\quad d \approx [1.148(0.02)]^{1/6} \approx 0.533\text{ ft},\quad \mathrm{Re} = \frac{4\rho Q}{\pi\mu d} \approx 665000,\quad \frac{\varepsilon}{d} \approx 0.00159$$

$$\text{Then}\quad f_{better} \approx 0.0224,\quad d_{better} \approx 0.543\text{ ft (converged)}$$

Result: $d_{best} \approx 0.543$ ft ≈ **6.5 in,** $\text{Cost}_{min} \approx \$14300_{pump} + \$45600_{pipe} \approx$ **\$60,000**. *Ans.*

6.72 Modify Prob. P6.57 by letting the diameter be unknown. Find the proper pipe diameter for which the pool will drain in about 2 hours flat.

Solution: Recall the data: Let $W = 5$ m, $Y = 8$ m, $h_o = 2$ m, $L = 15$ m, and $\varepsilon = 0$, with water, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. We apply the same theory as Prob. 6.57:

$$V = \sqrt{\frac{2gh}{1 + fL/D}}, \quad t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1 + f_{av}L/D)}{g}}, \quad f_{av} = fcn(\text{Re}_D) \quad \text{for a smooth pipe.}$$

For the present problem, $t_{\text{drain}} = 2$ hours and D is the unknown. Use an average value $h = 1$ m to find f_{av}. Enter these equations on EES (or you can iterate by hand) and the final results are

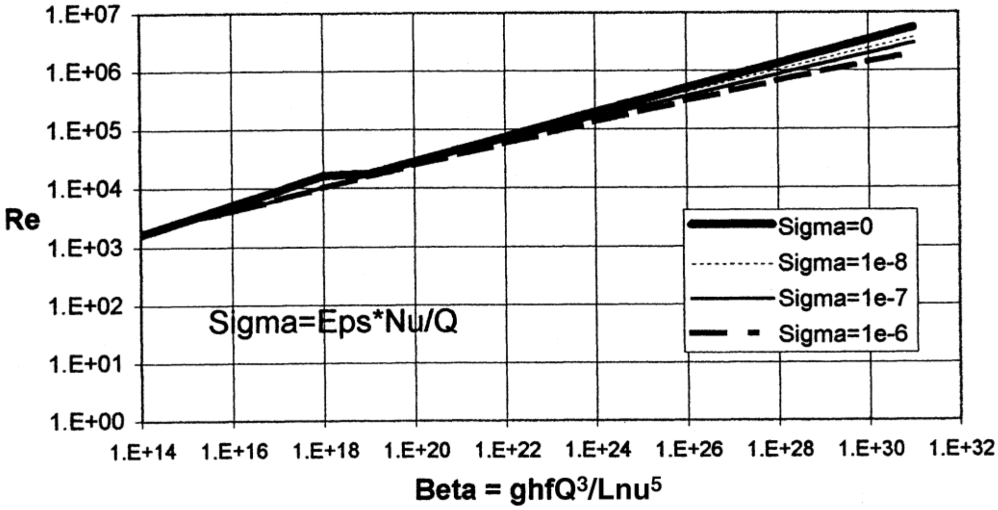
$$V = 2.36 \text{ m/s}; \quad \text{Re}_D = 217{,}000; \quad f_{\text{av}} \approx 0.0154; \quad D = 0.092 \text{ m} \approx \mathbf{9.2\ cm} \quad \textit{Ans.}$$

***6.73** The Moody chart, Fig. 6.13, is best for finding head loss (or Δp) when Q, V, d, and L are known. It is awkward for the "3rd" type of problem, finding d when h_f (or Δp) and Q are known (see Ex. 6.11). Prepare a modified Moody chart whose abscissa is independent of d, using as a parameter ε non-dimensionalized without d, from which one can immediately read the (dimensionless) ordinate to find d. Use your chart to solve Ex. 6.11.

Solution: An appropriate Pi group which does not contain d is $\beta = (ghfQ^3)/(L\nu^5)$. Similarly, an appropriate roughness parameter without d is $\sigma = (\varepsilon\nu/Q)$. After a lot of algebra, the Colebrook friction factor formula (6.48) becomes

$$Re_d^{5/2} = -2.0\left(\frac{128\beta}{\pi^3}\right)^{1/2} \log_{10}\left[\frac{\pi\sigma Re_d}{14.8} + \frac{2.51 Re_d^{3/2}}{(128\beta/\pi^3)^{1/2}}\right]$$

A plot of this messy relation is given below.



To solve Example 6.11, a 100-m-long, unknown-diameter pipe with a head loss of 8 m, flow rate of 0.342 m^3/s, and ε = 0.06 mm, we use that data to compute β = **9.8E21** and **σ = 3.5E–6**. The oil properties are ρ = 950 kg/m^3 and ν = 2E–5 m^2/s. Enter the chart above: let's face it, the scale is very hard to read, but we estimate, at β = 9.8E21 and σ = 3.5E–6, that 6E4 < Re$_d$ < 8E4, which translates to a diameter of **0.27 < *d* < 0.36 m**. *Ans.* (Example 6.11 gave d = 0.3 m.)

P6.74 Two reservoirs, which differ in surface elevation by 40 m, are connected by a new commercial steel pipe of diameter 8 cm. If the desired weight flow rate is 200 N/s of water at 20°C, what is the proper length of the pipe? Neglect minor losses.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. For commercial steel, ε = 0.046 mm, thus ε/d = 0.046mm/80mm = 0.000575. Find the velocity and the friction factor:

$$V = \frac{\dot{w}/(\rho g)}{(\pi/4)D^2} = \frac{200/[998(9.81)]}{(\pi/4)(0.08)^2} = 4.06\frac{m}{s}, \quad \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{998(4.06)(0.08)}{0.001} = 324{,}000$$

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right) \quad \text{yields} \quad f = 0.0185$$

Then we find the pipe length from the energy equation, which is simple in this case:

$$\Delta z = 40\,m = f\frac{L}{D}\frac{V^2}{2g} = (0.0185)\frac{L}{(0.08m)}\frac{(4.06)^2}{2(9.81)}, \quad \textit{Solve} \quad L \approx 205\,m \quad \textit{Ans.}$$

6.75 You wish to water your garden with 100 ft of $\frac{5}{8}$-in-diameter hose whose roughness is 0.011 in. What will be the delivery, in ft^3/s, if the gage pressure at the faucet is 60 lbf/in^2? If there is no nozzle (just an open hose exit), what is the maximum horizontal distance the exit jet will carry?

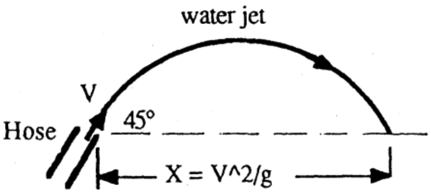


Fig. P6.75

Solution: For water, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. We are given $\varepsilon/d = 0.011/(5/8) \approx 0.0176$. For constant area hose, $V_1 = V_2$ and energy yields

$$\frac{p_{\text{faucet}}}{\rho g} = h_f, \quad \text{or:} \quad \frac{60 \times 144 \text{ psf}}{1.94(32.2)} = 138 \text{ ft} = f\frac{L}{d}\frac{V^2}{2g} = f\frac{100}{(5/8)/12}\frac{V^2}{2(32.2)},$$

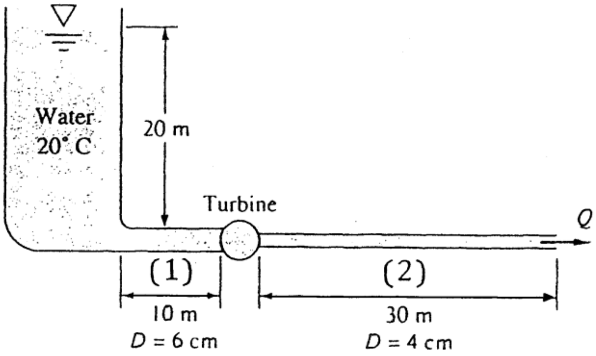
$$\text{or} \quad fV^2 \approx 4.64. \quad \text{Guess} \quad f \approx f_{\text{fully rough}} = 0.0463, \quad V \approx 10.0\ \frac{\text{ft}}{\text{s}}, \quad \text{Re} \approx 48400$$

$$\text{then} \quad f_{\text{better}} \approx 0.0472, \quad V_{\text{final}} \approx \mathbf{9.91\ ft/s} \text{ (converged)}$$

The hose delivery then is $Q = (\pi/4)(5/8/12)^2(9.91) = \mathbf{0.0211\ ft^3/s}$. *Ans.* (a)

From elementary particle-trajectory theory, the maximum horizontal distance X traveled by the jet occurs at $\theta = 45°$ (see figure) and is $\mathbf{X} = V^2/g = (9.91)^2/(32.2) \approx \mathbf{3.05\ ft}$ *Ans.* (b), which is pitiful. You need a *nozzle* on the hose to increase the exit velocity.

6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m^3/h. Why are there two solutions? Which is better?



Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d2 = 0.046/40 \approx 0.00115$. The energy equation, with $V1 \approx 0$ and $p1 = p2$, gives

$$z_1 - z_2 = 20\text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{\text{turbine}}, \quad h_{f1} = f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} \quad \text{and} \quad h_{f2} = f_2\frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$\text{Also,}\quad h_{\text{turbine}} = \frac{P}{\rho g Q} = \frac{400\text{ W}}{998(9.81)Q} \quad \text{and} \quad Q = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

If we rewrite the energy equation in terms of *Q* and multiply by *Q*, it is essentially a cubic polynomial, because for these rough walls the friction factors are almost constant:

$$Q\,h_{\text{turbine}} = \frac{400}{998(9.81)} = 20Q - \frac{8f_1L_1Q^3}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^3}{\pi^2 g d_2^5} - \frac{8Q^3}{\pi^2 g d_2^4}$$

Solve by EES or by iteration. There are three solutions, two of which are positive and the third is a meaningless negative value. The two valid (positive) solutions are:

(*a*) $Q = 0.00437\,m^3/s = \mathbf{15.7}\,m^3/hr$; $\text{Re}_1 = 92{,}500,\ f_1 = 0.0215$; $\text{Re}_1 = 138{,}800,\ f_1 = 0.0221$

(*b*) $Q = 0.00250\,m^3/s = \mathbf{9.0}\,m^3/hr$; $\text{Re}_1 = 52{,}900,\ f_1 = 0.0232$; $\text{Re}_1 = 79{,}400,\ f_1 = 0.0232$ **Ans.**

[The negative (meaningless) solution is $Q = -\,0.0069$ m^3/hr.] Both solutions (a) and (b) are valid mathematically. **Solution (*b*) is preferred** – the same power for 43% less water flow, and the turbine captures 16.3 m of the available 20 m head. Solution (*a*) is also unrealistic, because a real turbine's power increases with water flow rate. Turbine (*a*) would generate more than 400 W.

6.77 Modify Prob. 6.76 into an economic analysis, as follows. Let the 40 m of wrought-iron pipe have a uniform diameter *d*. Let the steady water flow available be $Q = 30\ \text{m}^3/\text{h}$. The cost of the turbine is \$4 per watt developed, and the cost of the piping is \$75 per centimeter of diameter. The power generated may be sold for \$0.08 per kilowatt hour. Find the proper pipe diameter for minimum *payback time*, i.e., minimum time for which the power sales will equal the initial cost of the system.

Solution: With flow rate known, we need only guess a diameter and compute power from the energy equation similar to Prob. 6.76:

$$P = \rho g Q h_t, \quad \text{where } h_t = 20\text{ m} - \frac{V^2}{2g}\left(1+f\frac{L}{d}\right) = 20 - \frac{8Q^2}{\pi^2 g d^4}\left(1+f\frac{L}{d}\right)$$

$$\text{Then}\quad \text{Cost} = \$4 * P + \$75(100d) \quad\text{and}\quad \text{Annual income} = \$0.08\left(\frac{P}{1000}\right)(24)(365)$$

The Moody friction factor is computed from $\text{Re} = 4\rho Q/(\pi\mu d)$ and $\varepsilon/d = 0.066/d(\text{mm})$. The payback time, in years, is then the cost divided by the annual income. For example,

$$\text{If } d = 0.1\text{ m},\quad \text{Re} \approx 106000,\quad f \approx 0.0200,\quad h_t \approx 19.48\text{ m},\quad P = 1589.3\text{ W}$$

$$\text{Cost} \approx \$7107 \quad \text{Income} = \$1{,}114/\text{year} \quad \text{Payback} \approx \mathbf{6.38\ years}$$

Since the piping cost is very small (<\$1000), both cost and income are nearly proportional to power, hence the payback will be nearly the same (6.38 years) regardless of diameter. There is an almost invisible minimum at **d ≈ 7 cm**, Re ≈ 151000, f ≈ 0.0201, $h_t \approx 17.0$ m, Cost ≈ \$6078, Income ≈ \$973, Payback ≈ 6.25 years. However, as diameter d decreases, we generate less power and gain little in payback time.

6.78 In Fig. P6.78 the connecting pipe is commercial steel 6 cm in diameter. Estimate the flow rate, in m^3/h, if the fluid is water at 20°C. Which way is the flow?

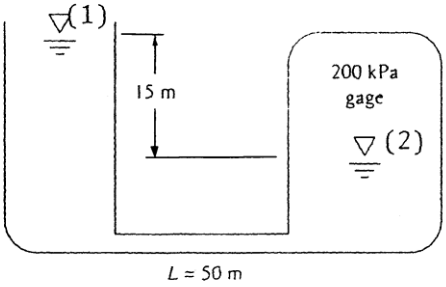


Fig. P6.78

Solution: For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. For commercial steel, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d = 0.046/60 \approx 0.000767$. With p_1, V_1, and V_2 all ≈ 0, the energy equation between surfaces (1) and (2) yields

$$0+0+z_1 \approx \frac{p_2}{\rho g}+0+z_2+h_f, \quad \text{or} \quad h_f = 15-\frac{200000}{998(9.81)} \approx -5.43\ \text{m}\ \textbf{(flow to left)} \leftarrow$$

$$\text{Guess turbulent flow:} \quad h_f = f\frac{L}{d}\frac{V^2}{2g} = f\frac{50}{0.06}\frac{V^2}{2(9.81)} = 5.43, \quad \text{or:} \quad fV^2 \approx 0.1278$$

$$\frac{\varepsilon}{d} = 0.00767, \quad \text{guess } f_{\text{fully rough}} \approx 0.0184, \quad V \approx \left(\frac{0.1278}{0.0184}\right)^{1/2} \approx 2.64\ \frac{m}{s}, \quad Re = 158000$$

$$f_{\text{better}} \approx 0.0204, \quad V_{\text{better}} = 2.50\ \frac{m}{s}, \quad Re_{\text{better}} \approx 149700, \quad f_{\text{3rd iteration}} \approx 0.0205\ (\text{converged})$$

The iteration converges to

$$f \approx 0.0205,\ V \approx 2.49\ \text{m/s},\ Q = (\pi/4)(0.06)^2(2.49) = 0.00705\ \text{m}^3/\text{s} = \mathbf{25\ m^3/h} \leftarrow \quad \textit{Ans.}$$

6.79 A garden hose is used as the return line in a waterfall display at the mall. In order to select the proper pump, you need to know the hose wall roughness, which is not supplied by the manufacturer. You devise a simple experiment: attach the hose to the drain of an above-ground pool whose surface is 3 m above the hose outlet. You estimate the minor loss coefficient in the entrance region as 0.5, and the drain valve has a minor-loss equivalent length of 200 diameters when fully open. Using a bucket and stopwatch, you open the valve and measure a flow rate of 2.0E–4 m^3/s for a hose of inside diameter 1.5 cm and length 10 m. Estimate the roughness height of the hose inside surface.

Solution: First evaluate the average velocity in the hose and its Reynolds number:

$$V = \frac{Q}{A} = \frac{2.0E\text{–}4}{(\pi/4)(0.015)^2} = 1.13\ \frac{m}{s}, \quad Re_d = \frac{\rho V d}{\mu} = \frac{998(1.13)(0.015)}{0.001} = 16940\ (\textit{turbulent})$$

Write the energy equation from surface (point 1) to outlet (point 2), assuming an energy correction factor $\alpha = 1.05$:

$$\frac{p_1}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+h_f+\Sigma h_{loss},\quad where\ \Sigma h_{loss}=\left(K_e+f\frac{L_{eq}}{d}\right)\frac{\alpha_2 V_2^2}{2g}$$

The unknown is the friction factor:

$$f=\frac{\dfrac{z_1-z_2}{V^2/2g}-\alpha_2-K_e}{(L+L_{eq})/d}=\frac{\dfrac{3m}{(1.13)^2/2(9.81)}-1.05-0.5}{(10/0.015+200)}=0.0514$$

For $f = 0.0514$ and $\mathrm{Re}_d = 16940$, the Moody chart (Eq. 6.48) predicts $\varepsilon/d \approx 0.0206$. Therefore the estimated hose-wall roughness is $\varepsilon = 0.0206(1.5\text{ cm}) =$ **0.031 cm** *Ans.*

6.80 The head-versus-flow-rate characteristics of a centrifugal pump are shown in Fig. P6.80. If this pump drives water at 20°C through 120 m of 30-cm-diameter cast-iron pipe, what will be the resulting flow rate, in m^3/s?

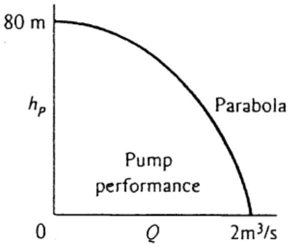


Fig. P6.80

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/300 \approx 0.000867$. The head loss must match the pump head:

$$h_f=f\frac{L}{d}\frac{V^2}{2g}=\frac{8fLQ^2}{\pi^2 gd^5}=h_{pump}\approx 80-20Q^2,\quad \text{with Q in m}^3/\text{s}$$

$$\text{Evaluate}\quad h_f=\frac{8f(120)Q^2}{\pi^2(9.81)(0.3)^5}=80-20Q^2,\quad \text{or:}\quad Q^2\approx\frac{80}{20+4080f}$$

$$\text{Guess}\quad f\approx 0.02,\quad Q=\left[\frac{80}{20+4080(0.02)}\right]^{1/2}\approx 0.887\ \frac{m^3}{s},\quad Re=\frac{4\rho Q}{\pi\mu d}\approx 3.76E6$$

$$\frac{\varepsilon}{d}=0.000867,\quad f_{better}\approx 0.0191,\quad Re_{better}\approx 3.83E6,\quad \text{converges to } \mathbf{Q\approx 0.905\ \frac{m^3}{s}}\quad \textit{Ans.}$$

6.81 The pump in Fig. P6.80 is used to deliver gasoline at 20°C through 350 m of 30-cm-diameter galvanized iron pipe. Estimate the resulting flow rate, in m^3/s. (Note that the pump head is now in meters of *gasoline*.)

Solution: For gasoline, take $\rho = 680\ kg/m^3$ and $\mu = 2.92E{-}4\ kg/m{\cdot}s$. For galvanized iron, take $\varepsilon \approx 0.15$ mm, hence $\varepsilon/d = 0.15/300 \approx 0.0005$. Head loss matches pump head:

$$h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8f(350)Q^2}{\pi^2(9.81)(0.3)^5} = 11901fQ^2 = h_{pump} \approx 80 - 20Q^2, \quad Q^2 = \frac{80}{20+11901f}$$

$$\text{Guess} \quad f_{rough} \approx 0.017, \quad Q \approx 0.600\ \frac{m^3}{s},$$

$$Re_{better} \approx 5.93E6, \quad \frac{\varepsilon}{d} = 0.0005, \quad f_{better} \approx 0.0168$$

This converges to $f \approx 0.0168$, $Re \approx 5.96E6$, $\mathbf{Q \approx 0.603\ m^3/s}$. *Ans.*

P6.82 Fluid at 20°C flows through a horizontal galvanized-iron pipe 20 m long and 8 cm in diameter. The wall shear stress is 90 Pa. Calculate the flow rate in m^3/h if the fluid is (*a*) glycerin; and (*b*) water.

Solution: (*a*) For glycerin, take $\rho = 1260\ kg/m^3$ and $\mu = 1.49\ kg/m{\cdot}s$. For galvanized iron, take $\varepsilon \approx 0.15$ mm, hence $\varepsilon/D = 0.15/80 \approx 0.001875$. But we are guessing this flow is ***laminar***:

$$\tau_{w,\text{laminar}} = 90\,Pa = \frac{8\mu V}{D} = \frac{8(1.49)V}{(0.08)}, \quad \text{solve} \quad V = 0.604\frac{m}{s}$$

$$Q = \frac{\pi}{4}D^2 V = \frac{\pi}{4}(0.08)^2(0.604) = 0.00304\ \frac{m^3}{s} = \mathbf{10.9}\ \frac{m^3}{h} \qquad Ans.(a)$$

$$Check \ \ Re_D = \frac{\rho VD}{\mu} = \frac{(1260)(0.604)(0.08)}{1.49} = 41 \text{ (yes, laminar)}$$

(*b*) For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001\ kg/m{\cdot}s$. For galvanized iron, take $\varepsilon \approx 0.15$ mm, hence $\varepsilon/D = 0.15/80 \approx 0.001875$. Now we are guessing this flow is ***turbulent***. At that roughness, the minimum friction factor is 0.023, which we can use for a first estimate:

$$\tau_w = 90\ Pa = f\frac{\rho V^2}{8} \approx (0.023)\frac{998V^2}{8}, \text{ or: } V \approx 5.6\frac{m}{s}, \ Re_D \approx 447{,}000$$

The Reynolds number estimate is certainly high enough, and into the fully-rough region of the Moody chart. Iterate briefly to the final result, only slightly different:

$$f = 0.0235 \;;\; \mathrm{Re}_D = 443{,}000 \;;\; V = 5.55\frac{m}{s} \;;\; Q = 0.0279\,\frac{m^3}{s} = \mathbf{100}\,\frac{m^3}{h} \quad \textit{Ans.(b)}$$

6.83 For the system of Fig. P6.55, let Δz = 80 m and *L* = 185 m of cast-iron pipe. What is the pipe diameter for which the flow rate will be 7 m^3/h?

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx$ 0.26 mm, but *d* is unknown. The energy equation is simply

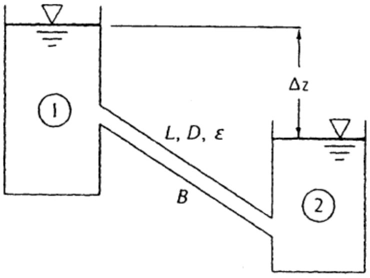


Fig. P6.55

$$\Delta z = 80\text{ m} = h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8f(185)(7/3600)^2}{\pi^2(9.81)d^5} = \frac{5.78E\text{–}5\,f}{d^5}, \quad \text{or} \quad d \approx 0.0591 f^{1/5}$$

$$\text{Guess} \quad f \approx 0.03, \quad d = 0.0591(0.03)^{1/5} \approx 0.0293\text{ m}, \quad \mathrm{Re} = \frac{4\rho Q}{\pi\mu d} \approx 84300, \quad \frac{\varepsilon}{d} \approx 0.00887$$

Iterate: $f_{better} \approx 0.0372$, $d_{better} \approx 0.0306$ m, $\mathrm{Re}_{better} \approx 80700$, $\varepsilon/d|_{better} \approx 0.00850$, etc. The process converges to f ≈ 0.0367, **d ≈ 0.0305 m**. *Ans.*

6.84 It is desired to deliver 60 m^3/h of water ($\rho = 998$ kg/m^3, $\mu = 0.001$ kg/m·s) at 20°C through a horizontal asphalted cast-iron pipe. Estimate the pipe diameter which will cause the pressure drop to be exactly 40 kPa per 100 meters of pipe length.

Solution: Write out the relation between Δp and friction factor, taking "L" = 100 m:

$$\Delta p = f\frac{L}{d}\frac{\rho}{2}V^2 = f\frac{100}{d}\frac{(998)}{2}\left[\frac{60/3600}{(\pi/4)d^2}\right]^2 = 40{,}000 = 22.48\frac{f}{d^5}, \quad or: \quad d^5 = 0.00562f$$

Knowing $\varepsilon = 0.12$ mm, then $\varepsilon/d = 0.00012/d$ and $\mathrm{Re}_d = 4\rho Q/(\pi\mu d) = 21178/d$. Use EES, or guess $f \approx 0.02$ and iterate until the proper diameter and friction factor are found. Final convergence: $f \approx 0.0216$; $\mathrm{Re}_d \approx 204{,}000$; $\boldsymbol{d = 0.104}$ **m**. *Ans.*

P6.85 For the system in Prob. P6.53, a pump, which delivers 15,000 W to the water, is used at night to refill the upper reservoir. The pipe diameter is increased from 12 cm to provide more flow. If the resultant flow rate is 90 m^3/h, estimate the new pipe size.

Solution: For water at 20°C, Table A.3, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Recall that $\Delta z = 60$ m and $L = 360$ m. Since the pressures and velocities cancel, the energy equation becomes

$$z_{lower} = z_{upper} + h_f - h_{pump}, \quad or: \quad h_p = \Delta z + h_f = 60m + f\frac{360m}{D}\frac{V^2}{2(9.81)}$$

$$\text{where} \quad \frac{1}{\sqrt{f}} \approx 2.0\log_{10}\left(\frac{\mathrm{Re}_D\sqrt{f}}{2.51}\right), \quad \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} \quad \frac{4(998)(90/3600)}{\pi(0.001)D},$$

$$Q = V\frac{\pi}{4}D^2 = 90\frac{m^3}{h} = 0.025\frac{m^3}{s}, \quad Power = \rho g Q h_p = (998)(9.81)Q h_p = 15{,}000\ W$$

You could solve by iteration, guessing values of D greater than 12 cm, until $Q = 90$ m^3/h. Or you could put the above equations into EES, which will report the answer:

$$\mathrm{Re}_D = 169{,}000 \ ; \ f = 0.0161 \ ; \ Q = 90m^3/h \quad \text{if} \quad D = \mathbf{0.188}m \quad \textit{Ans.}$$

Thus a 57% increase in diameter only produces a 13% increase in flow rate. Even with an indefinitely large diameter, because of the 60-meter elevation head to fight against, Q can never be greater than 92 m^3/h if $P = 15$ kW.

6.86 SAE 10 oil at 20°C flows at an average velocity of 2 m/s between two smooth parallel horizontal plates 3 cm apart. Estimate (a) the centerline velocity, (b) the head loss per meter, and (c) the pressure drop per meter.

Solution: For SAE 10 oil, take $\rho = 870\ \text{kg/m}^3$ and $\mu = 0.104$ kg/m·s. The half-distance between plates is called "*h*" (see Fig. 6.37). Check D_h and Re:

$$D_h = \frac{4A}{P} = 4h = 6\ \text{cm},\quad Re_{D_h} = \frac{\rho V D_h}{\mu} = \frac{870(2.0)(0.06)}{0.104} \approx 1004\ \text{(laminar)}$$

$$\text{Then}\quad u_{CL} = u_{max} = \frac{3}{2}V = \frac{3}{2}(2.0) \approx \mathbf{3.0\ \ m/s}\quad \textit{Ans. (a)}$$

The head loss and pressure drop per meter follow from laminar theory, Eq. (6.63):

$$\Delta p = \frac{3\mu VL}{h^2} = \frac{3(0.104)(2.0)(1.0)}{(0.015\ \text{m})^2} \approx \mathbf{2770\ \ Pa/m}\quad \textit{Ans. (c)}$$

$$h_f = \frac{\Delta p}{\rho g} = \frac{2770}{870(9.81)} \approx \mathbf{0.325\ \ m/m}\quad \textit{Ans. (b)}$$

6.87 A commercial-steel annulus 40 ft long, with *a* = 1 in and b = $\frac{1}{2}$ in, connects two reservoirs which differ in surface height by 20 ft. Compute the flow rate in ft^3/s through the annulus if the fluid is water at 20°C.

Solution: For water, take $\rho = 1.94\ \text{slug/ft}^3$ and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft. Compute the hydraulic diameter of the annulus:

$$D_h = \frac{4A}{P} = 2(a-b) = 1\ \text{inch};$$

$$h_f = 20\ \text{ft} = f\frac{L}{D_h}\frac{V^2}{2g} = f\left(\frac{40}{1/12}\right)\frac{V^2}{2(32.2)},\quad \text{or:}\quad fV^2 \approx 2.683$$

We can make a reasonable estimate by simply relating the Moody chart to *Dh*, rather than the more complicated "effective diameter" method of Eq. (6.77). Thus

$$\frac{\varepsilon}{D_h} = \frac{0.00015}{1/12} \approx 0.0018,\quad \text{Guess}\ f_{rough} \approx 0.023,\quad V = (2.683/0.023)^{1/2} \approx 10.8\ \frac{\text{ft}}{\text{s}}$$

$$Re = \frac{\rho V D_h}{\mu} = \frac{1.94(10.8)(1/12)}{2.09\text{E}{-5}} \approx 83550,\quad f_{better} \approx 0.0249,\quad V_{better} \approx 10.4\ \frac{\text{ft}}{\text{s}}$$

This converges to $f \approx 0.0250$, $V \approx 10.37$ ft/s, $Q = \pi(a^2 - b^2)V = \mathbf{0.17\ ft^3/s}$. *Ans.*

6.88 An oil cooler consists of multiple parallel-plate passages, as shown in Fig. P6.88. The available pressure drop is 6 kPa, and the fluid is SAE 10W oil at 20°C. If the desired total flow rate is 900 m^3/h, estimate the appropriate number of passages. The plate walls are hydraulically smooth.

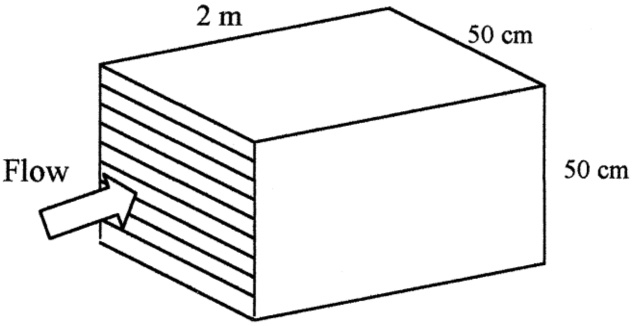


Fig. P6.88

Solution: For SAE 10W oil, $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The pressure drop remains 6 kPa no matter how many passages there are (ducts in parallel). Guess laminar flow, Eq. (6.63),

$$Q_{one\ passage} = \frac{bh^3}{3\mu}\frac{\Delta p}{L}$$

where h is the half-thickness between plates. If there are N passages, then $b = 50$ cm for all and $h = 0.5$ m/(2N). We find h and N such that $NQ = 900$ m^3/h for the full set of passages. The problem is ideal for EES, but one can iterate with a calculator also. We find that 18 passages are one too many—Q only equals 835 m^3/h. The better solution is:

$\mathbf{N = 17}$ **passages**, $Q_N = 935\ m^3/h$, $h = 1.47$ cm, $Re_{D_h} = 512$ (laminar flow)

6.89 An annulus of narrow clearance causes a very large pressure drop and is useful as an accurate measurement of viscosity. If a smooth annulus 1 m long with $a = 50$ mm and $b = 49$ mm carries an oil flow at 0.001 m^3/s, what is the oil viscosity if the pressure drop is 250 kPa?

Solution: Assuming laminar flow, use Eq. (6.73) for the pressure drop and flow rate:

$$Q = \frac{\pi}{8\mu}\frac{\Delta p}{L}\left[a^4 - b^4 - \frac{(a^2 - b^2)^2}{\ln(a/b)}\right], \quad \text{or, for the given data:}$$

$$0.001\ \text{m}^3/\text{s} = \frac{\pi}{8\mu}\left(\frac{250000}{1\ \text{m}}\right)\left[(0.05)^4 - (0.049)^4 - \frac{\{(0.05)^2 - (0.049)^2\}^2}{\ln(0.05/0.049)}\right]$$

Solve for $\mu \approx \mathbf{0.0065\ kg/m{\cdot}s}$ *Ans.*

P6.90 A rectangular sheet-metal duct is 200 ft long and has a fixed height *H* = 6 inches. The width *B*, however, may vary from 6 to 36 inches. A blower provides a pressure drop of 80 Pa of air at 20°C and 1 atm. What is the optimum width *B* that will provide the most airflow in ft^3/s?

Solution: For air at 20°C and 1 atm, take ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. The pressure drop is related to the hydraulic diameter of the duct. Convert *L* = 200 ft = 60.96 m. For sheet metal, from Table 6.1, the roughness ε = 0.05 mm.

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2, \quad \text{where } D_h = \frac{4A}{P} = \frac{2BH}{B+H}, \quad f \text{ related to } \mathrm{Re}_{Dh} \text{ and } \frac{\varepsilon}{D_h}$$

$$\text{Solve for } V = \sqrt{\frac{2\Delta p D_h}{f\rho L}} = \sqrt{\frac{2(80\,Pa)D_h}{f\,(1.20)(60.96)}} = 1.479\sqrt{\frac{D_h}{f}} \text{ which gives } V \text{ in } \frac{m}{s}$$

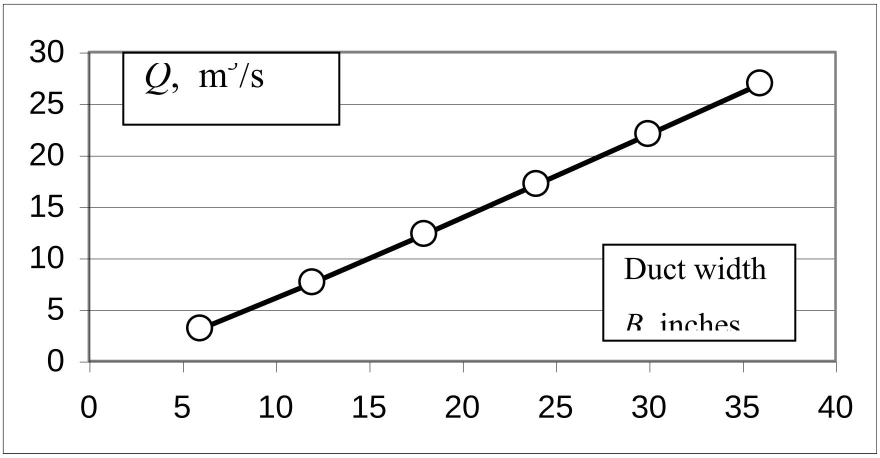
The duct area *A = 2BH* increases with *B* for a fixed *H*, and so does the hydraulic diameter. The Reynolds number ($\rho V D_h/\mu$) also increases, hence the friction factor *f* decreases. All of these factors make the flow rate *Q* increase with *H*. Therefore, without even making calculations, we conclude that **the widest *H* (36 inches) produces the most flow rate**. *Ans.*

We can calculate the actual flow rate for *H* = 36 inches = 0.9144 m:

$$H = 36\,in\,,\;\; D_h = 0.261\,m\,,\;\; \mathrm{Re}_{Dh} = 95{,}200\,,\;\; \frac{\varepsilon}{D_h} = 0.000191\,,\;\; f = 0.01914,$$

$$Giving\quad V = 5.46\,\frac{m}{s}\,,\;\; Q = V\,B\,H = 0.761\frac{m^3}{s} = \mathbf{27}\,\frac{ft^3}{s}\quad Ans.$$

Here is a plot of flow rate Q versus width H. It is almost exactly linear.



6.91 Heat exchangers often consist of many triangular passages. Typical is Fig. P6.91, with L = 60 cm and an isosceles-triangle cross section of side length a = 2 cm and included angle β = 80°. If the average velocity is V = 2 m/s and the fluid is SAE 10 oil at 20°C, estimate the pressure drop.

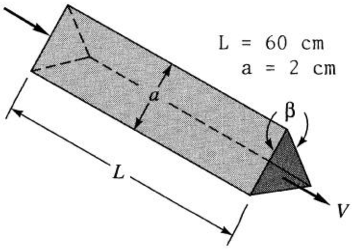


Fig. P6.91

Solution: For SAE 10 oil, take ρ = 870 kg/m^3 and μ = 0.104 kg/m·s. The Reynolds number based on side length a is Re = ρVa/μ ≈ **335**, so the flow is *laminar*. The bottom side of the triangle is 2(2 cm)sin40° ≈ *2.57* cm. Calculate hydraulic diameter:

$$A=\frac{1}{2}(2.57)(2\cos 40°)\approx 1.97\ \text{cm}^2;\quad \text{P}=6.57\ \text{cm};\quad \text{D}_\text{h}=\frac{4\text{A}}{\text{P}}\approx 1.20\ \text{cm}$$

$$\text{Re}_{\text{D}_\text{h}}=\frac{\rho \text{VD}_\text{h}}{\mu}=\frac{870(2.0)(0.0120)}{0.104}\approx 201;\quad \text{from Table 6.4,}\ \theta=40°,\quad \text{fRe}\approx 52.9$$

$$\text{Then}\quad \text{f}=\frac{52.9}{201}\approx 0.263,\quad \Delta\text{p}=\ \text{f}\frac{\text{L}}{\text{D}_\text{h}}\frac{\rho}{2}\text{V}^2=(0.263)\left(\frac{0.6}{0.012}\right)\left(\frac{870}{2}\right)(2)^2$$

$$\approx \mathbf{23000\ Pa}\quad \textit{Ans.}$$

6.92 A large room uses a fan to draw in atmospheric air at 20°C through a 30 cm by 30 cm commercial-steel duct 12 m long, as in Fig. P6.92. Estimate (a) the air flow rate in m^3/hr if the room pressure is 10 Pa vacuum; and (b) the room pressure if the flow rate is 1200 m^3/hr. Neglect minor losses.

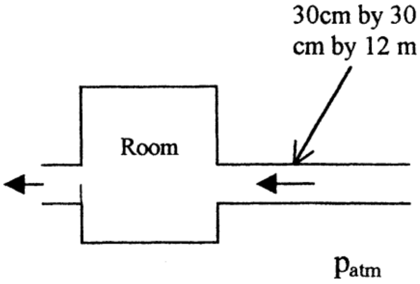


Fig. P6.92

Solution: For air, take ρ = 1.2 kg/m^3 and μ = 1.8E−5 kg/m·s. For commercial steel, ε = 0.046 mm. For a square duct, Dh = side-length = 30 cm, hence ε/d = 0.046/300 = 0.000153. The (b) part is easier, with flow rate known we can evaluate velocity, Reynolds number, and friction factor:

$$V=\frac{Q}{A}=\frac{1200/3600}{(0.3)(0.3)}=3.70\ \frac{m}{s},\quad Re_{D_h}=\frac{1.2(3.70)(0.3)}{1.8E\text{–}5}=74100,\quad \text{thus}\ f_{Moody}\approx 0.0198$$

Then the pressure drop follows immediately:

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = 0.0198\left(\frac{12}{0.3}\right)\left(\frac{1.2}{2}\right)(3.70)^2 = 6.53\ Pa,$$

or: $\mathbf{p_{room} = 6.5\ Pa}$ (*vacuum*) *Ans.* (b)

(a) If Δp = 10 Pa (vacuum) is known, we must iterate to find friction factor:

$$\Delta p = 10\ \ Pa = f\left(\frac{12}{0.3}\right)\left(\frac{1.2}{2}\right)V^2,\quad V = \frac{Q}{(0.3)^2},\quad f = fcn\left(\frac{1.2V(0.3)}{1.8E\text{–}5}, \frac{\varepsilon}{D_h} = 0.000153\right)$$

After iteration, the results converge to:

$V = 4.69$ m/s; Red = 93800; $f = 0.0190$; $\boldsymbol{Q} = 0.422\ \mathrm{m^3/s} = \mathbf{1520\ m^3/h}$ *Ans.* (a)

P6.93 In Moody's Example 6.6, the 6-inch diameter, 200-ft-long asphalted cast iron pipe has a pressure drop of about 280 lbf/ft^2 when the average water velocity is 6 ft/s. Compare this to an *annular* cast iron pipe with an inner diameter of 6 in and the same annular average velocity of 6 ft/s. (*a*) What outer diameter would cause the flow to have the same pressure drop of 280 lbf/ft^2? (*b*) How do the cross-section areas compare, and why? Use the hydraulic diameter approximation.

Solution: Recall the Ex. 6.6 data, $\varepsilon = 0.0004$ ft. For water at 68°F, take $\rho = 1.94$ slug/ft^3 and μ = 2.09E-5 slug/ft-sec. The hydraulic diameter of an annulus is $D_h = 2(R_o - R_i)$, where $R_i = 0.25$ ft. We know the pressure drop, hence the head loss is

$$h_f = f\frac{L}{D_h}\frac{V^2}{2g} = f\frac{200\,ft}{2(R_o - 0.25\,ft)}\frac{(6\,ft/s)^2}{32.2\,ft/s^2} = \frac{\Delta p}{\rho g} = \frac{280\,lbf/ft^2}{62.4\,lbf/ft^3} = 4.49\,ft$$

We do not know f or R_o. The additional relation is the Moody friction factor correlation:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\mathrm{Re}_{D_h}\sqrt{f}}\right) \quad \text{where} \quad \mathrm{Re}_{D_h} = \frac{\rho V D_h}{\mu} = \frac{(1.94)(6.0)[2(R_o - 0.25)]}{2.09E-5}$$

(*a*) For $\varepsilon = 0.0004$ ft, solve these two simultaneously, using EES or Excel, to obtain

$$f = 0.0199 \;;\; \mathrm{Re}_{D_{ho}} = 276{,}000 \;;\; R_o = \mathbf{0.498ft} \qquad \textit{Ans.}(a)$$

(*b*) The annular gap is 0.498 – 0.25 = 0.248 ft, just about equal to the inner radius. However, the annular area is **three times the area of Moody's pipe**! *Ans* .(*b*) The annular pipe has much more wall area than a hollow pipe, more friction, so more cross-section area is needed to match the pressure drop.

P6.94 Air at 20°C flows through a smooth duct of diameter 20 cm at an average velocity of 5 m/s. It then flows into a smooth square duct of side length *a*. Find the square duct size *a* for which the pressure drop per meter will be exactly the same as the circular duct?

Solution: For air at 20°C and 1 atm, take ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. Compute the pressure drop in the circular duct:

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(1.2)(5)(0.2)}{1.8\mathrm{E}-5} = 66{,}700 \;;\; f_{smooth} = 0.0196$$

$$\Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.0196)\left(\frac{1m}{0.2m}\right)\left(\frac{1.2kg/m^3}{2}\right)\left(5\frac{m}{s}\right)^2 = 1.47\frac{Pa}{m}$$

The square duct will have slightly different size, Reynolds number, and velocity:

$$D_h = \frac{4a^2}{4a} = a \;;\; \mathrm{Re}_{Dh} = \frac{\rho V_{square} a}{\mu} = \frac{(1.2)V_s\, a}{1.8E-5}$$

$$But \quad Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}(0.2)^2(5) = 0.157\frac{m^3}{s} = V_s\, a^2$$

Thus everything can be written in terms of the square duct size *a*:

$$\mathrm{Re}_{Dh} = \frac{1.2(0.157/a^2)a}{1.8\mathrm{E}-5} = \frac{10470}{a}\ ;\ \Delta p = 1.47\frac{Pa}{m} = f\frac{L}{D_h}\frac{\rho}{2}V_s^2 = f\frac{1m}{a}(\frac{1.2}{2})(\frac{0.157}{a^2})^2$$

$$\text{or: } 1.47 = \frac{0.0147 f}{a^5} \quad \text{or: } f = 99.5a^5$$

Guess *f* equal to, say, 0.02, find the improved Reynolds number and *f*, finally find *a*:

Vs = 4.70 m/s ; Re_{Dh} = 57,350 ; f = 0.0203 ; a = **0.183** m *Ans.*

P6.95 Although analytical solutions are available for laminar flow in many duct shapes [34], what do we do about ducts of arbitrary shape? Bahrami et al. [57] propose that a better approach to the pipe result, $f\,\mathrm{Re} = 64$, is achieved by replacing the hydraulic diameter D_h by $\sqrt{A}$, where *A* is the area of the cross section. Test this idea for the isosceles triangles of Table 6.4. If time is short, at least try 10°, 50°, and 80°. What do you conclude about this idea?

Solution: We can see for the triangles in Table 6.4 that the values of $f\,\mathrm{Re}_{Dh}$ are all less than 64, by as much as 25%. If we denote Bahrami's idea as $D_B = \sqrt{A}$, the new Reynolds number is based on

$$D_B = \frac{D_B}{D_h} D_h = \frac{\sqrt{A}}{4A/P} D_h = (\frac{P}{4\sqrt{A}})\, D_h$$

For an isosceles triangle of height h, $P = 2h\sec\theta + 2h\tan\theta$, $A = h^2 \tan\theta$

$$\text{Thus } D_B = (\frac{\sec\theta + \tan\theta}{2\sqrt{\tan\theta}})\, D_h$$

So we multiply the values of $f\,\mathrm{Re}_{Dh}$ by the above factor and see how we close it is to 64:

θ, degrees	10	20	30	40	50	60	70	80
$f\,\mathrm{Re}_B$	73	**63**	**61**	**62**	**65**	72	85	116

We see that intermediate angles, 20° to 50°, work well, sharper angles not so good. Bahrami et al. [57] make some suggestions to modify the idea for too-small or too-large angles.

P6.96 A fuel cell [Ref. 59] consists of air (or oxygen) and hydrogen micro ducts, separated by a membrane that promotes proton exchange for an electric current, as in Fig. P6.96. Suppose that the air side, at 20°C and approximately 1 atm, has five 1 mm by 1 mm ducts, each 1 m long. The total flow rate is 1.5E-4 kg/s. (*a*) Determine if the flow is laminar or turbulent. (*b*) Estimate the pressure drop.

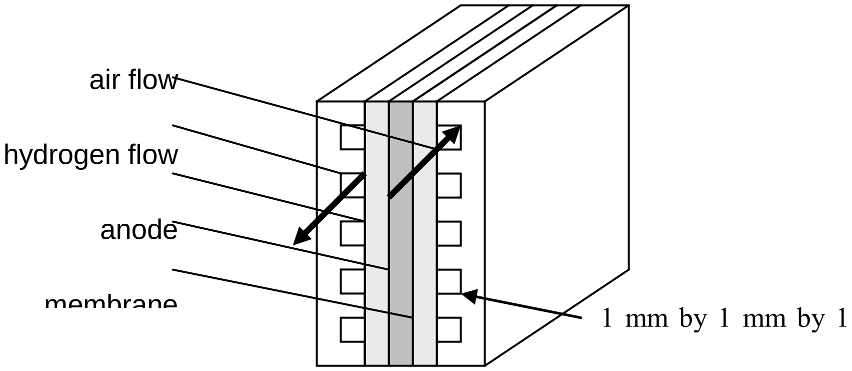


Fig. P6.96. Simplified diagram of an air-hydrogen fuel cell.
[*Problem courtesy of Dr. Pezhman Shirvanian*]

Solution: For air at 20°C and 1 atm, take ρ = 1.20 kg/m^3 and μ = 1.8E-5 kg/m-s. The hydraulic diameter of a square duct is easy, the side length a = 1 mm. The mass flow through one duct is

$$\dot{m}_{1duct} = \frac{1.5\text{E}-4\,kg/s}{5} = 0.3\text{E}-4\frac{kg}{s} = \rho AV = (1.20\frac{kg}{m^3})[(0.001m)^2]V$$

$$\text{Solve for } V = 25.0\frac{m}{s}, \text{ hence } \text{Re}_{Dh} = \frac{\rho V D_h}{\mu} = \frac{1.20(25.0)(0.001)}{0.000018} = 1667 \quad \text{(laminar)} \quad \textit{Ans.}(a)$$

(*b*) We could go with the simply circular-duct approximation, *f* = 64/Re, but we have a more exact laminar-flow result in Table 6.4 for a square duct:

$$f_{square} = \frac{56.91}{\mathrm{Re}_{Dh}} = \frac{56.91}{1667} = 0.0341$$

$$Then\ \Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.0341)(\frac{1m}{0.001m})(\frac{1.2kg/m^3}{2})(25\frac{m}{s})^2 = \mathbf{12{,}800\ Pa} \quad Ans.(b)$$

6.97 A heat exchanger consists of multiple parallel-plate passages, as shown in Fig. P6.97. The available pressure drop is 2 kPa, and the fluid is water at 20°C. If the desired total flow rate is 900 m^3/h, estimate the appropriate number of passages. The plate walls are hydraulically smooth.

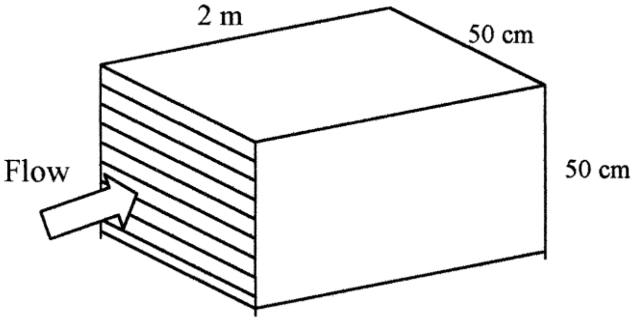


Fig. P6.97

Solution: For water, $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. Unlike Prob. 6.88, here we expect turbulent flow. If there are N passages, then b = 50 cm for all N and the passage thickness is $H = 0.5\ m/N$. The hydraulic diameter is $D_h = 2H$. The velocity in each passage is related to the pressure drop by Eq. (6.58):

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 \quad \text{where } f = f_{smooth} = fcn\left(\frac{\rho V D_h}{\mu}\right)$$

$$\text{For the given data,}\quad 2000\ Pa = f\frac{2.0\ m}{2(0.5\ m/N)}\frac{998\ kg/m^3}{2}V^2$$

Select N, find H and V and $Q_{total} = AV = b^2V$ and compare to the desired flow of 900 m^3/h. For example, guess $N = 20$, calculate $f = 0.0173$ and $Q_{total} = 2165\ m^3/h$. The converged result is

$$Q_{total} = 908\ m^3/h, \quad f = 0.028,$$

$$\mathrm{Re}_{D_h} = 14400, \quad H = 7.14\ mm, \quad \mathbf{N = 70\ passages} \quad Ans.$$

6.98 A rectangular heat exchanger is to be divided into smaller sections using sheets of commercial steel 0.4 mm thick, as sketched in Fig. P6.98. The flow rate is 20 kg/s of water at 20°C. Basic dimensions are L = 1 m, W = 20 cm, and H = 10 cm. What is the proper number of *square* sections if the overall pressure drop is to be no more than 1600 Pa?

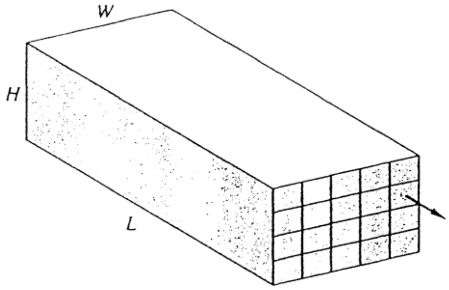


Fig. P6.98

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For commercial steel, $\varepsilon \approx 0.046$ mm. Let the short side (10 cm) be divided into "J" squares. Then the long (20 cm) side divides into "2J" squares and altogether there are N = 2J^2 squares. Denote the side length of the square as "a," which equals (10 cm)/J minus the wall thickness. The hydraulic diameter of a square exactly equals its side length, $D_h = a$. The total cross-section area is $A = N a^2$. Then the pressure drop relation becomes

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = f\frac{1.0}{a}\left(\frac{998}{2}\right)\left(\frac{Q}{Na^2}\right)^2 \le 1600 \text{ Pa}, \quad \text{where} \quad N = 2J^2 \text{ and } a = \frac{0.1}{J} - 0.0004$$

As a first estimate, neglect the 0.4-mm wall thickness, so $a \approx 0.1/J$. Then the relation for Δp above reduces to fJ ≈ 0.32. Since f ≈ 0.036 for this turbulent Reynolds number (Re ≈ 1E4) we estimate that J ≈ 9 and in fact this is not bad even including wall thickness:

$$J = 9, \quad N = 2(9)^2 = 162, \quad a = \frac{0.1}{9} - 0.0004 = 0.0107 \text{ m}, \quad V = \frac{20/998}{162(0.0107)^2} \approx 1.078 \ \frac{\text{m}}{\text{s}}$$

$$\text{Re} = \frac{\rho Va}{\mu} = \frac{998(1.078)(0.0107)}{0.001} \approx 11526, \quad \frac{\varepsilon}{a} = \frac{0.046}{10.7} \approx 0.00429, \quad f_{\text{Moody}} \approx 0.0360$$

$$\text{Then} \quad \Delta p = (0.036)\left(\frac{1.0}{0.0107}\right)\left(\frac{998}{2}\right)(1.078)^2 \approx 1950 \text{ Pa}$$

So the wall thickness increases V and decreases a so Δp is too large. Try J = 8:

$$J = 8, \quad N = 128, \quad a = 0.0121 \text{ m}, \quad V = 1.069 \ \frac{\text{m}}{\text{s}},$$

$$\text{Re} = 12913, \quad \frac{\varepsilon}{a} = 0.0038, \quad f \approx 0.0347$$

$$\text{Then} \quad \Delta p = f(L/a)(\rho/2)V^2 \approx \mathbf{1636 \text{ Pa}}. \quad \text{Close enough, } J = 8, \mathbf{N = 128} \quad \textit{Ans.}$$

[I suppose a practical person would specify J = 7, N = 98, to keep Δp < 1600 Pa.]

P6.99 In Sec. 6.11 it was mentioned that Roman aqueduct customers obtained extra water by attaching a diffuser to their pipe exits. Fig. P6.99 shows a simulation: a smooth inlet pipe, with and without a 15° diffuser expanding to a 5-cm-diameter exit. The pipe entrance is sharp-edged.

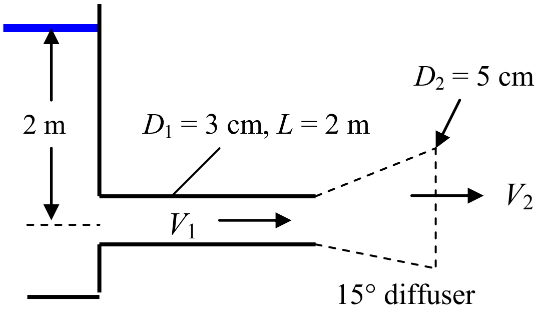


Fig. P6.99

Calculate the flow rate (*a*) without, and (*b*) with the diffuser.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. The energy equation between the aqueduct surface and the pipe exit yields

$$z_{surf} = z_2 + \frac{V_2^2}{2g} + h_f + \Sigma h_m = z_2 + \frac{V_2^2}{2g} + \frac{V_1^2}{2g}\left(f\frac{L}{D_1} + K_{entrance} + K_{diffuser}\right)$$

(*a*) Without the diffuser, $K_{\text{diff}} = 0$, and $V_1 = V_2$. For a sharp edge, take $K_{\text{ent}} = 0.5$. We obtain

$$2m = \frac{V_1^2}{2g}\left(1 + f\frac{2m}{0.03m} + 0.5\right)\ , \quad with \quad f = fcn(\text{Re} = \rho V_1 D_1/\mu)$$

$$Solve:\ \text{Re} = 115{,}000\ ;\ f = 0.0175\ ;\ V_1 = 4.48\,m/s\ ;\ Q_{without} = 0.00271\,m^3/s \quad Ans.(a)$$

(*b*) *With* the diffuser, from Fig. 6.23, for $D_1/D_2 = 3/5 = 0.6$ and $2\theta = 15°$, read $K_{\text{diffuser}} \approx 0.2$. From one-dimensional continuity, $V_2 = V_1(3/5)^2 = 0.36V_1$. The energy equation becomes

$$2m = \frac{(0.36V_1)^2}{2g} + \frac{V_1^2}{2g}\left(1 + f\frac{2m}{0.03m} + 0.5 + 0.2\right)$$

$$Solve\text{:}\quad \text{Re} = 134{,}000\ ;\ f = 0.0169\ ;\ V_1 = 3.84\,m/s\ ;\ Q_{with} = 0.00316\,m^3/s \qquad Ans.(b)$$

Adding the diffuser increases the flow rate by **17%**. [NOTE: Don't know if the Romans did this, but a well-rounded entrance, $K_{\text{ent}} = 0.05$, would increase the flow rate by another 15%.]

P6.100 Water at 68°F flows at 100 gal/min through a 2-inch-diameter commercial steel pipe that is 150 ft long. According to Table 6.5, how many 90° regular screwed elbows can be added to the system before the overall pressure drop exceeds 25 lbf/in^2?

Solution: For water at 68°F (20°C), $\rho = 1.94$ slug/ft^3 and $\mu = 0.0000209$ slug/ft-s. For commercial steel, $\varepsilon = 0.00015$ ft, hence $\varepsilon/D = 0.00015/(2/12) = 0.0009$. Compute the velocity, Reynolds number, and friction factor for a steady flow rate of 100 gal/min.

$$Q = 100\frac{\text{gal}}{\text{min}} = 0.223\frac{ft^3}{s}\ ;\ V = \frac{Q}{(\pi/4)D^2} = \frac{0.223\,ft^3 s}{(\pi/4)(2/12\,ft)^2} = 10.2\,\frac{ft}{s}$$

$$\text{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.94)(10.2)(2/12)}{(0.0000209)} = 158{,}000\ ;\ \frac{\varepsilon}{D} = 0.0009\ ;\ f_{Moody} = 0.0210$$

Compute the pressure drop of the pipe alone:

$$\Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.021)\left(\frac{150}{2/12}\right)\left(\frac{1.94}{2}\right)(10.2)^2 = 1910\frac{lbf}{ft^2} = 13.2\,\frac{lbf}{in^2}$$

Thus we can add elbows until their total minor loss = 25.0 – 13.2 = 11.8 lbf/in^2 = 1700 lbf/ft^2. From Table 6.5, each elbow has a loss $K = 0.95$.
The pressure loss of each elbow is

$$\Delta p_{elbow} = K_{elbow}\frac{\rho}{2}V^2 = (0.95)\left(\frac{1.94}{2}\right)(10.2)^2 = 96\ \frac{lbf}{ft^2}$$

$$\text{Thus the no. of elbows} = \frac{total\ elbow\ loss}{loss\ per\ elbow} = \frac{1700\,lbf/ft^2}{96\,lbf/ft^2} = \textbf{17 elbows max} \quad \textit{Ans.}$$

NOTE: IN PROBLEMS 6.100–6.110, MINOR LOSSES ARE *INCLUDED*.

6.101 In Fig. P6.101 a thick filter is being tested for losses. The flow rate in the pipe is 7 m^3/min, and the upstream pressure is 120 kPa. The fluid is air at 20°C. Using the water-manometer reading, estimate the loss coefficient *K* of the filter.

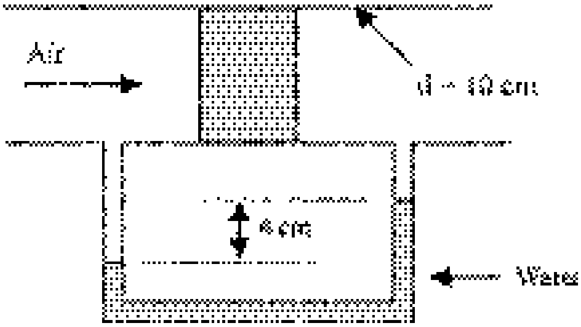


Fig. P6.101

Solution: The upstream density is $\rho_{air} = p/(RT) = 120000/[287(293)] = 1.43\ kg/m^3$. The average velocity *V* (which is used to correlate loss coefficient) follows from the flow rate:

$$V = \frac{Q}{A_{pipe}} = \frac{7/60\ m^3/s}{(\pi/4)(0.1\ m)^2} = 14.85\ m/s$$

The manometer measures the pressure drop across the filter:

$$\Delta p_{mano} = (\rho_w - \rho_a)gh_{mano} = (998 - 1.43\ kg/m^3)(9.81\ m/s^2)(0.04\ m) = 391\ Pa$$

This pressure is correlated as a loss coefficient using Eq. (6.78):

$$K_{filter} = \frac{\Delta p_{filter}}{(1/2)\rho V^2} = \frac{391\ Pa}{(1/2)(1.43\ kg/m^3)(14.85\ m/s)^2} \approx \mathbf{2.5} \quad \textit{Ans.}$$

6.102 A 70 percent efficient pump delivers water at 20°C from one reservoir to another 20 ft higher, as in Fig. P6.102. The piping system consists of 60 ft of galvanized-iron 2-in pipe, a reentrant entrance, two screwed 90° long-radius elbows, a screwed-open gate valve, and a sharp exit. What is the input power required in horsepower with and without a 6° well-designed conical expansion added to the exit? The flow rate is 0.4 ft^3/s.

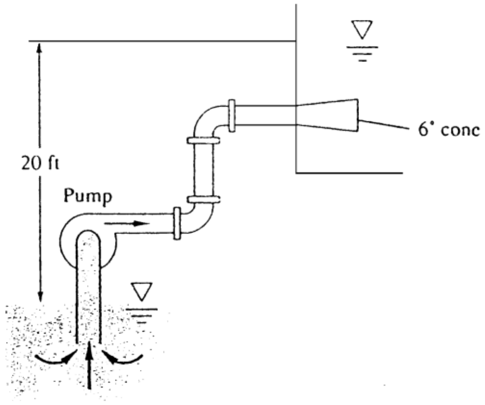


Fig. P6.102

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For galvanized iron, $\varepsilon \approx 0.0005$ ft, whence $\varepsilon/d = 0.0005/(2/12 \text{ ft}) \approx 0.003$. Without the 6° cone, the minor losses are:

$$K_{\text{reentrant}} \approx 1.0; \quad K_{\text{elbows}} \approx 2(0.41); \quad K_{\text{gate valve}} \approx 0.16; \quad K_{\text{sharp exit}} \approx 1.0$$

$$\text{Evaluate} \quad V = \frac{Q}{A} = \frac{0.4}{\pi(2/12)^2/4} = 18.3 \ \frac{\text{ft}}{\text{s}}; \quad \text{Re} = \frac{\rho V d}{\mu} = \frac{1.94(18.3)(2/12)}{2.09\text{E}{-5}} \approx 284000$$

At this Re and roughness ratio, we find from the Moody chart that f ≈ 0.0266. Then

$$\text{(a)} \quad h_{\text{pump}} = \Delta z + \frac{V^2}{2g}\left(f\frac{L}{d} + \Sigma K\right) = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + 0.82 + 0.16 + 1.0\right]$$

$$\text{or} \quad h_{\text{pump}} \approx 85.6 \text{ ft}, \quad \text{Power} = \frac{\rho g Q h_p}{\eta} = \frac{(62.4)(0.4)(85.6)}{0.70}$$

$$= 3052 \div 550 \approx \mathbf{5.55\ hp} \quad \textit{Ans.}\ \text{(a)}$$

(b) If we replace the sharp exit by a 6° conical diffuser, from Fig. 6.23, $K_{\text{exit}} \approx 0.3$. Then

$$h_p = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + .82 + .16 + 0.3\right] = 81.95 \text{ ft}$$

$$\text{then}\quad \text{Power} = (62.4)(0.4)(81.95)/0.7 \div 550 \approx \mathbf{5.31\ hp}\ (4\%\ \text{less})\quad \textit{Ans.}\ \text{(b)}$$

6.103 The reservoirs in Fig. P6.103 are connected by cast-iron pipes joined abruptly, with sharp-edged entrance and exit. Including minor losses, estimate the flow of water at 20°C if the surface of reservoir 1 is 45 ft higher than that of reservoir 2.

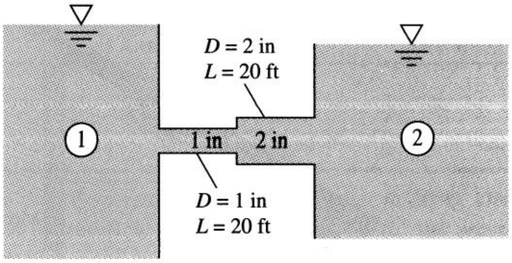


Fig. P6.103

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. Let "a" be the small pipe and "b" the larger. For wrought iron, $\varepsilon \approx$ 0.00015 ft, whence ε/da = 0.0018 and ε/db = 0.0009. From the continuity relation,

$$Q = V_a \frac{\pi}{4} d_a^2 = V_b \frac{\pi}{4} d_b^2 \quad \text{or, since}\ d_b = 2d_a, \quad \text{we obtain}\ V_b = \frac{1}{4} V_a$$

For pipe "a" there are two minor losses: a sharp entrance, K_1 = 0.5, and a sudden expansion, Fig. 6.22, Eq. (6.101), $K_2 = [1 - (1/2)^2]^2 \approx 0.56$. For pipe "b" there is one minor loss, the submerged exit, $K_3 \approx 1.0$. The energy equation, with equal pressures at (1) and (2) and near zero velocities at (1) and (2), yields

$$\Delta z = h_{f\text{-}a} + \sum h_{m\text{-}a} + h_{f\text{-}b} + \sum h_{m\text{-}b} = \frac{V_a^2}{2g}\left(f_a \frac{L_a}{d_a} + 0.5 + 0.56\right) + \frac{V_b^2}{2g}\left(f_b \frac{L_b}{d_b} + 1.0\right),$$

$$\text{or, since}\quad V_b = V_a/4,\quad \Delta z = 45\ \text{ft} = \frac{V_a^2}{2(32.2)}\left[240 f_a + 1.06 + \frac{120}{16} f_b + \frac{1.0}{16}\right]$$

where f_a and f_b are separately related to different values of Re and ε/d. Guess to start:

$$f_a \approx f_b \approx 0.02:\quad \text{then}\ V_a = 21.85\ \text{ft/s},\quad Re_a \approx 169000,\quad \varepsilon/d_a = 0.0018,\quad f_{a\text{-}2} \approx 0.0239$$

$$V_b = 5.46\ \text{ft/s},\quad Re_b \approx 84500,\quad \varepsilon/d_b = 0.0009,\quad f_{b\text{-}2} \approx 0.0222$$

$$\text{Converges to:}\quad f_a = 0.024,\quad f_b = 0.0224,\quad V_a \approx 20.3\ \text{ft/s},$$

$$Q = V_a A_a \approx \mathbf{0.111\ ft^3/s}.\quad \textit{Ans.}$$

P6.104 Water at 20°C flows through a smooth horizontal pipe at 12 m^3/h. The pipe diameter is 5 cm, and its length is 10 m. (*a*) Compute the pressure drop. (*b*) If a gate valve is added, what gate opening *h*/*D* will reduce the flow rate by 50% for the same pressure drop?

Solution: For water at 20°C, Table A.3, $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m-s. (*a*) First compute the Reynolds number from the known flow rate and then get the (smooth) friction factor:

$$V = \frac{Q}{A} = \frac{(12/3600)}{(\pi/4)(0.05)^2} = 1.70\frac{m}{s} \quad ; \quad \mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(998)(1.70)(0.05)}{0.001} = 84{,}700$$

$$\text{Eq.(6.38)}: \frac{1}{\sqrt{f}} = 2.0\log(84700\sqrt{f}) - 0.8 \quad \text{yields} \quad f \approx 0.0186$$

Then the pressure drop is given by

$$\Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.0186)\left(\frac{10m}{0.05m}\right)\left(\frac{998}{2}\right)(1.70)^2 = \mathbf{5350}\ Pa \qquad Ans.(a)$$

(*b*) The partly closed gate valve cuts the flow rate in half to 6 m^3/s, or $V = 0.85$ m/s. This cuts the Reynolds number in half, to 42,350, which slightly increases the (smooth) friction factor. From Eq. (6.38), the new friction factor is $f = 0.0217$. Add the *K* factor of the valve to Δp:

$$\Delta p = 5350\,Pa = \frac{\rho}{2}V^2\left(f\frac{L}{D} + K_{valve}\right) = \left(\frac{998}{2}\right)(0.85)^2\left[(0.0217)\left(\frac{10}{0.05}\right) + K_{valve}\right]$$

$$\text{or} \quad 14.8 = 4.34 + K_{valve} \quad , \quad \text{Solve for} \quad K_{valve} \approx \mathbf{10.5}$$

Figure 6.18*b* shows that, for the gate valve, a minor loss coefficient $K \approx 10.5$ is reached for a gate opening **$h/D \approx 0.35$**. *Ans.*(*b*)

6.105 The system in Fig. P6.105 consists of 1200 m of 5 cm cast-iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400 m, what gage pressure is required at point 1 to deliver 0.005 m^3/s of water at 20°C into the reservoir?

Solution: For water at 20°C, take $\rho = 998\ \text{kg/m}^3$ and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.0052$. With the flow rate known, we can compute V, Re:

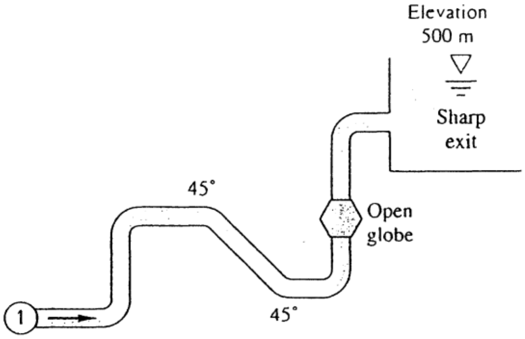


Fig. P6.105

$$V = \frac{Q}{A} = \frac{0.005}{(\pi/4)(0.05)^2} = 2.55\ \frac{m}{s}; \quad Re = \frac{998(2.55)(0.05)}{0.001} \approx 127000, \quad f_{Moody} \approx 0.0315$$

The minor losses may be listed as follows:

45° long-radius elbow: K ≈ 0.2; 90° long-radius elbow: K ≈ 0.3

Open flanged globe valve: K ≈ 8.5; submerged exit: K ≈ 1.0

Then the energy equation between (1) and (2—the reservoir surface) yields

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = 0 + 0 + z_2 + h_f + \sum h_m,$$

$$\text{or:}\quad p_1/(\rho g) = 500 - 400 + \frac{(2.55)^2}{2(9.81)}\left[0.0315\left(\frac{1200}{0.05}\right) + 0.5 + 2(0.2) + 4(0.3) + 8.5 + 1 - 1\right]$$

$$= 100 + 253 = 353\ \text{m}, \quad \text{or:}\quad \mathbf{p_1} = (998)(9.81)(353) \approx \mathbf{3.46\ MPa} \quad \textit{Ans.}$$

6.106 The water pipe in Fig. 6.106 slopes upward at 30°. The pipe is 1-inch diameter and *smooth*. The flanged globe valve is fully open. If the mercury manometer shows a 7-inch deflection, what is the flow rate in cubic feet per sec?

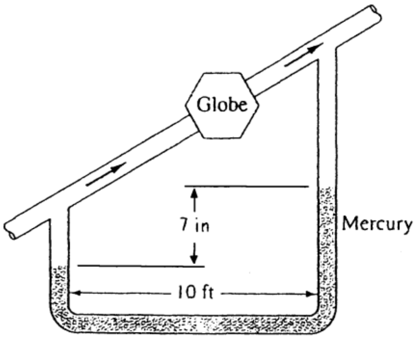


Fig. P6.106

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. The pipe length and elevation change are

$$L = \frac{10\text{ ft}}{\cos 30°} = 11.55\text{ ft};\quad z_2 - z_1 = 10\tan 30° = 5.77\text{ ft},\quad \text{Open 1}''\text{ globe valve: } K \approx 13$$

The manometer indicates the total pressure change between (1) and (2):

$$p_1 - p_2 = (\rho_{Merc} - \rho_w)gh\ + \rho_w g\Delta z = (13.6-1)(62.4)\left(\frac{7}{12}\right) + 62.4(5.77) \approx 819\text{ psf}$$

The energy equation yields

$$\frac{p_1 - p_2}{\rho g} = \Delta z + h_f + h_m = 5.77 + \frac{V^2}{2(32.2)}\left[f\frac{11.55}{1/12} + 13\right] \approx \frac{819\text{ lbf/ft}^2}{62.4\text{ lbf/ft}^3}$$

$$\text{or:}\quad V^2 \approx \frac{2(32.2)(7.35)}{(139f+13)}.\quad \text{Guess } f \approx 0.02,\ V \approx 5.48\ \frac{ft}{s},\ \text{Re} \approx 42400,\ f_{new} \approx 0.0217$$

Rapid convergence to f ≈ 0.0217, V ≈ 5.44 ft/s, Q = V(π/4)(1/12)^2 ≈ **0.0296 ft^3/s.** *Ans.* [NOTE that the manometer reading of 7 inches exactly balances the friction losses, and the hydrostatic pressure change $\rho g\Delta z$ cancels out of the energy equation.]

6.107 In Fig. P6.107 the pipe is galvanized iron. Estimate the percentage increase in the flow rate (a) if the pipe entrance is cut off flush with the wall and (b) if the butterfly valve is opened wide.

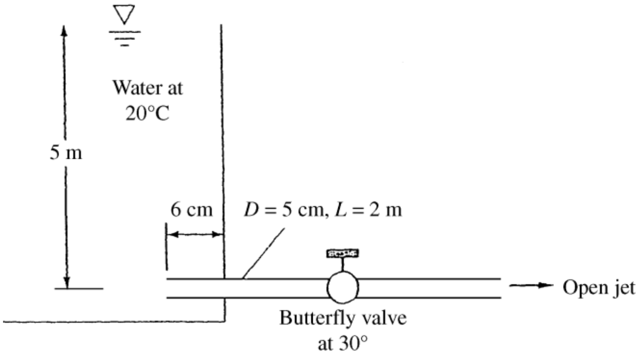


Fig. P6.107

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For galvanized iron, take $\varepsilon \approx 0.15$ mm, hence $\varepsilon/d = 0.003$. First establish minor losses as shown:

$$\text{Protruding entrance (Fig. 6.21a),} \quad \frac{L}{d} \approx 1.2, \ K \approx 1;$$

$$\text{Butterfly @ 30° (Fig 6.19)} \quad K \approx 80 \pm 20$$

The energy equation, with $p_1 = p_2$, yields:

$$\Delta z = \frac{V^2}{2g} + h_f + \sum h_m = \frac{V^2}{2g}\left[1 + f\frac{L}{d} + \sum K\right] = \frac{V^2}{2(9.81)}\left[1 + f\left(\frac{2}{0.05}\right) + 1.0 + 80 \pm 20\right] = 5 \text{ m}$$

$$\text{Guess} \quad f \approx 0.02, \quad V \approx 1.09 \ \frac{m}{s}, \quad Re \approx 54300, \quad \frac{\varepsilon}{d} = 0.003,$$

$$f_{new} \approx 0.0284, \quad V_{new} \approx 1.086 \ \frac{m}{s}$$

Thus the "base" flow, for our comparison, is $V_o \approx 1.086$ m/s, $Q_o \approx$ **0.00213** m^3/s.
If we cut off the entrance flush, we reduce K_{ent} from 1.0 to **0.5**; hardly a significant reduction in view of the huge butterfly valve loss $K_{valve} \approx 80$. The energy equation is

$$5 \text{ m} = \frac{V^2}{2(9.81)}[1 + 40f + 0.5 + 80 \pm 20], \quad \text{solve} \ V \approx 1.090 \ \frac{m}{s},$$

$$Q = \mathbf{0.00214} \ \frac{\mathbf{m^3}}{\mathbf{s}} \ (0.3\% \text{ more}) \quad \textit{Ans.} \text{ (a)}$$

If we open the butterfly wide, K_{valve} decreases from 80 to only **0.3**, a *huge* reduction:

$$5 \text{ m} = \frac{V^2}{2(9.81)}[1 + 40f + 1.0 + 0.3], \quad \text{solve} \ V \approx 5.4 \ \frac{m}{s},$$

$$Q = \mathbf{0.0106} \ \frac{\mathbf{m^3}}{\mathbf{s}} \ (5 \text{ times more}) \quad \textit{Ans.} \text{ (b)}$$

Obviously opening the valve has a dominant effect for this system.

6.108 The water pump in Fig. P6.108 maintains a pressure of 6.5 psig at point 1. There is a filter, a half-open disk valve, and two regular screwed elbows. There are 80 ft of 4-inch diameter commercial steel pipe. (a) If the flow rate is 0.4 ft^3/s, what is the loss coefficient of the filter? (b) If the disk valve is wide open and $K_{filter} = 7$, what is the resulting flow rate?

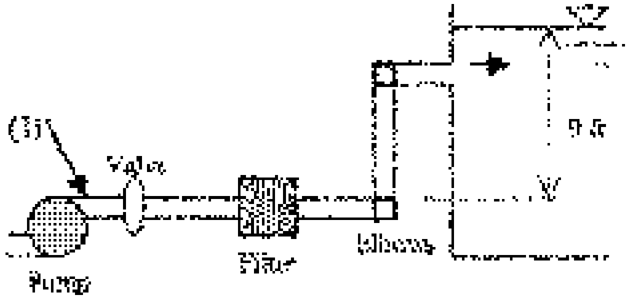


Fig. P6.108

Solution: For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. The energy equation is written from point 1 to the surface of the tank:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+\frac{V_1^2}{2g}[\frac{fL}{D}+K_{valve}+K_{filter}+2K_{elbow}+K_{exit}]$$

(a) From the flow rate, V_1 = Q/A = (0.4 ft^3/s)/[(π/4)(4/12 ft)^2] = 4.58 ft/s. Look up minor losses and enter into the energy equation:

$$\frac{(6.5)(144)\text{ lbf/ft}^2}{62.4\text{ lbf/ft}^3}+\frac{(4.58\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}+0$$

$$=0+0+9\text{ ft}+\frac{(4.58)^2}{2(32.2)}\left[f\frac{80\text{ ft}}{(4/12\text{ ft})}+2.8+K_{filter}+2(0.64)+1\right]$$

We can solve for K_{filter} if we evaluate f. Compute Re_D = (1.94)(4.58)(4/12)/(2.09E−5) = 141,700. For commercial steel, ε/D = 0.00015 ft/0.333 ft = 0.00045 and $V_1^2/2g$ = 0.326 ft. From the Moody chart, $f \approx 0.0193$, and fL/D = 4.62. The energy equation above becomes:

$$15.0\text{ ft}+0.326\text{ ft}=9\text{ ft}+0.326(4.62+2.8+K_{\text{filter}}+1.28+1)\text{ ft},$$

$$\text{Solve}\quad \boldsymbol{K_{\text{filter}}\approx 9.7}\quad \textit{Ans.}\text{ (a)}$$

(b) If K_{filter} = 7.0 and V is unknown, we must iterate for the velocity and flow rate. The energy equation becomes, with the disk valve wide open ($K_{Valve} \approx 0$):

$$15.0\text{ ft}+\frac{V^2}{2(32.2)}=9\text{ ft}+\frac{V^2}{2(32.2)}\left(f\frac{80}{1/3}+0+7.0+1.28+1\right)$$

$$\textit{Iterate to find}\quad f\approx 0.0189,\quad \text{Re}_D=169{,}000,\quad V=5.49\text{ ft/s},$$

$$\mathbf{Q=AV=0.48\ ft^3/s}\quad \textit{Ans.}\text{ (b)}$$

6.109 In Fig. P6.109 there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are three 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft^3/s of water at 20°C?

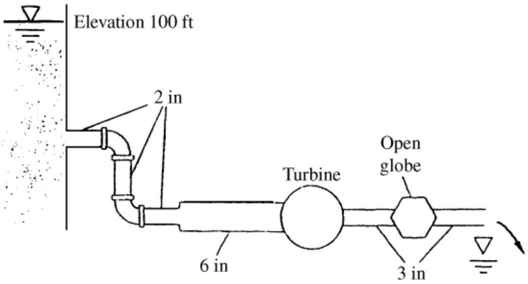


Fig. P6.109

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 2″, 6″, and 3″ pipes have, respectively,

$$\text{(a)}\quad L/d = 750,\ \varepsilon/d = 0.0051;\quad \text{(b)}\quad L/d = 150,\ \varepsilon/d = 0.0017;$$

$$\text{(c)}\quad L/d = 600,\ \varepsilon/d = 0.0034$$

The flow rate is known, so each velocity, Reynolds number, and *f* can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33\ \frac{ft}{s};\quad Re_a = \frac{1.94(7.33)(2/12)}{2.09E{-}5} = 113500,\quad f_a \approx 0.0314$$

$$\text{Also, } V_b = 0.82\text{ ft/s},\ Re_b = 37800,\ f_c \approx 0.0266;\ V_c = 3.26,\ Re_c = 75600,\ f_c \approx 0.0287$$

Finally, the minor loss coefficients may be tabulated:

sharp 2″ entrance: K = 0.5; three 2″ 90° elbows: K = 3(0.95)

2″ sudden expansion: K ≈ 0.79; 3″ open globe valve: K ≈ 6.3

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$h_t = \Delta z - \sum h_f - \sum h_m - V_c^2/(2g)$$

$$= 100 - \frac{(7.33)^2}{2(32.2)}[0.0314(750) + 0.5 + 3(0.95) + 0.79]$$

$$-\frac{(0.82)^2}{2(32.2)}(0.0266)(150) - \frac{(3.26)^2}{2(32.2)}[0.0287(600) + 6.3 + 1] \approx \mathbf{72.8\ ft}$$

The resulting turbine power = $\rho gQh_t = (62.4)(0.16)(72.8) \div 550 \approx$ **1.32 hp**. *Ans.*

6.110 In Fig. P6.110 the pipe entrance is sharp-edged. If the flow rate is 0.004 m^3/s, what power, in W, is extracted by the turbine?

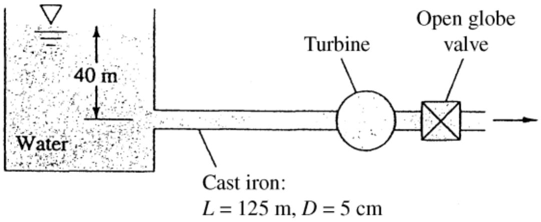


Fig. P6.110

Solution: For water at 20°C, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/50 \approx 0.0052$. The minor loss coefficients are Entrance: K ≈ 0.5; 5-cm(≈2″) open globe valve: K ≈ 6.9.

The flow rate is known, hence we can compute V, Re, and f:

$$V = \frac{Q}{A} = \frac{0.004}{(\pi/4)(0.05)^2} = 2.04\ \frac{m}{s},\quad Re = \frac{998(2.04)(0.05)}{0.001} \approx 102000,\quad f \approx 0.0316$$

The turbine head equals the elevation difference minus losses and exit velocity head:

$$h_t = \Delta z - h_f - \sum h_m - \frac{V^2}{2g} = 40 - \frac{(2.04)^2}{2(9.81)}\left[(0.0316)\left(\frac{125}{0.05}\right) + 0.5 + 6.9 + 1\right] \approx 21.5\ m$$

$$\text{Power} = \rho g Q h_t = (998)(9.81)(0.004)(21.5) \approx \mathbf{840\ W} \quad \textit{Ans.}$$

6.111 For the parallel-pipe system of Fig. P6.111, each pipe is cast iron, and the pressure drop $p1 - p2 = 3\ lbf/in^2$. Compute the total flow rate between 1 and 2 if the fluid is SAE 10 oil at 20°C.

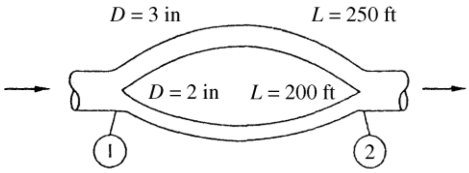


Fig. P6.111

Solution: For SAE 10 oil at 20°C, take $\rho = 1.69\ slug/ft^3$ and $\mu = 0.00217$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. Convert Δp = 3 psi = 432 psf and guess laminar flow in each:

$$\Delta p_a \overset{?}{=} \frac{128\mu L_a Q_a}{\pi d_a^4} = 432 = \frac{128(0.00217)(250)Q_a}{\pi(3/12)^4},$$

$$Q_a \approx 0.0763\ \frac{ft^3}{s}\cdot \quad \text{Check } Re \approx 300 \text{ (OK)}$$

$$\Delta p_b \overset{?}{=} \frac{128\mu L_b Q_b}{\pi d_b^4} = 432 = \frac{128(0.00217)(200)Q_b}{\pi(2/12)^4},$$

$$Q_b \approx 0.0188\ \frac{ft^3}{s}. \quad \text{Check } Re \approx 112 \text{ (OK)}$$

The total flow rate is $\mathbf{Q} = Q_a + Q_b = 0.0763 + 0.0188 \approx \mathbf{0.095\ ft^3/s}$. *Ans.*

6.112 If the two pipes in Fig. P6.111 are instead laid in **series** with the same total pressure drop of 3 psi, what will the flow rate be? The fluid is SAE 10 oil at 20°C.

Solution: For SAE 10 oil at 20°C, take ρ = 1.69 slug/ft^3 and μ = 0.00217 slug/ft·s. Again guess laminar flow. Now, instead of Δp being the same, Qa = Qb = Q:

$$\Delta p_a + \Delta p_b = 432\text{ psf} = \frac{128\mu L_a Q}{\pi d_a^4} + \frac{128\mu L_b Q}{\pi d_b^4} = \frac{128(0.00217)}{\pi} Q\left[\frac{250}{(3/12)^4} + \frac{200}{(2/12)^4}\right]$$

$$\text{Solve for}\quad \mathbf{Q \approx 0.0151\ ft^3/s}\quad \textit{Ans.}\quad \text{Check } Re_a \approx 60 \text{ (OK) and } Re_b \approx 90 \text{ (OK)}$$

In series, the flow rate is six times less than when the pipes are in parallel.

6.113 The parallel galvanized-iron pipe system of Fig. P6.113 delivers water at 20°C with a total flow rate of 0.036 m^3/s. If the pump is wide open and not running, with a loss coefficient K = 1.5, determine (a) the flow rate in each pipe and (b) the overall pressure drop.

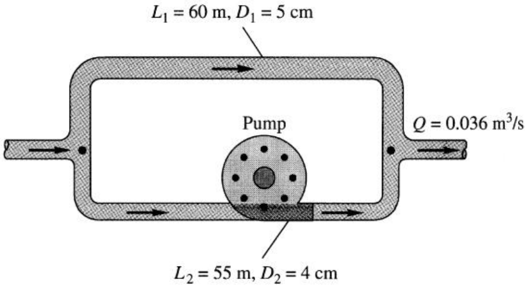


Fig. P6.113

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For galvanized iron, ε = 0.15 mm. Assume turbulent flow, with Δp the same for each leg:

$$h_{f1} = f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} = h_{f2} + h_{m2} = \frac{V_2^2}{2g}\left(f_2\frac{L_2}{d_2} + 1.5\right),$$

$$\text{and}\quad Q_1 + Q_2 = (\pi/4)d_1^2V_1 + (\pi/4)d_2^2V_2 = Q_{total} = 0.036\text{ m}^3/\text{s}$$

When the friction factors are correctly found from the Moody chart, these two equations may be solved for the two velocities (or flow rates). Begin by guessing f ≈ 0.020:

$$(0.02)\left(\frac{60}{0.05}\right)\frac{V_1^2}{2(9.81)} = \frac{V_2^2}{2(9.81)}\left[(0.02)\left(\frac{55}{0.04}\right) + 1.5\right],\quad \text{solve for } V_1 \approx 1.10V_2$$

$$\text{then}\quad \frac{\pi}{4}(0.05)^2(1.10V_2) + \frac{\pi}{4}(0.04)^2V_2 = 0.036.\quad \text{Solve } V_2 \approx 10.54\ \frac{m}{s},\ V_1 \approx 11.59\ \frac{m}{s}$$

$$\text{Correct } Re_1 \approx 578000,\quad f_1 \approx 0.0264,\quad Re_2 \approx 421000,\quad f_2 \approx 0.0282,\ \text{repeat.}$$

The 2nd iteration converges: $f_1 \approx 0.0264$, $V_1 = 11.69$ m/s, $f_2 \approx 0.0282$, $V_2 = 10.37$ m/s,

$$Q_1 = A_1V_1 = \mathbf{0.023\ m^3/s}, \quad Q_2 = A_2V_2 = \mathbf{0.013\ m^3/s}. \quad \textit{Ans.}\ \text{(a)}$$

The pressure drop is the same in either leg:

$$\Delta p = f_1 \frac{L_1}{d_1}\frac{\rho V_1^2}{2} = \left(f_2 \frac{L_2}{d_2} + 1.5\right)\frac{\rho V_2^2}{2} \approx \mathbf{2.16E6\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

***P6.114** A blower supplies standard air to a plenum that feeds two horizontal square sheet-metal ducts with sharp-edged entrances. One duct is 100 ft long, with a cross-section 6 in by 6 in. The second duct is 200 ft long. Each duct exhausts to the atmosphere. When the plenum pressure is 5.0 lbf/ft^2 gage, the volume flow in the longer duct is three times the flow in the shorter duct. Estimate both volume flows and the cross-section size of the longer duct.

Solution: For standard air, in BG units, take ρ = 0.00238 slug/ft^3 and μ = 3.78E-7 slug/ft-sec. For sheet metal, take ε = 0.00016 ft. The energy equation for this case is

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{p_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f + h_{entrance}\ ,\quad or:$$

$$\Delta p = \frac{1}{2}\rho V^2 \left(1 + f\frac{L}{D_h} + K_{ent}\right) \quad where\ K_{sharp-edged} \approx 0.5$$

We have abbreviated the duct velocity to V, without a subscript. For a square duct, the hydraulic diameter is the side length of the square. First compute the flow rate in the short duct:

$$5.0\frac{lbf}{ft^2} = \frac{0.00238\,slug/ft^3}{2}\{1 + f\frac{100\ ft}{0.5\ ft} + 0.5)\ ,\ f = fcn(\mathrm{Re}_{D_h}, \frac{\varepsilon}{D_h})$$

The Reynolds number for the short duct is Re = (0.00238)V(0.5)/(3.78E-7) = 3148V, and ε/D_h = 0.00016ft/0.5ft = 0.00032. The solution is

$$L{=}100\ ft:\ \mathrm{Re}_{D_h} = 87{,}000\ ;\ f = 0.0200\ ;\ V = 27.7\ ft/s\ ;\ Q_{short} = \mathbf{6.92ft^3/s}$$

For the longer duct, Re = $(0.00238)VD_h/(3.78E\text{-}7)$, and $\varepsilon/D_h = 0.00016\text{ft}/D_h$. We don't know D_h and must solve to make $Q_{long} = 3Q_{short}$. The solution is

$$L = 200\ ft:\ \text{Re}_{D_h} = 150{,}000\ ;\ f = 0.0177\ ;\ V = 27.5\ ft/s\ ;\ Q_{long} = 20.75\ ft^3/s$$

$$\text{Solve for}\quad D_{h,long} = \mathbf{0.87ft} \qquad \textit{Ans.}$$

NOTE: It is an interesting numerical quirk that, for these duct parameters, the velocities in each duct are almost identical, regardless of the magnitude of the pressure drop.

6.115 In Fig. P6.115 all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir (1) if valve C is (a) closed; and (b) open, with $K_{valve} = 0.5$.

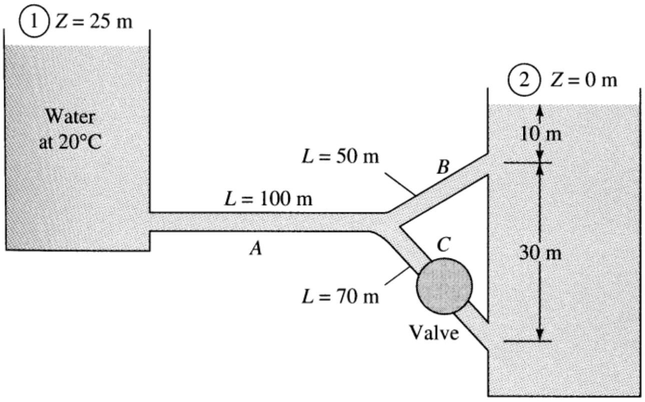


Fig. P6.115

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence $\varepsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$ (Table 6.5)

branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB},$$

$$\text{or:}\quad 25\text{ m} = \frac{V^2}{2(9.81)}\left[f\frac{100}{0.08} + 0.5 + 0.9 + f\frac{50}{0.08} + 1.0\right],\quad \text{where } f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Guess f ≈ f fully rough ≈ 0.027, then V ≈ 3.04 m/s, Re ≈ 998(3.04)(0.08)/(0.001) ≈ 243000, ε/d = 0.00325, then f ≈ 0.0273 (converged). Then the velocity through A and B is V = 3.03 m/s, and Q = (π/4)$(0.08)^2$(3.03) ≈ **0.0152 m^3/s**. *Ans.* (a).

If valve C is open, we have parallel flow through B and C, with $Q_A = Q_B + Q_C$ and, with *d* constant, $V_A = V_B + V_C$. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC},$$

$$\text{or:}\quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 0.9\right] + \frac{V_B^2}{2(9.81)}\left[f_B\frac{50}{0.08} + 1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 1.3\right] + \frac{V_C^2}{2(9.81)}\left[f_C\frac{70}{0.08} + 1.0\right]$$

plus the additional relation $V_A = V_B + V_C$. Guess f ≈ ffully rough ≈ 0.027 for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

$$\text{If } f \approx 0.027,\ \text{solve (laboriously)}\ V_A \approx 3.48\text{ m/s},\ V_B \approx 1.91\text{ m/s},\ V_C \approx 1.57\text{ m/s}.$$

$$\text{Compute } \text{Re}_A = 278000,\quad f_A \approx 0.0272,\quad \text{Re}_B = 153000,\quad f_B = 0.0276,$$
$$\text{Re}_C = 125000,\quad f_C = 0.0278$$

Repeat once for convergence: V_A ≈ 3.46 m/s, V_B ≈ 1.90 m/s, V_C ≈ 1.56 m/s. The flow rate from reservoir (1) is **Q_A** = (π/4)$(0.08)^2$(3.46) ≈ **0.0174 m^3/s**. (14% more) *Ans.* (b)

6.116 For the series-parallel system of Fig. P6.116, all pipes are 8-cm-diameter asphalted cast iron. If the total pressure drop $p_1 - p_2$ = 750 kPa, find the resulting flow rate *Q* m^3/h for water at 20°C. Neglect minor losses.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For asphalted cast iron, $\varepsilon \approx 0.12$ mm, hence $\varepsilon/d = 0.12/80 \approx 0.0015$ for all three pipes. The head loss is the same through AC and BC:

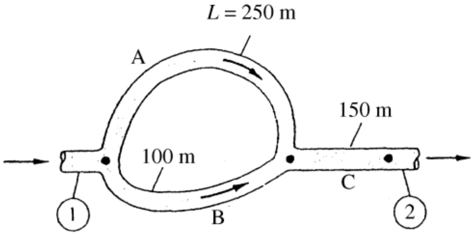


Fig. P6.116

$$\frac{\Delta p}{\rho g} = h_{fA} + h_{fC} = h_{fB} + h_{fC} = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_A + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_B + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C$$

Since d is the same, $V_A + V_B = V_C$ and f_A, f_B, f_C are found from the Moody chart. Cancel g and introduce the given data:

$$\frac{750000}{998} = f_A\frac{250}{0.08}\frac{V_A^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2} = f_B\frac{100}{0.08}\frac{V_B^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2}, \quad V_A + V_B = V_C$$

$$\text{Guess } f_{rough} \approx 0.022 \text{ and solve laboriously: } V_A \approx 2.09\ \frac{m}{s},\ V_B \approx 3.31\ \frac{m}{s},\ V_C \approx 5.40\ \frac{m}{s}$$

Now compute $Re_A \approx 167000$, $f_A \approx 0.0230$, $Re_B \approx 264000$, $f_B \approx 0.0226$, $Re_C \approx 431000$, and $f_C \approx 0.0222$. Repeat the head loss iteration and we converge: $V_A \approx 2.06$ m/s, $V_B \approx 3.29$ m/s, $V_C \approx 5.35$ m/s, $Q = (\pi/4)(0.08)^2(5.35) \approx$ **0.0269 m^3/s**. *Ans.*

6.117 A blower delivers air at 3000 m^3/h to the duct circuit in Fig. P6.117. Each duct is commercial steel and of square cross-section, with side lengths $a_1 = a_3 = 20$ cm and $a_2 = a_4 = 12$ cm. Assuming sea-level air conditions, estimate the power required if the blower has an efficiency of 75%. Neglect minor losses.

Solution: For air take ρ = 1.2 kg/m^3 and μ = 1.8E−5 kg/m·s. Establish conditions in each duct:

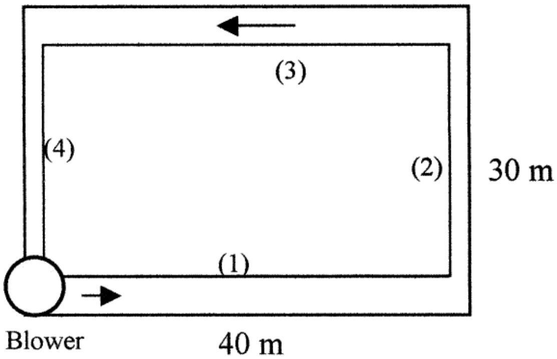


Fig. P6.117

$$Q=\frac{3000}{3600}=0.833\ \frac{\text{m}^3}{\text{s}};\ V_{1\&3}=\frac{0.833\ \text{m}^3/\text{s}}{(0.2\ \text{m})^2}=20.8\ \text{m/s};\ \text{Re}_{1\&3}=\frac{1.2(20.8)(0.2)}{1.8E\text{–}5}=278{,}000$$

$$V_{2\&4}=\frac{0.833\ \text{m}^3/\text{s}}{(0.12\ \text{m})^2}=57.8\ \text{m/s};\quad \text{Re}_{2\&4}=\frac{1.2(57.8)(0.12)}{1.8E\text{–}5}=463{,}000$$

For commercial steel (Table 6.1) ε = 0.046 mm. Then we can find the two friction factors:

$$\left.\frac{\varepsilon}{D}\right|_{1\&3}=\frac{0.046}{200}=0.00023;\quad \text{Re}_{1\&3}=278000;\quad \text{Moody chart: } f_{1\&3}\approx 0.0166$$

$$\left.\frac{\varepsilon}{D}\right|_{2\&4}=\frac{0.046}{120}=0.000383;\quad \text{Re}_{2\&4}=463000;\quad \text{Moody chart: } f_{1\&3}\approx 0.0170$$

$$\text{Then}\quad \Delta p_{1\&3}=\left(f\frac{L}{D}\frac{\rho V^2}{2}\right)_{1\&3}=(0.0166)\left(\frac{80}{0.2}\right)\frac{(1.2)(20.8)^2}{2}=1730\ Pa$$

$$\text{and}\quad \Delta p_{2\&4}=\left(f\frac{L}{D}\frac{\rho V^2}{2}\right)_{1\&3}=(0.0170)\left(\frac{60}{0.12}\right)\frac{(1.2)(57.8)^2}{2}=17050\ Pa$$

The total power required, at 75% efficiency, is thus:

$$Power=\frac{Q\Delta p}{\eta}=\frac{(0.833\ \text{m}^3/\text{s})(1730+17050\ \text{Pa})}{0.75}=\mathbf{20900\ W}\quad \textit{Ans.}$$

***6.118** For the piping system of Fig. P6.118, all pipes are concrete with a roughness of 0.04 inch. Neglecting minor losses, compute the overall pressure drop $p_1 - p_2$ in lbf/in^2. The flow rate is 20 ft^3/s of water at 20°C.

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. Since the pipes are all different make a little table of their respective *L*/*d* and ε/d:

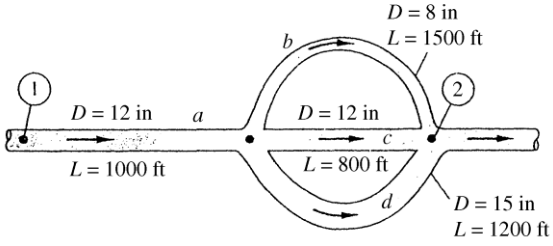


Fig. P6.118

(a)	L = 1000 ft,	d = 12 in,	L/d = *1000*,	ε/d = 0.00333
(b)	1500 ft	8 in	*2250*	0.00500
(c)	800 ft	12 in	*800*	0.00333
(d)	1200 ft	15 in	*960*	0.00267

With the flow rate known, we can find everything in pipe (a):

$$V_a = \frac{Q_a}{A_a} = \frac{20}{(\pi/4)(1\text{ ft})^2} = 25.5\ \frac{\text{ft}}{\text{s}},\quad Re_a = \frac{1.94(25.5)(1)}{2.09E{-}5} = 2.36E6,\quad f_a \approx \mathit{0.0270}$$

Then pipes (b,c,d) are in parallel, each having the same head loss and with flow rates which must add up to the total of 20 ft^3/s:

$$h_{fb} = \frac{8f_bL_bQ_b^2}{\pi^2 g d_b^5} = h_{fc} = \frac{8f_cL_cQ_c^2}{\pi^2 g d_c^5} = h_{fd} = \frac{8f_dL_dQ_d^2}{\pi^2 g d_d^5},\quad \text{and}\quad Q_b + Q_c + Q_d = 20\ \frac{\text{ft}^3}{\text{s}}$$

Introduce L_b, d_b, etc. to find that $Q_c = 3.77Q_b(f_b/f_c)^{1/2}$ and $Q_d = 5.38Q_b(f_b/f_d)^{1/2}$

Then the flow rates are iterated from the relation

$$\Sigma Q = 20\ \frac{\text{ft}^3}{\text{s}} = Q_b[1 + 3.77(f_b/f_c)^{1/2} + 5.38(f_b/f_d)^{1/2}]$$

$$\text{First guess:}\quad f_b = f_c = f_d:\quad Q_b \approx 1.97\text{ ft}^3/\text{s};\quad Q_c \approx 7.43\text{ ft}^3/\text{s};\quad Q_d \approx 10.6\text{ ft}^3/\text{s}$$

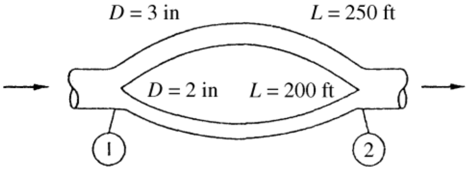
Improve by computing $Re_b \approx 349000$, $f_b \approx 0.0306$, $Re_c \approx 878000$, $f_c \approx 0.0271$, $Re_d \approx 1002000$, $f_d \approx 0.0255$. Repeat to find $Q_b \approx 1.835$ ft^3/s, $Q_c \approx 7.351$ ft^3/s, $Q_d \approx 10.814$ ft^3/s.

Repeat once more and quit: $Q_b \approx 1.833\ \text{ft}^3/\text{s}$, $Q_c \approx 7.349\ \text{ft}^3/\text{s}$, $Q_d \approx 10.819\ \text{ft}^3/\text{s}$, from which $V_b \approx 5.25$ ft/s, $V_c \approx 9.36$ ft/s, $V_d \approx 8.82$ ft/s. The pressure drop is

$$p_1 - p_2 = \Delta p_a + \Delta p_b = f_a \frac{L_a}{d_a}\frac{\rho V_a^2}{2} + f_b \frac{L_b}{d_b}\frac{\rho V_b^2}{2}$$

$$= 17000 + 1800 \approx 18800\ \text{psf} \approx \mathbf{131\ \frac{lbf}{in^2}} \quad \textit{Ans.}$$

P6.119 For the system of Prob. P6.111, again let the fluid be SAE 10W oil at 20°C, with both pipes cast iron. If the flow rate in the 2-inch pipe (*b*) is 3.0 ft^3/min, estimate the flow rate in the 3-inch pipe (*a*), in ft^3/min.



Solution: For SAE 10W oil at 20°C, ρ = 1.69 slug/ft^3 and μ = 0.00219 slug/ft-s. The oil is pretty viscous and the flow rate small, so we think Re_{Db} will be laminar:

$$\text{Re}_{Db} = \frac{\rho V_b D_b}{\mu} = \frac{4\rho Q_b}{\pi \mu D_b} = \frac{4(1.69)(3.0/60)}{\pi(0.00219)(2/12)} = 295 < 2300 \quad \text{Yes, Laminar}$$

$$\text{If we need it,}\ V_b = \frac{4Q_b}{\pi D_b^2} = \frac{4(3.0/60\,ft^3/s)}{\pi(2/12\ ft)^2} = 2.29\ \frac{ft}{s}$$

For laminar flow, compute the pressure drop Δp_b from Eq. (6.12) and set it equal to Δp_a:

$$\Delta p_b = \frac{128\mu L_b Q_b}{\pi D_b^4} = \frac{128(0.00219)(200)(3/60)}{\pi(2/12)^4} = 1156\ Pa$$

$$\text{Parallel Pipes: } \Delta p_a = \Delta p_b = 1156 = \frac{128\mu L_a Q_a}{\pi D_a^4} = \frac{128(0.00219)(250)Q_a}{\pi(3/12)^4}$$

$$\textit{Solve for}\quad Q_a = 0.2025\ \frac{ft^3}{s} = \mathbf{12.2}\ \frac{ft^3}{\text{min}} \qquad \textit{Ans.}$$

Check $\text{Re}_a = 4\rho Q/(\pi\mu D_a) = 4(1.69)(0.2025)/[\pi(0.00219)(0.25)] = 796$ Yes, laminar also.

6.120 Three cast-iron pipes are laid in parallel with these dimensions:

Pipe 1: $L1 = 800$ m $d1 = 12$ cm
Pipe 2: $L2 = 600$ m $d2 = 8$ cm
Pipe 3: $L3 = 900$ m $d3 = 10$ cm

The total flow rate is 200 m^3/h of water at 20°C. Determine (a) the flow rate in each pipe; and (b) the pressure drop across the system.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon = 0.26$ mm. Then, $\varepsilon/d1 = 0.00217$, $\varepsilon/d2 = 0.00325$, and $\varepsilon/d3 = 0.0026$. The head losses are the same for each pipe, and the flow rates add:

$$h_f = \frac{8f_1L_1Q_1^2}{\pi^2 g d_1^5} = \frac{8f_2L_2Q_2^2}{\pi^2 g d_2^5} = \frac{8f_3L_3Q_3^2}{\pi^2 g d_3^5}; \quad \text{and} \quad Q_1 + Q_2 + Q_3 = \frac{200}{3600}\frac{m^3}{s}$$

Substitute and combine: $Q_1[1 + 0.418(f_1/f_2)^{1/2} + 0.599(f_1/f_3)^{1/2}] = 0.0556\ m^3/s$

We could either go directly to EES or begin by guessing $f1 = f2 = f3$, which gives $Q1 = 0.0275$ m^3/s, $Q2 = 0.0115$ m^3/s, and $Q3 = 0.0165$ m^3/s. This is *very* close! Further iteration gives

$$\text{Re}_1 = 298000,\ f_1 = 0.0245;\ \text{Re}_2 = 177000,\ f_2 = 0.0275;\ \text{Re}_3 = 208000,\ f_3 = 0.0259$$

$$Q_1 = \mathbf{0.0281\ m^3/s}, \quad Q_2 = \mathbf{0.0111\ m^3/s}, \quad \text{and } Q_3 = \mathbf{0.0163\ m^3/s} \quad \textit{Ans.}\ \text{(a)}$$

$$h_f = 51.4\text{ m}, \quad \Delta p = \rho g h_f = (998\text{ kg/m}^3)(9.81\text{ m/s}^2)(51.4\text{ m}) = \mathbf{503{,}000\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

6.121 Consider the three-reservoir system of Fig. P6.121 with the following data:

$L1 = 95$ m $L2 = 125$ m $L3 = 160$ m
$z1 = 25$ m $z2 = 115$ m $z3 = 85$ m

All pipes are 28-cm-diameter unfinished concrete ($\varepsilon = 1$ mm). Compute the steady flow rate in all pipes for water at 20°C.

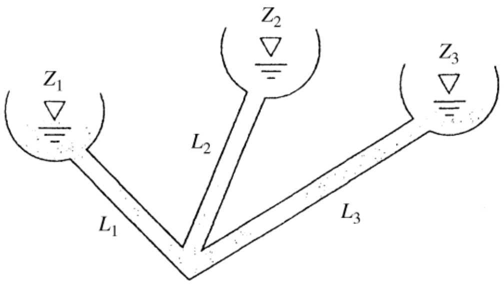


Fig. P6.121

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. All pipes have $\varepsilon/d = 1/280 = 0.00357$. Let the intersection be "a." The head loss at "a" is desired:

$$z_1 - h_a = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g}; \quad z_2 - h_a = f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g}; \quad z_3 - h_a = f_3 \frac{L_3}{d_3}\frac{V_3^2}{2g}$$

plus the requirement that $Q_1 + Q_2 + Q_3 = 0$ or, for same d, $V_1 + V_2 + V_3 = 0$

We guess h_a then iterate each friction factor to find V and Q and then check if $\sum Q = 0$.

$$h_a = 75 \text{ m:} \quad 25 - 75 = (-)50 = f_1\left(\frac{95}{0.28}\right)\frac{V_1^2}{2(9.81)}, \quad \text{solve } f_1 \approx 0.02754,\ V_1 \approx -10.25\ \frac{\text{m}}{\text{s}}$$

$$\text{Similarly,} \quad 115 - 75 = f_2(125/0.28)\left[V_2^2/2(9.81)\right] \quad \text{gives } f_2 \approx 0.02755. \quad V_2 \approx +7.99$$

$$\text{and} \quad 85 - 75 = f_3(160/0.28)\left[V_3^2/2(9.81)\right]$$

$$\text{gives} \quad f_3 \approx 0.02762,\ V_3 \approx +3.53\ \frac{\text{m}}{\text{s}}, \quad \sum V = \mathbf{+1.27}$$

Repeating for $h_a = 80$ m gives $V_1 = -10.75$, $V_2 = +7.47$, $V_3 = +2.49$ m/s, $\sum V = -0.79$.
Interpolate to $h_a \approx 78$ m, gives $V_1 = -10.55$ m/s, $V_2 = +7.68$ m/s, $V_3 = +2.95$ m/s, or:

$$\mathbf{Q_1 = -0.65\ m^3/s, \quad Q_2 = +0.47\ m^3/s, \quad Q_3 = +0.18\ m^3/s.} \quad \textit{Ans.}$$

6.122 Modify Prob. 6.121 by reducing the diameter to 15 cm, with $\varepsilon = 1$ mm. Compute the flow rate in each pipe. They all reduce, compared to Prob. 6.121, by a factor of about *5.2.* Can you explain this?

Solution: The roughness ratio increases to $\varepsilon/d = 1/150 = 0.00667$, and all L/d's increase. Guess $h_a = 75$ m: converges to $f_1 = 0.0333$, $f_2 = 0.0333$, $f_4 = 0.0334$

$$\text{and} \quad V_1 \approx -6.82 \text{ m/s}, \quad V_2 \approx +5.32 \text{ m/s}, \quad V_3 \approx +2.34 \text{ m/s}, \quad \sum V \approx \mathbf{+0.85}$$

We finally obtain $h_a \approx 78.2$ m, giving $V_1 = -7.04$ m/s, $V_2 = +5.10$ m/s, $V_3 = +1.94$ m/s,

$$\text{or:} \quad \mathbf{Q_1 = -0.124\ m^3/s}, \quad \mathbf{Q_2 = +0.090\ m^3/s}, \quad \mathbf{Q_3 = +0.034\ m^3/s}. \quad \textit{Ans.}$$

***6.123** Modify Prob. 6.121 as follows. Let z_3 be unknown and find its value such that the flow rate in pipe 3 is 0.2 m^3/s toward the junction. (This problem is best suited for computer iteration or EES.)

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. All pipes have $\varepsilon/d = 1/280 = 0.00357$. Let the intersection be "a." The head loss at "a" is desired for each in order to check the flow rate in pipe 3.

In Prob. 6.121, with z3 = 85 m, we found Q3 to be 0.18 m^3/s toward the junction, pretty close. We repeat the procedure with a few new values of z3, closing to $\sum Q = 0$ each time:

$$\begin{array}{rlllll} \text{Guess } z_3 = 85\text{ m:} & h_a = 78.19\text{ m}, & Q_1 = -0.6508, & Q_2 = +0.4718, & Q_3 = +0.1790\text{ m}^3/\text{s} \\ 90\text{ m:} & 80.65\text{ m}, & -0.6657, & +0.6657, & +0.2099\text{ m}^3/\text{s} \\ \text{Interpolate:} & h_a \approx 79.89, & Q_1 \approx -0.6611, & & \end{array}$$

$$Q_2 \approx +0.4608,\quad \mathbf{Q_3 \approx +0.200\ m^3/s},\quad \mathbf{z_3 \approx 88.4\ m}\quad \textit{Ans.}$$

6.124 The three-reservoir system in Fig. P6.124 delivers water at 20°C. The system data are as follows:

*D*1 = 8 in *D*2 = 6 in *D*3 = 9 in

*L*1 =1800 ft *L*2 = 1200 ft *L*3 = 1600 ft

All pipes are galvanized iron. Compute the flow rate in all pipes.

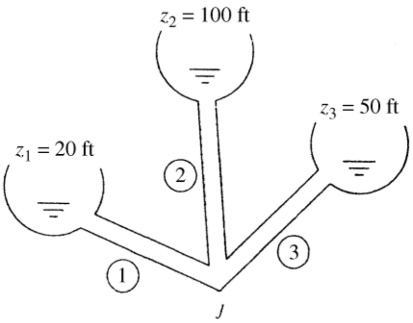


Fig. P6.124

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For galvanized iron, take $\varepsilon = 0.0005$ ft. Then the roughness ratios are

$$\varepsilon/d_1 = 0.00075 \quad \varepsilon/d_2 = 0.0010 \quad \varepsilon/d_3 = 0.00667$$

Let the intersection be "a." The head loss at "a" is desired:

$$z_1 - h_a = \frac{f_1 L_1}{d_1}\frac{V_1^2}{2g};\quad z_2 - h_a = \frac{f_2 L_2}{d_2}\frac{V_2^2}{2g};\quad z_3 - h_a = \frac{f_3 L_3}{d_3}\frac{V_3^2}{2g};\quad \text{plus } Q_1 + Q_2 + Q_3 = 0$$

We guess ha then iterate each friction factor to find V and Q and then check if ∑Q = 0.

$$\text{Guess} \quad h_a = 50 \text{ ft:} \quad 20 - 50 = (-)30 \text{ ft} = \frac{f_1(1800)V_1^2}{(8/12)2(32.2)},$$

$$\text{solve} \quad f_1 = 0.0194, \quad V_1 = -6.09 \ \frac{\text{ft}}{\text{s}}$$

$$\text{Similarly, } f_2 = 0.0204, V_2 \approx +8.11 \text{ ft/s and of course } V_3 = 0. \quad \text{Get } \sum Q = -0.54 \text{ ft}^3/\text{s}$$

Try again with a slightly lower ha to reduce Q1 and increase Q2 and Q3:

$$h_a = 48 \text{ ft:} \quad \text{converges to } Q_1 = -2.05 \ \frac{\text{ft}^3}{\text{s}}, \quad Q_2 = +1.62 \ \frac{\text{ft}^3}{\text{s}},$$

$$Q_3 = +0.76 \ \frac{\text{ft}^3}{\text{s}}, \quad \sum Q = +0.33$$

Interpolate to

$$h_a = 49.12 \text{ ft:} \quad \mathbf{Q_1 = -2.09\ ft^3/s, \quad Q_2 = +1.61\ ft^3/s, \quad Q_3 = +0.49\ ft^3/s} \quad \textit{Ans.}$$

6.125 Suppose that the three cast-iron pipes in Prob. 6.120 are instead connected to meet smoothly at a point B, as shown in Fig. P6.125. The inlet pressures in each pipe are: p1 = 200 kPa; p2 = 160 kPa; p3 = 100 kPa. The fluid is water at 20°C. Neglect minor losses. Estimate the flow rate in each pipe and whether it is toward or away from point B.

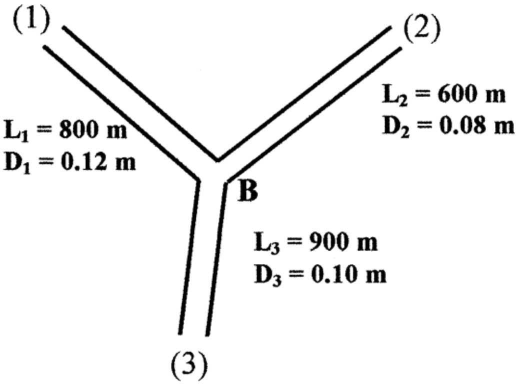


Fig. P6.125

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The pressure at point B must be a known (constant) value which makes the net flow rate equal to zero at junction B. The flow clearly goes from (1) to B, and from B to (3), but we are not sure about pipe (2). For cast iron (Table 6.1), ε = 0.26 mm. Each pipe has a flow rate based upon its pressure drop:

$$p_1 - p_B = f_1 \frac{L_1}{D_1}\frac{\rho V_1^2}{2}; \quad |p_2 - p_B| = f_2 \frac{L_2}{D_2}\frac{\rho V_2^2}{2}; \quad p_B - p_3 = f_3 \frac{L_3}{D_3}\frac{\rho V_3^2}{2}$$

where the f's are determined from the Moody chart for each pipe's ε/D and Re_D. The correct value of p_B makes the flow rates $Q_i = (\pi/4)D_i^2V_i$ balance at junction B. EES is excellent for this type of iteration, and the final results balance for p_B = 166.7 kPa:

$$f_1 = 0.0260;\ \ \mathrm{Re}_1 = 74300;\ \ \varepsilon/D_1 = 0.00217;\ \ \mathbf{Q_1 = +0.00701\ m^3/s}\ \text{(toward B)}$$

$$f_2 = 0.0321;\ \ \mathrm{Re}_2 = 18900;\ \ \varepsilon/D_2 = 0.00325;\ \ \mathbf{Q_2 = -0.00119\ m^3/s}\ \text{(away from B)}\quad \textit{Ans.}$$

$$f_3 = 0.0270;\ \ \mathrm{Re}_3 = 74000;\ \ \varepsilon/D_3 = 0.00260;\ \ \mathbf{Q_3 = -0.00582\ m^3/s}\ \text{(away from B)}$$

***6.126** Modify Prob. 6.124 as follows. Let all data be the same except that pipe 1 is fitted with a butterfly valve (Fig. 6.19*b*). Estimate the proper valve opening angle (in degrees) for the flow rate through pipe 1 to be reduced to 1.5 ft^3/s toward reservoir 1. (This problem requires iteration and is best suited to a digital computer or EES.)

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–4 slug/ft·s. For galvanized iron, take ε = 0.0005 ft. Then the roughness ratios are

$$\varepsilon/d_1 = 0.00075 \quad \varepsilon/d_2 = 0.0010 \quad \varepsilon/d_3 = 0.00667$$

For a butterfly valve loss coefficient "K" (to be found). Let the junction be "J." The head loss at "J" is desired and then to be iterated to give the proper flow rate in pipe (1):

$$z_1 - h_J = \frac{V_1^2}{2g}\left(f\frac{L}{d}+K\right)_1;\quad z_2 - h_J = \frac{V_2^2}{2g}\left(f\frac{L}{d}\right)_2;$$

$$z_3 - h_J = \frac{V_3^2}{2g}\left(f\frac{L}{d}\right)_3;\quad \text{and}\quad Q_1+Q_2+Q_3 = 0$$

We know z_1 = 20 ft, z_2 = 100 ft, and z_3 = 50 ft. From Prob. 6.124, where **K = 0**, the flow rate was 2.09 ft^3/s toward reservoir 1. Now guess a finite value of K and repeat:

$$K = 40:\ \ \text{converges to}\ \ h_J = 50.0,\quad Q_1 = -1.59\ \text{ft}^3/\text{s},\quad Q_2 = +1.59\ \text{ft}^3/\text{s};\quad Q_3 \approx 0$$

$$K = 50:\ \ \text{converges to}\ \ h_J = 50.03\ \text{ft},\quad Q_1 = -1.513\quad Q_2 = +1.591\quad Q_3 = -0.078$$

$$\mathbf{K = 52}:\ \ \text{gives}\ \ h_J = 50.04\ \text{ft},\quad \mathbf{Q_1 = -1.500\ ft^3/s}\quad Q_2 = 1.591\quad Q_3 = -0.091\quad \textit{Ans.}$$

From Fig. 6.19b, a butterfly valve coefficient K ≈ 52 occurs at $\theta_{\text{opening}} \approx$ **35°**. *Ans.*

***6.127** In the five-pipe horizontal network of Fig. P6.127, assume that all pipes have a friction factor f = 0.025. For the given inlet and exit flow rate of 2 ft^3/s of water at 20°C, determine the flow rate and direction in all pipes. If p_A = 120 lbf/in^2 gage, deter-mine the pressures at points *B*, *C*, and *D.*

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

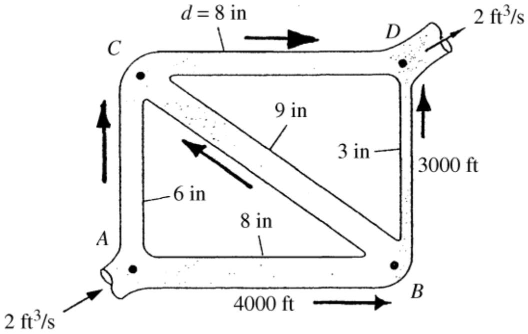


Fig. P6.127

$$\text{Pipe AC:}\quad h_f = \frac{8fLQ^2}{\pi^2 gd^5}\Big|_{AC} = \frac{8(0.025)(3000)Q_{AC}^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_{AC}^2,\quad \text{where } K_{AC} \approx 60.42$$

$$\text{Similarly,}\quad K_{AB} = 19.12,\quad K_{BC} = 13.26,\quad K_{CD} = 19.12,\quad K_{BD} = 1933.\quad \left(Q \text{ in } \frac{ft^3}{s}\right)$$

There are two triangular closed loops, and the total head loss must be zero for each. Using the flow directions assumed on the figure P6.127 above, we have

$$\text{Loop A-B-C:}\quad 19.12Q_{AB}^2 + 13.26Q_{BC}^2 - 60.42Q_{AC}^2 = 0$$
$$\text{Loop B-C-D:}\quad 13.26Q_{BC}^2 + 19.12Q_{CD}^2 - 1933.0Q_{BD}^2 = 0$$

And there are three independent junctions which have zero net flow rate:

$$\text{Junction A:}\quad Q_{AB} + Q_{AC} = 2.0;\quad \text{B: } Q_{AB} = Q_{BC} + Q_{BD};\quad \text{C: } Q_{AC} + Q_{BC} = Q_{CD}$$

These are five algebraic equations to be solved for the five flow rates. The answers are:

$$Q_{AB} = \mathbf{1.19},\quad Q_{AC} = \mathbf{0.81},\quad Q_{BC} = \mathbf{0.99},\quad Q_{CD} = \mathbf{1.80},\quad Q_{BD} = \mathbf{0.20}\ \frac{\mathbf{ft^3}}{\mathbf{s}}\quad \textit{Ans.}\text{ (a)}$$

The pressures follow by starting at A (120 psi) and subtracting off the friction losses:

$$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 120\times 144 - 62.4(19.12)(1.19)^2$$

$$p_B = 15590 \text{ psf} \div 144 = \mathbf{108\ \frac{lbf}{in^2}}$$

Similarly, $p_C \approx \mathbf{103\ psi}$ and $p_D \approx \mathbf{76\ psi}$ *Ans.* (b)

***6.128** Modify Prob. 6.127 above as follows: Let the inlet flow at A and the exit flow at D be unknown. Let $p_A - p_B = 100$ psi. Compute the flow rate in all five pipes.

Solution: Our head loss coefficients "K" from above are all the same. Head loss AB is known, plus we have two "loop" equations and two "junction" equations:

$$\frac{p_A - p_B}{\rho g} = \frac{100\times 144}{62.4} = 231 \text{ ft} = K_{AB}Q_{AB}^2 = 19.12Q_{AB}^2, \quad \text{or} \quad Q_{AB} = \mathbf{3.47\ ft^3/s}$$

$$\text{Two loops:} \quad 231 + 13.26Q_{BC}^2 - 60.42Q_{AC}^2 = 0$$

$$13.26Q_{BC}^2 + 19.12Q_{CD}^2 - 1933.0Q_{BD}^2 = 0$$

Two junctions: $Q_{AB} = 3.47 = Q_{BC} + Q_{BD}$; $Q_{AC} + Q_{BC} = Q_{CD}$

The solutions are in exactly the same ratio as the lower flow rates in Prob. 6.127:

$$Q_{AB} = \mathbf{3.47\ \frac{ft^3}{s}}, \quad Q_{BC} = \mathbf{2.90\ \frac{ft^3}{s}}, \quad Q_{BD} = \mathbf{0.58\ \frac{ft^3}{s}},$$

$$Q_{CD} = \mathbf{5.28\ \frac{ft^3}{s}}, \quad Q_{AC} = \mathbf{2.38\ \frac{ft^3}{s}} \quad \textit{Ans.}$$

6.129 In Fig. P6.129 all four horizontal cast-iron pipes are 45 m long and 8 cm in diameter and meet at junction *a*, delivering water at 20°C. The pressures are known at four points as shown:

$p_1 = 950$ kPa $\qquad p_2 = 350$ kPa
$p_3 = 675$ kPa $\qquad p_4 = 100$ kPa

Neglecting minor losses, determine the flow rate in each pipe.

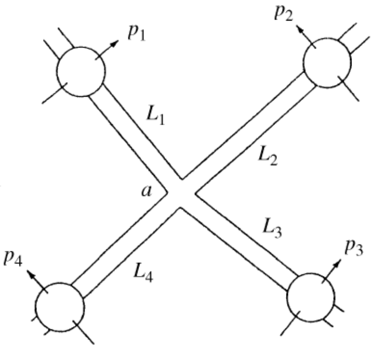


Fig. P6.129

Solution: For water at 20°C, take $\rho = 998\ \text{kg/m}^3$ and $\mu = 0.001$ kg/m·s. All pipes are cast iron, with ε/d = 0.26/80 = 0.00325. All pipes have L/d = 45/0.08 = 562.5. One solution method is to guess the junction pressure p_a, iterate to calculate the friction factors and flow rates, and check to see if the net junction flow is zero:

$$\text{Guess } p_a = 500\text{ kPa:}\quad h_{f1} = \frac{950000 - 500000}{998(9.81)} = 45.96\text{ m} = \frac{8f_1L_1Q_1^2}{\pi^2 g d_1^5} = 1.135\text{E}6 f_1 Q_1^2$$

$$\text{then guess } f_1 \approx 0.02,\quad Q_1 = 0.045\ \text{m}^3/\text{s},\quad \text{Re}_1 = 4\rho Q_1/(\pi\mu d_1) = 715000,\quad f_{1\text{-new}} \approx 0.0269$$

$$\text{converges to}\quad f_1 \approx 0.0270,\quad Q_1 \approx 0.0388\ \text{m}^3/\text{s}$$

$$\text{Iterate also to}\quad Q_2 = -0.0223\ \frac{\text{m}^3}{\text{s}}(\text{away from } a),\quad Q_3 = 0.0241,\quad Q_4 = -0.0365$$

$$\sum Q = +0.00403,\quad \text{so we have guessed } p_a \text{ a little low.}$$

Trying p_a = 530 kPa gives $\sum Q = -0.00296$, hence iterate to $p_a \approx 517$ kPa:

$$Q_1 = \mathbf{+0.0380}\ \frac{\mathbf{m^3}}{\mathbf{s}}\ (\text{toward a}),\quad Q_2 = \mathbf{-0.0236}\ \frac{\mathbf{m^3}}{\mathbf{s}},$$

$$Q_3 = \mathbf{+0.0229}\ \frac{\mathbf{m^3}}{\mathbf{s}},\quad Q_4 = \mathbf{-0.0373}\ \frac{\mathbf{m^3}}{\mathbf{s}}\quad \textit{Ans.}$$

***6.130** In Fig. P6.130 lengths *AB* and *BD* are 2000 and 1500 ft, respectively. The friction factor is 0.022 everywhere, and p_A = 90 lbf/in^2 gage. All pipes have a diameter of 6 in. For water at 20°C, determine the flow rate in all pipes and the pressures at points *B*, *C*, and *D*.

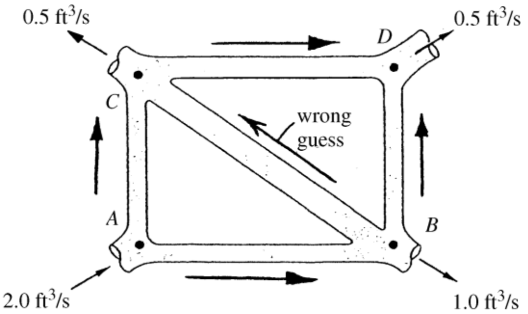


Fig. P6.130

Solution: For water at 20°C, take $\rho = 1.94\ \text{slug/ft}^3$ and $\mu = 2.09\text{E}{-5}$ slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

$$\text{Pipe AC:}\quad h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8(0.022)(1500)Q_{AC}^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_{AC}^2,\quad \text{where } K_{AC} \approx \mathbf{26.58}$$

Similarly, $K_{AB} = K_{CD} = 35.44$, $K_{BD} = 26.58$, and $K_{BC} = 44.30$.

The solution is similar to Prob. 6.127, except that (1) the K's are different; and (2) junctions B and C have additional flow leaving the network. The basic flow relations are:

$$\text{Loop ABC:}\quad 35.44Q_{AB}^2 + 44.3Q_{BC}^2 - 26.58Q_{AC}^2 = 0$$

$$\text{Loop BCD:}\quad 44.3Q_{BC}^2 + 35.44Q_{CD}^2 - 26.58Q_{BD}^2 = 0$$

$$\text{Junctions A,B,C:}\quad Q_{AB} + Q_{AC} = 2.0;$$

$$Q_{AB} = Q_{BC} + Q_{BD} + 1.0;\quad Q_{AC} + Q_{BC} = Q_{CD} + 0.5$$

In this era of PC "equation solvers" such as EES, it is probably not necessary to dwell upon any solution methods. For handwork, one might guess QAB, then the other four are obtained in sequence from the above relations, plus a check on the original guess for QAB. The assumed arrows are shown above. It turns out that we have guessed the direction incorrectly on QBC above, but the others are OK. The final results are:

$$Q_{AB} = \mathbf{0.949\ ft^3/s}\ \text{(toward B)};\quad Q_{AC} = \mathbf{1.051\ ft^3/s}\ \text{(toward C)}$$

$$Q_{BC} = \mathbf{0.239}\ \text{(toward B)};\quad Q_{CD} = \mathbf{0.312}\ \text{(toward D)};\quad Q_{BD} = \mathbf{0.188}\ \text{(to D)}\quad \textit{Ans.}\ \text{(a)}$$

The pressures start at A, from which we subtract the friction losses in each pipe:

$$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 90\times 144 - 62.4(35.44)(0.949)^2 = 10969\ \text{psf} \div 144 = \mathbf{76\ psi}$$

$$\text{Similarly, we obtain}\quad p_C = 11127\ \text{psf} = \mathbf{77\ psi};\quad p_D = 10911\ \text{psf} \approx \mathbf{76\ psi}\quad \textit{Ans.}\ \text{(b)}$$

6.131 A water-tunnel test section has a 1-m diameter and flow properties $V = 20$ m/s, $p =$ 100 kPa, and $T = 20$°C. The boundary-layer blockage at the end of the section is 9 percent. If a conical diffuser is to be added at the end of the section to achieve maximum pressure recovery, what should its angle, length, exit diameter, and exit pressure be?

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The Reynolds number is very high, Re = $\rho Vd/\mu$ = (998)(20)(1)/(0.001) ≈ 2.0E7; much higher than the diffuser data in Fig. 6.28b (Re ≈ 1.2E5). But what can we do (?) Let's use it anyway:

$$B_t = 0.09,\quad \text{read}\ C_{p,\max} \approx 0.71\quad \text{at } L/d \approx 25,\ 2\theta \approx 4°,\ AR \approx 8:$$

$$\text{Then}\quad \theta_{cone} \approx 2°,\quad L \approx 25d \approx \mathbf{25\ m},\quad D_{exit} = d(8)^{1/2} \approx \mathbf{2.8\ m}\quad \textit{Ans.}\ \text{(a)}$$

$$C_p \approx 0.71 = \frac{p_e - p_t}{(1/2)\rho V_t^2} = \frac{p_e - 100000}{(1/2)(998)(20)^2},\quad \text{or:}\quad p_{exit} \approx \mathbf{242000\ Pa}\quad \textit{Ans.}\ \text{(b)}$$

6.132 For Prob. 6.131, suppose we are limited by space to a total diffuser length of 10 meters. What should be the diffuser angle, exit diameter, and exit pressure for maximum recovery?

Solution: We are limited to L/D = 10.0. From Fig. 6.28b, read $C_{p,max} \approx 0.62$ at AR ≈ 4 and $2\theta \approx 6°$. *Ans.* The exit diameter and pressure are

$$D_e = d\sqrt{(AR)} = (1.0)(4.0)^{1/2} \approx \mathbf{2.0\ m} \quad \textit{Ans.}$$

$$C_{p,max} \approx 0.62 = (p_e - 100000)/[(1/2)(998)(20)^2], \quad \text{or:} \quad p_{exit} \approx \mathbf{224000\ Pa} \quad \textit{Ans.}$$

6.133 A wind-tunnel test section is 3 ft square with flow properties *V* = 150 ft/s, *p* = 15 lbf/in^2 absolute, and *T* = 68°F. Boundary-layer blockage at the end of the test section is 8 percent. Find the angle, length, exit height, and exit pressure of a flat-walled diffuser added onto the section to achieve maximum pressure recovery.

Solution: For air at 20°C and 15 psi, take $\rho = 0.00238$ slug/ft^3 and $\mu = 3.76E{-}7$ slug/ft·s. The Reynolds number is rather high, $Re = \rho Vd/\mu = (0.00238)(150)(3)/(3.76E{-}7) \approx 2.9E6$; much higher than the diffuser data in Fig. 6.28a (Re ≈ 2.8E5). But what can we do (?) Let's use it anyway:

$$B_t = 0.08, \quad \text{read } C_{p,max} \approx 0.70 \quad \text{at } L/W_1 \approx 17,\ 2\theta \approx 9.5°,\ AR \approx 3.75:$$

$$\text{Then} \quad \theta_{best} \approx \mathbf{4.75°}, \quad L \approx 17W_1 \approx \mathbf{51\ ft}, \quad W_2 \approx (AR)W_1 = 3.75(3) \approx \mathbf{11\ ft} \quad \textit{Ans.}$$

$$C_p \approx 0.70 = \frac{p_e - p_t}{(1/2)\rho V_1^2} = \frac{p_e - 15\times 144}{(1/2)(0.00238)(150)^2}, \quad \text{or:} \quad p_{exit} \approx \mathbf{2180\ \frac{lbf}{ft^2}} \quad \textit{Ans.}$$

6.134 For Prob. 6.133 above, suppose we are limited by space to a total diffuser length of 30 ft. What should the diffuser angle, exit height, and exit pressure be for maximum recovery?

Solution: We are limited to $L/W_1 = 10.0$. From Fig. 6.28a, read $C_{p,max} \approx 0.645$ at AR ≈ 2.8 and $2\theta \approx 10°$. *Ans.* The exit height and pressure are

$$W_{1,e} = (AR)W_1 = (2.8)(3.0) \approx \mathbf{8.4\ ft} \quad \textit{Ans.}$$

$$C_{p,max} \approx 0.645 = [p_e - (15)144]/[(1/2)(0.00238)(150)^2], \quad \text{or} \quad p_e = \mathbf{2180\ \frac{lbf}{ft^2}} \quad \textit{Ans.}$$

6.135 An airplane uses a pitot-static tube as a velocimeter. The measurements, with their uncertainties, are a static temperature of (−11 ± 3)°C, a static pressure of 60 ± 2 kPa, and a pressure difference $(p_o - p_s) = 3200 \pm 60$ Pa. (a) Estimate the airplane's velocity and its uncertainty. (b) Is a compressibility correction needed?

Solution: The air density is $\rho = p/(RT) = (60000\ \text{Pa})/[(287\ \text{m}^2/\text{s}^2\cdot\text{K})(262\ \text{K})] = 0.798\ \text{kg/m}^3$. (a) Estimate the velocity from the incompressible Pitot formula, Eq. (6.97):

$$V = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\Delta p}{p/(RT)}} = \sqrt{\frac{2(3200\ Pa)}{0.798\ \text{kg/m}^3}} = 90\ \frac{\text{m}}{\text{s}}$$

The overall uncertainty involves pressure difference, absolute pressure, and absolute temperature:

$$\frac{\delta V}{V} = \left[\left(\frac{1}{2}\frac{\delta \Delta p}{\Delta p}\right)^2 + \left(\frac{1}{2}\frac{\delta p}{p}\right)^2 + \left(\frac{1}{2}\frac{\delta T}{T}\right)^2\right]^{1/2} = \frac{1}{2}\left[\left(\frac{60}{3200}\right)^2 + \left(\frac{2}{60}\right)^2 + \left(\frac{3}{262}\right)^2\right]^{1/2} = 0.020$$

The uncertainty in velocity is 2%, therefore our final estimate is **$V \approx 90 \pm 2$ m/s** *Ans.* (a)
Check the Mach number. The speed of sound is $a = (kRT)^{1/2} = [1.4(287)(262)]^{1/2} = 324$ m/s. Therefore

$Ma = V/a = 90/324 = 0.28 < 0.3$. **No compressibility correction is needed.** *Ans.* (b)

6.136 For the pitot-static pressure arrangement of Fig. P6.136, the manometer fluid is (colored) water at 20°C. Estimate (a) the centerline velocity, (b) the pipe volume flow, and (c) the (smooth) wall shear stress.

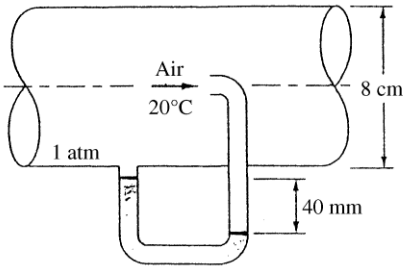


Fig. P6.136

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The manometer reads

$$p_o - p = (\rho_{water} - \rho_{air})gh = (998 - 1.2)(9.81)(0.040) \approx 391 \text{ Pa}$$

$$\text{Therefore} \quad V_{CL} = [2\Delta p/\rho]^{1/2} = [2(391)/1.2]^{1/2} \approx \mathbf{25.5\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

We can estimate the friction factor and then compute average velocity from Eq. (6.43):

$$\text{Guess} \quad V_{avg} \approx 0.85V_{CL} \approx 21.7\ \frac{m}{s}, \quad \text{then } Re_d = \frac{\rho Vd}{\mu} = \frac{1.2(21.7)(0.08)}{1.8E{-}5} \approx 115{,}700$$

$$\text{Then} \quad f_{smooth} \approx 0.0175, \quad V_{better} = \frac{25.5}{[1 + 1.33\sqrt{0.0175}]} \approx 21.69\ \frac{m}{s} \ \text{(converged)}$$

$$\text{Thus the volume flow is} \quad Q = (\pi/4)(0.08)^2(21.69) \approx \mathbf{0.109\ m^3/s}. \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Finally,} \quad \tau_w = \frac{f}{8}\rho V^2 = \frac{0.0175}{8}(1.2)(21.69)^2 \approx \mathbf{1.23\ Pa} \quad \textit{Ans.}\ \text{(c)}$$

6.137 For the 20°C water flow of Fig. P6.137, use the pitot-static arrangement to estimate (a) the centerline velocity and (b) the volume flow in the 5-in-diameter smooth pipe. (c) What error in flow rate is caused by neglecting the 1-ft elevation difference?

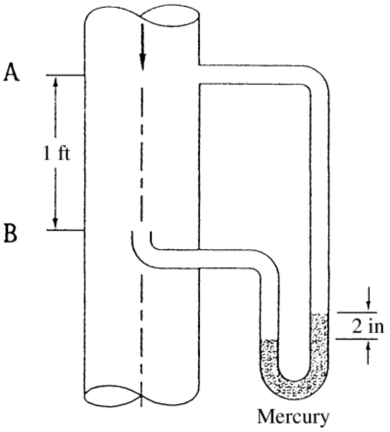


Fig. P6.137

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For the manometer reading h = 2 inches,

$$p_{oB} - p_A = (SG_{merc} - 1)(\rho g)_{water}h + \rho_{water}g(1\text{ ft}) \quad \text{but from the energy equation,}$$

$$p_A - p_B = \rho_{water}gh_{f\text{-}AB} - \rho_{water}g(1\text{ ft}) \quad \text{Therefore } p_{oB} - p_B = (SG - 1)\rho gh_{mano} + \rho gh_{f\text{-}AB}$$

$$\text{where friction loss } h_{f\text{-}AB} \approx f(\Delta L/d)(V^2/2g)$$

Thus the pitot tube reading equals the manometer reading (of about 130 psf) plus the friction loss between A and B (which is only about 3 psf), so there is only a small error:

$$(SG - 1)\rho gh = (13.56 - 1)(62.4)(2/12) \approx 130.6 \text{ psf}, \quad V_{CL} \approx \left[\frac{2\Delta p}{\rho}\right]^{1/2} = \left[\frac{2(130.6)}{1.94}\right]^{1/2}$$

$$\text{or}\quad V_{CL} \approx 11.6\ \frac{ft}{s},\quad \text{so}\ V_{avg} \approx 0.85V_{CL} \approx 9.9\ \frac{ft}{s},\quad Re = \frac{1.94(9.9)(5/12)}{2.09E{-}5} \approx 381500,$$

$$\text{so}\quad f_{smooth} \approx 0.0138,\quad \text{or}\quad \Delta p_{friction} = f(L/d)\rho V^2/2 \approx 3.2\ \text{lbf/ft}^2$$

If we now correct the pitot tube reading to $\Delta p_{pitot} \approx 130.6 + 3.2 = 133.8\ \text{psf},$ we may iterate and converge rapidly to the final estimate:

$$f \approx 0.01375,\ V_{CL} \approx \mathbf{11.75}\ \frac{\mathbf{ft}}{\mathbf{s}};\quad Q \approx \mathbf{1.39}\ \frac{\mathbf{ft^3}}{\mathbf{s}};\quad V_{avg} \approx \mathbf{10.17}\ \frac{\mathbf{ft}}{\mathbf{s}}\qquad \textit{Ans.}\ \text{(a, b)}$$

The error compared to our earlier estimate V ≈ 9.91 ft/s is about **2.6%** *Ans.* (c)

6.138 An engineer who took college fluid mechanics on a pass-fail basis has placed the static pressure hole far upstream of the stagnation probe, as in Fig. P6.138, thus contaminating the pitot measurement ridiculously with pipe friction losses. If the pipe flow is air at 20°C and 1 atm and the manometer fluid is Meriam red oil (SG = 0.827), estimate the air centerline velocity for the given manometer reading of 16 cm. Assume a smooth-walled tube.

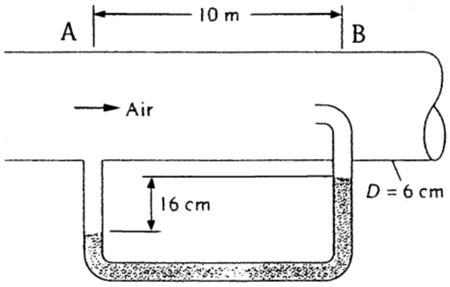


Fig. P6.138

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Because of the high friction loss over 10 meters of length, the manometer actually shows poB *less than* pA, which is a bit weird but correct:

$$p_A - p_{oB} = (\rho_{mano} - \rho_{air})gh = [0.827(998) - 1.2](9.81)(0.16) \approx 1294\ \text{Pa}$$

$$\text{Meanwhile,}\quad p_A - p_B = \rho g h_f = f\frac{L}{d}\frac{\rho V^2}{2},\quad \text{or}\quad p_{oB} - p_B = \frac{fL}{d}\frac{\rho V^2}{2} - 1294 = \frac{\rho}{2}V_{CL}^2$$

$$\text{Guess}\quad f \approx 0.02,\quad V \approx 0.85V_{CL},\quad \text{whence}\ 0.02\left(\frac{10}{0.06}\right)\left(\frac{1.2}{2}\right)V^2 - 1294 = \frac{1.2}{2}\left(\frac{V}{0.85}\right)^2$$

$$\text{Solve for}\quad V \approx 33.3\ \frac{m}{s},\quad Re_d = \frac{1.2(33.3)(0.06)}{1.8E{-}5} \approx 133000,\quad f_{better} \approx 0.0170,$$

$$V \approx V_{CL}/[1 + 1.33\sqrt{f}] \approx 0.852V_{CL},\quad \text{repeat to convergence}$$

Finally converges, $f \approx 0.0164$, $V \approx 39.87$ m/s, $V_{CL} = V/0.8546 \approx$ **46.65 m/s**. *Ans.*

6.139 Professor Walter Tunnel must measure velocity in a water tunnel. Due to budgetary restrictions, he cannot afford a pitot-static tube, so he inserts a total-head probe and a static-head probe, as shown, both in the mainstream away from the wall boundary layers. The two probes are connected to a manometer. (a) Write an expression for tunnel velocity V in terms of the parameters in the figure. (b) Is it critical that h1 be measured accurately? (c) How does part (a) differ from a pitot-static tube formula?

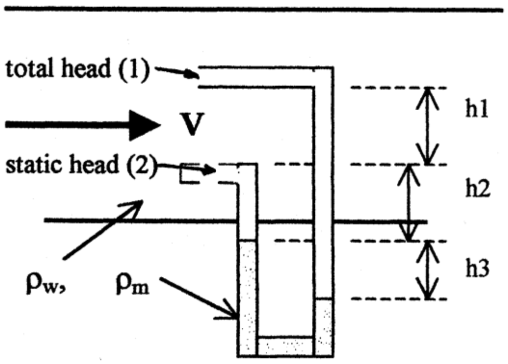


Fig. P6.139

Solution: Write Bernoulli from total-head inlet (1) to static-head inlet (2):

$$p_o + \rho_w g z_1 = p_s + \frac{\rho_w}{2}V^2 + \rho_w g z_2, \quad \text{Solve } V = \sqrt{\frac{2(p_o - p_s + \rho_w g h_1)}{\rho_w}}$$

Combine this with hydrostatics through the manometer:

$$p_s + \rho_w g h_2 + \rho_m g h_3 = p_o + \rho_w g h_1 + \rho_w g h_2 + \rho_w g h_3, \quad \text{cancel out } \rho_w g h_2$$

$$or: \quad p_o - p_s + \rho_w g h_1 = (\rho_m - \rho_w) g h_3$$

Introduce this into the expression for *V* above, for the final result:

$$V_{tunnel} = \sqrt{\frac{2(\rho_m - \rho_w) g h_3}{\rho_w}} \quad \textit{Ans.}\text{ (a)}$$

This is exactly the same as a pitot-static tube—*h*1 is not important. *Ans.* (b, c)

6.140 Kerosene at 20°C flows at 18 m^3/h in a 5-cm-diameter pipe. If a 2-cm-diameter thin-plate orifice with corner taps is installed, what will the measured pressure drop be, in Pa?

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 1.92E–3 kg/m·s. The orifice beta ratio is $\beta = 2/5 = 0.4$. The pipe velocity and Reynolds number are:

$$V = \frac{Q}{A} = \frac{18/3600}{(\pi/4)(0.05)^2} = 2.55\ \frac{m}{s}, \quad Re = \frac{804(2.55)(0.05)}{1.92E{-}3} = 53300$$

From Eqs. (6.112) and (6.113a) [corner taps], estimate $C_d \approx 0.6030$. Then the orifice pressure-drop formula predicts

$$Q = \frac{18}{3600} = 0.6030\frac{\pi}{4}(0.02)^2\sqrt{\frac{2\Delta p}{804[1-(0.4)^4]}},\quad \text{solve } \Delta p \approx \mathbf{273\ kPa} \quad \textit{Ans.}$$

***6.141** Gasoline at 20°C flows at 105 m^3/h in a 10-cm-diameter pipe. We wish to meter the flow with a thin-plate orifice and a differential pressure transducer that reads best at about 55 kPa. What is the proper β ratio for the orifice?

Solution: For gasoline at 20°C, take ρ = 680 kg/m^3 and μ = 2.92E–4 kg/m·s. This problem is similar to Example 6.21 in the text, but we don't have to be so precise because we don't know the exact geometry: corner taps, $D:\frac{1}{2}D$ taps, etc. The pipe velocity is

$$V_1 = \frac{Q}{A_1} = \frac{105/3600}{(\pi/4)(0.1)^2} = 3.71\ \frac{m}{s},\quad Re_D = \frac{680(3.71)(0.1)}{2.92E{-}4} \approx 865000$$

From Fig. 6.41, which is reasonable for all orifice geometries, read Cd ≈ 0.61. Then

$$V_{throat} = \frac{3.71\ m/s}{\beta^2} = C_d\sqrt{\frac{2(55000)}{680(1-\beta^4)}},\quad \text{or}\quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.478$$

$$\text{Solve for}\quad \beta \approx \mathbf{0.66} \quad \textit{Ans.}$$

Checking back with Fig. 6.41, we see that this is about right, so no further iteration is needed for this level of accuracy.

6.142 The shower head in Fig. P6.142 delivers water at 50°C. An orifice-type flow reducer is to be installed. The upstream pressure is constant at 400 kPa. What flow rate, in gal/min, results without the reducer? What reducer orifice diameter would decrease the flow by 40 percent?

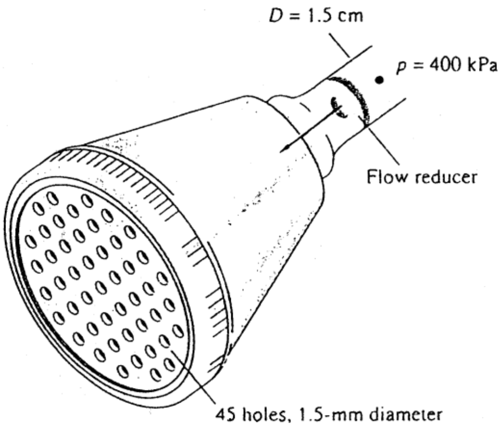


Fig. P6.142

Solution: For water at 50°C, take ρ = 988 kg/m^3 and μ = 0.548E–3 kg/m·s. Further assume that the shower head is a *poor diffuser*, so the pressure in the head is also about 400 kPa. Assume the outside pressure is sea-level standard, 101 kPa. From Fig. 6.41 for a 'typical' orifice, estimate Cd ≈ 0.61. Then, with $\beta \approx 0$ for the small holes, each hole delivers a flow rate of

$$Q_{1\ \text{hole}} \approx C_d A_{\text{hole}}\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \approx 0.61\left(\frac{\pi}{4}\right)(0.0015)^2\sqrt{\frac{2(400000-101000)}{988(1-0^4)}},$$

$$\text{or}\quad Q_{1\ \text{hole}} \approx 2.65\text{E–}5\ \text{m}^3/\text{s} \quad \text{and} \quad Q_{\text{total}} = 45Q_{1\ \text{hole}} \approx 0.00119\ \frac{\text{m}^3}{\text{s}}\left(\approx 19\ \frac{\text{gal}}{\text{min}}\right)$$

This is a *large* flow rate—a lot of expensive hot water. Checking back, the inlet pipe for this flow rate has ReD ≈ 183000, so Cd ≈ 0.60 would be slightly better and a repeat of the calculation would give **Qno reducer** ≈ 0.00117 m^3/s ≈ **18.6 gal/min**. *Ans.*

A 40% reduction would give Q = 0.6(0.00117) = 7.04E–4 m^3/s ÷ 45 = 1.57E–5 m^3/s for each hole, which corresponds to a pressure drop

$$Q_{1\ \text{hole}} = 1.57\text{E–}5 = 0.60\left(\frac{\pi}{4}\right)(0.0015)^2\sqrt{\frac{2\Delta p}{988}}, \quad \text{or} \quad \Delta p \approx 108000\ \text{Pa}$$

$$\text{or}\quad p_{\text{inside head}} \approx 101 + 108 \approx 209\ \text{kPa, the reducer must drop the inlet pressure to this.}$$

$$Q = 7.04\text{E–}4 \approx 0.61\left(\frac{\pi}{4}\right)(0.015\beta)^2\left[\frac{2(400000-209000)}{988(1-\beta^4)}\right]^{1/2}, \quad \text{or} \quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.332$$

$$\text{Solve for}\quad \beta \approx 0.56, \quad d_{\text{reducer}} \approx 0.56(1.5) \approx \mathbf{0.84\ cm} \quad \textit{Ans.}$$

6.143 A 10-cm-diameter smooth pipe contains an orifice plate with $D:\tfrac{1}{2}D$ taps and β = 0.5. The measured orifice pressure drop is 75 kPa for water flow at 20°C. Estimate the flow rate, in m^3/h. What is the nonrecoverable head loss?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. We know everything in the orifice relation, Eq. (6.104), except Cd, which we can estimate (as 0.61):

$$Q = C_d A_t\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d\frac{\pi}{4}(0.05)^2\sqrt{\frac{2(75000)}{998[1-(0.5)^4]}} = 0.0249C_d$$

$$\text{Guess}\quad C_d \approx 0.61, \quad Q \approx 0.0152\ \frac{\text{m}^3}{\text{s}}, \quad \text{Re}_D = \frac{4\rho Q}{\pi\mu D} \approx 193000, \quad C_d(\text{Eq. 6.112}) \approx 0.605$$

This is converged: $\mathbf{Q} = 0.0249(0.605) = 0.0150\ \text{m}^3/\text{s} \approx \mathbf{54\ m^3/h}$. *Ans.* (a)

(b) From Fig. 6.44, the non-recoverable head loss coefficient is K ≈ 1.8, based on Vt:

$$V_t = \frac{Q}{A_t} = \frac{0.0150}{\pi(0.025)^2} \approx 7.66\ \frac{\text{m}}{\text{s}},$$

$$\Delta p_{\text{loss}} = K\frac{\rho}{2}V_t^2 = 1.8\left(\frac{998}{2}\right)(7.66)^2 \approx \mathbf{53000\ Pa} \quad \textit{Ans. } \text{(b)}$$

6.144 Water at 20°C flows through the orifice in the figure, which is monitored by a mercury manometer. If d = 3 cm, (a) what is *h* when the flow is 20 m^3/h; and (b) what is *Q* when *h* = 58 cm?

Solution: (a) Evaluate V = Q/A = 2.83 m/s and $Re_D = \rho VD/\mu = 141{,}000$, $\beta = 0.6$, thus $C_d \approx 0.613$.

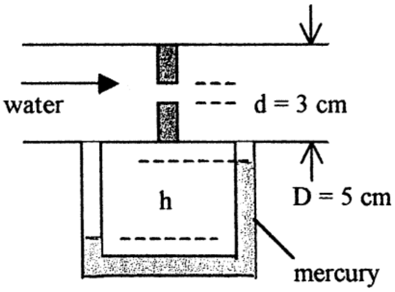


Fig. P6.144

$$Q = \frac{20}{3600} = C_d\frac{\pi}{4}d^2\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = (0.613)\frac{\pi}{4}(0.03)^2\sqrt{\frac{2(13550-998)(9.81)h}{998(1-0.6^4)}}$$

where we have introduced the manometer formula $\Delta p = (\rho_{\text{mercury}} - \rho_{\text{water}})gh$.

$$\text{Solve for:} \quad \boldsymbol{h} \approx 0.58\ \text{m} = \mathbf{58\ cm} \quad \textit{Ans. } \text{(a)}$$

Solve this problem when *h* = 58 cm is known and *Q* is the unknown. Well, we can see that the numbers are the same as part (a), and the solution is

$$\text{Solve for:} \quad \boldsymbol{Q} \approx 0.00556\ \text{m}^3/\text{s} = \mathbf{20\ m^3/h} \quad \textit{Ans. } \text{(b)}$$

.

6.145 The 1-m-diameter tank in Fig. P6.145 is initially filled with gasoline at 20°C. There is a 2-cm-diameter orifice in the bottom. If the orifice is suddenly opened, estimate the time for the fluid level *h*(t) to drop from 2.0 to 1.6 meters.

Solution: For gasoline at 20°C, take $\rho = 680\ \text{kg/m}^3$ and $\mu = 2.92\text{E}-4\ \text{kg/m·s}$. The orifice simulates "corner taps" with $\beta \approx 0$, so, from Eq. (6.112), Cd ≈ 0.596. From the energy equation, the pressure drop across the orifice is $\Delta p = \rho g h(t)$, or

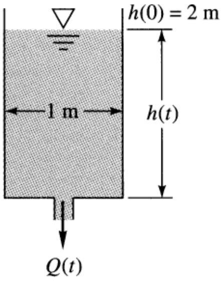


Fig. P6.145

$$Q = C_d A_t \sqrt{\frac{2\rho g h}{\rho(1-\beta^4)}} \approx 0.596\left(\frac{\pi}{4}\right)(0.02)^2\sqrt{2(9.81)h} \approx 0.000829\sqrt{h}$$

$$\text{But also}\quad Q = -\frac{d}{dt}(\upsilon_{\text{tank}}) = -A_{\text{tank}}\frac{dh}{dt} = -\frac{\pi}{4}(1.0\ \text{m})^2\frac{dh}{dt}$$

Set the *Q*'s equal, separate the variables, and integrate to find the draining time:

$$-\int_{2.0}^{1.6}\frac{dh}{\sqrt{h}} = 0.001056\int_0^{t_{\text{final}}} dt,\quad\text{or}\quad t_{\text{final}} = \frac{2[\sqrt{2}-\sqrt{1.6}]}{0.001056} = 283\ \text{s} \approx \mathbf{4.7\ min}\quad \textit{Ans.}$$

6.146 A pipe connecting two reservoirs, as in Fig. P6.146, contains a thin-plate orifice. For water flow at 20°C, estimate (a) the volume flow through the pipe and (b) the pressure drop across the orifice plate.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The energy equation should include the orifice head loss and the entrance and exit losses:

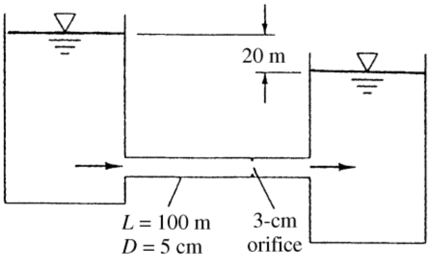


Fig. P6.146

$$\Delta z = 20\text{ m} = \frac{V^2}{2g}\left(f\frac{L}{d} + \Sigma K\right),\quad \text{where } K_{entr} \approx 0.5,\ K_{exit} \approx 1.0,\ K_{orifice}^{\beta=0.6} \approx 1.5 \text{ (Fig. 6.44)}$$

$$V^2 = \frac{2(9.81)(20)}{[f(100/0.05) + 0.5 + 1.0 + 1.5]} = \frac{392.4}{2000f + 3.0};\quad \text{guess } f \approx 0.02, V \approx 3.02 \text{ m/s}$$

$$\text{Iterate to}\quad f_{smooth} \approx 0.0162,\quad V \approx 3.33 \text{ m/s}$$

The final $Re = \rho VD/\mu \approx 166000$, and $Q = (\pi/4)(0.05)^2(3.33) \approx$ **0.00653 m^3/s** *Ans*. (a)
(b) The pressure drop across the orifice is given by the orifice formula:

$$Re_D = 166000,\quad \beta = 0.6,\quad C_d \approx 0.609 \text{ (Fig. 6.41):}$$

$$Q = 0.00653 = C_d A_t\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = 0.609\left(\frac{\pi}{4}\right)(0.03)^2\left[\frac{2\Delta p}{998(1-0.6^4)}\right]^{1/2},$$

$$\Delta p = \mathbf{100\,kPa}\quad \textit{Ans.(b)}$$

***6.147** Air flows through a 6-cm-diameter smooth pipe which has a 2 m-long per-forated section containing 500 holes (diameter 1 mm), as in Fig. P6.147. Pressure outside the pipe is sea-level standard air. If p_1 = 105 kPa and Q_1 = 110 m^3/h, estimate p_2 and Q_2, assuming that the holes are approximated by thin-plate orifices. *Hint:* A momentum control volume may be very useful.

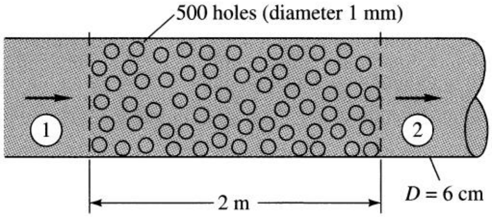


Fig. P6.147

Solution: For air at 20°C and 105 kPa, take ρ = 1.25 kg/m^3 and μ = 1.8E−5 kg/m·s. Use the entrance flow rate to estimate the wall shear stress from the Moody chart:

$$V_1 = \frac{Q_1}{A} = \frac{110/3600}{(\pi/4)(0.06)^2} = 10.8\ \frac{m}{s},\quad Re_1 = \frac{1.25(10.8)(0.06)}{1.8E\text{–}5} \approx 45000,\quad f_{smooth} \approx 0.0214$$

$$\text{then}\quad \tau_{wall} = \frac{f}{8}\rho V^2 = \frac{0.0214}{8}(1.25)(10.8)^2 \approx 0.390\ \text{Pa}$$

Further assume that the pressure does not change too much, so $\Delta p_{orifice} \approx 105000 - 101350 \approx 3650$ Pa. Then the flow rate from the orifices is, approximately,

$$\beta \approx 0,\ C_d \approx 0.61:\quad Q \approx 500 C_d A_t (2\Delta p/\rho)^{1/2} = 500(0.61)\left(\frac{\pi}{4}\right)(0.001)^2\left[\frac{2(3650)}{1.25}\right]^{1/2}$$

$$\text{or:}\quad Q \approx \mathbf{0.0183}\ \text{m}^3/\text{s},\quad \text{so}\ Q_2 = \frac{110}{3600} - 0.0183 \approx 0.01225\ \text{m}^3/\text{s}$$

Then $V_2 = Q_2/A_2 = 0.01225/[(\pi/4)(0.06)^2] \approx 4.33$ m/s. A control volume enclosing the pipe walls and sections (1) and (2) yields the x-momentum equation:

$$\sum F_x = p_1A - p_2A - \tau_w\pi DL = \dot{m}_2V_2 - \dot{m}_1V_1 = \rho AV_2^2 - \rho AV_1^2,\quad \text{divide by A:}$$

$$p_1 - p_2 = 0.390\left[\frac{\pi(0.06)(2.0)}{(\pi/4)(0.06)^2}\right] + 1.25(4.33)^2 - 1.25(10.8)^2 = 52 + 23 - 146 \approx -71\ \text{Pa}$$

Thus $p_2 = 105000 + 71 \approx$ **105 kPa** also and above is correct: Q_2 = **0.0123 m^3/s**. *Ans.*

6.148 A smooth pipe containing ethanol at 20°C flows at 7 m^3/h through a Bernoulli obstruction, as in Fig. P6.148. Three piezometer tubes are installed, as shown. If the obstruction is a thin-plate orifice, estimate the piezometer levels (a) h_2 and (b) h_3.

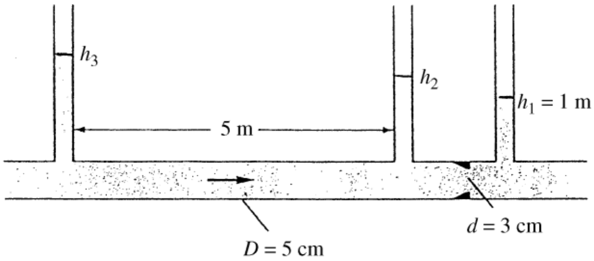


Fig. P6.148

Solution: For ethanol at 20°C, take $\rho = 789\ \text{kg/m}^3$ and $\mu = 0.0012$ kg/m·s. With the flow rate known, we can compute Reynolds number and friction factor, etc.:

$$V = \frac{Q}{A} = \frac{7/3600}{(\pi/4)(0.05)^2} = 0.99\ \frac{\text{m}}{\text{s}};\quad \text{Re}_D = \frac{789(0.99)(0.05)}{0.0012} = 32600,\quad f_{\text{smooth}} \approx 0.0230$$

From Fig. 6.44, at $\beta = 0.6$, $K \approx 1.5$. Then the head loss across the orifice is

$$\Delta h = h_2 - h_1 = K\frac{V_t^2}{2g} = (1.5)\left[\frac{\{0.99/(0.6)^2\}^2}{2(9.81)}\right] \approx 0.58\ \text{m},\quad \text{hence}\ \mathbf{h_2 \approx 1.58\ m}\quad \textit{Ans.}\ \text{(a)}$$

Then the piezometer change between (2) and (3) is due to Moody friction loss:

$$h_3 - h_2 = h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.023)\left(\frac{5}{0.05}\right)\frac{(0.99)^2}{2(9.81)} = 0.12\ \text{m},$$

$$\text{or}\quad h_3 = 1.58 + 0.12 \approx \mathbf{1.7\ m}\quad \textit{Ans.}\ \text{(b)}$$

6.149 In a laboratory experiment, air at 20°C flows from a large tank through a 2-cm-diameter smooth pipe into a sea-level atmosphere, as in Fig. P6.149. The flow is metered by a long-radius nozzle of 1-cm diameter, using a manometer with Meriam red oil (SG = 0.827). The pipe is 8 m long. The measurements of tank pressure and manometer height are as follows:

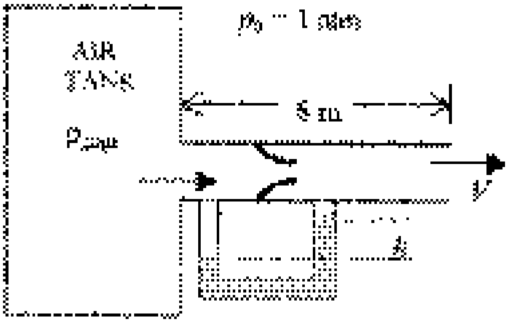


Fig. P6.149

*p*tank, (gage):	Pa	60	320	1200	2050	2470	3500	4900
*h*mano, mm:		6	38	160	295	380	575	820

Use this data to calculate the flow rates *Q* and Reynolds numbers Re*D* and make a plot of measured flow rate versus tank pressure. Is the flow laminar or turbulent? Compare the data with theoretical results obtained from the Moody chart, including minor losses. Discuss.

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 0.000015$ kg/m·s. With no elevation change and negligible tank velocity, the energy equation would yield

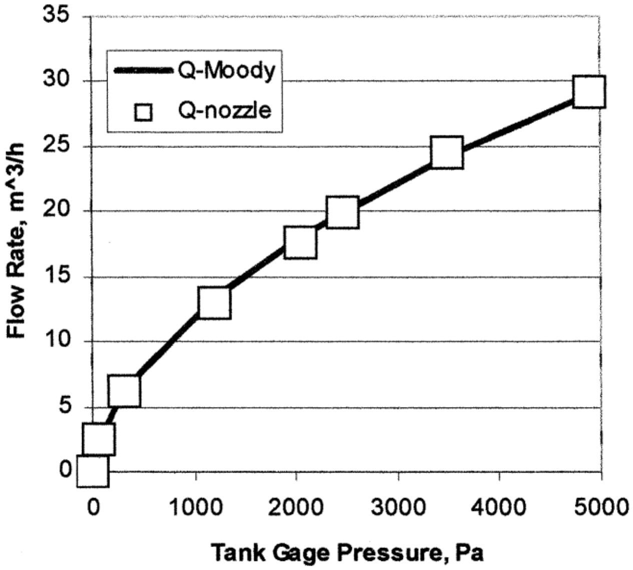
$$p_{\text{tank}} - p_{atm} = \frac{\rho V^2}{2}\left(1 + f\frac{L}{D} + K_{entrance} + K_{nozzle}\right), \quad K_{ent} \approx 0.5 \text{ and } K_{noz} \approx 0.7$$

Since Δp is given, we can use this expression plus the Moody chart to predict V and $Q = AV$ and compare with the flow-nozzle measurements. The flow nozzle formula is:

$$V_{throat} = C_d\sqrt{\frac{2\Delta p_{mano}}{\rho(1-\beta^4)}} \quad \text{where } \Delta p = (\rho_{oil} - \rho_{air})gh, \; C_d \text{ from Fig. 6.42 and } \beta = 0.5$$

The friction factor is given by the smooth-pipe Moody formula, Eq. (6.48) for $\varepsilon = 0$. The results may be tabulated as follows, and the plot on the next page shows excellent (too good?) agreement with theory.

p_{tank}, Pa:	60	320	1200	2050	2470	3500	4900
V, m/s (nozzle data):	2.32	5.82	11.9	16.1	18.2	22.3	26.4
Q, m^3/h (nozzle data):	2.39	6.22	12.9	17.6	19.9	24.5	29.1
Q, m^3/h (theory):	2.31	6.25	13.3	18.0	20.0	24.2	28.9
f_{Moody}:	0.0444	0.0331	0.0271	0.0252	0.0245	0.0234	0.0225



6.150 Gasoline at 20°C flows at 0.06 m^3/s through a 15-cm pipe and is metered by a 9-cm-diameter long-radius flow nozzle (Fig. 6.40a). What is the expected pressure drop across the nozzle?

Solution: For gasoline at 20°C, take $\rho = 680$ kg/m and $\mu = 2.92E{-}4$ kg/m·s. Calculate the pipe velocity and Reynolds number:

$$V = \frac{Q}{A} = \frac{0.06}{(\pi/4)(0.15)^2} = 3.40\ \frac{m}{s},\quad Re_D = \frac{680(3.40)(0.15)}{2.92E{-}4} \approx 1.19E6$$

The ISO correlation for discharge (Eq. 6.114) is used to estimate the pressure drop:

$$C_d \approx 0.9965 - 0.00653\left(\frac{10^6\beta}{Re_D}\right)^{1/2} = 0.9965 - 0.00653\left[\frac{10^6(0.6)}{1.19E6}\right]^{1/2} \approx \mathit{0.9919}$$

$$\text{Then}\quad Q = 0.06 = (0.9919)\left(\frac{\pi}{4}\right)(0.09)^2\sqrt{\frac{2\Delta p}{680(1-0.6^4)}},$$

$$\text{Solve}\quad \mathbf{\Delta p \approx 27000\ Pa}\quad \mathit{Ans.}$$

P6.151 An engineer needs to monitor a flow of 20°C gasoline at about 250±25 gal/min through a 4-in-diameter smooth pipe. She can use an orifice plate, a long-radius flow nozzle, or a venturi nozzle, all with 2-in-diameter throats. The only differential pressure gage available is accurate in the range 6 to 10 lbf/in^2. Disregarding flow losses, which device is best?

Solution: For gasoline at 20°C, take ρ = 680 kg/m^3 and μ = 2.92E-4 kg/m-s. We are given

β = 2/4 = 0.5. The flow rate is in the range $0.0142 < Q < 0.0174$ m^3/s. The pipe Reynolds number is in the range Re_D = 460,000 ± 10%. In SI units, the throat diameter is 0.0508 m, and its area is (π/4)(0.0508m)^2 = 0.00203 m^2. Our basic “obstruction” formula is Eq. (6.104):

$$Q = C_d A_t\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d(0.00203m^2)\sqrt{\frac{2\Delta p}{(680kg/m^3)\{1-(0.5)^4\}}} = 0.0158 \pm 0.0016\,\frac{m^3}{s}$$

It remains only to determine C_d for the three devices and then calculate Δp. The results are:

Orifice plate, D:1/2D taps: $C_d \approx 0.605$, Δp = **6.2 to 9.3** lbf/in^2 *Ans.*

Long-radius flow nozzle: $C_d \approx 0.99$, Δp = 2.3 to 3.5 lbf/in^2

Venturi nozzle: $C_d \approx 0.977$, Δp = 2.4 to 3.6 lbf/in^2

Only the orifice plate, with its high losses, is compatible with the available pressure gage.

6.152 Kerosene at 20°C flows at 20 m^3/h in an 8-cm-diameter pipe. The flow is to be metered by an ISA 1932 flow nozzle so that the pressure drop is 7 kPa. What is the proper nozzle diameter?

Solution: For kerosene at 20°C, take $\rho = 804$ kg/m^3 and $\mu = 1.92E{-}3$ kg/m·s. We cannot calculate the discharge coefficient exactly because we don't know β, so just estimate Cd:

$$\text{Guess}\quad C_d \approx 0.99,\quad \text{then}\ \ Q \approx 0.99\left(\frac{\pi}{4}\right)(0.08\beta)^2\sqrt{\frac{2(7000)}{804(1-\beta^4)}} = \frac{20}{3600}\frac{m^3}{s}$$

$$\text{or:}\quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.268,\quad \text{solve}\ \ \beta \approx 0.508,$$

$$Re_D = \frac{4(804)(20/3600)}{\pi(1.92E{-}3)(0.08)} \approx 37000$$

Now compute a better Cd from the ISA nozzle correlation, Eq. (6.115):

$$C_d \approx 0.99 - 0.2262\beta^{4.1} + (0.000215 - 0.001125\beta + 0.00249\beta^{4.7})\left(\frac{10^6}{Re_D}\right)^{1.15} \approx 0.9647$$

Iterate once to obtain a better $\beta \approx 0.515$, d = 0.515(8 cm) ≈ **4.12 cm** *Ans.*

6.153 Two water tanks, each with base area of 1 ft^2, are connected by a 0.5-in-diameter long-radius nozzle as in Fig. P6.153. If h = 1 ft as shown for *t* = 0, estimate the time for h(t) to drop to 0.25 ft.

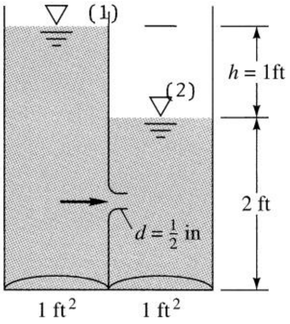


Fig. P6.153

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For a long-radius nozzle with $\beta \approx 0$, guess Cd ≈ 0.98 and Kloss ≈ 0.9 from Fig. 6.44. The elevation difference *h* must balance the head losses in the nozzle and submerged exit:

$$\Delta z = \sum h_{loss} = \frac{V_t^2}{2g}\sum K = \frac{V_t^2}{2(32.2)}(0.9_{nozzle} + 1.0_{exit}) = h, \quad \text{solve } V_t = 5.82\sqrt{h}$$

$$\text{hence} \quad Q = V_t\left(\frac{\pi}{4}\right)\left(\frac{1/2}{12}\right)^2 \approx 0.00794\sqrt{h} = -\frac{\mathbf{1}}{\mathbf{2}}A_{tank}\frac{dh}{dt} = -0.5\frac{dh}{dt}$$

The boldface factor **1/2** accounts for the fact that, as the left tank falls by dh, the right tank rises by the same amount, hence dh/dt changes twice as fast as for one tank alone. We can separate and integrate and find the time for h to drop from 1 ft to 0.25 ft:

$$\int_{0.25}^{1.0}\frac{dh}{\sqrt{h}} = 0.0159\int_0^{t_{final}} dt, \quad \text{or:} \quad t_{final} = \frac{2\left(\sqrt{1}-\sqrt{0.25}\right)}{0.0159} \approx \mathbf{63\ s} \quad \textit{Ans.}$$

P6.154 Gasoline at 20°C flows through a 6-cm-diameter pipe. It is metered by a modern venturi nozzle with *d* = 4 cm. The measured pressure drop is 6.5 kPa. Estimate the flow rate in gallons per minute.

Solution: For gasoline at 20°C, from Table A.3, ρ = 680 kg/m^3 and μ = 2.92E-4 kg/m-s. We are not sure of the Reynolds number, so assume for the present that Fig. 6.43 is valid, in which case, from Eq. (6.116),

$$C_{d,venturi} \approx 0.9858 - 0.196\beta^{4.5} = 0.9858 - 0.196\left(\frac{0.04}{0.06}\right)^{4.5} = 0.954$$

Then, from Eq. (6.104), the flow rate is

$$Q = C_d A_t \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = 0.954(\frac{\pi}{4})(0.04m)^2 \sqrt{\frac{2(8500\,Pa)}{680[1-(4/6)^4]}} = 0.00669\frac{m^3}{s} = \mathbf{106}\,\frac{\mathbf{gal}}{\mathbf{min}} \quad \textit{Ans.}$$

$$\text{Check} \quad \mathrm{Re}_D = \frac{4\rho Q}{\pi\mu D} = \frac{4(680)(0.00669)}{\pi(2.92\mathrm{E}-4)(0.06)} = 330,000 \quad \text{OK, within range.}$$

P6.155 It is desired to meter methanol at 20°C flowing through a 5-inch-diameter pipe. The expected flow rate is about 300 gal/min. Two flowmeters are available: a venturi nozzle and a thin-plate orifice, each with d = 2 in. The differential pressure gage on hand is most accurate at about 12-15 lbs/in^2. Which meter is better for this job?

Solution: For methanol at 20°C, from Table A.3, ρ = 791 kg/m^3 = 1.535 slug/ft^3 and μ = 5.98E-4 kg/m-s = 1.25E-5 slug/ft-s. Compute the average velocity in the pipe and find the discharge coefficients for each meter, for β = (2 in)/(5 in) = 0.4:

$$Q = 300\frac{\mathrm{gal}}{\mathrm{min}} = 0.668\frac{ft^3}{s} = \frac{\pi}{4}(\frac{5}{12}\,ft)^2\,V_{avg}\text{, solve for } V = 4.90\,\frac{ft}{s} \text{ or } V_t = 30.6\frac{ft}{s}$$

$$Orifice: \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.535)(4.90)(5/12)}{(0.0000125)} = 251,000,\ \beta{=}0.4,\ Fig.6.41: C_d \approx 0.601$$

$$Venturi\ nozzle: 1.5\mathrm{E}5 < \mathrm{Re}_D < 2\mathrm{E}6,\ OK,\ \ \beta{=}0.4,\ \ Fig.6.43:\ C_d \approx 0.983$$

Find the expected pressure drops from Eq. (6.104):

$$V_t = 30.6\frac{ft}{s} = C_d\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d\sqrt{\frac{2\Delta p}{(1.535)[1-(0.4)^4]}} = 1.156\,C_d\sqrt{\Delta p}$$

$$\text{Venturi nozzle:} \quad C_d = 0.983, \quad \text{solve} \quad \Delta p \approx 725\,\frac{lbf}{ft^2} \approx 5\,\frac{lbf}{in^2}$$

$$\textbf{Thin-plate orifice}: \ C_d = 0.601, \quad \text{solve} \quad \Delta p \approx 1940\,\frac{lbf}{ft^2} \approx \mathbf{14}\,\frac{lbf}{in^2}$$

The orifice plate is the better choice, for accuracy, although the head loss is much larger.

6.156 Ethanol at 20°C flows down through a modern venturi nozzle as in Fig. P6.156. If the mercury manometer reading is 4 in, as shown, estimate the flow rate, in gal/min.

Solution: For ethanol at 20°C, take $\rho = 1.53$ slug/ft^3 and $\mu = 2.51\text{E}{-5}$ slug/ft·s. Given $\beta = 0.5$, the discharge coefficient is

$$C_d = 0.9858 - 0.196(0.5)^{4.5} \approx 0.9771$$

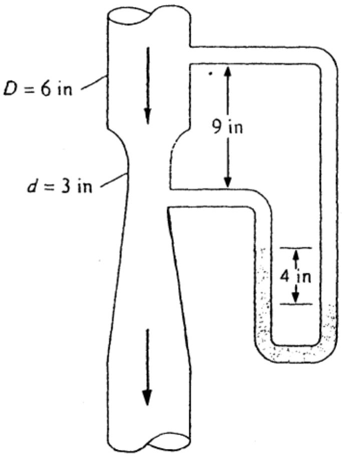


Fig. P6.156

The 9-inch displacement of manometer taps does not affect the pressure drop reading, because both legs are filled with ethanol. Therefore we proceed directly to Δp and Q:

$$\Delta p_{\text{nozzle}} = (\rho_{\text{merc}} - \rho_{\text{eth}})gh = (26.3 - 1.53)(32.2)(4/12) \approx 266 \text{ lbf/ft}^2$$

$$\text{Hence}\quad Q = C_d A_t\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = 0.9771\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2\sqrt{\frac{2(266)}{1.53(1-0.5^4)}} \approx 0.924\ \frac{\text{ft}^3}{\text{s}}\quad \textit{Ans.}$$

6.157 Modify Prob. 6.156 if the fluid is *air* at 20°C, entering the venturi at a pressure of 18 psia. Should a compressibility correction be used?

Solution: For air at 20°C and 18 psi, take $\rho = 0.00286$ slug/ft^3 and $\mu = 3.76\text{E}{-7}$ slug/ft·s. With β still equal to 0.5, Cd still equals 0.9771 as previous page. The manometer reading is

$$\Delta p_{\text{nozzle}} = (26.3 - 0.00286)(32.2)(4/12) \approx 282 \text{ lbf/ft}^2,$$

$$\text{whence}\quad Q = 0.9771\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2\sqrt{\frac{2(282)}{0.00286(1-0.5^4)}} \approx \mathbf{22.0}\ \frac{\mathbf{ft^3}}{\mathbf{s}}\quad \textit{Ans.}$$

From this result, the throat velocity Vt = Q/At ≈ 448 ft/s, quite high, the Mach number in the throat is approximately Ma = 0.4, a **(small) compressibility correction might be expected.** [Making a one-dimensional subsonic-flow correction, using the methods of Chap. 9, results in a throat volume flow estimate of Q ≈ 22.8 ft^3/s, about 4% higher.]

6.158 Water at 20°C flows in a long horizontal commercial-steel 6-cm-diameter pipe that contains a classical Herschel venturi with a 4-cm throat. The venturi is connected to a mercury manometer whose reading is *h* = 40 cm. Estimate (a) the flow rate, in m^3/h, and (b) the total pressure difference between points 50 cm upstream and 50 cm downstream of the venturi.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$kg/m·s. For commercial steel, $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d = 0.046/60 = 0.000767$. First *estimate* the flow rate:

$$\Delta p = (\rho_m - \rho_w)gh = (13560 - 998)(9.81)(0.40) \approx 49293 \text{ Pa}$$

$$\text{Guess} \quad C_d \approx 0.985, \quad Q = (0.985)\left(\frac{\pi}{4}\right)(0.04)^2\sqrt{\frac{2(49293)}{998[1-(4/6)^4]}} \approx 0.0137 \ \frac{m^3}{s}$$

$$\text{Check} \quad Re_D = \frac{4\rho Q}{\pi\mu D} \approx 291000$$

At this Reynolds number, we see from Fig. 6.42 that Cd does indeed ≈ 0.985 for the Herschel venturi. Therefore, indeed, Q = 0.0137 m^3/s ≈ **49 m^3/h**. *Ans.* (a)

(b) 50 cm upstream and 50 cm downstream are far enough that the pressure recovers from its throat value, and the total Δp is the sum of Moody pipe loss and venturi head loss. First work out the pipe velocity, V = Q/A = (0.0137)/[(π/4)(0.06)^2] ≈ 4.85 m/s. Then

$$Re_D = 291000, \quad \frac{\varepsilon}{d} = 0.000767, \quad \text{then } f_{Moody} \approx 0.0196; \quad \text{Fig. 6.44: } K_{venturi} \approx 0.2$$

$$\text{Then} \quad \Delta p = \Delta p_{Moody} + \Delta p_{venturi} = \frac{\rho V^2}{2}\left(f\frac{L}{d} + K\right)$$

$$= \frac{998(4.85)^2}{2}\left[0.0196\left(\frac{1.0}{0.06}\right) + 0.2\right] \approx \mathbf{6200\ Pa} \quad \textit{Ans. (b)}$$

6.159 A modern venturi nozzle is tested in a laboratory flow with water at 20°C. The pipe diameter is 5.5 cm, and the venturi throat diameter is 3.5 cm. The flow rate is measured by a weigh tank and the pressure drop by a water-mercury manometer. The mass flow rate and manometer readings are as follows:

$\dot{m}$, kg/s:	0.95	1.98	2.99	5.06	8.15
h, mm:	3.7	15.9	36.2	102.4	264.4

Use these data to plot a calibration curve of venturi discharge coefficient versus Reynolds number. Compare with the accepted correlation, Eq. (6.116).

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The given data of mass flow and manometer height can readily be converted to discharge coefficient and Reynolds number:

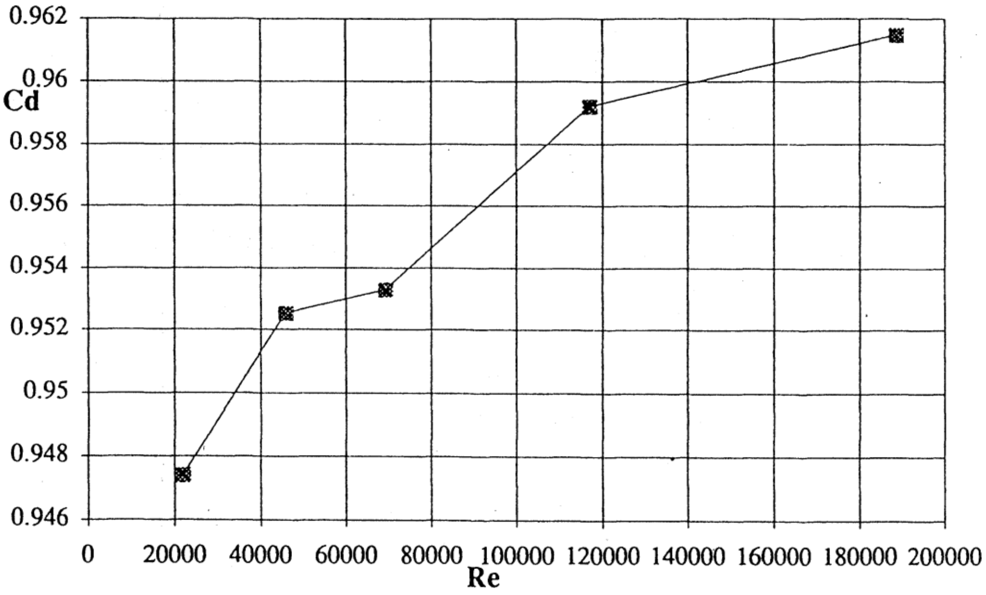
$$Q = \frac{\dot{m}}{998} = C_d\left(\frac{\pi}{4}\right)(0.035)^2\sqrt{\frac{2(13.56-1)\rho_w(9.81)h}{\rho_w[1-(3.5/5.5)^4]}}, \quad \text{or:} \quad C_d \approx \frac{\dot{m}\text{ (kg/s)}}{16.485\sqrt{h_{\text{meters}}}}$$

$$Re_D = \frac{4\dot{m}}{\pi\mu D} = \frac{4\dot{m}}{\pi(0.001)(0.055)} \approx 23150\,\dot{m}\text{ (kg/s)}$$

The data can then be converted and tabulated as follows:

h, m:	0.037	0.0159	0.0362	0.1024	0.2644
Cd:	0.947	0.953	0.953	0.959	0.962
ReD:	22000	46000	69000	117000	189000

These data are plotted in the graph below, similar to Fig. 6.42 of the text:



They closely resemble the “classical Herschel venturi,” but this data is actually for a *modern* venturi, for which we only know the value of Cd for 1.5E5 < ReD ≤ 2E5:

$$\text{Eq. (6.116)}\quad C_d \approx 0.9858 - 0.196\left(\frac{3.5}{5.5}\right)^{4.5} \approx \mathbf{0.960}$$

The two data points near this Reynolds number range are quite close to 0.960 ± 0.002.

P6.160 An instrument popular in the beverage industry is the *target flowmeter* in Fig. P6.x. A small flat disk is mounted in the center of the pipe, supported by a strong but thin rod.

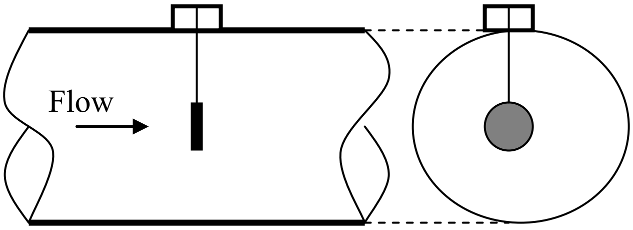


Fig. P6.x

(*a*) Explain how the flowmeter works.

(*b*) If the bending moment *M* of the rod is measured at the wall, derive a formula for the estimated velocity of the flow.
(*c*) List a few advantages and disadvantages of such an instrument.

Solution: (*a*) The flow creates a drag force *F* on the disk, approximately proportional to V^2.
(*b*) The bending moment is the drag force times the pipe radius. Thus the formula

$$M = F\,R\ ,\quad \text{where}\quad F = C_D\,\frac{\rho}{2}\,V^2\,A_{disk}$$

$$\text{Solve for}\quad V \approx \sqrt{\frac{2M}{C_D\,\rho\,A_{disk}\,R}}\qquad \textit{Ans.}(b)$$

We are neglecting the drag of the thin rod, especially if it is streamlined. In Chapter 7, Fig. 7.16, we learn that the drag coefficient of a disk is about 1.2 over a wide range of Reynolds number.
(*c*) *Advantages*: low cost; disk easy to keep clean; works for a wide variety of fluids; can measure flow in either direction; useful in moderately unsteady flows. *Disadvantages*: needs calibration because the formula is too simplified; very poor accuracy at low velocities, due to the square-root relationship; heavy flows can break the rod; gummy flows can coat the disk and change the calibration; the drag force causes a pipe head loss.

P6.161 An instrument popular in the water supply industry, sketched in Fig. P6.161, is the single jet water meter.

(*a*) How does it work?

(*b*) What do you think a typical calibration curve would look like?

(*c*) Can you cite further details, for example, reliability, head loss, cost [58]?

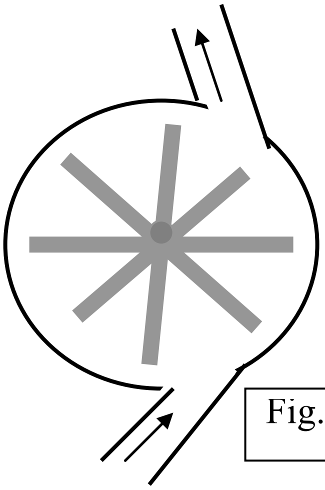


Fig. P6.161

Solution: (*a*) The single-jet meter is similar in principle to the standard turbine meter, Fig. 6.32 of the text, except that, instead of flowing axially through the turbine, it creates an in-plane jet that strikes the turbine blades and turns them. There are also multi-jet meters.
(*b*) Just as in a standard turbine meter, the single-jet meter turns the turbine at a rate nearly proportional to the flow rate. The manufacturer states a "*K* factor" of an equation for turning rate Ω versus volume flow rate *Q*, in the form $\Omega = K\,Q$. At very low flow rates, the Reynolds number is very low, and the manufacturer includes an *error curv*e (a few per cent deviation) for low rates.
(*c*) The single-jet meter is quite reliable and prized for its accuracy at low flow rates. It is not used for large pipe sizes because, due to its offset design, it would have to be huge.

6.162 Air flows at high speed through a Herschel venturi monitored by a mercury manometer, as shown in Fig. P6.161. The upstream conditions are 150 kPa and 80°C. If *h* = 37 cm, estimate the mass flow in kg/s. [HINT: The flow is compressible.]

Solution: The upstream density is $\rho_1 = p_1/(RT) = (150000)/[287(273 + 80)] = 1.48$ kg/m^3. The clue "high speed" means that we had better use the *compressible* venturi formula, Eq. (6.117):

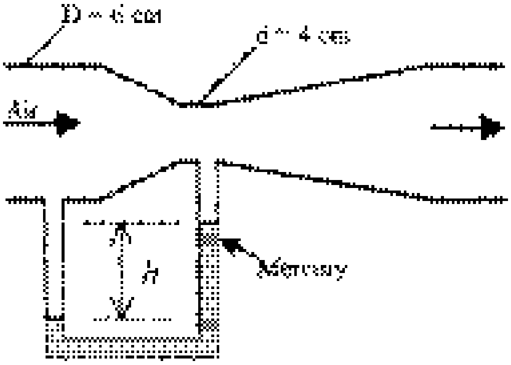


Fig. P6.162

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1-\beta^4}} \quad \text{where } \beta = 4/6 \text{ for this nozzle.}$$

The pressure difference is measured by the mercury manometer:

$$p_1 - p_2 = (\rho_{merc} - \rho_{air})gh = (13550 - 1.48\ kg/m^3)(9.81\ m/s^2)(0.37\ m) = 49200\ Pa$$

The pressure ratio is thus (150 – 49.2)/150 = 0.67 and, for $\beta = 2/3$, we read $Y \approx 0.76$ from Fig. 6.45. From Fig. 6.43 estimate $C_d \approx 0.985$. The (compressible) venturi formula thus predicts:

$$\dot{m} = 0.985(0.76)\left[\frac{\pi}{4}(0.04\ m)^2\right]\sqrt{\frac{2(1.48)(49200)}{1-(2/3)^4}} \approx 0.40\ \frac{kg}{s} \quad \textit{Ans.}$$

***6.163** Modify Prob. 6.162 as follows. Find the manometer reading h for which the mass flow through the venturi is approximately 0.4 kg/s. [HINT: The flow is compressible.]

Solution: This is, in fact, the answer to Prob. 6.162, but who knew? The present problem is intended as an iteration exercise, preferably with EES. We know the upstream pressure and density and the discharge coefficient, but we must iterate for Y and p_2 in the basic formula:

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1-\beta^4}} = 0.40\ kg/s$$

The answer should be $h = 0.37$ m, $Y \approx 0.76$, and $C_d \approx 0.985$, as in Prob. 6.162, but the problem is extremely sensitive to the value of h. A 10% change in h causes only a 2%

change in mass flow. The actual answer to Prob. 6.161 was a mass flow of 0.402 kg/s. EES reports that, for mass flow *exactly* equal to 0.400 kg/s, the required manometer height is ***h* = 0.361 m**. *Ans.*

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE 6.1 In flow through a straight, smooth pipe, the diameter Reynolds number for transition to turbulence is generally taken to be

(a) 1500 **(b) 2300** (c) 4000 (d) 250,000 (e) 500,000

FE 6.2 For flow of water at 20°C through a straight, smooth pipe at 0.06 m^3/h, the pipe diameter for which transition to turbulence occurs is approximately

(a) 1.0 cm (b) 1.5 cm (c) 2.0 cm (d) 2.5 cm (e) 3.0 cm

FE 6.3 For flow of oil ($\mu = 0.1$ kg/(m·s), SG = 0.9) through a long, straight, smooth 5-cm-diameter pipe at 14 m^3/h, the pressure drop per meter is approximately

(a) 2200 Pa **(b) 2500 Pa** (c) 10,000 Pa (d) 160 Pa (e) 2800 Pa

FE 6.4 For flow of water at a Reynolds number of 1.03E6 through a 5-cm-diameter pipe of roughness height 0.5 mm, the approximate Moody friction factor is

(a) 0.012 **(b)** 0.018 **(c) 0.038** (d) 0.049 (e) 0.102

FE 6.5 Minor losses through valves, fittings, bends, contractions etc. are commonly modeled as proportional to

(a) total head (b) static head **(c) velocity head** (d) pressure drop (e) velocity

FE 6.6 A smooth 8-cm-diameter pipe, 200 m long, connects two reservoirs, containing water at 20°C, one of which has a surface elevation of 700 m and the other with its surface elevation at 560 m. If minor losses are neglected, the expected flow rate through the pipe is

(a) 0.048 m^3/h (b) 2.87 m^3/h (c) 134 m^3/h **(d) 172 m^3/h** (e) 385 m^3/h

FE 6.7 If, in Prob. FE 6.6 the pipe is rough and the actual flow rate is 90 m^3/hr, then the expected average roughness height of the pipe is approximately

(a) 1.0 mm **(b) 1.25 mm** (c) 1.5 mm (d) 1.75 mm (e) 2.0 mm

FE 6.8 Suppose in Prob. FE 6.6 the two reservoirs are connected, not by a pipe, but by a sharp-edged orifice of diameter 8 cm. Then the expected flow rate is approximately

(a) 90 m^3/h **(b) 579 m^3/h** (c) 748 m^3/h (d) 949 m^3/h (e) 1048 m^3/h

FE 6.9 Oil ($\mu = 0.1$ kg/(m·s), SG = 0.9) flows through a 50-m-long smooth 8-cm-diameter pipe. The maximum pressure drop for which laminar flow is expected is approximately

(a) 30 kPa (b) 40 kPa (c) 50 kPa (d) 60 kPa **(e) 70 kPa**

FE 6.10 Air at 20°C and approximately 1 atm flows through a smooth 30-cm-square duct at 1500 cubic feet per minute. The expected pressure drop per meter of duct length is

(a) 1.0 Pa **(b) 2.0 Pa** (c) 3.0 Pa (d) 4.0 Pa (e) 5.0 Pa

FE 6.11 Water at 20°C flows at 3 cubic meters per hour through a sharp-edged 3-cm-diameter orifice in a 6-cm-diameter pipe. Estimate the expected pressure drop across the orifice.

(a) 440 Pa (b) 680 Pa (c) 875 Pa **(d) 1750 Pa** (e) 1870 Pa

FE 6.12 Water flows through a straight 10-cm-diameter pipe at a diameter Reynolds number of 250,000. If the pipe roughness is 0.06 mm, what is the approximate Moody friction factor?

(a) 0.015 (b) 0.017 **(c) 0.019** (d) 0.026 (e) 0.032

FE 6.13 What is the hydraulic diameter of a rectangular air-ventilation duct whose cross-section is 1 meter by 25 cm?

(a) 25 cm **(b) 40 cm** (c) 50 cm (d) 75 cm (e) 100 cm

FE 6.14 Water at 20°C flows through a pipe at 300 gal/min with a friction head loss of 45 ft. What is the power required to drive this flow?

(a) 0.16 kW (b) 1.88 kW **(c) 2.54 kW** (d) 3.41 kW (e) 4.24 kW

FE 6.15 Water at 20°C flows at 200 gal/min through a pipe 150 m long and 8 cm in diameter. If the friction head loss is 12 m, what is the Moody friction factor?

(a) 0.010 (b) 0.015 **(c) 0.020** (d) 0.025 (e) 0.030

COMPREHENSIVE PROBLEMS

C6.1 A pitot-static probe will be used to measure the velocity distribution in a water tunnel at 20°C. The two pressure lines from the probe will be connected to a U-tube manometer which uses a liquid of specific gravity 1.7. The maximum velocity expected in the water tunnel is 2.3 m/s. Your job is to select an appropriate U-tube from a manufacturer which supplies manometers of heights 8, 12, 16, 24 and 36 inches. The cost increases significantly with manometer height. Which of these should you purchase?

Solution: The pitot-static tube formula relates velocity to the difference between stagnation pressure p_o and static pressure p_s in the water flow:

$$p_o - p_s = \frac{1}{2}\rho_w V^2, \quad \text{where } \rho_w = 998\ \frac{\text{kg}}{\text{m}^3} \quad \text{and} \quad V_{max} = 2.3\ \frac{\text{m}}{\text{s}}$$

Meanwhile, the manometer reading *h* relates this pressure difference to the two fluids:

$$p_o - p_s = (\rho_{mano} - \rho_w)gh = \rho_w(SG_{mano} - 1)gh$$

$$\text{Solve for} \quad h_{max} = \frac{V_{max}^2}{2g(SG_{mano} - 1)} = \frac{(2.3)^2}{2(9.81)(1.7 - 1)} = 0.385\ m = \mathbf{15.2\ in}$$

It would therefore be most economical to ***buy the 16-inch manometer***. But be careful when you use it: a bit of overpressure will pop the manometer fluid out of the tube!

C6.2 A pump delivers a steady flow of water (ρ,μ) from a large tank to two other higher-elevation tanks, as shown. The same pipe of diameter *d* and roughness ε is used throughout. All minor losses *except through the valve* are neglected, and the partially-closed valve has a loss coefficient K_{valve}. Turbulent flow may be assumed with all kinetic energy flux correction coefficients equal to 1.06. The pump net head *H* is a known function of Q_A and hence also of $V_A = Q_A/A_{pipe}$, for example, $H = a - bV_A^2$, where *a* and *b* are constants. Subscript J refers to the junction point at the tee where branch A splits into B and C. Pipe length L_C is much longer than L_B. It is desired to predict the pressure at J, the three pipe velocities and friction factors, and the pump head. Thus there are 8 variables: H, V_A, V_B, V_C, f_A, f_B, f_C, p_J. Write down the eight equations needed to resolve this problem, but *do not solve*, since an elaborate iteration procedure, or an equation solver such as EES, would be required.

Solution: First, equation (1) is clearly the pump performance:

$$\mathrm{H = a - bV_A^2} \tag{1}$$

$$3\ \textit{Moody factors}:\quad f_A = fcn\left(V_A, \frac{\varepsilon}{d}\right) \tag{2}$$

$$f_B = fcn\left(V_B, \frac{\varepsilon}{d}\right) \tag{3}$$

$$f_C = fcn\left(V_C, \frac{\varepsilon}{d}\right) \tag{4}$$

Conservation of mass (constant area) at the junction J: $V_A = V_B + V_C$ (5)
Finally, there are three independent steady-flow energy equations:

$$(1)\ \textit{to}\ (2):\quad z_1 = z_2 - H + f_A \frac{L_A}{d}\frac{V_A^2}{2g} + f_B \frac{L_B}{d}\frac{V_B^2}{2g} \tag{6}$$

$$(1)\ \textit{to}\ (3):\quad z_1 = z_3 - H + f_A \frac{L_A}{d}\frac{V_A^2}{2g} + f_C \frac{L_C}{d}\frac{V_C^2}{2g} + K_{valve}\frac{V_C^2}{2g} \tag{7}$$

$$(J)\ \textit{to}\ (2):\quad \frac{p_J}{\rho g} + z_J = \frac{p_{atm}}{\rho g} + z_2 + f_B \frac{L_B}{d}\frac{V_B^2}{2g} \tag{8}$$

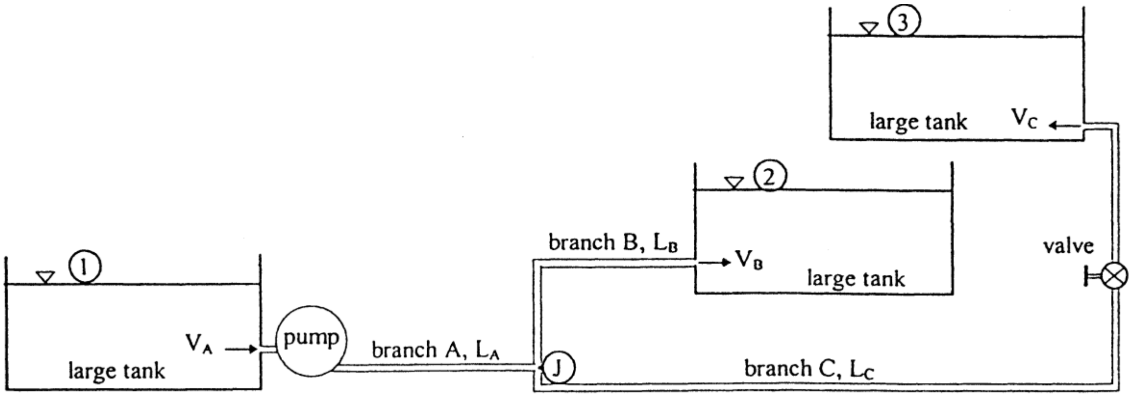
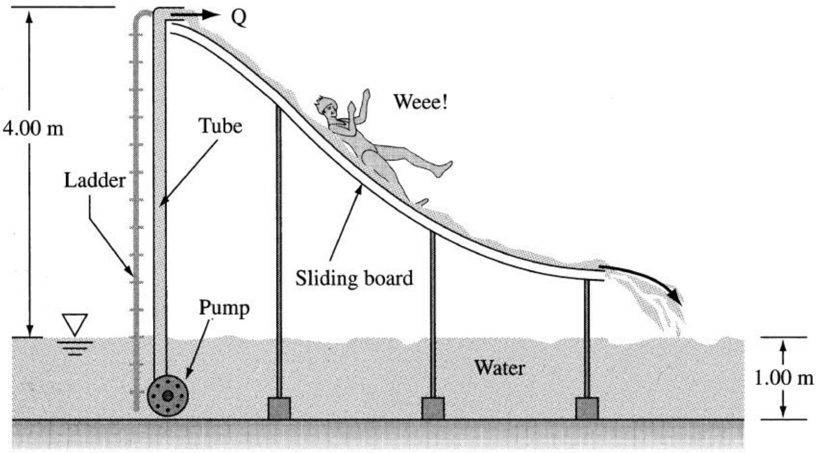


Fig. PC6.2

C6.3 The water slide in the figure is to be installed in a swimming pool. The manufacturer recommends a continuous water flow of 1.39E–3 m^3/s (about 22 gal/min) down the slide to ensure that customers do not burn their bottoms. An 80%-efficient pump under the slide, submerged 1 m below the water surface, feeds a 5-m-long, 4-cm-diameter hose, of roughness 0.008 cm, to the slide. The hose discharges the water at the top of the slide, 4 m above the water surface, as a free jet. Ignore minor losses and assume $\alpha = 1.06$. Find the brake horsepower needed to drive the pump.



FigC6.3

Solution: For water take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Write the steady-flow energy equation from the water surface (1) to the outlet (2) at the top of the slide:

$$\frac{p_a}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_a}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f - h_{pump}, \quad \text{where } V_2 = \frac{1.39E\text{–}3}{\pi(0.02)^2} = 1.106 \ \frac{m}{s}$$

$$\textit{Solve for} \quad h_{pump} = (z_2 - z_1) + \frac{V_2^2}{2g}\left(\alpha_2 + f\frac{L}{d}\right)$$

Work out Red = ρVd/μ = (998)(1.106)(0.04)/0.001 = 44200, ε/d = 0.008/4 = 0.002, whence f_{Moody} = 0.0268. Use these numbers to evaluate the pump head above:

$$h_{pump} = (5.0 - 1.0) + \frac{(1.106)^2}{2(9.81)}\left[1.06 + 0.0268\left(\frac{5.0}{0.04}\right)\right] = 4.27\text{ m},$$

$$\text{whence}\quad \mathbf{BHP_{required}} = \frac{\rho g Q h_{pump}}{\eta} = \frac{998(9.81)(1.39E\text{–}3)(4.27)}{0.8} = \mathbf{73\ watts}\quad \textit{Ans.}$$

C6.4 Suppose you build a house out in the 'boonies,' where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface.

(a) Find the total head loss from pipe inlet to exit, neglecting minor losses.

(b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe?

(c) Find the minimum required pipe diameter.

Solution: Convert 3.0 gal/min to 1.89E–4 m^3/s. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

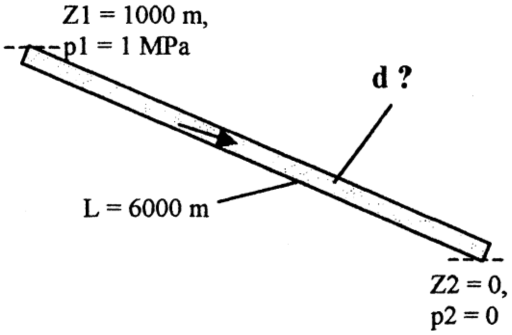


Fig. C6.4

$$\frac{p_{1gage}}{\rho g} + \cancel{\frac{\alpha_1 V_1^2}{2g}} + z_1 = \cancel{\frac{p_{2gage}}{\rho g}} + \cancel{\frac{\alpha_2 V_2^2}{2g}} + z_2 + h_f$$

$$\text{or:}\quad h_f = z_1 - z_2 + \frac{p_{1gage}}{\rho g} = 1000\text{ m} + \frac{1E6\text{ kPa}}{998(9.81)} = 1000 + 102 = 1102\text{ m}\quad \textit{Ans.}\text{ (a)}$$

(b) Thus, *elevation drop* of 1000 m is more important to head loss than $\Delta p/\rho g$ = 102 m.

(c) To find the minimum diameter, iterate between flow rate and the Moody chart:

$$h_f = f\frac{L}{d}\frac{V^2}{2g},\quad L = 6000\text{ m},\quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right),\quad V = \frac{Q}{\pi d^2/4},$$

$$Q = 1.89\text{E–}4\ \frac{\text{m}^3}{\text{s}},\quad \text{Re} = \frac{Vd}{\nu}$$

We are given hf = 1102 m and ν_{water} = 1.005E–6 m^2/s. We can iterate, if necessary, or use **EES**, which can swiftly arrive at the final result:

$$\text{f}_{\text{smooth}} = 0.0266;\quad \text{Re} = 17924;\quad \text{V} = 1.346\text{ m/s};\quad \mathbf{d_{min} = 0.0134\ m}\quad \textit{Ans.}\,\text{(c)}$$

C6.5 Water at 20°C flows, at the same flow rate Q = 9.4E–4 m^3/s, through two ducts, one a round pipe, and one an annulus, as shown. The cross-section area A of each duct is identical, and each has walls of commercial steel. Both are the same length. In the cross-sections shown, R = 15 mm and a = 25 mm.

(a) Calculate the correct radius b for the annulus.

(b) Compare head loss per unit length for the two ducts, first using the hydraulic diameter and second using the 'effective diameter' concept.

(c) If the losses are different, why? Which duct is more 'efficient'? Why?

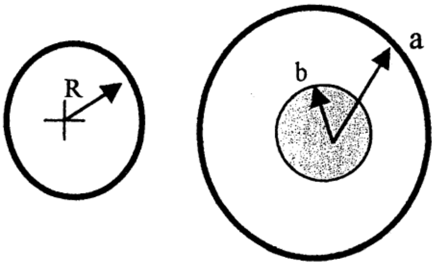


Fig. C6.5

Solution: (a) Set the areas equal:

$$A = \pi R^2 = \pi(a^2 - b^2),\quad \textit{or}:\quad b = \sqrt{a^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20\text{ mm}\quad \textit{Ans.}\text{ (a)}$$

(b) Find the round-pipe head loss, assuming ν = 1.005E–6 m^2/s:

$$V = \frac{Q}{A} = \frac{9.4E\text{–}4\ \text{m}^3/\text{s}}{\pi(0.015\text{ m})^2} = 1.33\ \frac{\text{m}}{\text{s}};\quad \text{Re} = \frac{(1.33)(0.030)}{1.005E\text{–}6} = 39700;$$

$$\frac{\varepsilon}{d} = 0.00153,\, f_{Moody} = 0.0261$$

Thus hf/L = (f/d)(V^2/2g) = (0.0261/0.03)(1.33^2)/2/9.81 = **0.0785** (*round*) *Ans.* (b)

Annulus: $D_h = 4A/P = 2(a-b) = 20$ mm, same V = 1.33 m/s:

$$\mathrm{Re}_{Dh} = \frac{VD_h}{\nu} = 26500,\quad \frac{\varepsilon}{D_h} = 0.0023,\ f_{Moody} = 0.0291,$$

$$h_f/L \approx \left(\frac{f}{D_h}\frac{V^2}{2g}\right) \approx \mathbf{0.131} \quad (annulus) \quad Ans.\ \text{(b)}$$

Effective-diameter concept: b/a = 0.8, Table 6.3: $D_{eff} = 0.667D_h = 13.3$ mm. Then

$$\mathrm{Re}_{Deff} = 17700,\quad \frac{\varepsilon}{D_{eff}} = 0.00345,\ f_{Moody} = 0.0327,$$

$$\frac{h_f}{L} = \frac{f}{D_h}\frac{V^2}{2g} = \mathbf{0.147} \quad (annulus—D_{eff}) \quad Ans.\ \text{(b)}$$

NOTE: Everything here uses D_{eff} except h_f, which by definition uses D_h.

We see that the annulus has about 85% more head loss than the round pipe, for the same area and flow rate! This is because the annulus has more wall area, thus more friction. *Ans.* (c)

C6.6 John Laufer (*NACA Tech. Rep.* 1174, 1954) gave velocity data for 20°C airflow in a smooth 24.7-cm-diameter pipe at Re ≈ 5 E5:

u/u_{CL}:	1.0	0.997	0.988	0.959	0.908	0.847	0.818	0.771	0.690
r/R:	0.0	0.102	0.206	0.412	0.617	0.784	0.846	0.907	0.963

The centerline velocity u_{CL} was 30.5 m/s. Determine (a) the average velocity by numerical integration and (b) the wall shear stress from the log-law approximation. Compare with the Moody chart and with Eq. (6.43).

Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 0.00018 kg/m·s. The average velocity is defined by the (dimensionless) integral

$$V = \frac{1}{\pi R^2}\int_0^R u(2\pi r)dr, \quad \text{or:} \quad \frac{V}{u_{CL}} = \int_0^1 \frac{u}{u_{CL}} 2\eta d\eta, \quad \text{where } \eta = \frac{r}{R}$$

Prepare a spreadsheet with the data and carry out the integration by the trapezoidal rule:

$$\int_0^1 \frac{u}{u_c}2\eta d\eta \approx [(u/u_c)_2\eta_2 + (u/u_c)_1\eta_1](\eta_2 - \eta_1) + [(u/u_c)_3\eta_3 + (u/u_c)_2\eta_2](\eta_3 - \eta_2) + \cdots$$

The integral is evaluated on the spreadsheet below. The result is $V/u_{CL} \approx 0.8356$,

$$\text{or} \quad V \approx (0.8356)(30.5) \approx \mathbf{25.5\ m/s}. \quad Ans.\ \text{(a)}$$

The wall shear stress is estimated by fitting the log-law (6.28) to each data point:

$$\text{For each (u,y),}\quad \frac{u}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \kappa \approx 0.41 \quad \text{and} \quad B \approx 5.0$$

We know ν for air and are given u and y from the data, hence we can solve for u*. The spreadsheet gives u* ≈ 1.1 m/s ± 1%, or $\tau_w = \rho u^{*2} = (1.2)(1.1)^2 \approx$ **1.45 Pa**. *Ans.* (b)

y/R	r/R	u/uCL	$\int$u/uCL 2πr/R dr/R	u*
1.000	0.000	1.000	.0000	—
0.898	0.102	0.997	.0104	1.126
0.794	0.206	0.988	.0421	1.128
0.588	0.412	0.959	.1654	1.126
0.383	0.617	0.908	.3613	1.112
0.216	0.784	0.847	.5657	1.099
0.154	0.846	0.818	.6498	1.101
0.093	0.907	0.771	.7347	1.098
0.037	0.963	0.690	.8111	1.097
0.000	1.000	0.000	**.8356**	—

We make similar estimates from the Moody chart by evaluating Re and *f* and iterating:

$$\text{Guess } V \approx 25 \text{ m/s}, \quad \text{then } Re = \frac{1.2(25)(0.247)}{0.00018} \approx 412000, \; f_{smooth} \approx 0.0136$$

$$V_{better} = u_{CL}/[1 + 1.3\sqrt{f}] \approx 26.5, \quad \text{whence } Re \approx 436000, \; f_{better} \approx 0.0135$$

This converges to **V ≈ 26.5 m/s** *Ans.* and $\boldsymbol{\tau_w} = (f/8)\rho V^2 \approx$ **1.42 Pa**. *Ans.*

C6.7 Consider energy exchange in fully-developed laminar flow between parallel plates, as in Eq. (6.63). Let the pressure drop over a length L be Δp. Calculate the rate of work done by this pressure drop on the fluid in the region ($0 < x < L, -h < y < +h$) and compare with the integrated energy dissipated due to the viscous function Φ from Eq. (4.50) over this same region. The two should be equal. Explain why this is so. Can you relate the viscous drag force and the wall shear stress to this energy result?

Solution: From Eq. (6.63), the velocity profile between the plates is parabolic:

$$u = \frac{3}{2}V\left(1 - \frac{y^2}{h^2}\right) \quad \text{where } V = \frac{h^2}{3\mu}\frac{\Delta p}{L} \text{ is the average velocity}$$

Let the width of the flow be denoted by b. The work done by pressure drop Δp is:

$$\dot{W}_{pressure} = \Delta p V A = \left(\frac{3\mu L V}{h^2}\right)(V)(2hb) = \frac{6\mu L b V^2}{h}$$

Meanwhile, from Eq. (4.50), the viscous dissipation function for this fully-developed flow is:

$$\Phi = \mu\left(\frac{\partial u}{\partial y}\right)^2 = \mu\left(\frac{3Vy}{h^2}\right)^2 = \frac{9\mu V^2 y^2}{h^4}$$

Integrate this to get the total dissipated energy over the entire flow region of dimensions L by b by $2h$:

$$\dot{E}_{dissipated} = Lb\int_{-h}^{+h}\left(\frac{9\mu V^2 y^2}{h^4}\right)dy = \frac{6\mu L b V^2}{h} = \dot{W}_{pressure}! \quad \textit{Ans.}$$

The two energy terms are equal. There is no work done by the wall shear stresses (where $u = 0$), so the pressure work is entirely absorbed by viscous dissipation within the flow field. *Ans.*

C6.8 This text has presented the traditional correlations for turbulent smooth-wall friction factor, Eq. (6.38), and the law-of-the-wall, Eq. (6.28). Recently, groups at Princeton and Oregon [56] have made new friction measurements and suggest the following smooth-wall friction law:

$$\frac{1}{\sqrt{f}} = 1.930\log_{10}(\mathrm{Re}_D\sqrt{f}) - 0.537$$

In earlier work, they also report that better values for the constants κ and B in the log-law, Eq. (6.28), are $\kappa \approx 0.421 \pm 0.002$ and $B \approx 5.62 \pm 0.08$. (*a*) Calculate a few values of f in the range $1E4 \le \mathrm{Re}_D \le 1E8$ and see how the two formulas differ. (*b*) Read Ref. 56 and briefly check the five papers in its bibliography. Report to the class on the general results of this work.

Solution: The two formulas are practically identical except as the Reynolds number is very high or very low. The new formula was fit to new, and extensive, friction data in Ref. 56 and can thus be said to be slightly more accurate. Here is a table of calculations.

Re_D	$f_{Prandtl}$	$f_{Ref.56}$	Difference
3000	0.04353	0.04251	-2.41%
10000	0.03089	0.0305	-1.10%
30000	0.02349	0.02344	-0.18%
100000	0.01799	0.01811	0.62%
300000	0.01447	0.01464	1.22%
1000000	0.01165	0.01186	1.76%
3.0E+06	0.009722	0.009938	2.17%
1.0E+07	0.008104	0.008316	2.56%
3.0E+07	0.006949	0.007153	2.86%
1.0E+08	0.005941	0.006134	3.15%

They differ by no more than 3%.

C6.9 A pipeline has been proposed to carry natural gas 1715 miles from Alaska’s North Slope to Calgary, Alberta, Canada. The (smooth) pipe diameter will be 52 inches. The gas will be at high pressure, averaging 2500 lbs/in^2. (*a*) Why? The proposed flow rate is 4 billion cubic feet per day at sea-level conditions. (*b*) What volume flow rate, at 20°C, would carry the same mass at the high pressure? (*c*) If natural gas is assumed to be methane (CH_4), what is the total pressure drop? (*d*) If each pumping station can deliver 12,000 hp to the flow, how many stations are needed?

Solution: From Table A.4, for CH_4, $R = 518$ m^2/(s^2-K) and $\mu = 1.03\text{E-}5$ kg/m-s. Sea-level density is $\rho_o = p/RT = 101350/[518(288)] = 0.679$ kg/m^3. The proposed mass flow rate is

$$\upsilon ol = (4.0\text{E}9\ ft^3)(0.3048 m/ft)^3 = 1.13\text{E}8\ m^3\ ;$$

$$Q = \upsilon ol/time = (1.13\text{E}8\,m^3)/(24\times 3600 s) = 1311 m^3/s$$

$$\dot{m} = \rho_o Q = (0.679\,kg/m^3)(1311 m^3/s) = 890\ kg/s$$

(*a*) The high pressure means that the gas is compacted about 170 times and thus ***much less volume needs to be pumped***. *Ans.*(*a*)

(*b*) At 2500 lbs/in^2 = 1.72E7 Pa and 20°C, the average gas density would be

$$\rho = \frac{p}{RT} = \frac{1.72\text{E}7\,Pa}{(518\,m^2/s^2K)(293K)} = 114\frac{kg}{m^3}$$

To match the mass flows, we would have

$$\dot{m}_{\text{standard}} = 890\frac{kg}{s} = \dot{m}_{pipeline} = (114\frac{kg}{m^3})Q_{pipeline}\ ,\ \ hence\ \ Q_{pipeline} = \mathbf{7.84}\frac{m^3}{s}\quad Ans.(b)$$

(*c*) For the pressure drop, first find the Moody (smooth-wall) friction factor:

$$V_{pipeline} = \frac{Q}{A} = \frac{7.84\,m^3/s}{(\pi/4)(1.32m)^2} = 5.73\frac{m}{s}\ ;\ \text{Re}_{pipeline} = \frac{\rho VD}{\mu} = \frac{(114)(5.73)(1.32)}{0.0000103} = 8.35\text{E}7$$

$$\frac{1}{\sqrt{f}} \approx 2.0\log(\frac{\text{Re}_D\sqrt{f}}{2.51}) \qquad yields \qquad f_{smooth} \approx 0.00608$$

Convert 1715 miles = 2760 km. Then the Darcy formula gives the (horizontal) pressure drop:

$$\Delta p_{total} = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.00608)(\frac{2760000m}{1.32m})(\frac{114}{2})(5.73)^2 = \mathbf{2.37E7}\,Pa = 3440\ \frac{lbf}{in^2}\quad Ans.(c)$$

(*d*) Total horsepower = $Q\,\Delta p$ = (7.84)(2.37E7) = 1.86E8 W ÷ 745.7 = 249,000 hp total. If we divide this by 12,000 hp per pump, we get 20.76 or **21 pump stations minimum**. *Ans*. (*d*)

Each station must increase the gas pressure by 3440/21 = 164 psi, only 7% of the average pressure in the pipeline.

Chapter 7 • Flow Past Immersed Bodies

P7.1 An ideal gas, at 20°C and 1 atm, flows at 12 m/s past a thin flat plate. At a position 60 cm downstream of the leading edge, the boundary layer thickness is 5 mm. Which of the 13 gases in Table A.4 is this likely to be?

Solution: We are looking for the kinematic viscosity. For a gas at low velocity and a short distance, we can guess *laminar* flow. Then we can begin by trying Eq. (7.1*a*):

$$\frac{\delta}{x} = \frac{0.005m}{0.6m} = \frac{5.0}{\sqrt{\mathrm{Re}_x}} = \frac{5.0}{\sqrt{Vx/\nu}} = \frac{5.0\sqrt{\nu}}{\sqrt{(12m/s)(0.6m)}}$$

$$Solve\ \ for \qquad \nu = 2.0E-5\ m^2/s$$

The only gas in Table A.4 which matches this viscosity is the last one, $\mathbf{CH_4}$. *Ans*.

But wait! Is it laminar? Check Re_x = (12)(0.6)/(2.0E-5) = 360,000. Yes, OK.

7.2 A gas at 20°C flows at 8 ft/s past a smooth, sharp flat plate. At x = 206 cm, the boundary layer thickness is 5 cm. Which of the gases in Table A.4 is this most likely to be?

Solution: Velocity fairly slow, U = 8 ft/s = 2.44 m/s. Distance x fairly long, but let's begin by guessing a *laminar* boundary layer:

$$\frac{\delta}{x} = \frac{5cm}{206cm} = 0.0243 \approx \frac{5}{\sqrt{\mathrm{Re}_x}}\ ,\quad \text{solve}\quad \mathrm{Re}_x \approx 42,400 \quad \text{(OK for laminar flow)}$$

$$\text{If this is correct,}\quad \mathrm{Re}_x = 42,400 = \frac{Ux}{\nu} = \frac{(2.44)(2.06)}{\nu}\ ,\quad \text{solve}\ \nu \approx \mathbf{1.18E-4}\ \frac{m^2}{s}$$

This value of ν exactly matches helium in Table A.4. It is not far from the value for hydrogen, but the **helium result is right on the money**. *Ans*.

Guessing turbulent flow, δ/x = 0.0243 = 0.16/$\mathrm{Re}_x^{1/7}$, solve $\mathrm{Re}_x \approx$ 541,000 (too small for transition to turbulence on a smooth wall). This would give $\nu \approx$ 9.3E-6, about 15% greater than the kinematic viscosity of CO_2. But the Reynolds number is too low, so I reject this answer.

P7.3 Equation (7.1*b*) assumes that the boundary layer on the plate is turbulent from the leading edge onward. Devise a scheme for determining the boundary-layer thickness more accurately when the flow is laminar up to a point $\mathrm{Re}_{x,\mathrm{crit}}$ and turbulent thereafter. Apply this scheme to computation of the boundary-layer thickness at $x = 1.5$ m in 40 m/s flow of air at 20°C and 1 atm past a flat plate. Compare your result with Eq. (7.1*b*). Assume $\mathrm{Re}_{x,\mathrm{crit}} \approx 1.2\mathrm{E}6$.

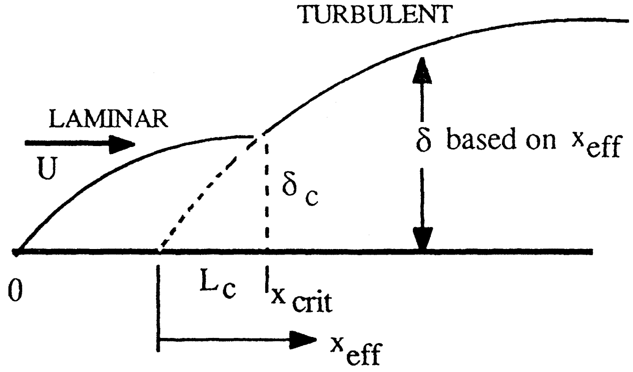


Fig. P7.3

Solution: Given the transition point x_{crit}, $\mathrm{Re}_{\mathrm{crit}}$, calculate the laminar boundary layer thick-ness δ_c at that point, as shown above, $\delta_c/x_c \approx 5.0/\mathrm{Re}_{\mathrm{crit}}^{1/2}$. Then find the "apparent" distance upstream, L_c, which gives the same *turbulent* boundary layer thickness, $\delta_c/L_c \approx 0.16/\mathrm{Re}_{L_c}^{1/7}$. Then begin $x_{\mathrm{effective}}$ at this "apparent origin" and calculate the remainder of the turbulent boundary layer as $\delta/x_{\mathrm{eff}} \approx 0.16/\mathrm{Re}_{\mathrm{eff}}^{1/7}$. Illustrate with a numerical example as requested. For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\mathrm{E}{-5}$ kg/m·s.

$$\mathrm{Re}_{\mathrm{crit}} = 1.2\mathrm{E}6 = \frac{1.2(40)x_c}{1.8\mathrm{E}{-5}} \quad \text{if } x_c = 0.45 \text{ m}, \quad \text{then } \delta_c = \frac{5.0(0.45)}{(1.2\mathrm{E}6)^{1/2}} \approx 0.00205 \text{ m}$$

$$\text{Compute} \quad L_c = \left(\frac{\delta_c}{0.16}\right)^{7/6}\left(\frac{\rho U}{\mu}\right)^{1/6} = \left(\frac{0.00205}{0.16}\right)^{7/6}\left[\frac{1.2(40)}{1.8\mathrm{E}{-5}}\right]^{1/6} \approx 0.0731 \text{ m}$$

Finally, at x = 1.5 m, compute the effective distance and the effective Reynolds number:

$$x_{\mathrm{eff}} = x + L_c - x_c = 1.5 + 0.0731 - 0.45 = 1.123 \text{ m}, \quad \mathrm{Re}_{\mathrm{eff}} = \frac{1.2(40)(1.123)}{1.8\mathrm{E}{-5}} \approx 2.995\mathrm{E}6$$

$$\delta|_{1.5\text{ m}} \approx \frac{0.16 x_{\mathrm{eff}}}{\mathrm{Re}_{\mathrm{eff}}^{1/7}} = \frac{0.16(1.123)}{(2.995\mathrm{E}6)^{1/7}} \approx \mathbf{0.0213\ m} \quad \textit{Ans.}$$

Compare with a straight all-turbulent-flow calculation from Eq. (7.1*b*):

$$\mathrm{Re_x} = \frac{1.2(40)(1.5)}{1.8\mathrm{E}{-}5} \approx 4.0\mathrm{E}6, \quad \text{whence } \delta\big|_{1.5\ \mathrm{m}} \approx \frac{0.16(1.5)}{(4.0\mathrm{E}6)^{1/7}} \approx \mathbf{0.027\ m} \quad (25\%\ \text{higher}) \quad \textit{Ans.}$$

P7.4 A smooth ceramic sphere (SG = 2.6) is immersed in a flow of water at 20°C and 25 cm/s. What is the sphere diameter if it is encountering (a) creeping motion, Re*d* = 1; or (b) transition to turbulence, Re*d* = 250,000?

Solution: For water, take $\rho = 998\ \mathrm{kg/m^3}$ and $\mu = 0.001$ kg/m·s.
(a) Set Re*d* equal to 1:

$$\mathrm{Re}_d = 1 = \frac{\rho V d}{\mu} = \frac{(998\ \mathrm{kg/m^3})(0.25\ \mathrm{m/s})d}{0.001\ \mathrm{kg/m{\cdot}s}}$$

Solve for **d = 4E–6 m** = 4 μm *Ans.* (a)

(b) Similarly, at the transition Reynolds number,

$$\mathrm{Re}_d = 250000 = \frac{(998\ \mathrm{kg/m^3})(0.25\ \mathrm{m/s})d}{0.001\ \mathrm{kg/m{\cdot}s}}, \quad \text{solve for } \mathbf{d = 1.0\ m} \quad \textit{Ans.}\ \text{(b)}$$

P7.5 SAE 30 oil at 20°C flows at 1.8 $\mathrm{ft^3/s}$ from a reservoir into a 6-in-diameter pipe. Use flat-plate theory to estimate the position *x* where the pipe-wall boundary layers meet in the center. Compare with Eq. (6.5), and give some explanations for the discrepancy.

Solution: For SAE 30 oil at 20°C, take $\rho = 1.73\ \mathrm{slug/ft^3}$ and $\mu = 0.00607$ slug/ft·s. The average velocity and pipe Reynolds number are:

$$V_{avg} = \frac{Q}{A} = \frac{1.8}{(\pi/4)(6/12)^2} = 9.17\ \frac{\mathrm{ft}}{\mathrm{s}}, \quad \mathrm{Re_D} = \frac{\rho V D}{\mu} = \frac{1.73(9.17)(6/12)}{0.00607} = 1310\ \text{(laminar)}$$

Using Eq. (7.1a) for laminar flow, find “x_e” where $\delta = D/2 = 3$ inches:

$$x_e \approx \frac{\delta^2 \rho V}{25\mu} = \frac{(3/12)^2(1.73)(9.17)}{25(0.00607)} \approx \mathbf{6.55\ ft} \quad \textit{Ans.}\ \text{(flat-plate boundary layer estimate)}$$

This is far from the truth, much too short. Equation (6.5) for laminar pipe flow predicts

$$x_e = 0.06D\,Re_D = 0.06(6/12\text{ ft})(1310) \approx \mathbf{39\ ft} \quad \textit{Alternate Ans.}$$

The entrance flow is **accelerating**, a favorable pressure gradient, as the core velocity increases from V to 2V, and the accelerating **boundary layer is much thinner** and takes much longer to grow to the center. *Ans.*

P7.6 For the laminar parabolic boundary-layer profile of Eq. (7.6), compute the shape factor "H" and compare with the exact Blasius-theory result, Eq. (7.31).

Solution: Given the profile approximation $u/U \approx 2\eta - \eta^2$, where $\eta = y/\delta$, compute

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \delta\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta = \frac{2}{15}\delta$$

$$\delta^* = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy = \delta\int_0^1 (1-2\eta+\eta^2)\,d\eta = \frac{1}{3}\delta$$

Hence $H = \delta^*/\theta = (\delta/3)/(2\delta/15) \approx \mathbf{2.50}$ (compared to *2.59* for Blasius solution)

P7.7 Air at 20°C and 1 atm enters a 40-cm-square duct as in Fig. P7.7. Using the "displacement thickness" concept of Fig. 7.4, estimate (a) the mean velocity and (b) the mean pressure in the core of the flow at the position *x* = 3 m. (c) What is the average gradient, in Pa/m, in this section?

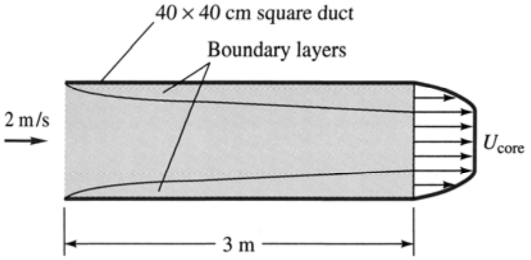


Fig. P7.7

Solution: For air at 20°C, take $\rho = 1.2\ \text{kg/m}^3$ and $\mu = 1.8\text{E}{-}5$ kg/m·s. Using laminar boundary-layer theory, compute the displacement thickness at x = 3 m:

$$Re_x = \frac{\rho Ux}{\mu} = \frac{1.2(2)(3)}{1.8\text{E}{-}5} = 4\text{E}5 \text{ (laminar)}, \quad \delta^* = \frac{1.721x}{Re_x^{1/2}} = \frac{1.721(3)}{(4\text{E}5)^{1/2}} \approx 0.0082\text{ m}$$

Then, by continuity, $$V_{exit} = V\left(\frac{L_o}{L_o - 2\delta^*}\right)^2 = (2.0)\left(\frac{0.4}{0.4 - 0.0164}\right)^2$$

$$\approx \mathbf{2.175\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

The pressure change in the (frictionless) core flow is estimated from Bernoulli's equation:

$$p_{exit} + \frac{\rho}{2}V_{exit}^2 = p_o + \frac{\rho}{2}V_o^2, \quad \text{or:} \quad p_{exit} + \frac{1.2}{2}(2.175)^2 = 1\ \text{atm} + \frac{1.2}{2}(2.0)^2$$

Solve for $p|_{x=3m} = 1\ \text{atm} - 0.44\ \text{Pa} = -0.44$ **Pa** (*gage*) *Ans.* (b)

The average pressure gradient is Δp/x = (–0.44 Pa/3.0 m) ≈ **–0.15 Pa/m** *Ans.* (c)

P7.8 Air, ρ =1.2 kg/m^3 and μ = 1.8E–5 kg/m·s, flows at 10 m/s past a flat plate. At the trailing edge of the plate, the following velocity profile data are measured:

y, mm:	0	0.5	1.0	2.0	3.0	4.0	5.0	6.0
u, m/s:	0	1.75	3.47	6.58	8.70	9.68	10.0	10.0
$u(U-u)$, m^2/s:	0	14.44	22.66	22.50	11.31	3.10	0.0	0.0

If the upper surface has an area of 0.6 m^2, estimate, using momentum concepts, the friction drag, in newtons, on the upper surface.

Solution: Make a numerical estimate of drag from Eq. (7.2): $F = \rho b \int u(U-u)dy$. We have added the numerical values of $u(U-u)$ to the data above. Using the trapezoidal rule between each pair of points in this table yields

$$\int_0^\delta u(U-u)\,dy \approx \frac{1}{1000}\left[0.5\left(\frac{0+14.44}{2}\right) + \left(\frac{14.44+22.66}{2}\right) + \cdots\right] \approx 0.061\ \frac{m^3}{s}$$

The drag is approximately $F = 1.2b(0.061) = 0.073b$ newtons or **0.073 N/m**. *Ans.*

7.9 Repeat the flat-plate momentum analysis of Sec. 7.2 by replacing Eq. (7.6) with the simple but unrealistic linear velocity profile suggested by Schlichting [1]:

$$\frac{u}{U} \approx \frac{y}{\delta} \qquad \text{for} \quad 0 \le y \le \delta$$

Compute momentum-integral estimates of c_f, θ/x, δ^*/x, and H.

Solution: Carry out the same integrations as Section 7.2. Results are less accurate:

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy = \frac{\delta}{6}\ ; \quad \delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \frac{\delta}{2}\ ; \ H = \frac{\delta/2}{\delta/6} = 3.0$$

$$\tau_w = \mu\frac{U}{\delta} = \rho U^2\frac{d\theta}{dx} = \rho U^2\frac{d(\delta/6)}{dx}\ ; \quad Integrate: \frac{\delta}{x} \approx \frac{\sqrt{12}}{\sqrt{\mathrm{Re}_x}} \approx \frac{3.64}{\sqrt{\mathrm{Re}_x}}$$

Substitute these results back for the following inaccurate estimates:

$$c_f = \frac{\theta}{x} = \frac{0.577}{\sqrt{\mathrm{Re}_x}}\ ; \qquad \frac{\delta^*}{x} = \frac{1.732}{\sqrt{\mathrm{Re}_x}}\ ; \qquad H = 3.0 \qquad Ans.(a,b,c,d)$$

P7.10 Repeat Prob. 7.9, using the polynomial profile suggested by K. Pohlhausen in 1921:

$$\frac{u}{U} \approx 2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}$$

Does this profile satisfy the boundary conditions of laminar flat-plate flow?

Solution: Pohlhausen's quadratic profile satisfies no-slip at the wall, a smooth merge with u → U as y → δ, and, further, the boundary-layer curvature condition at the wall. From Eq. (7.19b),

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}\right)_{\text{wall}} = 0, \quad \text{or:} \quad \frac{\partial^2 u}{\partial y^2}\Big|_{\text{wall}} = 0 \quad \text{for flat-plate flow} \ \left(\frac{\partial p}{\partial x} = 0\right)$$

This profile gives the following integral approximations:

$$\theta \approx \frac{37}{315}\delta; \quad \delta^* \approx \frac{3}{10}\delta; \quad \tau_w \approx \mu\frac{2U}{\delta} \approx \rho U^2 \frac{d}{dx}\left(\frac{37}{315}\delta\right), \quad \text{integrate to obtain:}$$

$$\frac{\delta}{x} \approx \frac{\sqrt{(1260/37)}}{\sqrt{Re_x}} \approx \frac{\mathbf{5.83}}{\sqrt{\mathbf{Re_x}}}; \quad C_f = \frac{\theta}{x} \approx \frac{\mathbf{0.685}}{\sqrt{\mathbf{Re_x}}};$$

$$\frac{\delta^*}{x} \approx \frac{\mathbf{1.751}}{\sqrt{\mathbf{Re_x}}}; \quad H \approx \mathbf{2.554} \quad \textit{Ans.}\ \text{(a, b, c, d)}$$

P7.11 Air at 20°C and 1 atm flows at 2 m/s past a sharp flat plate. Assuming that the Kármán parabolic-profile analysis, Eqs. (7.6–7.10), is accurate, estimate (a) the local velocity *u*; and (b) the local shear stress τ at the position $(x, y) = (50\text{ cm}, 5\text{ mm})$.

Solution: For air, take $\rho = 1.2\text{ kg/m}^3$ and $\mu = 1.8\text{E}{-5}$ kg/m·s. First compute Re_X = (1.2)(2)(0.5)/(1.8E-5) = 66667, and $\delta(x) \approx (0.5\text{m})(5.5)/(66667)^{1/2} = 0.01065$ m. The location we want is $y/\delta = 5\text{ mm}/10.65\text{ mm} = 0.47$, and Eq. (7.6) predicts local velocity:

$$u(0.5\text{ m, 5 mm}) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) = (2\text{ m/s})[2(0.47) - (0.47)^2] \approx 1.44\ \mathbf{m/s} \quad \textit{Ans.}\ \text{(a)}$$

The local shear stress at this *y* position is estimated by differentiating Eq. (7.6):

$$\tau(0.5\text{ m, 5 mm}) = \mu\frac{\partial u}{\partial y} \approx \frac{\mu U}{\delta}\left(2 - \frac{2y}{\delta}\right) = \frac{(1.8E\text{–}5\text{ kg/m·s})(2\text{ m/s})}{0.01065\text{ m}}[2 - 2(0.47)]$$

$$= \mathbf{0.0036\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

P7.12 The velocity profile shape $u/U \approx 1 - \exp(-4.605y/\delta)$ is a smooth curve with $u = 0$ at $y = 0$ and $u = 0.99U$ at $y = \delta$ and thus would seem to be a reasonable substitute for the parabolic flat-plate profile of Eq. (7.3). Yet when this new profile is used in the integral analysis of Sec. 7.3, we get the lousy result $\delta/x \approx 9.2/Re_x^{1/2}$, which is 80 percent high. What is the reason for the inaccuracy? [*Hint:* The answer lies in evaluating the laminar boundary-layer momentum equation (7.19*b*) at the wall, $y = 0$.]

Solution: This profile satisfies no-slip at the wall and merges very smoothly with u → U at the outer edge, but it does *not* have the right shape for flat-plate flow. It does not satisfy the zero curvature condition at the wall (see Prob. 7.10 for further details):

$$\text{Evaluate} \quad \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} \approx -\left(\frac{4.605}{\delta}\right)^2 U \approx -\frac{21.2U}{\delta^2} \neq 0 \quad \text{by a long measure!}$$

The profile has a **strong negative curvature** at the wall and simulates a **favorable pressure gradient shape**. Its momentum and displacement thickness are much too small.

P7.13 Derive modified forms of the laminar boundary-layer equations for flow along the outside of a circular cylinder of constant R, as in Fig. P7.13. Consider the two cases (a) $\delta << R$; and (b) $\delta \approx R$. What are the boundary conditions?

Solution: The Navier-Stokes equations for cylindrical coordinates are given in Appendix D, with "x" in the Fig. P7.13 denoting the axial coordinate "z." Assume "axisymmetric" flow, that is, $v_\theta = 0$ and $\partial/\partial\theta = 0$ everywhere. The boundary layer assumptions are:

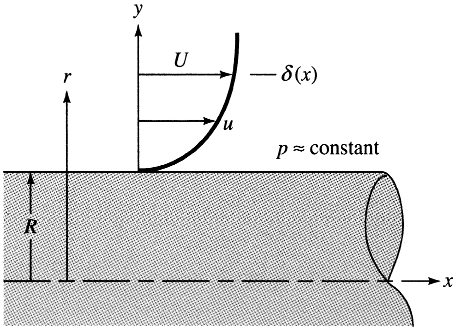


Fig. P7.13

$$v_r << u; \quad \frac{\partial u}{\partial x} << \frac{\partial u}{\partial r}; \quad \frac{\partial v_r}{\partial x} << \frac{\partial v_r}{\partial r}; \quad \text{hence r-momentum (Eq. D-5) becomes} \quad \boldsymbol{\frac{\partial p}{\partial r} \approx 0}$$

Thus $p \approx p(x)$ only, and for a long straight cylinder, $p \approx$ constant and $U \approx$ constant

Then, with $\partial p/\partial x = 0$, the x-momentum equation (D-7 in the Appendix) becomes

$$\boldsymbol{\rho u \frac{\partial u}{\partial x} + \rho v_r \frac{\partial u}{\partial r} \approx \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)} \quad \text{when } \delta \approx R \quad \textit{Ans.} \text{ (b)}$$

$$\text{plus continuity:} \quad \boldsymbol{\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r v_r) \approx 0} \quad \text{when } \delta \approx R \quad \textit{Ans.} \text{ (b)}$$

For thick boundary layers (part *b*) the radial geometry is important.

If, however, the boundary layer is very thin, $\delta << R$, then $r = R + y \approx R$ itself, and we can use (x, y) coordinates:

$$\text{Continuity:}\quad \frac{\partial u}{\partial x}+\frac{\partial v_r}{\partial y}\approx 0 \quad \text{if } \delta << R \quad \textit{Ans.}\text{ (a)}$$

$$\text{x-momentum:}\quad \rho u\frac{\partial u}{\partial x}+\rho v_r\frac{\partial u}{\partial y}\approx \mu\frac{\partial^2 u}{\partial y^2} \quad \text{if } \delta << R \quad \textit{Ans.}\text{ (a)}$$

Thus a thin boundary-layer on a cylinder is exactly the same as flat-plate (Blasius) flow.

P7.14 Show that the two-dimensional laminar-flow pattern with dp/dx = 0,

$$u = U_o(1 - e^{Cy}) \qquad v = v_0 < 0$$

is an exact solution to the boundary-layer equations (7.19). Find the value of the constant *C* in terms of the flow parameters. Are the boundary conditions satisfied? What might this flow represent?

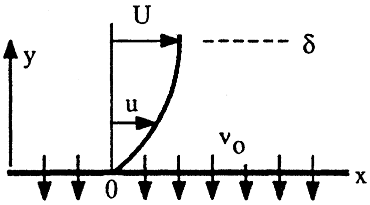


Fig. P7.14

Solution: Substitute these (u,v) into the x-momentum equation (7.19*b*) with $\partial u/\partial x = 0$:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}\approx \mu\frac{\partial^2 u}{\partial y^2},\quad \text{or:}\quad 0+\rho(v_o)\left(-CU_oe^{Cy}\right)\approx \mu\left(-C^2U_oe^{Cy}\right),$$

$$\text{or:}\quad \mathbf{C = \rho v_o/\mu = constant < 0}$$

If the constant is *negative*, u does not go to ∞ and the solution represents laminar boundary-layer **flow past a flat plate with wall suction**, $v_o \le 0$ (see figure). It satisfies

at y = 0: u = 0 (no slip) and $v = v_o$ (suction); as $y \to \infty$, $u \to U_o$ (freestream)

The thickness δ, where $u \approx 0.99U_o$, is defined by $\exp(\rho v_o\delta/\mu) = 0.01$, or $\delta = -4.6\mu/\rho v_o$.

P7.15 Discuss whether fully developed laminar incompressible flow between parallel plates, Eq. (4.143) and Fig. 4.16*b*, represents an exact solution to the boundary-layer equations (7.19) and the boundary conditions (7.20). In what sense, if any, are duct flows also boundary-layer flows?

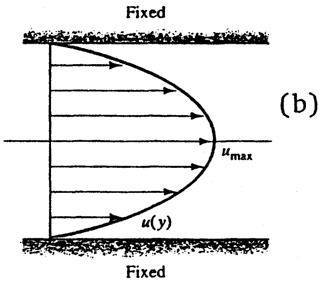


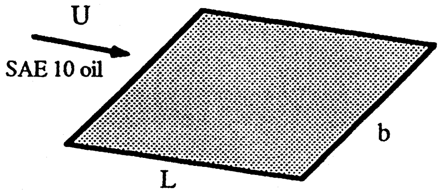
Fig. 4.16

Solution: The analysis for flow between parallel plates leads to Eq. (4.143):

$$u = \left(\frac{dp}{dx}\right)\frac{h^2}{2\mu}\left(1-\frac{y^2}{h^2}\right); \quad v = 0; \quad \frac{dp}{dx} = \text{constant} < 0; \quad \frac{dp}{dy} = 0, \quad u(\pm h) = 0$$

It is indeed a "boundary layer," with $v << u$ and $\partial p/\partial y \approx 0$. The "freestream" is the centerline velocity, $u_{max} = (-dp/dx)(h^2/2\mu)$. The boundary layer does not grow because it is constrained by the two walls. The entire duct is filled with boundary layer. *Ans.*

P7.16 A thin flat plate 55 by 110 cm is immersed in a 6-m/s stream of SAE 10 oil at 20°C. Compute the total friction drag if the stream is parallel to (a) the long side and (b) the short side.



Solution: For SAE 30 oil at 20°C, take $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/m·s.

$$\text{(a) } L = 110\text{ cm}, \quad Re_L = \frac{891(6.0)(1.1)}{0.29} = 20300 \text{ (laminar)}, \quad C_D = \frac{1.328}{(20300)^{1/2}} \approx 0.00933$$

$$F = C_D\left(\frac{\rho}{2}\right)U^2(2bL) = 0.00933\left(\frac{891}{2}\right)(6)^2[2(0.55)(1.1)] \approx \mathbf{181\ N} \quad \textit{Ans. (a)}$$

The drag is 41% *more* if we align the flow with the *short* side:

$$\text{(b) } L = 55\text{ cm}, \quad Re_L = 10140, \quad C_D = 0.0132, \quad F \approx \mathbf{256\ N} \text{ (41\% more)} \quad \textit{Ans. (b)}$$

7.17 Consider laminar flow past a flat plate of width *b* and length *L*. What percentage of the friction drag on the plate is carried by the rear half of the plate?

Solution: The formula for laminar boundary drag on a plate is Eq. (7.26):

$$D(x) = 0.664\, b\, \rho^{1/2}\, \mu^{1/2}\, U^{3/2}\, x^{1/2} = (const)\; x^{1/2}$$

At $x = L$, we obtain a force equal to (const) $L^{1/2}$. At $x = L/2$, we obtain a force equal to (const) $L^{1/2}/\sqrt{2}$, which is 70.7% of the total force. Thus the force on the trailing half of the plate is only (100 – 70.7) = **29.3%** of the total force on the plate.

P7.18 The approximate answers to Prob. 7.11 are $u \approx 1.44$ m/s and $\tau \approx 0.0036$ Pa at x = 50 cm and y = 5 mm. [Do not reveal this to your friends who are working on Prob. 7.11.] Repeat that problem by using the exact Blasius flat-plate boundary-layer solution.

Solution: (a) Calculate the Blasius variable η (Eq. 7.21), then find $f' = u/U$ at that position:

$$\eta = y\sqrt{\frac{U}{\nu x}} = (0.005\text{ m})\sqrt{\frac{2\text{ m/s}}{(0.000015\text{ m}^2/\text{s})(0.5\text{ m})}} = 2.58,$$

$$\textit{Table 7.1:}\quad \frac{u}{U} \approx 0.768, \quad \therefore\; u \approx \mathbf{1.54\ m/s} \quad \textit{Ans.}\text{ (a)}$$

(b) Differentiate Eq. (7.21) to find the local shear stress:

$$\tau = \mu\frac{\partial u}{\partial y} = \mu\frac{\partial}{\partial y}[Uf'(\eta)] = \mu U\sqrt{\frac{U}{\nu x}}\, f''(\eta). \quad \text{At}\;\; \eta = 2.58, \quad \text{estimate}\;\; f''(\eta) \approx 0.217$$

$$\text{Then}\quad \tau \approx (0.000018)(2.0)\sqrt{\frac{(2.0)}{(0.000015)(0.5)}}\,(0.217) \approx \mathbf{0.0040\ Pa} \quad \textit{Ans.}\text{ (b)}$$

P7.19 Air at 20°C and 1 atm flows at 50 ft/s past a thin flat plate whose area (bL) is 24 ft^2. If the total friction drag is 0.3 lbf, what are the length and width of the plate?

Solution: For air at 20°C and 1 atm, take ρ = 0.00238 slug/ft^3 and μ = 3.76E-7 slug/ft-s. Low speed air, not too big a plate: Guess *laminar* flow and check this later. Use Eq. (7.27):

$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}}\;\; (\textit{one side}) \quad \textit{hence}\;\; F = C_D\frac{\rho}{2}V^2\, 2bL\;, \quad \textit{where}\;\; bL = 24\, ft^2$$

$$\textit{Apply data}:\; F = 0.3\, lbf = \frac{1.328\sqrt{3.76E-7}}{\sqrt{(0.00238)(50)L}}\left(\frac{0.00238}{2}\right)(50)^2\, 2bL$$

$$\textit{Solve}:\; b\sqrt{L} = 21.36 = \frac{24}{L}\sqrt{L}\;, \quad \textit{or}:\; \sqrt{L} = 1.12 \qquad L = \mathbf{1.26}\, ft,\; b = \mathbf{19.0}\, ft \quad \textit{Ans.}$$

Check the Reynolds number: $Re_L = (0.00238)(50)(1.26)/(3.76E\text{-}7) = 399{,}000$. **Laminar**, OK.

P7.20 Air at 20°C and 1 atm flows at 20 m/s past the flat plate in Fig. P7.20. A pitot stagnation tube, placed 2 mm from the wall, develops a manometer head *h* = 16 mm of Meriam red oil, SG = 0.827. Use this information to estimate the downstream position x of the pitot tube. Assume laminar flow.

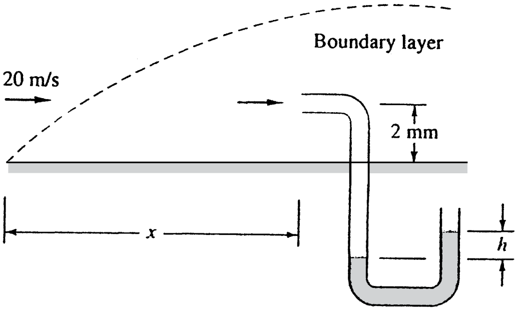


Fig. P7.20

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Assume constant stream pressure, then the manometer can be used to estimate the local velocity u at the position of the pitot inlet:

$$\Delta p_{mano} = p_o - p_\infty = (\rho_{oil} - \rho_{air})gh_{mano} = [0.827(998) - 1.2](9.81)(0.016) \approx 129 \text{ Pa}$$

$$\text{Then} \quad u_{\text{pitot inlet}} \approx [2\Delta p/\rho]^{1/2} = [2(129)/1.2]^{1/2} \approx 14.7 \text{ m/s}$$

Now, with u known, the Blasius solution uses u/U to determine the position η:

$$\frac{u}{U} = \frac{14.7}{20} = 0.734, \quad \text{Table 7.1 read } \eta \approx 2.42 = y(U/\nu x)^{1/2}$$

$$\text{or:} \quad x = (U/\nu)(y/\eta)^2 = (20/1.5E{-}5)(0.002/2.42)^2 \approx \mathbf{0.908\ m} \quad \textit{Ans.}$$

Check $Re_x = (20)(0.908)/(1.5E{-}5) \approx 1.21E6$, OK, laminar if the flow is very smooth.

P7.21 For the experimental set-up of Fig. P7.20, suppose the stream velocity is unknown and the pitot stagnation tube is traversed across the boundary layer of air at 1 atm and 20°C. The manometer fluid is Meriam red oil, and the following readings are made:

y, mm:	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
h, mm:	1.2	4.6	9.8	15.8	21.2	25.3	27.8	29.0	29.7	29.7

Using this data only (not the Blasius theory) estimate (a) the stream velocity, (b) the boundary layer thickness, (c) the wall shear stress, and (d) the total friction drag between the leading edge and the position of the pitot tube.

Solution: As in Prob. 7.20, the air velocity $u = [2(\rho_{oil} - \rho_{air})gh/\rho_{air}]^{1/2}$. For the oil, take $\rho_{oil} = 0.827(998) = 825$ kg/m^3. For air, $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-}5$ kg/m·s. (a, b) We see that h levels out to 29.7 mm at $y = 4.5$ mm. Thus

$$U_\infty = [2(825-1.2)(9.81)(0.0297)/1.2]^{1/2} = \mathbf{20.0\ m/s} \quad \textit{Ans.}\text{ (a)} \quad \delta = \mathbf{4.5\ mm} \quad \textit{Ans.}\text{ (b)}$$

(c) The wall shear stress is estimated from the derivative of velocity at the wall:

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} \approx \mu\frac{\Delta u}{\Delta y} \approx (1.8E\text{–}5)\left(\frac{4.02-0}{0.0005-0}\right) \approx \mathbf{0.14\ Pa} \quad \textit{Ans.}\text{ (c)}$$

where we have calculated $u_{\text{near-wall}} = [2(825 - 1.2)(9.81)(0.0012)/1.2]^{1/2} = 4.02$ m/s.
(d) To estimate drag, first see if the boundary layer is laminar. Evaluate Re_δ:

$$Re_\delta = \frac{\rho U\delta}{\mu} = \frac{1.2(20)(0.0045)}{1.8E\text{–}5} \approx 6000, \quad \text{which implies } Re_{x,laminar} \approx 1.44E6$$

This is a little high, maybe, but let us assume a *smooth* wall, therefore laminar, in which case the drag is *twice the local shear stress times the wall area*. From Prob. 7.20, we estimated the distance x to be 0.908 m. Thus

$$\mathbf{F} \approx 2\tau_w xb = 2(0.14\text{ Pa})(0.908\text{ m})(1.0) \approx \mathbf{0.25\ N}\text{ per meter of width.} \quad \textit{Ans.}\text{ (d)}$$

P7.22 For the Blasius flat-plate problem, Eqs. (7.21) to (7.23), does a two-dimensional stream function $\psi(x, y)$ exist? If so, determine the correct *dimensionless* form for ψ, assuming that $\psi = 0$ at the wall, $y = 0$.

Solution: A stream function $\psi(x, y)$ **does exist** because the flow satisfies the two-dimensional equation of continuity, Eq. (7.19*a*). That is, $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Given the "Blasius" form of u, we may integrate to find ψ:

$$u = \frac{\partial\psi}{\partial y},\quad \text{thus}\quad \psi = \int u\,dy\Big|_{x=\text{const}} = \int_0^y \left(U\frac{df}{d\eta}\right)dy = \int_0^\eta \left(U\frac{df}{d\eta}\right)d\eta\left(\sqrt{\nu x/U}\right)$$

$$\text{or}\quad \psi = (\nu x U)^{1/2}\int_0^\eta df = \mathbf{(\nu x U)^{1/2} f}\quad \textit{Ans.}$$

The integration assumes that $\psi = 0$ at $y = 0$, which is very convenient.

P7.23 Suppose you buy a 4 × 8-ft sheet of plywood and put it on your roof rack, as in the figure. You drive home at 35 mi/h.
(a) If the board is perfectly aligned with the airflow, how thick is the boundary layer at the end? (b) Estimate the drag if the flow remains laminar. (c) Estimate the drag for (smooth) turbulent flow.

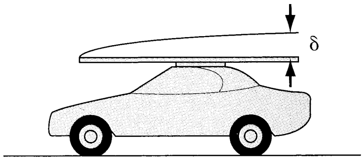


Fig. P7.23

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Convert L = 8 ft = 2.44 m and U = 35 mi/h = 15.6 m/s. Evaluate the Reynolds number, is it laminar or turbulent?

$$Re_L = \frac{\rho UL}{\mu} = \frac{1.2(15.6)(2.44)}{1.8E{-}5} = 2.55E6\quad \textit{probably laminar + turbulent}$$

(a) Evaluate the range of boundary-layer thickness between laminar and turbulent:

$$\textit{Laminar}:\ \frac{\delta}{L} = \frac{\delta}{2.44\ m} \approx \frac{5.0}{\sqrt{2.55E6}} = 0.00313,\quad \textit{or}:\ \delta \approx 0.00765\ m = \mathbf{0.30\ in}$$

$$\textit{Turbulent}:\ \frac{\delta}{2.44} \approx \frac{0.16}{(2.55E6)^{1/7}} = 0.0195,\quad \textit{or}:\ \delta \approx 0.047\ m = \mathbf{1.9\ in}\quad \textit{Ans.}\ \text{(a)}$$

(b, c) Evaluate the range of boundary-layer drag for both laminar and turbulent flow. Note that, for flow over both sides, the appropriate area A = 2bL:

$$F_{lam} = C_D \frac{\rho}{2}U^2 A \approx \left(\frac{1.328}{\sqrt{2.55E6}}\right)\frac{1.2}{2}(15.6)^2(2.44\times 1.22\times 2\ sides) = \mathbf{0.73\ N} \quad Ans.\ \text{(b)}$$

$$F_{turb} \approx \left(\frac{0.031}{(2.55E6)^{1/7}}\right)\frac{1.2}{2}(15.6)^2(2.44\times 1.22\times 2\ sides) = \mathbf{3.3\ N} \quad Ans.\ \text{(c)}$$

We see that the turbulent drag is about 4 times larger than laminar drag.

P7.24 Air at 20°C and 1 atm flows past the flat plate in Fig. P7.24. The two pitot tubes are each 2 mm from the wall. The manometer fluid is water at 20°C. If U = 15 m/s and L = 50 cm, determine the values of the manometer readings h1 and h2 in cm. Assume laminar boundary-layer flow.

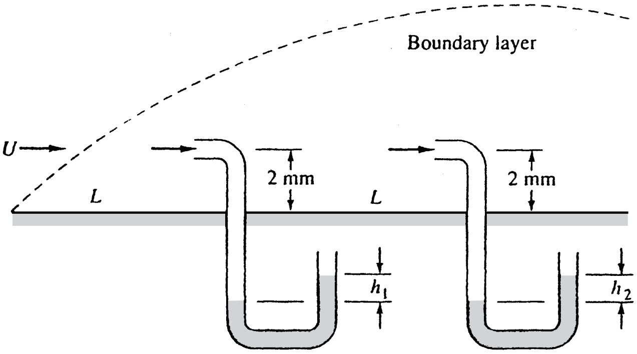


Fig. P7.24

Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. The velocities *u* at each pitot inlet can be estimated from the Blasius solution:

$$(1)\quad \eta_1 = y[U/\nu x_1]^{1/2} = (0.002)\{15/[1.5E\text{–}5(0.5)]\}^{1/2} = 2.83,\quad \text{Table 7.1: read } f' \approx 0.816$$

$$\text{Then}\quad u_1 = Uf' = 15(0.816) \approx 12.25\ \text{m/s}$$

$$(2)\quad \eta_2 = y[U/\nu x_2]^{1/2} = 2.0,\quad f' \approx 0.630,\quad u_2 = 15(0.630) \approx 9.45\ \text{m/s}$$

Assume constant stream pressure, then the manometers are a measure of the local velocity *u* at each position of the pitot inlet, so we can find Δp across each manometer:

$$\Delta p_1 = \frac{\rho}{2}u_1^2 = \frac{1.2}{2}(12.25)^2 = 90\ \text{Pa} = \Delta\rho g h_1 = (998-1.2)(9.81)h_1,\quad \mathbf{h_1 \approx 9.2\ mm}$$

$$\Delta p_2 = \frac{\rho}{2}u_2^2 = \frac{1.2}{2}(9.45)^2 = 54\text{ Pa} = (998-1.2)(9.81)h_2, \quad \text{or: } \mathbf{h_2 \approx 5.5\ mm} \quad \textit{Ans.}$$

P7.25 Consider the smooth square 10 by 10 cm duct in Fig. P7.25. The fluid is air at 20°C and 1 atm, flowing at V_{avg} = 24 m/s. It is desired to increase the pressure drop over the 1-m length by adding sharp 8-mm-long flat plates across the duct, as shown. (a) Estimate the pressure drop if there are no plates. (b) Estimate how many plates are needed to generate an additional 100 Pa of pressure drop.

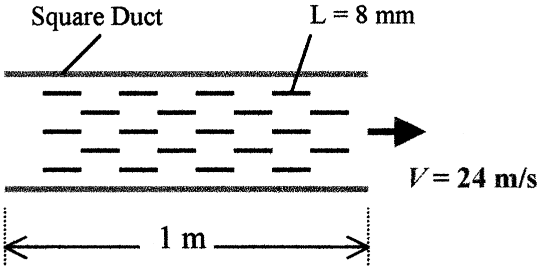


Fig. P7.25

Solution: For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. (a) Compute the duct Reynolds number and hence the Moody-type pressure drop. The hydraulic diameter is 10 cm, thus

$$\text{Re}_{Dh} = \frac{VD_h}{\nu} = \frac{(24\text{ m/s})(0.1\text{ m})}{0.000015\text{ m}^2/\text{s}} = 160000\ (\textit{turbulent}) \qquad f_{smooth} = 0.0163$$

$$\Delta p_{Moody} = f\frac{L}{D_h}\frac{\rho V^2}{2} = (0.0163)\left(\frac{1.0\text{ m}}{0.1\text{ m}}\right)\frac{(1.2\text{ kg/m}^3)(24\text{ m/s})^2}{2} = \mathbf{56\ Pa} \quad \textit{Ans.}\text{ (a)}$$

(b) To estimate the plate-induced pressure drop, first calculate the drag on one plate:

$$\text{Re}_L = \frac{(24)(0.008)}{0.000015} = 12800, \quad C_D = \frac{1.328}{\sqrt{12800}} = 0.0117,$$

$$F = C_D\frac{\rho}{2}V^2 bL(2\ \textbf{sides}) = (0.0117)\frac{1.2}{2}(24)^2(0.1)(0.008)(2) = 0.00649\ N$$

Since the duct walls must support these plates, the effect is an additional pressure drop:

$$\Delta p_{extra} = 100\ Pa = \frac{FN_{plates}}{A_{duct}} = \frac{(0.00649\ N)N_{plates}}{(0.1\text{ m})^2}, \quad or\text{: } \mathbf{N_{plates} \approx 154} \quad \textit{Ans.}\text{ (b)}$$

P7.26 Consider laminar flow past the square-plate arrangements in the figure below. Compared to the drag of a single plate (1), how much larger is the drag of four plates together as in configurations (a) and (b)? Explain your results.

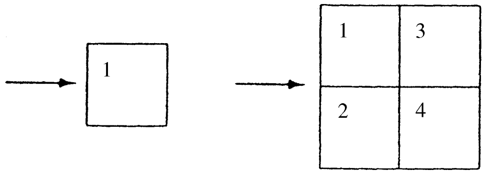


Fig. P7.26 (a)

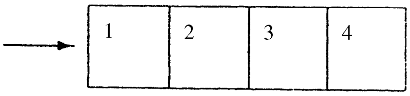


Fig. P7.26 (b)

Solution: The laminar formula $C_D = 1.328/Re_L^{1/2}$ means that $C_D \propto L^{-1/2}$. Thus:

$$\text{(a)}\ \ F_a = \frac{const}{\sqrt{2L_1}}(4A_1) = \sqrt{8}F_1 = \mathbf{2.83F_1} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)}\ \ F_b = \frac{const}{\sqrt{4L_1}}(4A_1) = \mathbf{2.0F_1} \quad \textit{Ans.}\ \text{(b)}$$

The plates near the trailing edge have less drag because their boundary layers are thicker and their wall shear stresses are less. These configurations do ***not*** quadruple the drag.

7.27 Air at 20°C and 1 atm flows at 3 m/s past a sharp flat plate 2 m wide and 1 m long. (*a*) What is the wall shear stress at the end of the plate? (*b*) What is the air velocity at a point 4.5 mm normal to the end of the plate? (*c*) What is the total friction drag on the plate?

Solution: For at 20°C and 1 atm, take ρ = 1.2 kg/m3 and μ = 1.8E-5 kg/m-s. Check the Reynolds number to see if the flow is laminar or turbulent:

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{(1.2)(3.0)(1.0)}{1.8\mathrm{E}-5} = 200{,}000 \qquad \textbf{Laminar}$$

We can proceed with our laminar-flow formulas:

$$c_{f,x=L} = \frac{0.664}{\sqrt{\mathrm{Re}_L}} = \frac{0.664}{\sqrt{200000}} = 0.00148;\ \tau_w = c_f\frac{\rho}{2}U^2 = (0.00148)(\frac{1.2}{2})(3)^2 = \mathbf{0.0080}\,Pa\ \ Ans.(a)$$

$$\text{At } y = 4.5\,mm, \text{ the Blasius } \eta = y\sqrt{\frac{U}{\nu x}} = (0.0045m)\sqrt{\frac{3.0}{(1.5E-5)(1.0)}} = 2.01$$

$$\text{Table 7.1: at } \eta = 2.0,\ \text{read } \frac{u}{U} \approx 0.63,\quad \text{hence } u = (0.63)(3.0) \approx \mathbf{1.89}\frac{m}{s}\ \ Ans.(b)$$

Finally, compute the drag for both sides of the plate, $A = 2bL$:

$$C_D = \frac{1.328}{\sqrt{200,000}} = 0.00297,$$

$$\text{or: } F = C_D\frac{\rho}{2}U^2(2bL) = (0.00297)(\frac{1.2}{2})(3.0)^2[2(2.0)(1.0)] = \mathbf{0.064}\ N\quad Ans.(c)$$

NOTE: For part *(b)*, we never had to compute the boundary layer thickness, $\delta \approx 11.2$ mm.

P7.28 Flow straighteners are arrays of narrow ducts placed in wind tunnels to remove swirl and other in-plane secondary velocities. They can be idealized as square boxes constructed by vertical and horizontal plates, as in Fig. P7.28. The cross section is *a* by *a*, and the box length is *L*. Assuming laminar flat-plate flow and an array of $N \times N$ boxes, derive a formula for (a) the total drag on the bundle of boxes and (b) the effective pressure drop across the bundle.

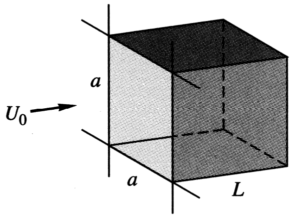


Fig. P7.28

Solution: For laminar flow over any one wall of size *a* by *L*, we estimate

$$\frac{F_{\text{one wall}}}{(1/2)\rho U^2 aL} \approx \frac{1.328}{\sqrt{(\rho UL/\mu)}}, \quad \text{or} \quad F_{\text{one wall}} \approx 0.664\,(\rho\mu L)^{1/2} U^{3/2} a$$

Thus, for **4** walls and $\mathbf{N^2}$ boxes, $F_{\text{total}} \approx \mathbf{2.656\,N^2\,(\rho\mu L)^{1/2}\,U^{3/2}\,a}$ *Ans.* (a)

The pressure drop across the array is thus

$$\Delta p_{\text{array}} = \frac{F_{\text{total}}}{(Na)^2} \approx \mathbf{\frac{2.656}{a}(\rho\mu L)^{1/2}U^{3/2}} \quad \textit{Ans. (b)}$$

This is *completely* different from the predicted Δp for laminar flow through a long square duct, as in Section 6.6:

$$\Delta p_{\text{duct}} = f\frac{L}{D_h}\frac{\rho}{2}U^2 = \left(\frac{56.91\mu}{\rho Ua}\right)\left(\frac{L}{a}\right)\frac{\rho}{2}U^2 \approx \frac{28.5\mu LU}{a^2} \quad (?)$$

This has almost no relation to *Answer* (b) above, being the Δp for a long square duct filled with boundary layer. Answer (b) is for a very short duct with thin wall boundary layers.

P7.29 Let the flow straighteners in Fig. P7.28 form an array of 20 × 20 boxes of size a = 4 cm and L = 25 cm. If the approach velocity is U_o = 12 m/s and the fluid is sea-level standard air, estimate (a) the total array drag and (b) the pressure drop across the array. Compare with Sec. 6.6.

Solution: For sea-level air, take ρ = 1.205 kg/m^3 and μ = 1.78E−5 kg/m·s. The analytical formulas for array drag and pressure drop are given above. Hence

$$F_{\text{array}} = 2.656N^2(\rho\mu L)^{1/2}U^{3/2}a = 2.656(20)^2[1.205(1.78\text{E}{-5})(0.25)]^{1/2}(12)^{3/2}(0.04)$$

$$\text{or:} \quad \mathbf{F \approx 4.09\ N} \quad (Re_L = 203000, \text{OK, laminar}) \quad \textit{Ans. (a)}$$

$$\Delta p_{\text{array}} = \frac{F}{(Na)^2} = \frac{4.09}{[20(0.04)]^2} \approx \mathbf{6.4\ Pa} \quad \textit{Ans. (b)}$$

This is a far cry from the (much lower) estimate we would have by assuming the array is a bunch of long square ducts as in Sect. 6.6 (as shown in Prob. 7.28):

$$\Delta p_{\text{long duct}} \approx \frac{28.5\mu LU}{a^2} = \frac{28.5(1.78\text{E}{-5})(0.25)(12)}{(0.04)^2} \approx \mathbf{0.95\ Pa} \quad \text{(not accurate)} \quad \textit{Ans.}$$

P7.30 In Ref. 56 of Ch. 6, McKeon et al. propose new, supposedly more accurate values for the turbulent log-law constants, $\kappa = 0.421$ and $B = 5.62$. Use these constants, and the one-seventh power-law, to repeat the analysis that led to the formula for turbulent boundary layer thickness, Eq. (7.42). What is the approximate percent shift in δ/x compared to the textbook's formula? Comment.

Solution: We can start with Eq. (7.37), modified for the new constants:

$$\left(\frac{2}{c_f}\right)^{1/2} = \frac{1}{0.421}\ln\left[\mathrm{Re}_\delta\left(\frac{c_f}{2}\right)^{1/2}\right] + 5.62$$

Calculate and list a few values for Re_δ in the range 10^4 to 10^7:

Re_δ	10^4	10^5	10^6	10^7
c_f	0.00483	0.00313	0.00217	0.00159

These new values fit, reasonably, the least-squares power-law $c_f \approx 0.0203\ \mathrm{Re}_\delta^{-0.160}$. Then Eq. (7.41) modifies to

$$c_f = 0.0203\mathrm{Re}_\delta^{-0.160} = 2\frac{d}{dx}\left(\frac{7}{72}\delta\right),\ or:\ \mathrm{Re}_\delta^{-0.160} = 9.58\frac{d(\mathrm{Re}_\delta)}{d(\mathrm{Re}_x)}$$

$$\text{Integrate to}\quad \frac{\delta}{x} \approx \frac{0.162}{\mathrm{Re}_x^{0.138}} \qquad \textit{Ans.}$$

This is very similar to Eq. (7.42), so the change is marginal. Actual calculations for δ/x in the range of $\mathrm{Re}_x = 10^6$ to 10^9 show that the new formula **averages ten per cent higher thickness**.

P7.31 The centerboard on a sailboat is 3 ft long parallel to the flow and protrudes 7 ft down below the hull into seawater at 20°C. Using flat-plate theory for a smooth surface, estimate its drag if the boat moves at 10 knots. Assume $\mathrm{Re}_{x,tr} = 5\mathrm{E}5$.

Solution: For seawater, take $\rho = 1.99$ slug/ft^3 and $\mu = 2.23\text{E}{-5}$ slug/ft·s. Evaluate Re$_L$ and the drag. Convert 10 knots to 16.9 ft/s.

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{(1.99\ \text{slug/ft}^3)(16.9\ \text{ft/s})(3\ \text{ft})}{0.0000223\ \text{slug/ft·s}} = 4.52E6\ \ \text{(turbulent)}$$

$$\text{From Eq. (7.49a),}\quad C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} - \frac{1440}{\mathrm{Re}_L} = \frac{0.031}{(4.52E6)^{1/7}} - \frac{1440}{4.52E6}$$

$$= 0.00347 - 0.00032 = 0.00315$$

$$F_{drag} = C_D\frac{\rho}{2}V^2bL(2\ \text{sides}) = 0.00315\left(\frac{1.99}{2}\right)(16.9)^2(3\ \text{ft})(7\ \text{ft})(2\ \text{sides}) = \mathbf{38\ lbf}\quad \textit{Ans.}$$

P7.32 A flat plate of length L and height δ is placed at a wall and is parallel to an approaching boundary layer, as in Fig. P7.32. Assume that the flow over the plate is fully turbulent and that the approaching flow is a one-seventh-power law

$$u(y) = U_o\left(\frac{y}{\delta}\right)^{1/7}$$

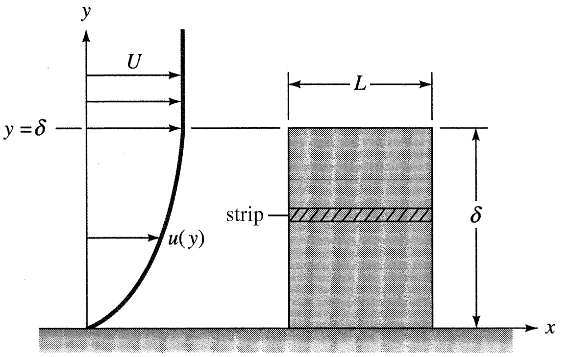


Fig. P7.32

Using strip theory, derive a formula for the drag coefficient of this plate. Compare this result with the drag of the same plate immersed in a uniform stream U_o.

Solution: For a ‘strip’ of plate *dy* high and *L* long, subjected to flow *u*(y), the force is

$$dF = C_D \frac{\rho}{2} u^2 (L\ dy)(2\ \text{sides}), \quad \text{where } C_D \approx \frac{0.031}{(\rho u L/\mu)^{1/7}}, \quad \text{combine into dF and integrate:}$$

$$dF = 0.031\rho\nu^{1/7}L^{6/7}u^{13/7}dy, \quad \text{or} \quad F = 0.031\rho\nu^{1/7}L^{6/7}\int_0^{\delta}\left[U_o(y/\delta)^{1/7}\right]^{13/7}dy$$

$$\text{The result is} \quad \mathbf{F = 0.031(49/62)}\boldsymbol{\rho\nu}^{\mathbf{1/7}}\mathbf{L^{6/7}U_O{}^{13/7}}\boldsymbol{\delta} \quad \textit{Ans.}$$

This drag is (49/62), or 79%, of the force on the same plate immersed in a uniform stream.

P7.33 An alternate analysis of turbulent flat-plate flow was given by Prandtl in 1927, using a wall shear-stress formula from pipe flow

$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Show that this formula can be combined with Eqs. (7.32) and (7.40) to derive the following relations for turbulent flat-plate flow.

$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}} \qquad c_f = \frac{0.0577}{\mathrm{Re}_x^{1/5}} \qquad C_D = \frac{0.072}{\mathrm{Re}_L^{1/5}}$$

These formulas are limited to Re_x between 5 × 10^5 and 10^7.

Solution: Use Prandtl’s correlation for the left hand side of Eq. (7.32) in the text:

$$\tau_w \approx 0.0225\rho U^2(\nu/U\delta)^{1/4} = \rho U^2\frac{d\theta}{dx} \approx \rho U^2\frac{d}{dx}\left(\frac{7}{72}\delta\right), \quad \text{cancel } \rho U^2 \text{ and rearrange:}$$

$$\delta^{1/4}d\delta = 0.2314(\nu/U)^{1/4}dx, \quad \text{Integrate:} \quad \frac{4}{5}\delta^{5/4} = 0.2314(\nu/U)^{1/4}x$$

Take the $(5/4)^{\text{th}}$ root of both sides and rearrange for the final thickness result:

$$\delta \approx 0.37(\nu/U)^{1/5}x^{4/5}, \quad \text{or:} \quad \boldsymbol{\frac{\delta}{x} \approx \frac{0.37}{\mathrm{Re}_x^{1/5}}} \quad \textit{Ans. (a)}$$

$$\text{Substitute } \delta(x) \text{ into } \tau_w: \quad C_f \approx \frac{2(0.0225)}{(0.37)^{1/4}}\left(\frac{\nu}{Ux}\right)^{1/5}, \quad \text{or} \quad \mathbf{C_f \approx \frac{0.0577}{Re_x^{1/5}}} \quad \textit{Ans. (b)}$$

$$\text{Finally,} \quad C_D = \int_0^1 C_f \, d\left(\frac{x}{L}\right) = \frac{5}{4} C_f(\text{at } x = L) \approx \mathbf{\frac{0.072}{Re_L^{1/5}}} \quad \textit{Ans. (c)}$$

7.34 Consider turbulent flow past a flat plate of width *b* and length *L*. What percentage of the friction drag on the plate is carried by the rear half of the plate?

Solution: The formula for turbulent boundary drag on a plate is Eq. (7.45):

$$C_D = \frac{2D(x)}{\rho U^2 bx} \approx \frac{0.031}{\mathrm{Re}_x^{1/7}} = \frac{0.031\mu^{1/7}}{(\rho U x)^{1/7}}, \quad \text{or}: \quad D(x) = (const)\, x^{6/7}$$

At $x = L$, we obtain a force equal to (const) $L^{6/7}$. At $x = L/2$, we obtain a force equal to (const) $L^{6/7}/2^{6/7}$ = (const)(0.552) $L^{6/7}$, which is 55.2% of the total force. Thus the force on the trailing half of the plate is only (100 – 55.2) = **44.8%** of the total force on the plate. Unlike laminar flow (29.3%), this is nearly half of the total, since turbulent shear drops off much slower with *x*.

P7.35 Repeat Problem 7.26 for *turbulent* flow. Explain your results.

Solution: The turbulent formula $C_D = 0.031/\mathrm{Re}_L^{1/7}$ means that $C_D \propto L^{-1/7}$. Thus:

$$\text{(a)} \quad F_a = \frac{const}{(2L_1)^{1/7}}(4A_1) = \mathbf{3.62F_1} \quad \textit{Ans. (a)}$$

$$\text{(b)} \quad F_b = \frac{const}{(4L_1)^{1/7}}(4A_1) = \mathbf{3.28F_1} \quad \textit{Ans. (b)}$$

The trailing areas have *slightly* less shear stress, hence we are *nearly* quadrupling drag.

P7.36 A ship is 125 m long and has a wetted area of 3500 m^2. Its propellers can deliver a maximum power of 1.1 MW to seawater at 20°C. If all drag is due to friction, estimate the maximum ship speed, in kn.

Solution: For seawater at 20°C, take $\rho = 1025$ kg/m^3 and $\mu = 0.00107$ kg/m·s. Evaluate

$$\mathrm{Re_L} = \frac{\rho UL}{\mu} = \frac{1025V(125)}{0.00107} \text{ (surely turbulent)}, \quad C_D = \frac{0.031}{\mathrm{Re_L^{1/7}}} = \frac{0.00217}{V^{1/7}}$$

$$\text{Power} = FV = \left[\frac{0.0217}{V^{1/7}}\left(\frac{1025}{2}\right)V^2(3500)\right]V = 1.1E6 \text{ watts}, \quad \text{or} \quad V^{20/7} \approx 282.0$$

$$\text{Solve for} \quad V = 7.2 \text{ m/s} \approx \mathbf{14\ knots}. \quad \textit{Ans.}$$

Check $\mathrm{Re_L} = (1025)(7.2)(125)/(0.00107) = 8.6E8$, typical of ships.

P7.37 Air at 20°C and 1 atm flows past a long flat plate, at the end of which is placed a narrow scoop, as shown in Fig. P7.37. (a) Estimate the height *h* of the scoop if it is to extract 4 kg/s per meter of width into the paper. (b) Find the drag on the plate up to the inlet of the scoop, per meter of width.

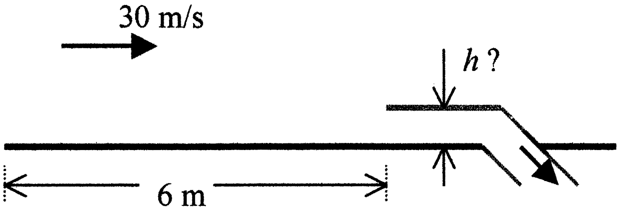


Fig. P7.37

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. We assume that the scoop does not alter the boundary layer at its entrance. (a) Compute the displacement thickness at $x = 6$ m:

$$\mathrm{Re}_x = \frac{Ux}{\nu} = \frac{(30 \text{ m/s})(6 \text{ m})}{0.000015 \text{ m}^2/\text{s}} = 1.2E7, \quad \frac{\delta^*}{x} \approx \frac{1}{8}\left(\frac{0.16}{\mathrm{Re}_x^{1/7}}\right) = \frac{0.020}{(1.2E7)^{1/7}} = 0.00195$$

$$\delta^*|_{x=6\text{ m}} = (6 \text{ m})(0.00195) = 0.0117 \text{ m}$$

If δ^* were zero, the flow into the scoop would be uniform: 4 kg/s/m = $\rho U h$ = (1.2)(30)h, which would make the scoop h_o = 0.111 m high. However, we lose the near-wall mass flow $\rho U \delta^*$, so the proper scoop height is equal to

$$h = h_o + \delta^* = 0.111\text{ m} + 0.0117\text{ m} \approx \mathbf{0.123\ m} \quad \textit{Ans.}\text{ (a)}$$

(b) Assume Re_{tr} = 5E5 and use Eq. (7.49a) to estimate the drag:

$$\mathrm{Re}_x = 1.2E7, \quad C_d = \frac{0.031}{\mathrm{Re}_x^{1/7}} - \frac{1440}{\mathrm{Re}_x} = 0.00302 - 0.00012 = 0.00290$$

$$F_{drag} = C_d \frac{\rho}{2} V^2 bL = 0.0029\left(\frac{1.2\text{ kg/m}^3}{2}\right)(30\text{ m/s})^2(1\text{ m})(6\text{ m}) = \mathbf{9.4\ N} \quad \textit{Ans.}\text{ (b)}$$

P7.38 Atmospheric boundary layers are very thick but follow formulas very similar to those of flat-plate theory. Consider wind blowing at 10 m/s at a height of 80 m above a smooth beach. Estimate the wind shear stress, in Pa, on the beach if the air is standard sea-level conditions. What will the wind velocity striking your nose be if (a) you are standing up and your nose is 170 cm off the ground; (b) you are lying on the beach and your nose is 17 cm off the ground?

Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Assume a *smooth* beach and use the log-law velocity profile, Eq. (7.34), given u = 10 m/s at y = 80 m:

$$\frac{u}{u^*} = \frac{10\text{ m/s}}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B = \frac{1}{0.41}\ln\left(\frac{80u^*}{1.5E\text{–}5}\right) + 5.0, \quad \text{solve } u^* \approx 0.254\text{ m/s}$$

$$\text{Hence} \quad \tau_{\text{surface}} = \rho u^{*2} = (1.2)(0.254)^2 \approx \mathbf{0.0772\ Pa} \quad \textit{Ans.}$$

The log-law should be valid as long as we stay above y such that $yu^*/\nu > 50$:

$$\text{(a) } y = 1.7\text{ m:} \quad \frac{u}{0.254} \approx \frac{1}{0.41}\ln\left[\frac{1.7(0.254)}{1.5E\text{–}5}\right] + 5, \quad \text{solve } u_{1.7\text{ m}} \approx \mathbf{7.6\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } y = 17\text{ cm:} \quad \frac{u}{0.254} \approx \frac{1}{0.41}\ln\left[\frac{0.17(0.254)}{1.5E\text{–}5}\right] + 5, \quad \text{solve } u_{17\text{ cm}} \approx \mathbf{6.2\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

The (b) part seems very close to the surface, but $yu^*/\nu \approx 2800 > 50$, so the log-law is OK.

P7.39 A hydrofoil 50 cm long and 4 m wide moves at 28 kn in seawater at 20°C. Using flat-plate theory with Re_{tr} = 5E5, estimate its drag, in N, for (a) a smooth wall and (b) a rough wall, $\varepsilon = 0.3$ mm.

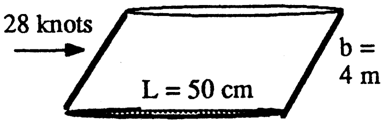


Fig. P7.39

Solution: For seawater at 20°C, take ρ = 1025 kg/m^3 and μ = 0.00107 kg/m·s. Convert 28 knots = 14.4 m/s. Evaluate Re_L = (1025)(14.4)(0.5)/(0.00107) ≈ 6.9E6 (turbulent). Then

$$\text{Smooth, Eq. (7.49a):}\quad C_D = \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L} \approx 0.00306$$

$$\text{Drag} = C_D\left(\frac{\rho}{2}\right)U^2 bL(2\text{ sides}) = (0.00306)\left(\frac{1025}{2}\right)(14.4)^2(4)(0.5)(2) \approx \mathbf{1300\ N} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Rough,}\quad \frac{L}{\varepsilon} = \frac{500}{0.3} = 1667, \quad \text{Fig. 7.6 or Eq. (7.48b):}\ C_D \approx 0.00742$$

$$\text{Drag} = (0.00742)\left(\frac{1025}{2}\right)(14.4)^2(4)(0.5)(2\text{ sides}) \approx \mathbf{3150\ N} \quad \textit{Ans.}\text{ (b)}$$

P7.40 Hoerner (Ref. 12) plots the drag of a flag in winds, based on total surface area $2bL$, in the figure at right. A linear approximation is $C_D \approx 0.01 + 0.05L/b$, as shown. Test Reynolds numbers were 1E6 or greater. (a) Explain why these values are greater than for a flat plate. (b) Assuming sea-level air at 50 mi/h, with area bL = 4 m^2, find the proper flag dimensions for which the total drag is approximately 400 N.

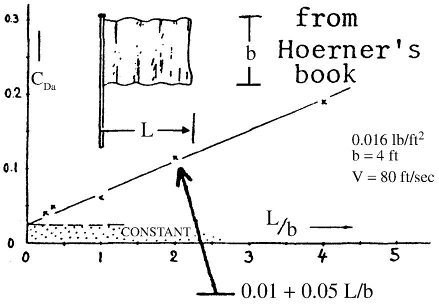


Fig. P7.40

Solution: (a) The drag is greater because the fluttering of the flag causes additional *pressure drag* on the corrugated sections of the cloth. *Ans.* (a)

(b) For air take $\rho = 1.225$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. Convert $U = 50$ mi/h = 22.35 m/s. Evaluate the drag force from the force coefficient:

$$F = C_D \frac{\rho}{2} U^2 A = \left(0.01 + 0.05\frac{L}{b}\right)\left(\frac{1.225}{2}\right)(22.35)^2(2 \times 4.0 \text{ m}^2) = 400 \text{ N}$$

$$\text{Solve for} \quad C_D = 0.163 \quad or \quad L/b \approx 3.07$$

Combine this with the fact that $bL = 4$ m^2 and we obtain

$$L \approx \mathbf{3.51\,m} \quad \text{and} \quad b \approx \mathbf{1.14\,m} \quad \textit{Ans.}\text{ (b)}$$

P7.41 Repeat Prob. 7.20 with the sole change that the pitot probe is now **10 mm** from the wall (5 times higher). Show that the flow there cannot possibly be laminar, and use smooth-wall turbulent-flow theory to estimate the position *x* of the probe, in m.

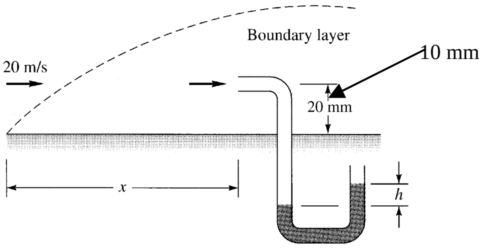


Fig. P7.20

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. For U = 20 m/s, it is *not possible* for a laminar boundary-layer to grow to a thickness of 10 mm. Even at the largest possible laminar Reynolds number of 3E6, the laminar thickness is only

$$\text{Re}_x = 3\text{E}6 = 1.2(20)x/1.8\text{E-}5, \quad \text{or} \quad x = 2.25 \text{ m},$$

$$\delta \approx \frac{5x}{\text{Re}_x^{1/2}} = \frac{5(2.25)}{(3\text{E}6)^{1/2}} \approx 0.0065\,m = 6.5 \text{ mm} < 10 \text{ mm! } \quad \textit{Ans.}$$

Therefore the flow must be turbulent. Recall from Prob. 7.20 that the manometer reading was $h = 16$ mm of Meriam red oil, SG = 0.827. Thus

$$\Delta p_{\text{mano}} = \Delta\rho g h = [0.827(998) - 1.2](9.81)(0.016) \approx 129 \text{ Pa}, \quad u_{\text{pitot}} = \sqrt{\frac{2\Delta p}{\rho}} \approx 14.7 \ \frac{\text{m}}{\text{s}}$$

$$\text{Then, at y = 20 mm,} \quad \frac{u}{U} = \frac{14.7}{20} \approx 0.734 \approx \left(\frac{y}{\delta}\right)^{1/7} = \left(\frac{10 \text{ mm}}{\delta}\right)^{1/7}, \quad \text{or} \quad \delta \approx 87 \text{ mm}$$

Thus, crudely, $\delta/x = 0.0.087\text{m}/x \approx 0.16/\mathrm{Re}_x^{1/7}$, solve for $\boldsymbol{x \approx 5.15}$ **m**. *Ans.*

7.42 A light aircraft flies at 30 m/s (67 mi/h) in air at 20°C and 1 atm. Its wing is an NACA 0009 airfoil, with a chord length of 150 cm and a very wide span (neglect aspect ratio effects). Estimate the drag of this wing, per unit span length, (*a*) by flat plate theory; and (*b*) using the data from Fig. 7.25 for $\alpha = 0°$.

Solution: For air at 20°C and 1 atm, $\rho = 1.2\ \text{kg/m}^3$ and $\mu = 1.8\text{E-5 kg/m-s}$. First find the Reynolds number, based on chord length, to see where we are:

$$\mathrm{Re}_c = \frac{\rho U c}{\mu} = \frac{(1.2\,kg/m^3)(30\,m/s)(1.5\,m)}{1.8\mathrm{E}-5\,kg/m-s} = 3\times 10^6 \qquad \text{turbulent}$$

(*a*) For flat-plate theory, use Eq. (7.49*a*), which assumes transition at $\mathrm{Re}_x = 500{,}000$:

$$C_d = \frac{0.031}{\mathrm{Re}_c^{1/7}} - \frac{1440}{\mathrm{Re}_c} = \frac{0.031}{(3\mathrm{E}6)^{1/7}} - \frac{1440}{3\mathrm{E}6} = 0.00368 - 0.00048 = 0.0032$$

$$Drag = C_d\frac{\rho}{2}U^2(2bc) = (0.0032)(\frac{1.2}{2})(30)^2[2(1.0)(1.5)] = \mathbf{5.2}\ \frac{N}{m} \quad Ans.(a)$$

(*b*) For the actual NACA 0009 airfoil, at $\mathrm{Re}_c = 3\mathrm{E}6$, in Fig. 7.25, read $C_d \approx 0.0065$. Then

$$Drag = C_d\frac{\rho}{2}U^2(bc) = (0.0065)(\frac{1.2}{2})(30)^2[(1.0)(1.5)] = \mathbf{5.3}\ \frac{N}{m} \quad Ans.(b)$$

The two are quite close. A thin airfoil at low angles is similar to a flat plate.

P7.43 In the flow of air at 20°C and 1 atm past a flat plate in Fig. P7.43, the wall shear is to be determined at position *x* by a *floating element* (a small area connected to a strain-gage force measurement). At *x* = 2 m, the element indicates a shear stress of 2.1 Pa. Assuming turbulent flow from the leading edge, estimate (a) the stream velocity *U*, (b) the boundary layer thickness δ at the element, and (c) the boundary-layer velocity *u*, in m/s, at 5 mm above the element.

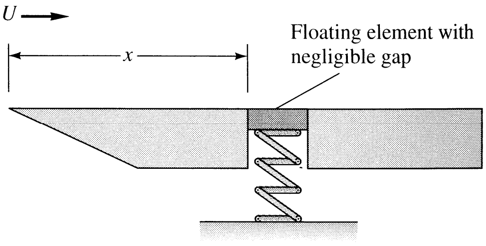


Fig. P7.43

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-}5$ kg/m·s. The shear stress is

$$\tau_w = 2.1\ \text{Pa} = C_f\frac{\rho}{2}U^2 = \frac{0.027}{(\rho Ux/\mu)^{1/7}}\left(\frac{\rho U^2}{2}\right) = \frac{0.027}{[1.2U(2)/1.8\text{E}{-}5]^{1/7}}\left(\frac{1.2U^2}{2}\right)$$

Solve for $U \approx \mathbf{34\ \frac{m}{s}}$ *Ans.* (a) Check $\text{Re}_x \approx 4.54\text{E}6$ (OK, turbulent)

With the local Reynolds number known, solve for local thickness:

$$\delta \approx \frac{0.16x}{\text{Re}_x^{1/7}} = \frac{0.16(2\ \text{m})}{(4.54\text{E}6)^{1/7}} \approx 0.036\ \text{m} \approx \mathbf{36\ mm} \quad \textit{Ans.}\ \text{(b)}$$

The log-law, Eq. (7.34), is best for estimating the velocity at y = 5 mm above the element. The friction velocity is $u^* = (\tau_w/\rho)^{1/2} = (2.1/1.2)^{1/2} = 1.32$ ft/s. Enter the log-law:

$$u/u^* = u/1.32 = (1/\kappa)\ln(yu^*/\nu) + B =$$

$$= (1/0.41)\ \ln[(0.005)(1.32)/1.5E-5] + 5.0, \quad \text{solve for } u \approx \mathbf{26.3}\, m/s \quad \textit{Ans.}(c)$$

P7.44 Extensive measurements of wall shear stress and local velocity for turbulent airflow on the flat surface of the University of Rhode Island wind tunnel have led to the following proposed correlation:

$$\frac{\rho y^2 \tau_w}{\mu^2} \approx 0.0207\left(\frac{uy}{\nu}\right)^{1.77}$$

Thus, if y and $u(y)$ are known at a point in a flat-plate boundary layer, the wall shear may be computed directly. If the answer to part (c) of Prob. 7.43 is $u \approx 26.3$ m/s, determine whether the correlation is accurate for this case.

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E–}5$ kg/m·s. The shear stress is given as 2.1 Pa, and part (c) was y = 5 mm. Check each side of the proposed correlation:

$$\frac{\rho y^2\tau_w}{\mu^2} = \frac{1.2(0.005)^2(2.1)}{(1.8\text{E–}5)^2} \approx 194000;$$

$$0.0207\left(\frac{uy}{\nu}\right)^{1.77} = 0.0207\left[\frac{26.3(0.005)}{1.5\text{E–}5}\right]^{1.77} \approx \ 197000 \quad \text{(1.6\% more)}$$

The **correlation is good** and comparable to the approximate calculations in P7.43.

P7.45 A thin sheet of fiberboard weighs 90 N and lies on a rooftop, as shown in the figure. Assume ambient air at 20°C and 1 atm. If the coefficient of solid friction between board and roof is $\sigma = 0.12$, what wind velocity will generate enough friction to dislodge the board?

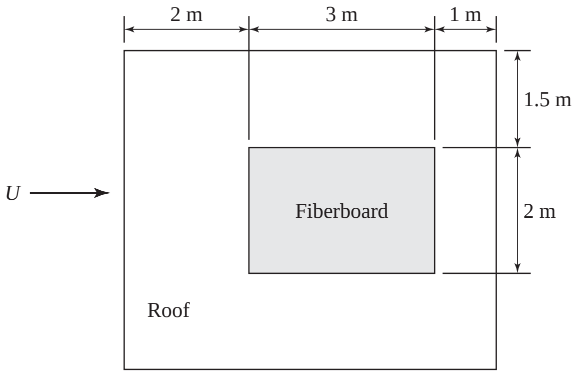


Fig. P7.45

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E–}5$ kg/m·s. Our first problem is to evaluate the drag when the leading edge is *not* at x = 0. Since the dimensions are large, we will assume that the flow is *turbulent* and check this later:

$$F = \int_{x_1}^{x_2} \tau_w\, dA = \int_{x_1}^{x_2}\left[\frac{0.027(\rho/2)U^2}{(\rho U x/\mu)^{1/7}}\right] b\,dx = \left(\frac{0.031 b\rho U^2}{2}\right)\left(\frac{\mu}{\rho U}\right)^{1/7}\left(x_2^{6/7} - x_1^{6/7}\right)$$

Set this equal to the dislodging friction force $F = \sigma W = 0.12(90) = 10.8$ N:

$$\frac{0.031}{2}(1.2)(2.0)U^2\left(\frac{1.8E\text{–}5)}{1.2U}\right)^{1/7}(5.0^{6/7}-2.0^{6/7}) = 10.8\text{ N}$$

Solve this for U = **33 m/s ≈ 73 mi/h** *Ans.*

$\mathrm{Re}_{x_1} = 4.4\text{E}6$: turbulent, OK.

P7.46 A ship is 150 m long and has a wetted area of 5000 m^2. If it is encrusted with barnacles, the ship requires 7000 hp to overcome friction drag when moving in seawater at 15 kn and 20°C. What is the average roughness of the barnacles? How fast would the ship move with the same power if the surface were smooth? Neglect wave drag.

Solution: For seawater at 20°C, take $\rho = 1025\text{ kg/m}^3$ and $\mu = 0.00107$ kg/m·s. Convert 15 kn = 7.72 m/s. Evaluate $\mathrm{Re_L} = (1025)(7.72)(150)/(0.00107) \approx 1.11\text{E}9$ (turbulent). Then

$$\mathrm{F} = \frac{\text{Power}}{\mathrm{U}} = \frac{5.22\text{E}6\text{ W}}{7.72} = 6.76\text{E}5\text{ N},\quad \mathrm{C_D} = \frac{2\mathrm{F}}{\rho \mathrm{U}^2\mathrm{A}} = \frac{2(6.76\text{E}5)}{1025(7.72)^2(5000)} \approx 0.00443$$

Fig. 7.6 or Eq. (7.48b):

$$\frac{\mathrm{L}}{\varepsilon} \approx 16800,\quad \varepsilon_{\text{barnacles}} = \frac{150}{16800} \approx \mathbf{0.0089\ m}\quad \textit{Ans.}\text{ (a)}$$

If the surface were smooth, we could use Eq. (7.45) to predict a higher ship speed:

$$\mathrm{P} = \mathrm{FU} = \left[\mathrm{C_D}\frac{\rho \mathrm{U}^2}{2}\mathrm{A}\right]\mathrm{U} = \left\{\frac{0.031}{[1025\mathrm{U}(150)/.00107]^{1/7}}\right\}\left(\frac{1025}{2}\right)\mathrm{U}^2(5000)\mathrm{U},$$

$$\text{or:}\quad \mathrm{P} = 5.22\text{E}6\text{ watts} = 5428\mathrm{U}^{20/7},\quad \text{solve for } \mathrm{U} = 11.1\text{ m/s} \approx \mathbf{22\ knots}\quad \textit{Ans.}\text{ (b)}$$

7.47 Local boundary layer effects, such as shear stress and heat transfer, are best correlated with local variables, rather using distance *x* from the leading edge. The momentum thickness θ is often used as a length scale. Use the analysis of turbulent flat-plate flow to write local wall shear stress τ_w in terms of dimensionless θ and compare with the formula recommended by Schlichting [1]: $C_f \approx 0.033\,\mathrm{Re}_\theta^{-0.268}$.

Solution: Our turbulent flat-plate theory, Eqs. (7.40) to (7.43), has expressions for C_f and θ in terms of Re_x. Eliminate Re_x to solve for C_f in terms of Re_θ

$$\frac{\theta}{\delta} = \frac{\mathrm{Re}_\theta}{\mathrm{Re}_\delta} = \frac{7}{72} = \frac{\mathrm{Re}_\theta}{(0.16\,\mathrm{Re}_x^{6/7})} \quad \text{Eq. (7.42)}, \quad \text{and} \quad \mathrm{Re}_x = \left(\frac{0.027}{C_f}\right)^7 \quad \text{Eq. (7.43)}$$

$$\text{Eliminate } \mathrm{Re}_x \text{ to obtain } \mathrm{Re}_\theta = \frac{7(0.16)}{72}\left(\frac{0.027}{C_f}\right)^6 \quad :$$

$$\text{and, finally,} \quad C_f \approx 0.0135\,\mathrm{Re}_\theta^{-1/6} \qquad \textit{Ans.}$$

The exponent (-1/6) is not as steep as Schlichting's exponent (-0.268), but the two formulas agree in the range where Schlichting's formula applies, $1E6 < \mathrm{Re}_x < 1E7$.

P7.48 In 1957 H. Görtler proposed the adverse-gradient test cases

$$U = \frac{U_o}{(1 + x/L)^n}$$

and computed separation for laminar flow at $n = 1$ to be $x_{sep}/L = 0.159$. Compare with Thwaites' method, assuming $\theta_o = 0$.

Solution: Introduce this stream velocity ($n = 1$) into Eq. (7.54), with $\theta_o = 0$, and integrate:

$$\theta^2 = \frac{0.45\nu}{U_o^6}\left(1+\frac{x}{L}\right)^6 \int_0^x U_o^5\left(1+\frac{x}{L}\right)^{-5} dx, \quad \text{or:} \quad \lambda = \frac{\theta^2}{\nu}\frac{dU}{dx} = \frac{0.45}{4}\left[1-\left(1+\frac{x}{L}\right)^4\right]$$

$$\text{Separation:} \quad \lambda = -0.09 \quad \text{if} \quad \left(\frac{x}{L}\right)_{sep} \approx \mathbf{0.158} \quad (\le 1\%\ \text{error}) \quad \textit{Ans.}$$

P7.49 Based on your understanding of boundary layers, which flow direction (left or right) for the foil shape in the figure will have *less* total drag?

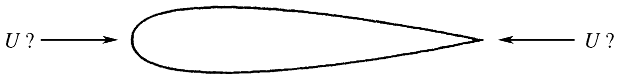


Fig. P7.49

Solution: Flow to the left has a long run of mild favorable gradient and then a short run of *strong* adverse gradient—separation and a broad wake will occur, **high pressure drag**. Flow to the right has a long run of *mild* adverse gradient—less separation, **low pressure drag**.

P7.50 Consider the flat-walled diffuser in Fig. P7.50, which is similar to that of Fig. 6.26*a* with constant width *b*. If *x* is measured from the inlet and the wall boundary layers are thin, show that the core velocity *U*(*x*) in the diffuser is given approximately by

$$U = \frac{U_o}{1+(2x\tan\theta)/W}$$

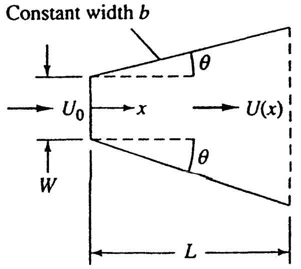


Fig. P7.50

where *W* is the inlet height. Use this velocity distribution with Thwaites' method to compute the wall angle θ for which laminar separation will occur in the exit plane when diffuser length $L = 2W$. Note that the result is independent of the Reynolds number.

Solution: We can approximate U(x) by the one-dimensional continuity relation:

$$\mathrm{U_oWb = U(W + 2x\tan\theta)b,\ \ or:\ \ U(x) \approx U_o/[1 + 2x\tan\theta/W]}\quad \text{(same as Görtler, Prob. 7.48)}$$

We return to the solution from Görtler's ($n = 1$) distribution in Prob. 7.48:

$$\lambda = -0.09 \quad \text{if} \quad \frac{2\mathrm{x}\tan\theta}{\mathrm{W}} = 0.159 \quad \text{(separation)}, \quad \text{or} \quad \mathrm{x = L = 2W},$$

$$\tan\theta_{\text{sep}} = \frac{0.159}{4} = 0.03975, \quad \boldsymbol{\theta_{\square\square\square}} \approx 2.3° \quad \textit{Ans.}$$

[This laminar result is much less than the turbulent value $\theta_{\text{sep}} \approx 8°$–$10°$ in Fig. 6.26c.]

P7.51 The derivation of Eq. (7.42) for δ/x in turbulent flow used very simple velocity correlations, Eqs. (7.38) and (7.39). A unified law-of-the-wall was given by Spalding [54]:

$$y^+ = u^+ + e^{-\kappa B}[e^{\kappa u^+} - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2} - \frac{(\kappa u^+)^3}{6}]$$

where $u^+ = u/u^*$ and $y^+ = yu^*/\nu$, while κ and B are the log-law constants in Eq. (7.34). This clever inverse formula fits all the data in Fig. 6.10 out to the edge of the logarithmic region.

How can Spalding's formula be used to improve Eqs. (7.38) and (7.40) and thus lead to improved turbulent flat-plate-flow relations?

Solution: First, about Eq. (7.38). This came from applying the log-law to $u = U$ at $y = \delta$. If we do the same thing with Spalding's complex formula, we get almost *exactly the same thing*, namely, Eq. (7.38). So Spalding's law is no help there. Second, we could, in principle, integrate Spalding's formula to get a much better estimate of momentum thickness than Eq. (7.40). In fact, no rational person would attempt such a complex integration, and the result would be Reynolds number dependent anyway and laborious to implement. Thus, Spalding's formula is **no help in improving the analysis of Eqs. (7.38) and (7.40)**. *Ans.*

Interestingly, Spalding's formula *can* be used, by staying in (u^+, y^+) coordinates, to generate an easy, accurate, alternative formula for turbulent friction and drag. [See Ch. 6 of Ref. 2.]

P7.52 Clift et al. [46] give the formula $F \approx (6\pi/5)(4 + a/b)\mu Ub$ for the drag of a prolate spheroid in *creeping motion*, as shown in Fig. P7.52. The half-thickness b is 4 mm. See also [49]. If the fluid is SAE 50W oil at 20°C, (a) check that $\mathrm{Re}_b < 1$; and (b) estimate the spheroid length if the drag is 0.02 N.

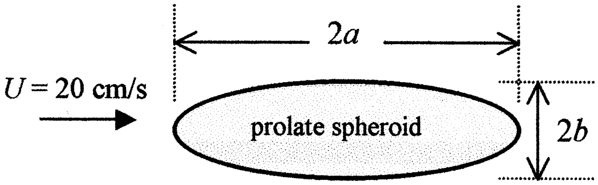


Fig. P7.52

Solution: For SAE 50W oil, take $\rho = 902$ kg/m^3 and $\mu = 0.86$ kg/m·s. (a) The Reynolds number based on half-thickness is:

$$\mathrm{Re}_b = \frac{\rho U b}{\mu} = \frac{(902\ \text{kg/m}^3)(0.2\ \text{m/s})(0.004\ \text{m})}{0.86\ \text{kg/m·s}} = \mathbf{0.84} < 1 \quad \textit{Ans.}\ \text{(a)}$$

(b) With a given force and creeping-flow force formula, we can solve for the half-length a:

$$F = 0.02\ N = \frac{6\pi}{5}\left(4+\frac{a}{b}\right)\mu U b = \frac{6\pi}{5}\left(4+\frac{a}{0.004}\right)(0.86\ \text{kg/m·s})(0.20\ \text{m/s})(0.004\ \text{m})$$

$$\textit{Solve for}\quad a = 0.0148\ \text{m},\quad \textbf{\textit{Spheroid length}} = 2a = \mathbf{0.030\ m} \quad \textit{Ans.}\ \text{(b)}$$

P7.53 From Table 7.2, the drag coefficient of a wide plate normal to a stream is approximately 2.0. Let the stream conditions be U_∞ and p_∞. If the average pressure on the front of the plate is approximately equal to the free-stream stagnation pressure, what is the average pressure on the rear?

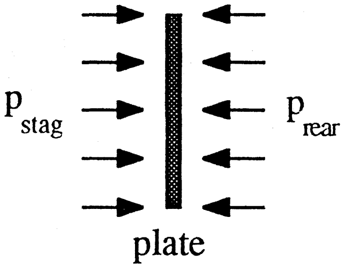


Fig. P7.53

Solution: If the drag coefficient is 2.0, then our approximation is

$$\mathrm{F_{drag}} = 2.0\frac{\rho}{2}\mathrm{U}_\infty^2\mathrm{A_{plate}} \overset{?}{\approx} (\mathrm{p_{stag}} - \mathrm{p_{rear}})\mathrm{A_{plate}}, \quad \text{or:} \quad \mathrm{p_{rear}} \approx \mathrm{p_{stag}} - \rho\mathrm{U}_\infty^2$$

Since, from Bernoulli, $\mathrm{p_{stag}} = \mathrm{p}_\infty + \frac{\rho}{2}\mathrm{U}_\infty^2$, we obtain $\mathbf{p_{rear} \approx p_\infty - \frac{\rho}{2}U_\infty^2}$ *Ans.*

P7.54 If a missile takes off vertically from sea level and leaves the atmosphere, it has zero drag when it starts and zero drag when it finishes. It follows that the drag must be a maximum somewhere in between. To simplify the analysis, assume a constant drag coefficient, C_D, and a constant vertical acceleration, a. Let the density variation be modeled by the troposphere relation, Eq. (2.20). Find an expression for the altitude z^* where the drag is a maximum. Comment on your result.

Solution: For constant acceleration and C_D, the drag follows simple formulas:

$$F = \text{Drag} = C_D\frac{\rho}{2}V^2 A \ , \ V = at \ , \ z = \frac{1}{2}at^2, \ \ V^2 = a^2t^2 = 2az$$

where A is the missile reference area and z is the altitude. The density is given by Eq. (2.20):

$$\frac{\rho}{\rho_o} = (1 - \frac{Bz}{T_o})^n \ , \quad \textit{where} \quad n = \frac{g}{RB} - 1 \approx 4.26$$

Combine these, noting that only ρ and V vary, and write the result in terms of z:

$$F = C_D \frac{\rho_o}{2}(1-\frac{Bz}{T_o})^n (2az)A = Kz(1-\frac{Bz}{T_o})^n, \quad K = C_D \rho_o a A$$

$$\text{Minimize: } \frac{dF}{dz} = 0 = K(1-\frac{Bz}{T_o})^n + Kz(1-\frac{Bz}{T_o})^{n-1}(-\frac{B}{T_o})$$

$$\text{Simplify and rearrange: } z_{\text{max drag}} = z^* = \frac{T_o}{B(n+1)} \qquad \textit{Ans.}$$

For these simplifications, the point of maximum drag, z*, is dependent only upon ground temperature T_o, the lapse rate B, and the exponent n. For the standard atmosphere of Eq. (2.20), we obtain z^* = (288.16 K)/[0.0065 K/m)(4.26+1)] = **8,430** meters.

P7.55 A ship tows a submerged cylinder, 1.5 m in diameter and 22 m long, at U = 5 m/s in fresh water at 20°C. Estimate the towing power in kW if the cylinder is (a) parallel, and (b) normal to the tow direction.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s.

$$\text{(a) Parallel, } \frac{L}{D} \approx 15, \quad Re_L = \frac{998(5)(22)}{0.001} = 1.1E8, \quad \text{Table 7.3: estimate } C_{D,\text{frontal}} \approx 1.1$$

$$F = 1.1\left(\frac{998}{2}\right)(5)^2\left(\frac{\pi}{4}\right)(1.5)^2 \approx 24000 \text{ N}, \quad \text{Power} = FU \approx \mathbf{120\ kW} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) Normal, } Re_D = \frac{998(5)(1.5)}{0.001} = 7.5E6, \quad \text{Fig. 7.16a: } C_{D,\text{frontal}} \approx 0.4$$

$$F = 0.4\left(\frac{998}{2}\right)(5)^2(1.5)(22) \approx 165000 \text{ N}, \quad \text{Power} = FU \approx \mathbf{800\ kW} \quad \textit{Ans.}\text{ (b)}$$

P7.56 A delivery vehicle carries a long sign on top, as in Fig. P7.56. If the sign is very thin and the vehicle moves at 65 mi/h, (a) estimate the force on the sign with no crosswind. (b) Discuss the effect of a crosswind.

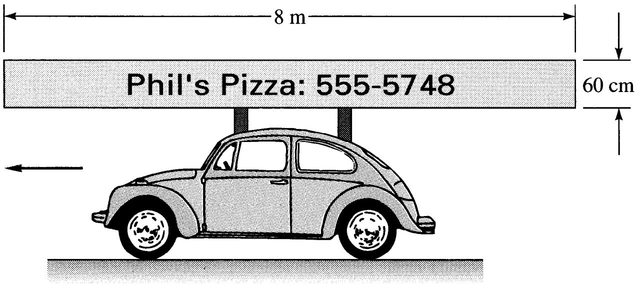


Fig. P7.56

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}-5$ kg/m·s. Convert 65 mi/h = 29.06 m/s. (a) If there is no crosswind, we may estimate the drag force by flat-plate theory:

$$\mathrm{Re}_L = \frac{1.2(29.06)(8)}{1.8E\text{–}5} = 1.55E7\ (turbulent), \quad C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{0.031}{(1.55E7)^{1/7}} \approx 0.00291$$

$$F_{drag} = C_D\left(\frac{\rho}{2}\right)V^2bL(2\ sides) = 0.00291\left(\frac{1.2}{2}\right)(29.06)^2(0.6)(8)(2\ sides) = \mathbf{14\ N} \quad Ans.\ \text{(a)}$$

(b) A crosswind will cause a large side force on the sign, greater than the flat-plate drag. The sign will act like an airfoil. For example, if the 29 m/s wind is at an angle of only 5° with respect to the sign, from Eq. (7.70), $C_L \approx 2\pi\sin(5°)/(1 + 2/.075) \approx 0.02$. The lift on the sign is then about

$$\text{Lift} = C_L(\rho/2)V^2bL \approx (0.02)(1.2/2)(29.06)^2(0.6)(8) \approx \mathbf{50\ N} \quad Ans.\ \text{(b)}$$

P7.57 The main cross-cable between towers of a coastal suspension bridge is 60 cm in diameter and 90 m long. Estimate the total drag force on this cable in crosswinds of 50 mi/h. Are these laminar-flow conditions?

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. Convert 50 mi/h = 22.35 m/s. Check the Reynolds number of the cable:

$$\text{Re}_\text{D} = \frac{1.2(22.34)(0.6)}{1.8\text{E}{-5}} \approx 894000 \text{ (turbulent flow)} \quad \text{Fig. 7.16a: } \text{C}_\text{D} \approx 0.3$$

$$\text{F}_\text{drag} = \text{C}_\text{D}\frac{\rho}{2}\text{U}^2\text{DL} = 0.3\left(\frac{1.2}{2}\right)(22.35)^2(0.6)(90) \approx \mathbf{5000\ N} \quad (\textbf{not laminar}) \quad \textit{Ans.}$$

***P7.58** Modify Prob. P7.54 to be more realistic by accounting for missile drag during ascent. Assume constant thrust T and missile weight W. Neglect the variation of g with altitude. Solve for the altitude z* in the Standard Atmosphere where the drag is a maximum, for T = 40,000 N, W = 8,000 N, and $C_\text{D}A = 0.4$ m^2. The writer does not believe an analytic solution is practical.

Solution: Summation of vertical forces gives the (variable) acceleration:

$$\sum F_z = T - W - F_{drag} = m\,a = \frac{W}{g}\frac{dV}{dt}\ ,$$

$$\text{or: } \frac{dV}{dt} = g\left(\frac{T}{W} - 1\right) - \frac{gC_D A\rho_o}{2W}\left(1 - \frac{Bz}{T_o}\right)^n V^2\ , \text{ where } V = \frac{dz}{dt}$$

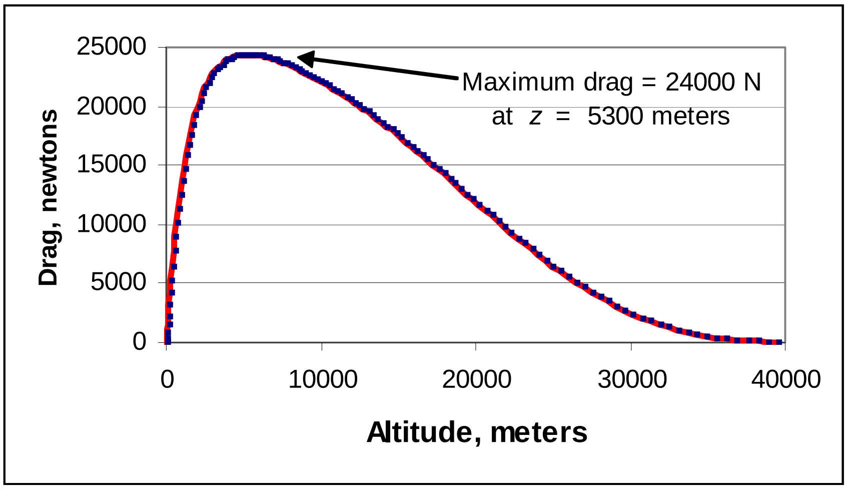
The power of the z term makes this a cumbersome ordinary differential equation. Even if we rewrite it in terms of z alone, the writer cannot handle it. Enter the given numerical

$$\frac{dV}{dt} = 9.81\left[\left(\frac{40000}{8000} - 1\right) - \frac{0.4(1.2255)}{2(8000)}\left(1 - \frac{0.0065\,z}{288.16}\right)^{4.26} V^2\right]$$

$$= 39.24\frac{m}{s^2} - 0.003006\left(1 - \frac{0.0065\,z}{288.16}\right)^{4.26} V^2$$

data:

Solve numerically with $V = 0$ at $z = 0$. The results are shown in the plot below. The maximum drag is about 24,000 newtons at an altitude of 5300 meters. Since the drag contributes to the now-variable acceleration, the simple result of Prob. 7.54 is not accurate.



***P7.59** Joe can pedal his bike at 10 m/s on a straight, level road with no wind. The bike rolling resistance is 0.80 N/(m/s), i.e. 0.8 N per m/s of speed. The drag area C_DA of Joe and his bike is 0.422 m^2. Joe's mass is 80 kg and the bike mass is 15 kg. He now encounters a head wind of 5.0 m/s. (a) Develop an equation for the speed at which Joe can pedal into the wind. (*Hint*: A cubic equation.) (b) Solve for V for this head wind. (c) Why is the result not simply V = 10 – 5 = 5 m/s, as one might first suspect?

Solution: Evaluate force and power with the drag based on *relative* velocity V + V_{wind}:

$$\sum F = F_{rolling} + F_{drag} = C_{RR}V + C_DA\frac{\rho}{2}(V + V_{wind})^2$$

$$Power = V\sum F = C_{RR}V^2 + C_DA\frac{\rho}{2}V(V + V_{wind})^2$$

Let V_{nw} (=10 m/s) be the bike speed with no wind and denote $V_{rel} = V + V_{wind}$. Joe's power output will be the same with or without the headwind:

$$P_{nw} = C_{RR}V_{nw}^2 + C_DA\frac{\rho}{2}V_{nw}^3 = P = C_{RR}V^2 + C_DA\frac{\rho}{2}VV_{rel}^2,$$

$$or:\quad V^3 + \left(2V_w + \frac{2C_{RR}}{\rho C_DA}\right)V^2 + \left(V_w^2\right)V - \left(V_{nw}^3 + \frac{2C_{RR}}{\rho C_DA}V_{nw}^2\right) = 0 \quad Ans. \text{ (a)}$$

For our given numbers, assuming ρ_{air} = 1.2 kg/m^3, the result is the cubic equation

$$V^3 + 13.16V^2 + 25V - 1316 = 0, \quad \textit{solve for } \mathbf{V \approx 7.4\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(b)}$$

Since drag is proportional to V_{rel}^2, a linear transformation V = Vnw − Vwind is ***not*** possible. Even if there were no rolling resistance, V ≈ *7.0* m/s, not 5.0 m/s. *Ans.* (c)

P7.60 A fishnet consists of 1-mm-diameter strings overlapped and knotted to form 1- by 1-cm squares. Estimate the drag of 1 m^2 of such a net when towed normal to its plane at 3 m/s in 20°C seawater. What horsepower is required to tow 400 ft^2 of this net?

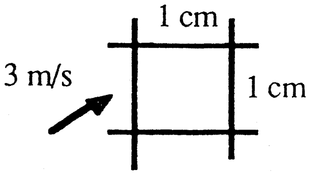


Fig. P7.60

Solution: For seawater at 20°C, take $\rho = 1025$ kg/m^3 and $\mu = 0.00107$ kg/m·s. Neglect the *knots* at the net's intersections. Estimate the drag of a single one-centimeter strand:

$$\mathrm{Re_D} = \frac{1025(3)(0.001)}{0.00107} \approx 2900; \quad \text{Fig. 7.16a or Fig. 5.3a: } \mathrm{C_D} \approx 1.0$$

$$\mathrm{F_{one\ strand}} = \mathrm{C_D}\frac{\rho}{2}\mathrm{U^2DL} = (1.0)\left(\frac{1025}{2}\right)(3)^2(0.001)(0.01) \approx 0.046 \text{ N/strand}$$

$$\text{one m}^2 \text{ contains } \textit{20,000} \text{ strands:} \quad \mathrm{F_{1\ sq\ m}} \approx 20000(0.046) \approx \mathbf{920\ N} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{To tow } 400 \text{ ft}^2 = 37.2 \text{ m}^2 \text{ of net,} \quad \mathrm{F} = 37.2(920) \approx 34000 \text{ N} \approx 7700 \text{ lbf}$$

$$\text{If } \mathrm{U} = 3\ \frac{\mathrm{m}}{\mathrm{s}} = 9.84\ \frac{\mathrm{ft}}{\mathrm{s}}, \quad \text{Tow Power} = \mathrm{FU} = (7700)(9.84) \div 550 \approx \mathbf{140\ hp} \quad \textit{Ans.}\ \text{(b)}$$

P7.61 A filter may be idealized as an array of cylindrical fibers normal to the flow, as in Fig. P7.61. Assuming that the fibers are uniformly distributed and have drag coefficients given by Fig 7.16*a*, derive an approximate expression for the pressure drop Δp through a filter of thickness L.

Solution: Consider a filter section of height H and width b and thickness L. Let N be the number of fibers of diameter D per unit area HL of filter. Then the drag of all these filters must be balanced by a pressure Δp across the filter:

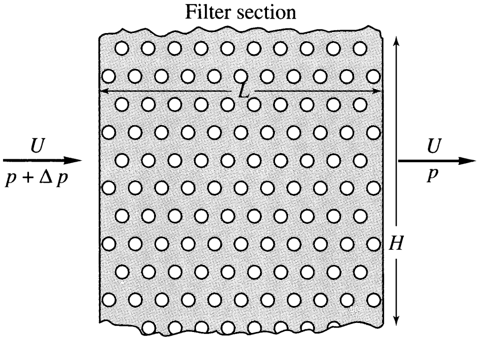


Fig. P7.61

$$\Delta pHb = \sum F_{fibers} = NHLC_D\frac{\rho}{2}U^2Db, \quad \text{or:} \quad \Delta p_{filter} \approx \mathbf{NLC_D\frac{\rho}{2}U^2D} \quad \textit{Ans.}$$

This simple expression does not account for the *blockage* of the filters, that is, in cylinder arrays one must increase "U" by $1/(1-\sigma)$, where σ is the solidity ratio of the filter.

P7.62 A sea-level smokestack is 52 m high and has a *square* cross-section. Its supports can withstand a maximum side force of 90 kN. If the stack is to survive 90 mi/h hurricane winds, what is its maximum possible (square) width?

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E–5 kg/m·s. Convert 90 mi/h = 40.2 m/s. We cannot compute Re without knowing the side length a, so we assume that Re > 1E4 and that Table 7.2 is valid. The worst case drag is when the square cylinder has its *flat* face forward, $C_D \approx 2.1$. Then the drag force is

$$F = C_D\frac{\rho}{2}U^2aL = 2.1\left(\frac{1.225}{2}\right)(40.2)^2a(52) \overset{?}{=} 90000\text{ N},\quad \text{solve } \mathbf{a \approx 0.83\ m} \quad \textit{Ans.}$$

$$\text{Check } Re_a = (1.225)(40.2)(0.83)/(1.78\text{E}-5) \approx 2.3\text{E}6 > 1\text{E}4,\ \text{OK.}$$

P7.63 For those who think electric cars are sissy, Keio University in Japan has tested a 22-ft long prototype whose eight electric motors generate a total of 590 horsepower. The "Kaz" cruises at 180 mi/h (see *Popular Science*, August 2001, p. 15). If the drag coefficient is 0.35 and the frontal area is 26 ft^2, what percent of this power is expended against sea-level air drag?

Solution: For air, take ρ = 0.00237 slug/ft^3. Convert 180 mi/h to 264 ft/s. The drag is

$$F = C_D\frac{\rho}{2}V^2A_{frontal} = (0.35)\left(\frac{0.00237\text{ slug/ft}^3}{2}\right)(264\text{ ft/s})^2(26\text{ ft}^2) = 752\text{ lbf}$$

$$Power = FV = (752\text{ lbf})(264\text{ ft/s})/(550\text{ ft}\cdot\text{lbf/hp}) = \mathbf{361\ hp}$$

The horsepower to overcome drag is **61% of the total 590** horsepower available. *Ans.*

P7.64 A parachutist jumps from a plane, using an 8.5-m-diameter chute in the standard atmosphere. The total mass of chutist and chute is 90 kg. Assuming a fully open chute in quasisteady motion, estimate the time to fall from 2000 to 1000 m.

Solution: For the standard altitude (Table A-6), read ρ = 1.112 kg/m^3 at 1000 m altitude and ρ = 1.0067 kg/m^3 at 2000 meters. Viscosity is not a factor in Table 7.3, where we read $C_D \approx 1.2$ for a low-porosity chute. If acceleration is negligible,

$$W = C_D\frac{\rho}{2}U^2\frac{\pi}{4}D^2,\quad or:\quad 90(9.81)\ N = 1.2\left(\frac{\rho}{2}\right)U^2\frac{\pi}{4}(8.5)^2,\quad or:\quad U^2 = \frac{25.93}{\rho}$$

$$\text{Thus}\quad U_{1000\ m} = \sqrt{\frac{25.93}{1.1120}} = \mathbf{4.83\ \frac{m}{s}} \quad\text{and}\quad U_{2000\ m} = \sqrt{\frac{25.93}{1.0067}} = \mathbf{5.08\ \frac{m}{s}}$$

Thus the change in velocity is very small (an average deceleration of only –0.001 m/s^2) so we can reasonably estimate the time-to-fall using the average fall velocity:

$$\Delta t_{fall} = \frac{\Delta z}{V_{avg}} = \frac{2000-1000}{(4.83+5.08)/2} \approx \mathbf{202\ s} \quad Ans.$$

P7.65 As soldiers get bigger and packs get heavier, a parachutist and load can weigh as much as 400 lbf. The standard 28-ft parachute may descend too fast for safety. For heavier loads, the U.S. Army Natick Center has developed a 28-ft, higher drag, less porous XT-11 parachute (see the URL http://www.paraflite.com/html/advancedparachute.html). This parachute has a sea-level descent speed of 16 ft/s with a 400-lbf load. (a) What is the drag coefficient of the XT-11? (b) How fast would the standard chute descend at sea-level with such a load?

Solution: For sea-level air, take $\rho = 0.00237$ slug/ft^3. (a) Everything is known except C_D:

$$F = C_D \frac{\rho}{2} V^2 A = 400\ \text{lbf} = C_D \frac{0.00237\ \text{slug/ft}^3}{2} (16\ \text{ft/s})^2 \frac{\pi}{4} (28\ \text{ft})^2$$

$$Solve\ for \quad C_{D,new\ chute} = \mathbf{2.14} \quad Ans.\ \text{(a)}$$

(b) From Table 7.3, a standard chute has a drag coefficient of about 1.2. Then solve for *V*:

$$F = C_D \frac{\rho}{2} V^2 A = 400\ \text{lbf} = (1.2) \frac{0.00237\ \text{slug/ft}^3}{2} V^2 \frac{\pi}{4} (28\ \text{ft})^2$$

$$Solve\ for \quad V_{old\ chute} = \mathbf{21.4\ ft/s} \quad Ans.\ \text{(b)}$$

***P7.66** A sphere of density ρ_s and diameter *D* is dropped from rest in a fluid of density ρ and viscosity μ. Assuming a constant drag coefficient C_{d_o}, derive a differential equation for the fall velocity *V*(*t*) and show that the solution is

$$V = \left[\frac{4gD(S-1)}{3C_{d_o}}\right]^{1/2} \tanh Ct$$

$$C = \left[\frac{3gC_{d_o}(S-1)}{4S^2 D}\right]^{1/2}$$

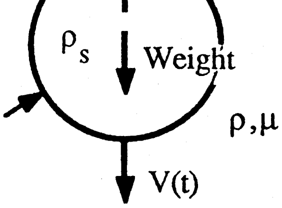


Fig. P7.66

where $S = \rho_s/\rho$ is the specific gravity of the sphere material.

Solution: Newton's law for downward motion gives

$$\sum F_{down} = ma_{down}, \quad \text{or:} \quad W - B - C_D\frac{\rho}{2}V^2A = \frac{W}{g}\frac{dV}{dt}, \quad \text{where } A = \frac{\pi}{4}D^2$$

$$\text{and} \quad W - B = \rho(S-1)g\frac{\pi}{6}D^3. \quad \text{Rearrange to } \frac{dV}{dt} = \beta - \alpha V^2,$$

$$\beta = g\left(1 - \frac{1}{S}\right) \quad \text{and} \quad \alpha = \frac{\rho g C_D A}{2W}$$

Separate the variables and integrate from rest, V = 0 at t = 0: $\int dt = \int dV/(\beta - \alpha V^2)$,

$$\text{or:} \quad V = \sqrt{\frac{\beta}{\alpha}}\tanh\left(t\sqrt{\alpha\beta}\right) = \mathbf{V_{final}tanh(Ct)} \quad \textit{Ans.}$$

$$\text{where} \quad V_{final} = \left[\frac{4gD(S-1)}{3C_D}\right]^{1/2} \quad \text{and} \quad C = \left[\frac{3gC_D(S-1)}{4S^2D}\right]^{1/2}, \quad S = \frac{\rho_s}{\rho} > 1$$

P7.67 A world-class bicycle rider can generate one-half horsepower for long periods. If racing at sea-level, estimate the velocity which this cyclist can maintain. Neglect rolling friction.

Solution: For sea-level air, take $\rho = 1.22$ kg/m^3. From Table 7.3 for a bicycle with a rider in the racing position, $C_DA \approx 0.30$ m^2. With power known, we can solve for speed:

$$Power = FV = \left(C_DA\frac{\rho}{2}V^2\right)V = 0.5 \text{ hp} = 373\ W = (0.3 \text{ m}^2)\frac{1.22 \text{ kg/m}^3}{2}V^3$$

$$\textit{Solve for} \quad V = \mathbf{12.7\ m/s\ \ (\textit{about}\ 28\ mi/h)} \quad \textit{Ans.}$$

P7.68 The Mars roving-laboratory parachute, in the Chap. 5 opener photo, is a 55-ft-diameter disk-gap-band chute, with a measured drag coefficient of 1.12 [59]. Mars has very low density, about 2.9E-5 slug/ft^3, and its gravity is only 38% of earth gravity. If the mass of payload and chute is 2400 kg, estimate the terminal fall velocity of the parachute.

Solution: Convert D = 55 ft = 16.8 m and r = 2.9E-5 slug/ft^3 = 0.015 kg/m^3. At terminal velocity, the parachute weight is balanced by chute drag:

$$W = mg = (2400)[0.38(9.81)] = 8950\,N = C_D \frac{\rho}{2} V^2 \frac{\pi}{4} D^2 = (1.12)(\frac{0.015}{2}) V^2 \frac{\pi}{4} (16.8)^2$$

$$\text{or:} \quad 8950\,N = 1.86 V^2, \qquad \text{solve} \quad V_{\text{terminal}} = \mathbf{69}\,\frac{m}{s} = 155\,\frac{mi}{h} \quad \textit{Ans.}$$

This is very fast (!), but, after all, Mars atmosphere is very thin. After reaching this fall velocity, the payload is further decelerated by retrorockets.

P7.69 Two baseballs, of diameter 7.35 cm, are connected to a rod 7 mm in diameter and 56 cm long, as in Fig. P7.69. What power, in W, is required to keep the system spinning at 400 r/min? Include the drag of the rod, and assume sea-level standard air.

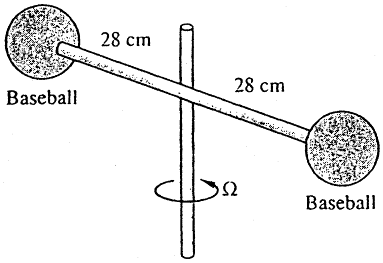


Fig. P7.69

Solution: For sea-level air, take $\rho = 1.225$ kg/m^3 and $\mu = 1.78E{-}5$ kg/m·s. Assume a laminar drag coefficient $C_D \approx 0.47$ from Table 7.3. Convert $\Omega = 400$ rpm × $2\pi/60 = 41.9$ rad/s. Each ball moves at a centerline velocity

$$V_b = \Omega r_b = (41.9)(0.28 + 0.0735/2) \approx 13.3 \text{ m/s}$$

$$\text{Check} \quad \text{Re} = 1.225(13.3)(0.0735)/(1.78E{-}5) \approx 67000; \quad \text{Table 7.3: } C_D \approx 0.47$$

Then the drag force on each baseball is approximately

$$F_b = C_D \frac{\rho}{2} V_b^2 \frac{\pi}{4} D^2 = 0.47\left(\frac{1.225}{2}\right)(13.3)^2 \frac{\pi}{4}(0.0735)^2 \approx 0.215 \text{ N}$$

Make a similar approximate estimate for the drag of each rod:

$$V_r = \Omega r_{avg} = 41.9(0.14) \approx 5.86 \frac{\text{m}}{\text{s}}, \quad \text{Re} = \frac{1.225(5.86)(0.007)}{1.78E{-}5} \approx 2800, \quad C_D \approx 1.2$$

$$F_{rod} \approx C_D\left(\frac{\rho}{2}\right)V_r^2 DL = 1.2\left(\frac{1.225}{2}\right)(5.86)^2(0.007)(0.28) \approx 0.0495 \text{ N}$$

Then, with two balls and two rods, the total driving power required is

$$P = 2F_bV_b + 2F_rV_r = 2(0.215)(13.3) + 2(0.0495)(5.86) = 5.71 + 0.58 \approx \mathbf{6.3\ W} \quad \textit{Ans.}$$

P7.70 The Army's new ATPS personnel parachute is said to be able to bring a 400-lbf load, trooper plus pack, to ground at 16 ft/s in "mile-high" Denver, Colorado. If we assume that Table 7.3 is valid, what is the approximate diameter of this new parachute?

Solution: Assume that Denver is 5280 ft = 1609 m standard altitude. From Table A.6, interpolate $\rho = 1.046$ kg/m^3 = 0.00203 slug/ft^3. From Table 7.3, $C_D \approx 1.2$ for a parachute. Then the force balance is

$$W = 400\,lbf = Drag = C_D \frac{\rho}{2}V^2 \frac{\pi}{4}D^2 = 1.2\left(\frac{0.00203 slug/ft^3}{2}\right)\left(16\frac{ft}{s}\right)^2 \frac{\pi}{4}D^2$$

$$\textit{Solve for} \quad D \approx \mathbf{40\ ft} \qquad \textit{Ans.}$$

P7.71 The 2009 Ford Escape Hybrid has an empty weight of 3669 lbf, a drag-area C_DA = 11.6 ft^2 [21], and a rolling resistance coefficient RRC (without brakes) of 0.88 lbf per mi/h of speed. Estimate the maximum velocity this vehicle can attain when rolling freely, at sea-level conditions, down a 5-degree slope.

Solution: For sea-level air, take ρ = 0.00238 slug/ft^3. Convert the RRC to English units: 0.88 lbf/(mi/h) = 0.70 lbf/(ft/s). At maximum (terminal) velocity, the rolling resistance and the air drag balance the vehicle weight component along the 5° slope:

$$W \sin\theta \;=\; (RRC)\,V \;+\; (C_D A)\frac{\rho}{2}V^2 \;\;, \;\; \text{or:}$$

$$(3669\,lbf)\sin(5^o) \;=\; (0.70\frac{lbf}{ft/s})V \;+\; (11.6\;ft^2)(\frac{0.00238\,slug/ft^3}{2})\,V^2$$

$$Solve\;\; for \qquad V \;=\; 129\;ft/s \;=\; \mathbf{88}\;mi/h \qquad\qquad Ans.$$

At this (surprisingly high) terminal speed, the drag force is still double the rolling resistance.

P7.72 A settling tank for a municipal water supply is 2.5 m deep, and 20°C water flows through continuously at 35 cm/s. Estimate the minimum length of the tank which will ensure that all sediment (SG = 2.55) will fall to the bottom for particle diameters greater than (a) 1 mm and (b) 100 μm.

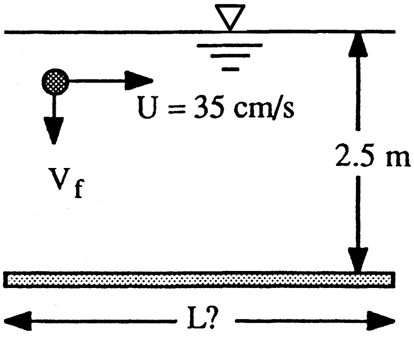


Fig. P7.72

Solution: For water at 20°C, take $\rho = 998\text{ kg/m}^3$ and $\mu = 0.001$ kg/m·s. The particles travel with the stream flow U = 35 cm/s (no horizontal drag) and fall at speed V_f with drag equal to their net weight in water:

$$W_{net} = (SG-1)\rho_w g\frac{\pi}{6}D^3 = Drag = C_D\frac{\rho_w}{2}V_f^2\frac{\pi}{4}D^2, \quad \text{or:} \quad V_f^2 = \frac{4(SG-1)gD}{3C_D}$$

where $C_D = fcn(Re_D)$ from Fig. 7.16b. Then $L = Uh/V_f$ where $h = 2.5$ m.

$$\text{(a)}\ \ D = 1\text{ mm: } V_f^2 = \frac{4(2.55-1)(9.81)(0.001)}{3C_D}, \quad \text{iterate Fig. 7.16b to } C_D \approx 1.0,$$

$$Re_D \approx 140,\ V_f \approx 0.14\text{ m/s}, \quad \text{hence } L = Uh/V_f = \frac{(0.35)(2.5)}{0.14} \approx \mathbf{6.3\ m} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\ \ D = 100\ \mu\text{m: } V_f^2 = \frac{4(2.55-1)(9.81)(0.0001)}{3C_D}, \quad \text{iterate Fig. 7.16b to } C_D \approx 36,$$

$$Re_D \approx 0.75, \quad V_f \approx 0.0075\text{ m/s}, \quad L = \frac{0.35(2.5)}{0.0075} \approx \mathbf{120\ m} \quad \textit{Ans.}\text{ (b)}$$

P7.73 A balloon is 4 m in diameter and contains helium at 125 kPa and 15°C. Balloon material and payload weigh 200 N, not including the helium. Estimate (a) the terminal ascent velocity in sea-level standard air; (b) the final standard altitude (neglecting winds) at which the balloon will come to rest; and (c) the minimum diameter (<4 m) for which the balloon will just barely begin to rise in sea-level air.

Solution: For sea-level air, take $\rho = 1.225\text{ kg/m}^3$ and $\mu = 1.78\text{E}{-5}$ kg/m·s. For helium $R = 2077$ J/kg·°K. Sea-level air pressure is 101350 Pa. For upward motion V,

$$\textit{Net buoyancy} = \textit{weight} + \textit{drag}, \quad \textit{or:} \quad (\rho_{air} - \rho_{He})g\frac{\pi}{6}D^3 = W + C_D\frac{\rho}{2}V^2\frac{\pi}{4}D^2$$

$$\textit{or:} \quad \left[1.225 - \frac{125000}{2077(288)}\right](9.81)\frac{\pi}{6}(4)^3 = 200 + C_D\left(\frac{1.225}{2}\right)V^2\frac{\pi}{4}(4)^2$$

Guess *turbulent* flow: $C_D \approx 0.2$: Solve for **V ≈ 9.33 m/s** *Ans.* (a)

Check $Re_D = 2.6\text{E}6$: OK, turbulent flow.

(b) If the balloon comes to rest, buoyancy will equal weight, with no drag:

$$\left[\rho_{air} - \frac{125000}{2077(288)}\right](9.81)\ \frac{\pi}{6}(4)^3 = 200,$$

$$\text{Solve:}\quad \rho_{air} \approx 0.817\ \frac{\text{kg}}{\text{m}^3},\quad \mathbf{Z_{Table\ A6} \approx 4000\ m}\quad \textit{Ans.}\ \text{(b)}$$

(c) If it just begins to rise at sea-level, buoyancy will be slightly greater than weight:

$$\left[1.225 - \frac{125000}{2077(288)}\right](9.81)\frac{\pi}{6}D^3 > 200,\quad \textit{or}:\quad \mathbf{D > 3.37\ m}\quad \textit{Ans.}\ \text{(c)}$$

P7.74 It is difficult to define the "frontal area" of a motorcycle due to its complex shape. One then measures the *drag-area*, that is, C_DA, in area units. Hoerner [12] reports the drag-area of a typical motorcycle, including the (upright) driver, as about 5.5 ft^2. Rolling friction is typically about 0.7 lbf per mi/h of speed. If that is the case, estimate the maximum sea-level speed (in mi/h) of the new Harley-Davidson *V-Rod*™ cycle, whose liquid-cooled engine produces 115 hp.

Solution: For sea-level air, take ρ = 0.00237 slug/ft^3. Convert 0.7 lbf per mi/h rolling friction to 0.477 lbf per ft/s of speed. Then the power relationship for the cycle is

$$Power = (F_{dr} + F_{roll})V = \left(C_D A\frac{\rho}{2}V^2 + C_{roll}V\right)V,$$

$$\text{or:}\quad 115{*}550\ \frac{\text{ft-lbf}}{\text{s}} = \left[(5.5\ \text{ft}^2)\frac{0.00237\ \text{slug/ft}^3}{2}V^2 + \left(0.477\ \frac{\text{lbf}}{\text{ft/s}}\right)V\right]V$$

Solve this cubic equation, by iteration or EES, to find V_{max} ≈ 192 ft/s ≈ **131 mi/h**. *Ans.*

P7.75 The helium-filled balloon in Fig. P7.75 is tethered at 20°C and 1 atm with a string of negligible weight and drag. The diameter is 50 cm, and the balloon material weighs 0.2 N, not including the helium. The helium pressure is 120 kPa. Estimate the tilt angle θ if the airstream velocity U is (a) 5 m/s or (b) 20 m/s.

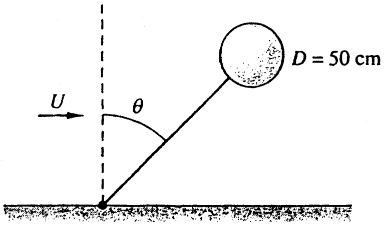


Fig. P7.75

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. For helium, R = 2077 J/kg·°K. The helium density = (120000)/[2077(293)] ≈ 0.197 kg/m^3.

The balloon net buoyancy is independent of the flow velocity:

$$B_{net} = (\rho_{air} - \rho_{He})g\frac{\pi}{6}D^3 = (1.2 - 0.197)(9.81)\frac{\pi}{6}(0.5)^3 \approx 0.644 \text{ N}$$

The net upward force is thus Fz = (Bnet – W) = 0.644 – 0.2 = *0.444* N. The balloon drag *does* depend upon velocity. At 5 m/s, we expect laminar flow:

$$\text{(a) } U = 5\ \frac{m}{s}:\ \ \mathrm{Re}_D = \frac{1.2(5)(0.5)}{1.8E\text{–}5} = 167000; \quad \textit{Table } 7.3:\ C_D \approx 0.47$$

$$Drag = C_D\frac{\rho}{2}U^2\frac{\pi}{4}D^2 = 0.47\left(\frac{1.2}{2}\right)(5)^2\frac{\pi}{4}(0.5)^2 \approx 1.384 \text{ N}$$

$$\text{Then} \quad \theta_a = \tan^{-1}\left(\frac{Drag}{F_z}\right) = \tan^{-1}\left(\frac{1.384}{0.444}\right) = \mathbf{72°} \quad \textit{Ans.}\text{ (a)}$$

(b) At 20 m/s, Re = 667000 (*turbulent*), Table 7.3: C_D ≈ 0.2:

$$Drag = 0.2\left(\frac{1.2}{2}\right)(20)^2\frac{\pi}{4}(0.5)^2 = 9.43 \text{ N}, \quad \theta_b = \tan^{-1}\left(\frac{9.43}{0.444}\right) = \mathbf{87°} \quad \textit{Ans.}\text{ (b)}$$

These angles are too steep—the balloon needs more buoyancy and/or less drag.

P7.76 The 2005 movie *The World's Fastest Indian* tells the story of Burt Munro, a New Zealander who, in 1937, set a motorcycle record of 201 mi/h on the Bonneville Salt Flats. Using the data of Prob. P7.74, (a) estimate the horsepower needed to drive this fast. (*b*) What horsepower would have gotten Burt up to 250 mi/h?

Solution: Prob. P7.74 suggests C_DA = 5.5 ft^2 and F_{rolling} = 0.7 lbf per mi/h of speed. Convert 201 mi/hr to 295 ft/s. Bonneville is at 4300 ft altitude, so take ρ = 1.0784 kg/m^3 = 0.00209 slug/ft^3 from Table A.6. Now compute the total resistance force:

$$F = F_{drag} + F_{rolling} = (C_DA)\frac{\rho}{2}V^2 + 0.7V_{mi/h}$$

$$= (5.5\,ft^2)\left(\frac{0.00209 slug/ft^3}{2}\right)(295\,ft/s)^2 + 0.7(201) = 500 + 141 = 641 lbf$$

$$Power = FV = (641 lbf)(295\,ft/s) = 189{,}000\ ft-lbf/s \div 550 = \mathbf{343\,hp} \quad \textit{Ans.}(a)$$

A lot of power! Presumably Burt did some streamlining to reduce drag.

(*b*) Repeat this for V = 250 mi/h = 367 ft/s to get F = 914 lbf, Power = **610 hp**. *Ans*. (*b*)

P7.77 To measure the drag of an upright person, without violating human-subject protocols, a life-sized mannequin is attached to the end of a 6-m rod and rotated at Ω = 80 rev/min, as in Fig. P7.77. The power required to maintain the rotation is 60 kW. By including rod-drag power, which is significant, estimate the drag-area C_DA of the mannequin, in m^2.

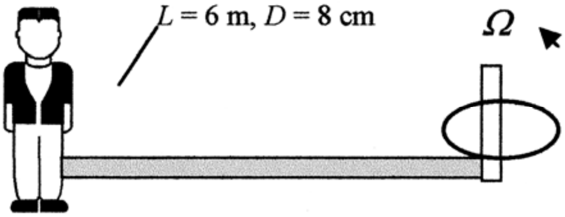


Fig. P7.77

Solution: For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. The mannequin velocity is $V_m = \Omega L$ = [(80 × 2π/60)rad/s](6m) ≈ (8.38 rad/s)(6 m) ≈ 50.3 m/s. The velocity at mid-span of the rod is $\Omega L/2$ = 25 m/s. Crudely estimate the power to rotate the rod:

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(25\text{ m/s})(0.08\text{ m})}{0.000015\text{ m}^2/\text{s}} = 133000, \quad \textit{Table } 7.2\text{: } C_{D,rod} \approx 1.2$$

$$P_{rod} \approx \int_0^L \left(C_D \frac{\rho}{2}\Omega^2 r^2 D\, dr \right) r \approx C_D \frac{\rho}{2}\Omega^2 D \frac{L^4}{4}$$

$$\text{Input the data:} \quad P_{rod} = (1.2)\frac{1.2}{2}(8.38)^2(0.08)\frac{6.0^4}{4} \approx 1300\ W$$

Rod power is thus only about 2% of the total power. The total power relation is:

$$P = P_{rod} + (C_DA)_{man}\left(\frac{\rho}{2}\right)V^3 = 1300 + (C_DA)_{man}\left(\frac{1.2}{2}\right)(50.3)^3 = 60000\ W$$

Solve for $\mathbf{(C_DA)_{mannequin} \approx 0.77\ m^2}$ *Ans.*

P7.78 On April 24, 2007, a French bullet train set a new record, for rail-driven trains, of 357.2 mi/h, beating the old record by 12 per cent. Using the data in Table 7.3, estimate the sea-level horsepower required to drive this train at such a speed.

Solution: Take sea-level density as 1.2255 kg/m^3. Convert 357.2 mi/h to 159.7 m/s. From Table 7.3, the drag-area of a high-speed train is about $C_D A \approx 8.5$ m^2. Thus the drag force is

$$F = (C_D A)\,\frac{\rho}{2}V^2 = (8.5\ m^2)\left(\frac{1.2255\,kg/m^3}{2}\right)\left(159.7\,\frac{m}{s}\right)^2 = 133{,}000\,N$$

$$\text{Power required} = F\,V = (133{,}000)(159.7) = 21.2\ MW/745.7 = \mathbf{28{,}400\,hp} \quad \textit{Ans.}$$

P7.79 A radioactive dust particle approximate a sphere with a density of 2400 kg/m^3. How long, in days, will it take the particle to settle to sea level from 12 km altitude if the particle diameter is (a) 1 μm; (b) 20 μm?

Solution: For such small particles, tentatively assume that Stokes' law prevails:

$$F_{drag} \approx 3\pi\mu DV = W_{net} = (\rho_p - \rho_{air})g\frac{\pi}{6}D^3 = (2400 - 1 \text{ or so})(9.81)\frac{\pi}{6}D^3 \approx 12320D^3$$

$$\text{Thus} \quad V_{fall} \approx (12320D^3)/[3\pi D\mu] \approx 1307D^2/\mu = -dZ/dt, \quad \text{where } Z = \text{altitude}$$

Thus the time to fall varies inversely as D^2 and depends on an average viscosity in the air:

$$\Delta t_{fall} = \frac{1}{1307D^2}\int_0^{12000}\mu\,dZ = \frac{12000}{1307D^2}\mu_{avg}|_{0\text{-}12000}, \quad \text{Table A-6 gives } \mu_{avg} \approx 1.61E\text{–}5\ \frac{kg}{m\cdot s}$$

Try our two different diameters and check the Reynolds number for Stokes' flow:

$$\text{(a) } D = 1\ \mu m: \quad \Delta t = \frac{12000(1.61E\text{–}5)}{1307(1E\text{–}6)^2} \approx 1.48E8\text{ s} \approx \mathbf{1710\ days} \quad \textit{Ans.}\text{ (a)}$$

$$Re_{max} \approx 5E\text{–}6 << 1,\ OK$$

$$\text{(b) } D = 20\ \mu m: \quad \Delta t = \frac{12000(1.61E\text{–}5)}{1307(2E\text{–}5)^2} \approx 3.70E5\text{ s} \approx \mathbf{4.3\ days} \quad \textit{Ans.}\text{ (b)}$$

$$\mathrm{Re}_{\max} \approx 0.04 << 1, \text{ OK}$$

P7.80 A heavy sphere attached to a string should hang at an angle θ when immersed in a stream of velocity U, as in Fig. P7.80. Derive an expression for θ as a function of the sphere and flow properties. What is θ if the sphere is steel (SG = 7.86) of diameter 3 cm and the flow is sea-level standard air at $U = 40$ m/s? Neglect the string drag.

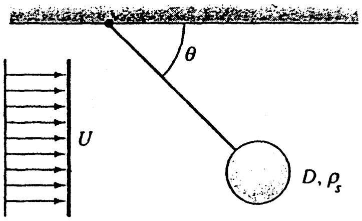
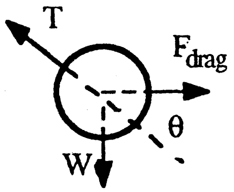


Fig. P7.80

Solution: For sea-level air, take $\rho = 1.225\ \text{kg/m}^3$ and $\mu = 1.78\text{E–}5$ kg/m·s. The sphere should hang so that string tension balances the resultant of drag and net weight:



$$\tan\theta = \frac{W_{net}}{Drag}, \quad \text{or} \quad \theta = \tan^{-1}\left[\frac{(\rho_s - \rho)g(\pi/6)D^3}{(\pi/8)C_D\rho U^2 D^2}\right] \quad \textit{Symbolic answer.}$$

For the given numerical data, first check Re and the drag coefficient, then find the angle:

$$\text{Re}_D = \frac{1.225(40)(0.03)}{1.78\text{E–}5} \approx 83000, \quad \text{Fig. 7.16b: } C_D \approx \mathbf{0.5}$$

$$F = \frac{\pi}{8}(0.5)(1.225)(40)^2(0.03)^2 \approx 0.346\ \text{N};$$

$$W = [7.86(998) - 1.225](9.81)\frac{\pi}{6}(0.03)^3 \approx 1.09\ \text{N} \quad \therefore \quad \theta = \tan^{-1}(1.09/0.346) \approx \mathbf{72°} \quad \textit{Ans.}$$

P7.81 A typical U.S. Army parachute has a projected diameter of 28 ft. For a payload mass of 80 kg, (a) what terminal velocity will result at 1000-m standard altitude? For the same velocity and payload, what size drag-producing "chute" is required if one uses a square flat plate held (b) vertically; and (c) horizontally? (Neglect the fact that flat shapes are not dynamically stable in free fall.) Neglect plate weight.

Solution: For air at 1000 meters, from Table A-3, $\rho \approx 1.112\ \text{kg/m}^3$. Convert D = 28 ft = 8.53 m. Convert $W = mg = 80(9.81) = 785$ N. From Table 7-3 for a parachute, read $C_D \approx 1.2$, Then, for part (a),

$$W = 785\ \text{N} = Drag = 1.2\left(\frac{1.112}{2}\right)U^2\frac{\pi}{4}(8.53)^2, \quad \text{solve for } U \approx \mathbf{4.53\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

(c) From Table 7-3 for a square plate normal to the stream, read $C_D \approx 1.18$. Then

$$W = 785\text{ N} = Drag = 1.18\left(\frac{1.112}{2}\right)(4.53)^2 L^2, \quad \text{solve for } L \approx 7.63\ m = \mathbf{25\ ft} \quad \textit{Ans.}\text{ (c)}$$

This is a comparable size to the parachute, but a square plate is ungainly and unstable.

(b) For a square plate parallel to the stream use (*turbulent*) flat plate theory. We need the viscosity—at 1000 meters altitude, estimate $\mu_{\text{air}} \approx 1.78\text{E}{-}5$ kg/m·s. Then

$$W = 785\text{ N} = Drag = \frac{0.031}{[1.112(4.53)L/1.78E{-}5]^{1/7}}\left(\frac{1.112}{2}\right)(4.53)^2 L^2 (2\ sides)$$

$$\text{Solve for} \quad L \approx 114\text{ m} = \mathbf{374\ ft} \quad \textit{Ans.}\text{ (b)}$$

This is ridiculous, as it was meant to be. A plate parallel to the stream is a *low-drag* device. You would need a plate the size of a football field.

P7.82 Skydivers, flying over sea-level ground, typically jump at about 8000 ft altitude and free-fall spread-eagled until they open their chutes at about 2000 ft. They take about 10 s to reach terminal velocity. Estimate how many seconds of free-fall they enjoy if (*a*) they fall spread-eagled; or (*b*) they fall feet first? Assume a total skydiver weight of 220 lbf.

Solution: From Table 7.3 use $C_DA = 9\text{ ft}^2$ spread-eagled and 1.2 ft^2 when feet first. From Table A.6, $\rho = 0.001869\text{ slug/ft}^3$ at 8000 ft standard altitude and $0.002242\text{ slug/ft}^3$ at 2000 ft. (*a*) Compute each terminal velocity spread-eagled ($C_DA = 9\text{ ft}^2$):

$$At\ 8000\,ft:\ W = 220\,lbf = (C_DA)(\rho/2)V^2 = (9)(0.001869/2)V^2,\ V = 162\ ft/s = 110\,mi/h$$

$$At\ 2000\,ft:\ W = 220\,lbf = (C_DA)(\rho/2)V^2 = (9)(0.002242/2)V^2,\ V = 148\ ft/s = 101\,mi/h$$

The difference is less than 10%, so just assume an average velocity of (162+148)/2 = 155 ft/s. For the first 10 s, assume an average velocity of 162/2 times 10 s = 810 ft before reaching terminal speed. That leaves (6000-810) divided by 155 ft/s of terminal fall ≈ **46 s** *Ans.*(*a*)

(*b*) Compute each terminal velocity falling feet first ($C_DA = 1.2\text{ ft}^2$):

$$At\ 8000\,ft:\ W = 220\,lbf = (C_DA)(\rho/2)V^2 = (1.2)(0.001869/2)V^2,\ V = 443\ ft/s = 302\,mi/h$$

$$At\ 2000\,ft:\ W = 220\,lbf = (C_DA)(\rho/2)V^2 = (1.2)(0.002242/2)V^2,\ V = 404\ ft/s = 276\,mi/h$$

Ridiculously fast! The average is 424 ft/s. For the first 10 s, assume an average velocity of 443/2 times 10 s = 2215 ft before reaching terminal speed. That leaves (6000-2215) divided by 424 ft/s of terminal fall ≈ only **9 s** *Ans.*(*b*) [Not 'enjoyable', in the writer's opinion .]

P7.83 A high-speed car has a drag coefficient of 0.3 and a frontal area of 1.0 m^2. A parachute is to be used to slow this 2000-kg car from 80 to 40 m/s in 8 s. What should the chute diameter be? What distance will be travelled during deceleration? Assume sea-level air.

Solution: For sea-level air, take $\rho = 1.225\ kg/m^3$ and $\mu = 1.78E{-}5$ kg/m·s. The solution this problem follows from Eq. (1) of Example 7.7.

$$V = V_o/[1 + (K/m)V_o t] \quad \text{where} \quad K = \frac{\rho}{2}(C_{Dcar}A_{car} + C_{Dchute}A_{chute})$$

Take C_{Dchute} = **1.2**. Enter the given data at t = 8 sec and find the desired value of K:

$$V = 40 = \frac{80}{1 + (K/2000)(80)(8\ s)}, \quad \text{solve for} \quad K \approx 3.125 = \frac{1.225}{2}\left[0.3(1) + 1.2\frac{\pi}{4}D^2\right]$$

$$\text{Solve for} \quad D \approx \mathbf{2.26\ m} \quad \textit{Ans.}\ \text{(a)}$$

The distance travelled is given as Eq. (2) of Ex. 7.7:

$$\alpha = \frac{K}{m}V_o = \frac{3.125}{2000}(80) = 0.125\ s^{-1}, \quad S = \frac{V_o}{\alpha}\ln(1 + \alpha t) = \frac{80}{0.125}\ln[1 + 0.125(8)]$$

$$\text{or} \quad S \approx \mathbf{440\ m} \quad \textit{Ans.}\ \text{(b)}$$

P7.84 A Ping-Pong ball weighs 2.6 g and has a diameter of 3.8 cm. It can be supported by an air jet from a vacuum cleaner outlet, as in Fig. P7.84. For sea-level standard air, what jet velocity is required?

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E−5 kg/m·s. The ball weight must balance its drag:

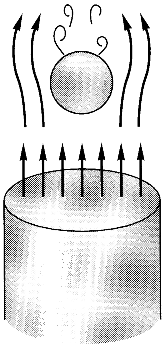


Fig. P7.84

$$W = 0.0026(9.81) = 0.0255\ \text{N} = C_D \frac{\rho}{2} V^2 \frac{\pi}{4} D^2 = C_D \frac{1.225}{2} V^2 \frac{\pi}{4}(0.038)^2, \quad C_D = \text{fcn(Re)}$$

$$C_D V^2 = 36.7,\ \text{Use Fig. 7.16b, converges to } C_D \approx 0.47,\ \text{Re} \approx 23000,\ V \approx 9\ \mathbf{m/s} \quad \textit{Ans.}$$

P7.85 In this era of expensive fossil fuels, many alternatives have been pursued. One idea from SkySails, Inc., shown in Fig. P7.85, is the assisted propulsion of a ship by a large tethered kite. The tow force of the kite assists the ship propeller and is said to reduce annual fuel consumption by 10%-35%. For a typical example, let the ship be 120 m long, with a wetted area of 2800 m^2. The kite area is 330 m^2 and has a force coefficient of 0.8. The kite cable makes an angle of 25° with the horizontal. Let V_{wind} = 30 mi/h. Neglect ship wave drag. Estimate the ship speed (*a*) due to the kite only; and (*b*) if the propeller delivers 1,250 hp to the water. [*Hint*: The kite sees the *relative* velocity of the wind.]

Fig. P7.85. Ship propulsion assisted by a large kite. [*Copyright SkySails, Inc.*]

Solution: Assume sea level air density, ρ_a = 1.2255 kg/m^3. For seawater, take ρ = 1025 kg/m^3 and μ = 0.00107 kg/m-s. The wind velocity is 30 mi/h = 13.4 m/s. (*a*) For a wind V_{air} and a ship speed V, the kite force equals the friction drag of the ship:

Note that the kite sees the wind velocity *relative* to the moving ship, ($V_{air} - V$). The horizontal kite force and the ship drag are both equal to 17,400 N.

$$F_{kite} = C_D \frac{\rho_a}{2} A_{kite} (V_{air} - V)^2 \cos 25^o = F_{ship} = C_{d,friction} \frac{\rho}{2} A_{ship} V^2 \ ,$$

$$\text{where} \quad C_{d,friction} = \frac{0.031}{\mathrm{Re}_L^{1/7}} = [\frac{0.031}{(1025 * V * 120/.00107)^{1/7}}]$$

$$\text{Thus} \quad (0.8)(\frac{1.2255}{2})(330)(13.4 - V)^2 \cos 25^o = C_{d,friction} (\frac{1025}{2})(2800)\ V^2$$

$$\text{Solve by iteration or EES:} \quad V = V_{ship} = 2.52\ m/s = \mathbf{4.9}\ \text{knots} \quad \mathit{Ans.(a)}$$

(*b*) The propeller power is equivalent to a propulsion force, F_p = Power/V. This force, added to the kite horizontal force, must equal the ship drag. Convert 1250 hp to (1250)(745.7) = 932 kW.

$$\frac{932{,}000\ \text{W}}{V} + (0.8)(\frac{1.2255}{2})(330)(13.4 - V)^2 \cos 25^o = C_{d,friction}(\frac{1025}{2})(2800)\ V^2$$

$$\text{where} \ C_{d,friction} = \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{0.031}{(1025 * V * 120/.00107)^{1/7}}$$

Solve by iteration or EES to obtain $V = 7.44$ m/s $=$ **14.5 knots**. *Ans.*(*b*)
This is fast, about the average speed of a cargo ship. At this speed, the relative wind on the kite is small, only 6 m/s. Hence the kite force is only 5200 N, while the propeller force is 125,000 N.
It might be better to cut the power, for example, by 50% to 625 hp, which would result in $V = 6.0$ m/s = 11.6 knots, which is only a 20% reduction in ship speed.

P7.86 Hoerner [Ref. 12 of Chap. 7, p. 3–25] states that the drag coefficient of a flag of 2:1 aspect ratio is 0.11 based on planform area. URI has an aluminum flagpole 25 m high and 14 cm in diameter. It flies equal-sized national and state flags together. If the fracture stress of aluminum is 210 MPa, what is the maximum flag size that can be used yet avoids breaking the flagpole in hurricane (75 mi/h) winds? Neglect the drag of the flagpole.

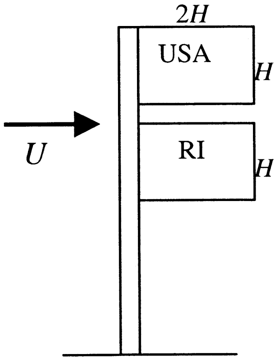


Fig. P7.86

Solution: URI is approximately sea-level, $\rho = 1.225$ kg/m^3. Convert 75 mi/h = 33.5 m/s. We will use the most elementary strength of materials formula, without even a stress-concentration factor, since this is just a *fluid mechanics* book:

$$\sigma = \frac{My}{I} = 210E6\ \text{Pa} = \frac{M(0.07\ \text{m})}{(\pi/4)(0.07\ \text{m})^4}, \quad \text{solve for } M_{fracture} = 56600\ \text{N·m}$$

Assume flags are at the top (see figure) with no space between. Each flag is "*H*" by "2*H*."

Then,

$$M = 56600\ \text{N·m} = F_{USA}\left(25\ \text{m} - \frac{H}{2}\right) + F_{RI}\left(25\ \text{m} - \frac{3H}{2}\right),$$

$$\text{where} \quad F_{USA} = F_{RI} = 0.11\left(\frac{1.225}{2}\right)(33.5)^2 H(2H)$$

Iterate or use EES: $F = 1281\ \text{N}$, **H = 2.91 m**, Flag length = **2H = 5.82 m** *Ans.*

P7.87 A tractor-trailer truck has a drag area $C_DA = 8\ \text{m}^2$ bare and $C_DA = 6.7\ \text{m}^2$ with a deflector added (Fig. 7.18b). Its rolling resistance is 50 N for each mi/h of speed. Calculate the total horsepower required if the truck moves at (a) 55 mi/h; and (b) 75 mi/h.

Solution: For sea-level air, take $\rho = 1.225\ \text{kg/m}^3$ and $\mu = 1.78\text{E–}5\ \text{kg/m·s}$. Convert V = 55 mi/h = *24.6* m/s and 75 mi/h = *33.5* m/s. Take each speed in turn:

$$\text{(a)} \quad 55\ \frac{\text{mi}}{\text{h}}: \quad F_{\text{bare}} = (8\ \text{m}^2)\left(\frac{1.225}{2}\right)(24.6)^2 + 50(55) = 2962 + 2750 = 5712\ \text{N}$$

$$\text{Power required} = \text{FV} = (5712)(24.6) = 140\ \text{kW} \approx \mathbf{188\ hp}\ \text{(bare)}$$

$$\text{with a deflector,} \quad \text{F} \approx 2481 + 2750 = 5231\ \text{N}, \quad \text{Power} = 129\ \text{kW} \approx \mathbf{172\ hp}\ (-8\%)$$

$$\text{(b)} \quad 75\ \frac{\text{mi}}{\text{h}}: \quad \text{F} = 8\left(\frac{1.225}{2}\right)(33.5)^2 + 50(75) = 9258\ \text{N},$$

$$\text{Power} = 310\ \text{kW} \approx \mathbf{416\ hp}\ \text{(bare)}$$

$$\text{With deflector,} \quad \text{F} = 8363\ \text{N}, \quad \text{Power} = 280\ \text{kW} \approx \mathbf{376\ hp}\ (-10\%)$$

P7.88 A pickup truck has a clean drag-area C_DA of 35 ft^2. Estimate the horsepower required to drive the truck at 55 mi/h (a) clean and (b) with the 3- by 6-ft sign in Fig. P7.88 installed if the rolling resistance is 150 lbf at sea level.

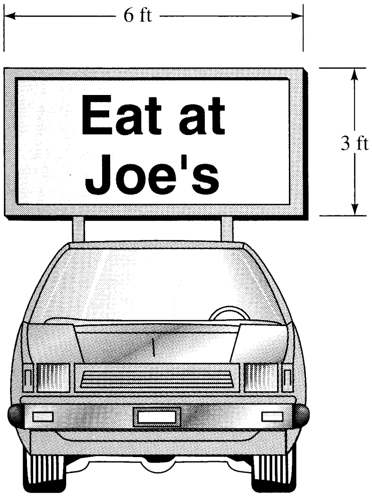


Fig. P7.88

Solution: For sea-level air, take ρ = 0.00238 $slug/ft^3$ and μ = 3.72E–7 slug/ft·s. Convert V = 55 mi/h = *80.7* ft/s. Calculate the drag without the sign:

$$F = F_{rolling} + C_DA\frac{\rho}{2}V^2$$
$$= 150 + 35(0.00238/2)(80.7)^2 \approx 421\text{ lbf}$$

$$\text{Horsepower} = (421)(80.7) \div 550 \approx \mathbf{62\ hp}\ \text{(clean)} \quad \textit{Ans.}\text{ (a)}$$

With a sign added, b/h = 2.0, read $C_D \approx$ **1.19** from Table 7.3. Then

$$F = 421_{clean} + 1.19\left(\frac{0.00238}{2}\right)(80.7)^2(6)(3) \approx 587\text{ lbf},$$

$$\text{Power} = FV \approx 86\ \mathbf{hp} \quad \textit{Ans.}\text{ (b)}$$

P7.89 The new AMTRAK high-speed *Acela* train can reach 150 mi/h, which presently it seldom does, because of the curvy coastline tracks in New England. If 75% of the power expanded at this speed is due to air drag, estimate the total horsepower required by the *Acela*.

Solution: For sea-level air, take ρ = 1.22 kg/m^3. From Table 7.3, the drag-area C_DA of a streamlined train is approximately 8.5 m^2. Convert 150 mi/h to 67.1 m/s. Then

$$0.75P_{train} = \left[(C_DA)\frac{\rho}{2}V^2\right]V = (8.5\text{ m}^2)\left(\frac{1.22\text{ kg/m}^3}{2}\right)(67.1\text{ m/s})^3 = 1.56E6\ \textbf{\textit{watts}}$$

$$\text{Solve for} \quad P_{train} = 2.08E6\text{ W} = \mathbf{2800\ hp} \quad \textit{Ans.}$$

P7.90 In the great hurricane of 1938, winds of 85 mi/h blew over a boxcar in Providence, Rhode Island. The boxcar was 10 ft high, 40 ft long, and 6 ft wide, with a 3-ft clearance above tracks 4.8 ft apart. What wind speed would topple a boxcar weighing 40,000 lbf?

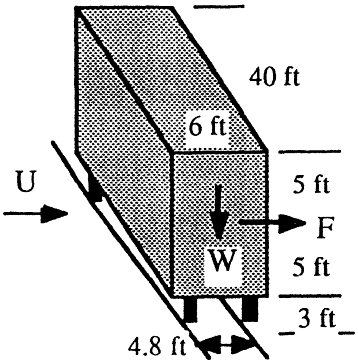


Fig. P7.90

Solution: For sea-level air, take ρ = 0.00238 slug/ft^3 and μ = 3.72E–7 slug/ft·s. From Table 7.3 for b / h = 4, estimate $C_D \approx$ 1.2. The estimated drag force F on the left side of the box car is thus

$$F = C_D \frac{\rho}{2} V^2 bh = 1.2\left(\frac{0.00238}{2}\right) V^2 (40)(10) \approx 0.5712V^2 \text{ (in ft/s)}$$

$$\text{Sum moments about right wheels: } (0.5712V^2)(8\text{ ft}) - (40000\text{ lbf})(2.4\text{ ft}) = 0,\ V^2 = 21008$$

$$\text{Solve}\quad V_{\text{overturn}} = 145\text{ ft/s} \approx \mathbf{99\ mi/h}\quad \textit{Ans.}$$

[The 1938 wind speed of 85 mi/h would overturn the car for a car weight of 29600 lbf.]

P7.91 A cup anemometer uses two 5-cm-diameter hollow hemispheres connected to two 15-cm rods, as in Fig. P7.91. Rod drag is neglected, and the central bearing has a retarding torque of 0.004 N·m. With simplifying assumptions, estimate and plot rotation rate Ω versus wind velocity in the range 0 < U < 25 m/s.

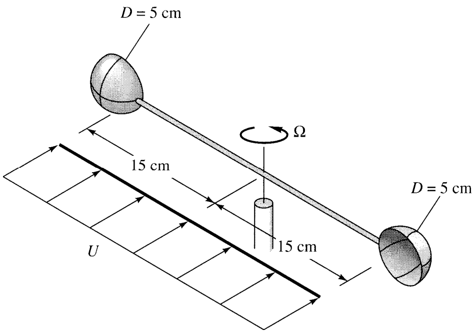
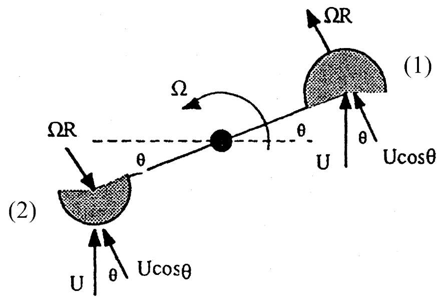


Fig. P7.91

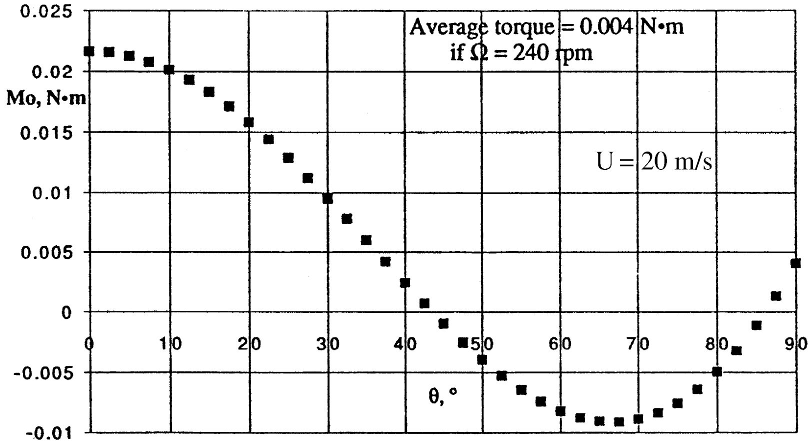
Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E–5 kg/m·s. For any instantaneous angle θ, as shown, the drag forces are assumed to depend on the relative velocity normal to the cup:

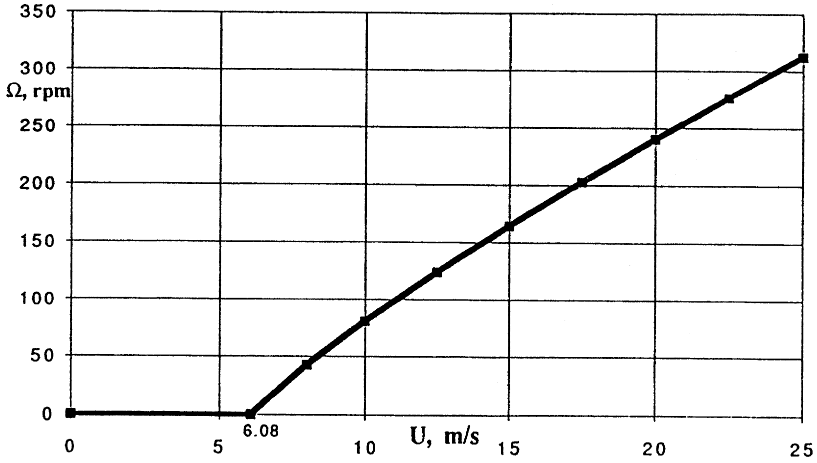


$$M_o = R\left[C_{D1}\frac{\rho}{2}(U\cos\theta - \Omega R)^2 - C_{D2}\frac{\rho}{2}(U\cos\theta + \Omega R)^2\right],$$

$$C_{D1} \approx 1.4,\quad C_{D2} \approx 0.4$$

For a given wind velocity 0 < U < 25 m/s, we find the rotation rate Ω (here in rad/s) for which the average torque over a 90° sweep is exactly equal to the frictional torque of 0.004 N·m. [The torque given by the formula mirrors itself over 90° increments.] For U = 20 m/s, the torque variation given by the formula is shown in the graph below. We do this for the whole range of U values and then plot Ω (in rev/min) versus U below. We see that the anemometer will not rotate until U ≥ 6.08 m/s. Thereafter the variation of Ω with U is approximately linear, making this a popular wind-velocity instrument.





***P7.92** A 1500-kg automobile uses its drag-area, $C_DA = 0.4\ m^2$, plus brakes and a parachute, to slow down from 50 m/s. Its brakes apply 5000 N of resistance. Assume sea-level standard air. If the automobile must stop in 8 seconds, what diameter parachute is appropriate?

Solution: For sea-level air take $\rho = 1.225$ kg/m^3. From Table 7.3 for a parachute, read $C_{Dp} \approx 1.2$. The force balance during deceleration is, with $V_o = 50$ m/s,

$$\sum F = -F_{roll} - F_{drag} = -5000 - \frac{1.225}{2}\left(0.4 + 1.2\frac{\pi}{4}D_p^2\right)V^2 = (ma)_{car} = 1500\frac{dV}{dt}$$

Note that, if drag = 0, the car slows down linearly and stops in 50(1500)/(5000) = **15 s**, not fast enough—so we definitely need the drag to cut it down to 8 seconds. The first-order differential equation above has the form

$$\frac{dV}{dt} = -b - aV^2, \quad \text{where } a = \frac{1.225}{2}\left(\frac{0.4 + 1.2\pi D_p^2/4}{1500}\right) \quad \text{and} \quad b = \frac{5000}{1500}$$

Separate the variables and integrate, with $V = V_o = 50$ m/s at $t = 0$:

$$\int_{V_o}^{0} \frac{dV}{b + aV^2} = -\int_0^t dt, \quad \text{Solve: } t = \frac{1}{\sqrt{ab}}\tan^{-1}\left(V_o\sqrt{\frac{a}{b}}\right) = 8\ s?$$

The unknown is D_p, which lies within *a*! Iteration is needed—an ideal job for EES! Well, anyway, you will find that $D_p = 3$ m is too small ($t \approx 9.33$ s) and $D_p = 4$ m is too large ($t \approx 7.86$ s). We may interpolate (or EES will quickly report):

$$\mathbf{D_{parachute(t=8\ s)} \approx 3.9\ m} \quad \textit{Ans.}$$

P7.93 A hot-film probe is mounted on a cone-and-rod system in a sea-level airstream of 45 m/s, as in Fig. P7.93. Estimate the maximum cone vertex angle allowable if the flow-induced bending moment at the root of the rod is not to exceed 30 N·cm.

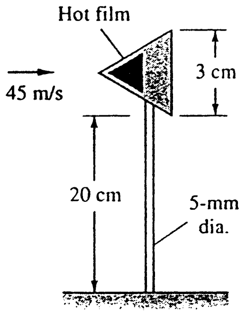


Fig. P7.93

Solution: For sea-level air take $\rho = 1.225$ kg/m^3 and $\mu = 1.78E{-}5$ kg/m·s. First figure the rod's drag and moment, assuming it is a smooth cylinder:

$$Re_{D,rod} = \frac{1.225(45)(0.005)}{1.78E{-}5} = 15500, \quad \textit{Fig.7.16a: read } C_{D,rod} \approx 1.2$$

$$F_{rod} = 1.2\left(\frac{1.225}{2}\right)(45)^2(0.005)(0.2) = 1.49\ \text{N}, \quad M_{base,rod} = 1.49(10) = 14.9\ \text{N·cm}$$

Then add in the drag-moment of the cone about the base:

$$\sum M_{base} = 30 = 14.9 + C_{D,cone}\left(\frac{1.225}{2}\right)(45)^2\left(\frac{\pi}{4}\right)(0.03)^2(21.5\text{ cm})$$

$$\text{Solve for}\quad C_{D,cone} \approx 0.80,\quad \text{Table 7.3: read}\ \ \theta_{cone} \approx \mathbf{60°}\quad \textit{Ans.}$$

P7.94 A rotary mixer consists of two 1-m-long half-tubes rotating around a central arm, as in Fig. P7.94. Using the drag from Table 7.2, derive an expression for the torque *T* required to drive the mixer at angular velocity Ω in a fluid of density *ρ*. Suppose that the fluid is water at 20°C and the maximum driving power available is 20 kW. What is the maximum rotation speed Ω r/min?

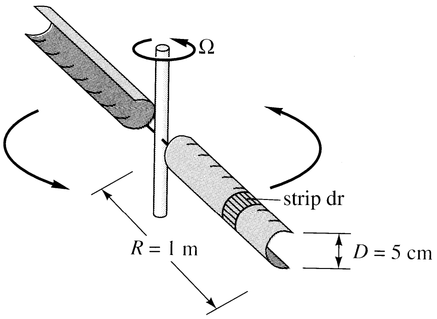


Fig. P7.94

Solution: Consider a *strip* of half-tube of width dr, as shown in Fig. P7.94 above. The local velocity is U = Ωr, and the strip frontal area is Ddr. The total torque (2 tubes) is

$$T_o = 2\int_{tube} r\,dF = 2\int_0^R r\left[C_D\frac{\rho}{2}(\Omega r)^2 D\,dr\right] \approx \mathbf{\frac{1}{4}C_D\rho\Omega^2DR^4}\quad \textit{Ans.}\text{ (a)}$$

(b) For water at 20°C, take $\rho = 998\text{ kg/m}^3$ and $\mu = 0.001$ kg/m·s. Assume that the half-tube shape has the drag coefficient $C_D \approx 2.3$ as in Table 7.2. Then, with power known,

$$P = 20000\text{ W} = T_o\Omega = \left[\frac{1}{4}(2.3)(998)\Omega^2(0.05)(1.0)^4\right]\Omega = 28.7\Omega^3$$

$$\text{Solve for}\quad \Omega_{max} = 8.87\ \frac{\text{rad}}{\text{s}}\times\frac{60}{2\pi} \approx \mathbf{85\ \frac{rev}{min}}\quad \textit{Ans.}\text{ (b)}$$

P7.95 An airplane weighing 28 kN, with a drag-area $C_DA = 5\text{ m}^2$, lands at sea level at 55 m/s and deploys a drag parachute 3 m in diameter. No other brakes are applied. (a) How long will it take the plane to slow down to 20 m/s? (b) How far will it have traveled in that time?

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E−5 kg/m·s. The analytical solution to this deceleration problem was given in Example 7.8 of the text: Take $C_{D,chute}$ = 1.2.

$$V = \frac{V_o}{1+\alpha t}, \quad \alpha = \frac{\rho g V_o}{2W}\sum C_D A = \frac{1.225(9.81)(55)}{2(28000)}\left[5\text{ m}^2 + 1.2\frac{\pi}{4}(3\text{ m})^2\right] = 0.159\text{ s}^{-1}$$

(a) Then the time required to slow down from 55 m/s to 20 m/s, without brakes, is

$$20\text{ m/s} = \frac{55\text{ m/s}}{1+0.159t}, \quad \text{solve for } \mathbf{t = 11.0\text{ s}} \quad \textit{Ans. (a)}$$

(b) The distance traveled was also derived in Example 7.8:

$$S = \frac{V_o}{\alpha}\ln(1+\alpha t) = \frac{55}{0.159}\ln[1+0.159(11.0)] \approx \mathbf{350\text{ m}} \quad \textit{Ans. (b)}$$

***P7.96** A Savonius rotor (see Fig. 6.29*b*) can be approximated by the two open half-tubes in Fig. P7.96 mounted on a central axis. If the drag of each tube is similar to that in Table 7.2, derive an approximate formula for the rotation rate Ω as a function of U, D, L, and the fluid properties ρ and μ.

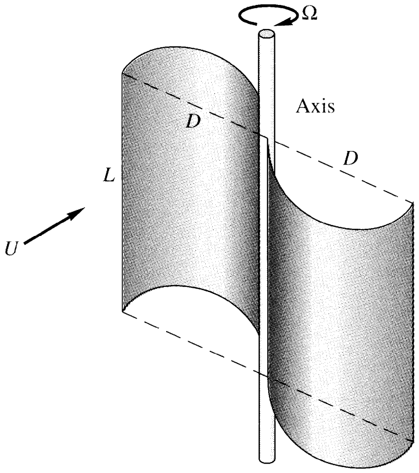


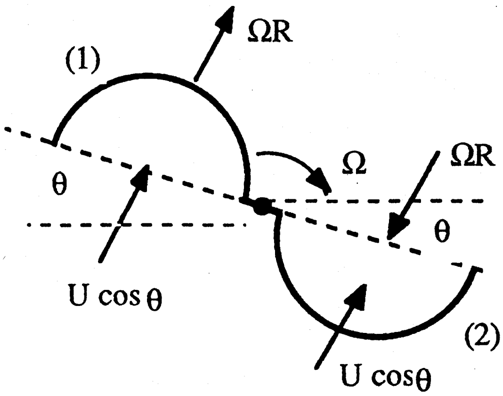
Fig. P7.96

Solution: The analysis is similar to Prob. 7.91 (the cup anemometer). At any arbitrary angle as shown, the net torque caused by the relative velocity on each half-tube is set to zero (assuming a frictionless bearing):

$$T_o = 0 = \frac{D}{2}(F_1 - F_2) \quad \text{where}$$

$$F_1 = C_{D1}(\rho/2)(U\cos\theta - \Omega D/2)^2 DL$$

$$F_2 = C_{D2}(\rho/2)(U\cos\theta + \Omega D/2)^2 DL$$



This pattern of torque repeats itself every 90°. Thus the torque is an average value:

$$T_{o,avg} = 0 \quad \text{if} \quad F_{1,avg} = F_{2,avg}$$

$$\text{or}\quad \frac{\rho}{2}DLC_{D1}\left(U\cos\theta-\Omega\frac{D}{2}\right)^2_{avg}\approx\frac{\rho}{2}DLC_{D2}\left(U\cos\theta+\Omega\frac{D}{2}\right)^2_{avg}$$

$$\text{or:}\quad U\cos\theta(1-\zeta)_{avg}\approx\Omega\frac{D}{2}(1+\zeta)_{avg},\quad \zeta=\sqrt{\frac{C_{D2}}{C_{D1}}}=\sqrt{\frac{1.2}{2.3}}\approx 0.722$$

where C_{D1} = 2.3 and C_{D2} = 1.2 are taken from Table 7.2. The average value of cosθ over 0 to 90° is $2/\pi \approx 0.64$. Then a simple approximate expression for rotation rate is

$$\Omega_{\mathbf{avg}}\approx\frac{2U}{D}\left[\cos\theta\frac{1-\zeta}{1+\zeta}\right]_{avg}=\frac{2U}{D}\left[\frac{2}{\pi}\left(\frac{1-0.722}{1+0.722}\right)\right]\approx\mathbf{0.21\frac{U}{D}}\quad \textit{Ans.}$$

***P7.97** A simple measurement of automobile drag can be found by an unpowered *coastdown* on a level road with no wind. Assume constant rolling resistance. For an automobile of mass 1500 kg and frontal area 2 m^2, the following velocity-versus-time data are obtained during a coastdown:

t, s:	0.00	10.0	20.0	30.0	40.0
V, m/s:	27.0	24.2	21.8	19.7	17.9

Estimate (a) the rolling resistance and (b) the drag coefficient. This problem is well suited for digital-computer analysis but can be done by hand also.

Solution: For air, take ρ = 1.2 kg/m^3 and μ = 1.8E−5 kg/m·s. Assuming that rolling friction is linearly proportional to the car velocity. Then the equation of motion is

$$\sum F_x = m\frac{dV}{dt} = -F_{rolling}-F_{drag} = -KV - C_DA\frac{\rho}{2}V^2.$$

$$\text{Separate and integrate:}\quad \int_0^t dt = -\int_{V_o}^{V}\frac{dV}{(K/m)V+(C_DA\rho/2m)V^2},$$

$$\text{or}\quad \mathbf{t=\frac{m}{K}ln\left[\frac{K+C_DA\rho V/2}{V}\frac{V_o}{K+C_DA\rho V_o/2}\right]}$$

This is the formula which we must fit to the data. Introduce numerical values to get

$$t=\frac{-1500}{K}\ln\left[\frac{K+32.4C_D}{27}\frac{V}{K+1.2VC_D}\right]\quad \text{solve by least squares for } \mathbf{K} \text{ and } \mathbf{C_D}.$$

The least-squares results are **K ≈ 9.1 N·s/m** and **C_D ≈ 0.24**. *Ans.*

These two values give *terrific* accuracy with respect to the data—deviations of less than ±0.06%! Actually, the data are not sensitive to K or C_D, at least if the two are paired nicely. Any *K* from 8 to 10 N·s/m, paired with C_D from 0.20 to 0.28, gives excellent accuracy. We need more data points to discriminate between parameters.

***P7.98** A buoyant ball of specific gravity SG < 1, dropped into water at inlet velocity V_o, will penetrate a distance *h* and then pop out again, as in Fig. P7.98. (a) Make a dynamic analysis, assuming a constant drag coefficient, and derive an expression for *h* as a function of system properties. (b) How far will a 5-cm-diameter ball, with SG = 0.5 and C_D = 0.47, penetrate if it enters at 10 m/s?

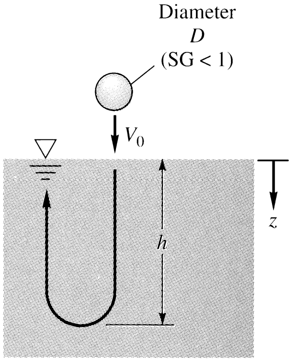


Fig. P7.98

Solution: The buoyant force is up, $W_{net} = (1 - SG)\rho g(\pi/6)D^3$, and with z down as shown, the equation of motion of the ball is

$$\sum F_z = m\frac{dV}{dt} = -W_{net} - C_D\frac{\rho}{2}V^2A, \quad A = \frac{\pi}{4}D^2.$$

$$\text{Separate and integrate:} \quad -\int_0^t dt = \int_{V_o}^{V} \frac{dV}{(W_{net}/m) + (C_D\rho A/2m)V^2},$$

$$\text{or} \quad V = V_f \tan\left[\tan^{-1}\left(\frac{V_o}{V_f}\right) - t\frac{W_{net}}{mV_f}\right] \quad \text{where } V_f = \sqrt{2W_{net}/(\rho C_D A)} \quad \text{for short.}$$

The total distance travelled until the ball stops (at V= 0) and turns back upwards is

$$\mathbf{h} = \int_0^{t(V=0)} V\,dt = \frac{\mathbf{m}}{\boldsymbol{\rho}\mathbf{C_D A}}\mathbf{ln[1 + (V_o/V_f)^2]} \quad \textit{Ans.}\text{ (a)}$$

(b) Apply the specific data to find the depth of penetration for a numerical example. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s.

$$W_{net} = (1 - 0.5)(9790)\frac{\pi}{6}(0.05)^3 \approx 0.320 \text{ N}; \quad m = 0.5(998)\frac{\pi}{6}(0.05)^3 \approx 0.0327 \text{ kg}$$

$$V_f = \sqrt{\frac{2(0.320 \text{ N})}{998(0.47)(\pi/4)(0.05)^2}} \approx 0.834\ \frac{m}{s}, \quad \text{check } Re_f = \frac{998(0.834)(0.05)}{0.001} \approx 42000 \text{ OK}$$

Then the formula predicts total penetration depth of

$$\mathbf{h} = \frac{0.0327\ \text{kg}}{998(0.47)(\pi/4)(0.05)^2}\ln\left[1+\left(\frac{10.0}{0.834}\right)^2\right] \approx \mathbf{0.18\ m} \quad \textit{Ans.}\ \text{(b)}$$

NOTE: We have neglected “hydrodynamic” mass of the ball (Section 8.8).

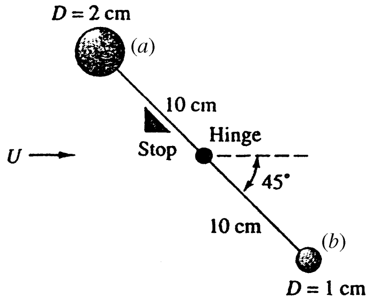


Fig. P7.99

P7.99 Two steel balls (SG = 7.86) are connected by a thin hinged rod of negligible weight and drag, as shown in Fig. P7.99. A stop prevents counter-clockwise rotation. Estimate the sea-level air velocity U for which the rod will first begin to rotate clockwise.

Solution: For sea-level air, take $\rho = 1.225$ kg/m^3 and μ = 1.78E–5 kg/m·s. Let “a” and “b” denote the large and small balls, respectively, as shown. The rod begins to rotate when the moments of drag and weight are balanced. The (clockwise) moment equation is

$$\Sigma M_O = F_a(0.1\sin 45°) - W_a(0.1\cos 45°) - F_b(0.1\sin 45°) + W_b(0.1\cos 45°) = 0$$

For 45°, there are nice cancellations to obtain $\therefore\ F_a - F_b = W_a - W_b$, or:

$$C_{Da}\frac{\rho}{2}U^2\frac{\pi}{4}D_a^2 - C_{Db}\frac{\rho}{2}U^2\frac{\pi}{4}D_b^2 \overset{?}{=} (SG)\rho_{water}g\frac{\pi}{6}D_a^3 - (SG)\rho_{water}g\frac{\pi}{6}D_b^3$$

Assuming that $C_{Da} = C_{Db} \approx 0.47$ (Re < 250000), we may easily solve for air velocity:

$$U^2(0.47)\left(\frac{1.225}{2}\right)\frac{\pi}{4}[(0.02)^2 - (0.01)^2] = (7.86)(9790)\frac{\pi}{6}[(0.02)^3 - (0.01)^3]$$

$$\text{Solve for} \quad U = \sqrt{4158} \approx \mathbf{64\ m/s} \quad \textit{Ans.}$$

We may check that $Re_{max} = 1.225(64)(0.02)/1.78E{-}5 \approx$ *89000*, OK, $C_D \approx 0.47$.

P7.100 A tractor-trailer truck is coasting freely, with no brakes, down an 8° slope at 1000-m standard altitude. Rolling resistance is 120 N for every m/s of speed. Its frontal area is 9 m^2, and the weight is 65 kN. Estimate the terminal coasting velocity, in mi/h, for (a) no deflector; and (b) a deflector installed.

Solution: For air at 100-m altitude, $\rho = 1.112\ \text{kg/m}^3$. From Table 7.3, $C_D = 0.96$ without and 0.76 with a deflector. Summing forces along the roadway gives:

$$W\sin\theta = F_{drag} + F_{roll} = C_D\frac{\rho}{2}V^2A_{frontal} + C_{roll}V$$

(a, b) Applying the given data results in a quadratic equation:

$$\text{No deflector:}\quad (65000\ \text{N})\sin 8^\circ = (0.96)\left(\frac{1.112}{2}\right)(9.0)V^2 + 120V,$$

$$\text{or:}\quad V^2 + 24.98V - 1883 = 0 \quad \text{Solve } V = 32.7\ \text{m/s} = \mathbf{73\ mi/h} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{With deflector:}\quad (65000\ \text{N})\sin 8^\circ = (0.76)\left(\frac{1.112}{2}\right)(9.0)V^2 + 120V,$$

$$\text{or:}\quad V^2 + 31.55V - 2379 = 0 \quad \text{Solve } V = 35.5\ \text{m/s} = \mathbf{79\ mi/h} \quad \textit{Ans.}\ \text{(b)}$$

P7.101 Icebergs can be driven at substantial speeds by the wind. Let the iceberg be idealized as a large, flat cylinder, $D \ll L$, with one-eighth of its bulk exposed, as in Fig. P7.101. Let the seawater be at rest. If the upper and lower drag forces depend upon relative velocities between the berg and the fluid, derive an approximate expression for the steady iceberg speed V when driven by wind velocity U.

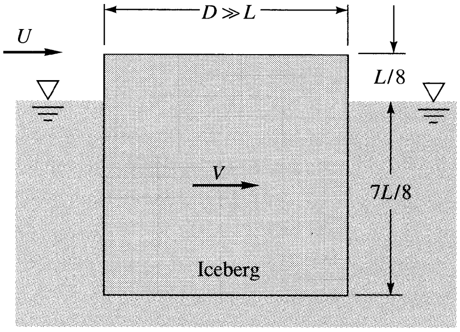
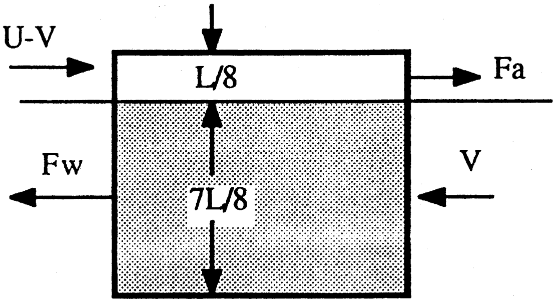


Fig. P7.101

Solution: Assuming steady drifting (no acceleration), the berg sees a water current V coming from the front and a relative air velocity U – V coming from behind. Ignoring moments (the berg will merely tilt slightly), the two forces must balance:



$$F_{air} = C_{D,air}\frac{1}{2}\rho_{air}(U-V)^2 D\frac{L}{8}$$

$$= F_{water} = C_{D,water}\frac{1}{2}\rho_{water}V^2\frac{7L}{8}D$$

This has the form of a quadratic equation:

$$U^2 - 2UV + V^2 = \alpha V^2, \quad \text{or} \quad V_{\text{berg}} = U_{\text{wind}}\left(\frac{\sqrt{\alpha}-1}{\alpha-1}\right) \quad \text{where } \alpha = \frac{7\rho_w C_{Dw}}{\rho_a C_{Da}} << 1$$

Since $\alpha = \mathcal{O}(5000)$, we approximate $V_{berg} \approx U/\sqrt{\alpha}$.

P7.102 Sand particles (SG = 2.7), approximately spherical with diameters from 100 to 250 μm, are introduced into an upward-flowing water stream. What is the minimum water velocity to carry all the particles upward?

Solution: Clearly the largest particles need the most water speed. Set net weight = drag:

$$W_{net} = (\rho_p - \rho_w)g\frac{\pi}{6}D^3 = C_D\frac{\rho_w}{2}V^2\frac{\pi}{4}D^2, \quad \text{solve } V^2 = \frac{4gD(SG-1)}{3C_D}$$

$$\text{or:} \quad C_D V^2 = \frac{4}{3}(9.81)(0.00025)(2.7-1) = \mathbf{0.00555} \quad \left(with\ V\ in\ \frac{\text{m}}{\text{s}}\right)$$

Iterate in Figure 7.16b: $\text{Re}_D \approx 10$, $C_D \approx 4$, $\mathbf{V_{min} \approx 0.04\ m/s}$ *Ans.*

P7.103 When immersed in a uniform stream, a heavy rod hinged at A will hang at *Pode's angle θ*, after L. Pode (1951). Assume the cylinder has normal drag coefficient C_{Dn} and tangential coefficient C_{Dt}, related to V_n and V_t, respectively. Derive an expression for θ as a function of system parameters. Compute θ for a steel rod, L = 40 cm, D = 1 cm, hanging in sea-level air at V = 35 m/s.

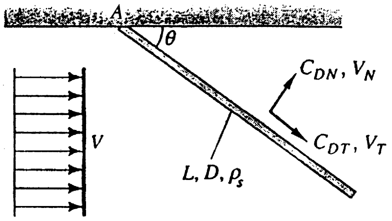
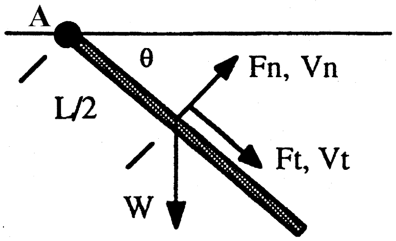


Fig. P7.103

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E–5 kg/m·s. The tangential drag force passes right through A, so the moment balance is



$$\sum M_A = F_n\frac{L}{2} - W\frac{L}{2}\cos\theta = C_{Dn}\frac{\rho}{2}(V\sin\theta)^2 DL\frac{L}{2} - (\rho_s - \rho)g\frac{\pi}{4}D^2L\frac{L}{2}\cos\theta,$$

$$\text{Solve for Pode's angle} \quad \boldsymbol{\frac{\sin^2\theta}{\cos\theta} = \frac{(\rho_s-\rho)g(\pi/2)D}{\rho C_{Dn}V^2}} \quad \textit{Ans.}\ \text{(a)}$$

For the numerical data, take SG(steel) = 7.86, $\mathrm{Re}_n \approx 17000$ (laminar), $C_{Dn} \approx 1.2$:

$$\frac{\sin^2\theta}{\cos\theta} = \frac{[7.86(998)-1.225](9.81)(\pi/2)(0.01)}{(1.225)(1.2)(35)^2} = 0.671, \quad \text{solve } \boldsymbol{\theta_{\mathrm{Pode}} \approx 44°} \quad \textit{Ans.}\text{ (b)}$$

P7.104 The Russian Typhoon-class submarine is 170 m long, with a maximum diameter of 23 m. Its propulsor can deliver up to 80,000 hp to the seawater. Model the sub as an 8:1 ellipsoid and estimate the maximum speed, in knots, of this ship.

Solution: For seawater, take $\rho = 1025\ \mathrm{kg/m^3}$. The flow is surely turbulent ($\mathrm{Re_L} > 2\mathrm{E}9$) so use the "turbulent" value $C_D \approx 0.08$ in Table 7.3 for an 8:1 ellipsoid. The power is cubic in V:

$$Power = 80{,}000\,hp = 5.97E7\ W \ = \ FV = (C_D\frac{\rho}{2}V^2\frac{\pi}{4}D^2_{\max})V \ = \ (C_D\frac{\rho}{2}\frac{\pi}{4}D^2_{\max})V^3$$

$$Plug\ in:\ 5.97E7\ W \ = \ (0.08)(\frac{1025}{2})\frac{\pi}{4}(23m)^2V^3 \ ;\ Solve\ V = 15.2\frac{m}{s} = \ \mathbf{29.5\,knots} \quad Ans.$$

P7.105 A ship 50 m long, with a wetted area of 800 m^2, has the hull shape of Fig. 7.19, with no bow or stern bulbs. Total propulsive power is 1 MW. For seawater at 20°C, plot the ship's velocity V (in knots) versus power P for $0 < P < 1$ MW. What is the most efficient setting?

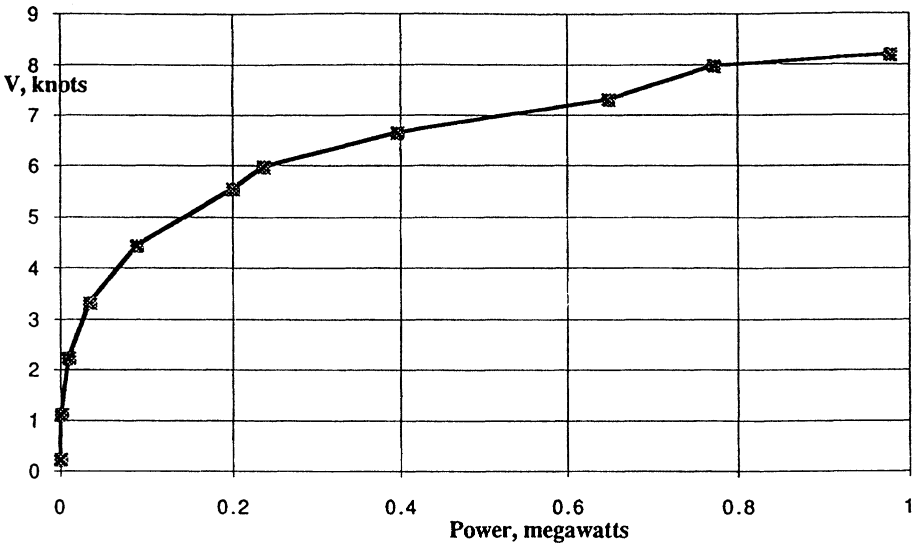
Solution: For seawater at 20°C, take $\rho = 1025\ \mathrm{kg/m^3}$ and $\mu = 0.00107$ kg/m·s. The drag is taken to be the sum of friction and wave drag—which are defined differently:

$$\mathrm{F} = \mathrm{F_{frict}} + \mathrm{F_{wave}} = \mathrm{C_{D,frict}}\frac{\rho}{2}\mathrm{V^2A_{wet}} + \mathrm{C_{D,wave}}\frac{\rho}{2}\mathrm{V^2L^2},$$

$$\text{with} \quad \mathrm{C_{D,wave}} \text{ from Fig. 7.19 and } \mathrm{C_{D,frict}} \approx 0.031/\mathrm{Re_L^{1/7}} \quad \text{(turbulent flat plate formula)}$$

$$\text{Here,}\ \ \mathrm{F} = \mathrm{C_{Dfrict}}\frac{1025}{2}\mathrm{V^2}(800) + \mathrm{C_{Dwave}}\frac{1025}{2}\mathrm{V^2}(50)^2, \quad \text{with V in m/s}\left(1\ \frac{\mathrm{m}}{\mathrm{s}} = 1.94\ \mathrm{kn}\right)$$

Assume different values of V, calculate friction and wave drag (the latter depending upon the Froude number V/√(gL) = V/√[9.81(50)] ≈ 0.0452V(m/s). Then compute the power in watts from P = FV, with F in newtons and V in m/s. Plot P versus V in knots on the graph below. The results show that, below 4 knots, wave drag is negligible and sharp increases in ship speed are possible with small increases in power. Wave drag limits the maximum speed to about 8 knots. There are two good high-velocity, "high slope" regions—at 6 knots and at 7.5 knots—where speed increases substantially with power.



P7.106 For the kite-assisted ship of Prob. P7.85, again neglect wave drag and let the wind velocity be 30 mi/h. Estimate the kite area that would tow the ship, unaided by the propeller, at a ship speed of 8 knots.

Solution: Since the only unknown is the kite area, this problem is simpler than Prob. P7.85. Convert 8 knots to 4.12 m/s. Assume sea level air density, $\rho_a = 1.2255$ kg/m^3. For seawater, take $\rho = 1025$ kg/m^3 and $\mu = 0.00107$ kg/m-s. The kite force balances the ship drag:

$$F_{kite} = C_D \frac{\rho_a}{2} A_{kite} (V_{air} - V)^2 \cos 25^{\mathrm{o}} = F_{ship} = C_{d,\,friction} \frac{\rho}{2} A_{ship} V^2$$

$$\text{Evaluate } \mathrm{Re}_L = \frac{\rho V L}{\mu} = \frac{(1025)(4.12)(120)}{0.00107} = 4.73E8$$

$$C_{d,\,friction} = \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{0.031}{(4.73E8)^{1/7}} = 0.00179$$

$$\text{Finally, } (0.8)\left(\frac{1.2255}{2}\right) A_{kite} (13.4 - 4.12)^2 \cos 25^{\mathrm{o}} = (0.00179)\left(\frac{1025}{2}\right)(2800)(4.12)^2$$

$$\text{Solve for} \quad A_{kite} = \mathbf{1130}\ m^2 \quad Ans.$$

P7.107 The largest flag in Rhode Island stands outside Herb Chambers' auto dealership, on the edge of Route I-95 in Providence. The flag is 50 ft long, 30 ft wide, weighs 250 lbf, and takes four strong people to raise it or lower it. Using Prob. P7.40 for input, estimate (a) the wind speed, in mi/h, for which the flag drag is 1000 lbf; and (*b*) the flag drag when the wind is a low-end category 1 hurricane, 74 mi/h. [HINT: Providence is at sea level.]

Solution: Prob. P7.40 suggests a drag coefficient $C_D \approx 0.02 + 0.1(L/b)$, based on flag area Lb. Thus, for this big flag, $C_D = 0.02 + 0.1(50\text{ ft})/(30\text{ ft}) \approx 0.187$. From Table A.3, sea level density is 1.2255 kg/m^3 = 0.00238 slug/ft^3. Then a drag of 1000 lbf occurs when

$$F = 1000\,lbf = C_D \frac{\rho}{2} V^2 L b = (0.187)\left(\frac{0.00238}{2}\right) V^2 (50\,ft)(30\,ft)$$

$$Solve\ for\ V^2 = 2996 \frac{ft^2}{s^2}, \quad V = 54.7\ ft/s = \mathbf{37}\ mi/h \qquad Ans.(a)$$

(*b*) Convert 74 mi/h = 108.5 ft/s. Then compute the hurricane drag:

$$F = C_D \frac{\rho}{2} V^2 L b = (0.187)\left(\frac{0.00238}{2}\right)(108.5)^2 (50)(30) \approx \mathbf{3900}\ lbf \qquad Ans.(b)$$

P7.108 The data in Fig. P7.108 are for lift and drag of a *spinning* sphere from Ref. 12, pp. 7–20. Suppose a tennis ball (W ≈ 0.56 N, D ≈ 6.35 cm) is struck at sea level with initial velocity V_o = 30 m/s, with "topspin" (front of the ball rotating downward) of 120 rev/sec. If the initial height of the ball is 1.5 m, estimate the horizontal distance travelled before it strikes the ground.

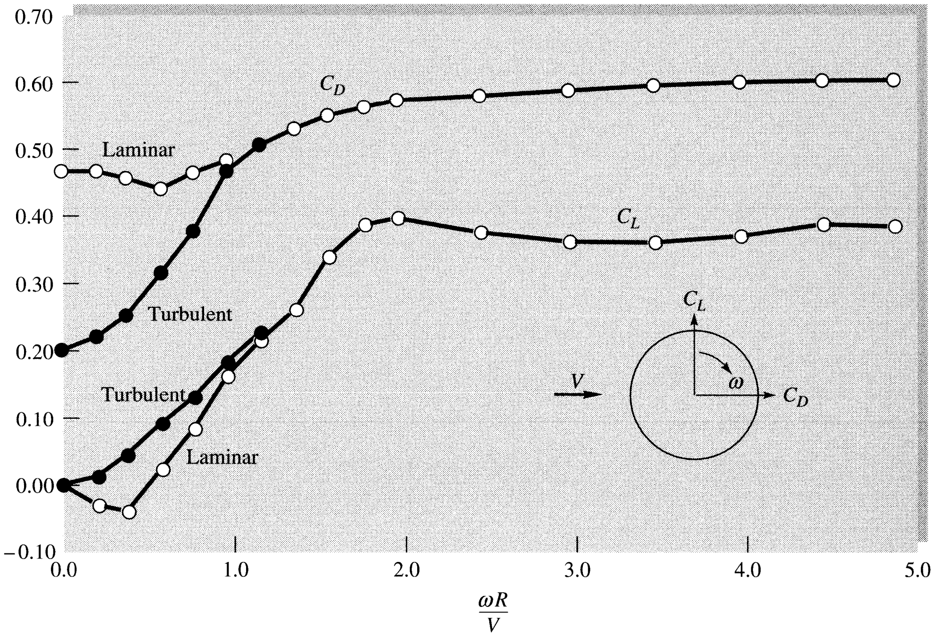
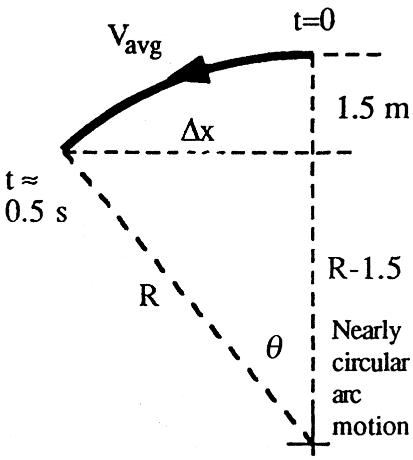


Fig. P7.108

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E−5 kg/m·s. For this short distance, the ball travels in nearly a circular arc, as shown at right. From Figure P7.108 we read drag and lift:



$$\omega = 120(2\pi) = 754\ \frac{\text{rad}}{\text{s}},$$

$$\frac{\omega R}{V} = \frac{754(0.03175)}{30} \approx 0.80,$$

$$\text{Read}\quad C_D \approx \mathbf{0.47},\quad C_L \approx \mathbf{0.12}$$

Initially, the accelerations in the horizontal and vertical directions are (*z* up, *x* to left)

$$a_{x,0} = -\frac{\text{drag}}{m} = -\frac{0.47(1.225/2)(30)^2(\pi/4)(0.0635)^2}{0.56/9.81} \approx \mathbf{-14.4\ m/s^2}$$

$$a_{z,0} = -g - \frac{\text{lift}}{m} = -9.81 - \frac{0.12(1.225/2)(30)^2(\pi/4)(0.0635)^2}{0.56/9.81} \approx \mathbf{-13.5\ m/s^2}$$

The term a_x serves to slow down the ball from 30 m/s, when hit, to about 24 m/s when it strikes the floor about 0.5 s later. The average velocity is (30 + 24)/2 = **27** m/s. The term a_z causes the ball to curve in its path, so one can estimate the radius of curvature and the angle of turn for which Δz = 1.5 m. Then, finally, one estimates Δx as desired:

$$\frac{V^2_{avg}}{R} = a_z,\quad \text{or:}\quad R \approx \frac{(27)^2}{13.5} \approx 54\ \text{m};\quad \theta = \cos^{-1}\left(\frac{54-1.5}{54}\right) \approx 13.54°$$

$$\text{Finally,}\quad \Delta x_{ball} = R\sin\theta = (54)\sin(13.54°) \approx \mathbf{12.6\ m}\quad \textit{Ans.}$$

A more exact numerical integration of the equations of motion (not shown here) yields the result Δx ≈ 13.0 m at t ≈ 0.49 s.

P7.109 The world record for automobile mileage, 12,665 miles per gallon, was set in 2005 by the PAC-CAR II in Fig. P7.109, built by students at the Swiss Federal Institute of Technology in Zurich [52]. This little car, with an empty weight of 64 lbf and a height of only 2.5 ft, traveled a 21-km course at 30 km/hr to set the record. It has a reported drag coefficient of 0.075 (comparable to an airfoil), based upon a frontal area of 3 ft^2. (*a*) What is the drag of this little car when on the course? (*b*) What horsepower is required to propel it? (*c*) Do a bit of research and explain why a value of miles per gallon is completely misleading in this particular case.

Fig. P7.109. The world's best mileage, from the PAC-Car II of ETH Zurich.

Solution: For air, assuming sea-level, take $\rho = 0.00238$ slug/ft^2. Convert $V = 30$ km/h to 27.34 ft/s. (*a*) Then the car's drag on the course, in lbf, is

$$F = C_D \frac{\rho}{2} V^2 A = (0.075)\frac{0.00238 slug/ft^3)}{2}(27.34\, ft/s)^2 (3\, ft^2) = \mathbf{0.20\ lbf} \quad Ans.(a)$$

Pretty small! Probably the rolling resistance is larger than this.
(*b*) The power required to overcome drag is simply

$$P = FV = (0.20 lbf)(27.34\, ft/s) = 5.47 \frac{ft-lbf}{s} \div 550 = \mathbf{0.010\,hp} \quad Ans.(b)$$

Pretty small! Not much of an engine is required. (*c*) The actual propulsor for this car was a very small hydrogen fuel cell. Thus "miles per gallon" does not make much sense.

P7.110 A baseball pitcher throws a curveball with an initial velocity of 65 mi/h and a spin of 6500 r/min about a vertical axis. A baseball weighs 0.32 lbf and has a diameter of 2.9 in. Using the data of Fig. P7.108 for turbulent flow, estimate how far such a curveball will have deviated from its straightline path when it reaches home plate 60.5 ft away.

Solution: For sea-level air, take $\rho = 0.00238$ slug/ft^3 and $\mu = 3.72\text{E}{-7}$ slug/ft·s. Again, for this short distance, the ball travels

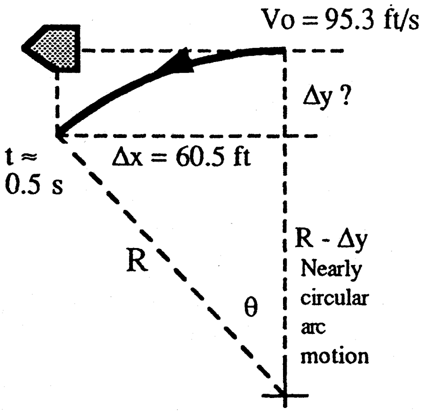


Fig. P7.110

in nearly a circular arc, as shown above. However, gravity is *not* involved in this curved *horizontal* path. First evaluate the lift and drag:

$$V_o = 65\ \frac{\text{mi}}{\text{h}} = 95\ \frac{\text{ft}}{\text{s}},\quad \omega = 6500\left(\frac{2\pi}{60}\right) = 681\ \frac{\text{rad}}{\text{s}},\quad \frac{\omega R}{V} = \frac{681(2.9/24)}{95} \approx 0.86$$

$$\text{Fig. P7.108:}\quad \text{Read } C_D \approx \mathbf{0.44},\ C_L \approx \mathbf{0.17}$$

The initial accelerations in the *x*- and *y*-directions are

$$a_{x,0} = -\frac{\text{drag}}{m} = -\frac{0.44(0.00238/2)(95)^2(\pi/4)(2.9/12)^2}{0.32/32.2\ \text{slug}} \approx \mathbf{-22.0\ \frac{ft}{s^2}}$$

$$a_{y,0} = -\frac{\text{lift}}{m} = -\frac{0.17(0.00238/2)(95)^2(\pi/4)(2.9/12)^2}{0.32/32.2} \approx \mathbf{-8.5\ \frac{ft}{s^2}}$$

The ball is in flight about 0.5 sec, so a_x causes it to slow down to about 85 ft/s, with an average velocity of (95 + 85)/2 ≈ **90** ft/s. Then one can use these numbers to estimate R:

$$R = \frac{V^2_{avg}}{|a_y|} = \frac{(90)^2}{8.5} \approx 954 \text{ ft}; \quad \theta = \sin^{-1}\left(\frac{\Delta x}{R}\right) = \sin^{-1}\left(\frac{60.5}{954}\right) \approx 3.63^\circ$$

Finally, $\Delta y_{\text{home plate}} = R(1-\cos\theta) = 954(1-\cos 3.63^\circ) \approx \mathbf{1.9\ ft}$ *Ans.*

P7.111 A table tennis ball has a mass of 2.6 g and a diameter of 3.81 cm. It is struck horizontally at an initial velocity of 20 m/s while it is 50 cm above the table, as in Fig. P7.111. For sea-level air, what topspin (as shown), in r/min, will cause the ball to strike the opposite edge of the table, 4 m away? Make an analytical estimate, using Fig. P7.108, and account for the fact that the ball decelerates during flight.

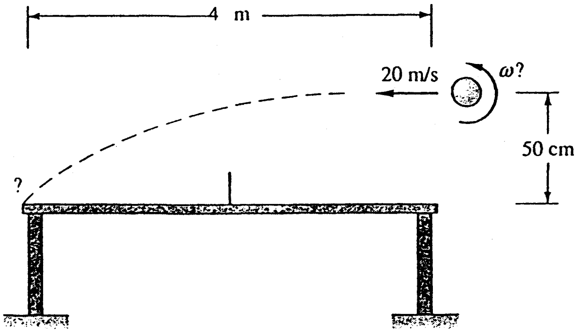


Fig. P7.111

NOTE: The table length is 4 **meters**.

Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E−5 kg/m·s. This problem is **difficult** because *the ball is so light and will decelerate greatly* during its trip across the table. For the last time, as in Prob. 7.108, for this short distance, we assume the ball travels in nearly a *circular arc*, as analyzed there. First, from the geometry of the table, Δx = 4 m, Δz = 0.5 m, the required radius of curvature is known:

$$R(1-\cos\theta) = 0.5 \text{ m}; \quad R\sin\theta = 4 \text{ m}; \quad \text{solve for } R = \mathit{16.25} \text{ m}, \ \theta = \mathit{14.25}^\circ$$

Then the centripetal acceleration should be estimated from R and the average velocity during the flight. Estimate, from Fig. P7.108, that CD ≈ 0.5. Then compute

$$a_{x,o} = -\frac{drag}{mass} = -\frac{0.5(1.225/2)(20)^2(\pi/4)(0.0381)^2}{0.0026} = -53 \ \frac{\text{m}}{\text{s}^2}$$

This reduces the ball speed from 20 m/s to about 12 m/s during the 0.25-s flight. Taking our average velocity as (20 + 12)/2 ≈ 16 m/s, we compute the vertical acceleration:

$$a_{z,avg} = \frac{V_{avg}^2}{R} = \frac{(16)^2}{16.25} = 15.75\ \frac{\text{m}}{\text{s}^2} = 9.81 + \frac{C_L(1.225/2)(16)^2(\pi/4)(0.0381)^2}{0.0026}$$

$$\text{Solve for}\quad C_{L,avg} \approx \mathbf{0.086}$$

From Fig. P7.108, this value of C_L (probably laminar) occurs at about $\omega R/V \approx 0.6$, or $\omega = 0.6(16)/(0.0381/2) \approx 500$ rad/s ≈ **4800 rev/min**. *Ans.*

P7.112 A smooth wooden sphere (SG = 0.65) is connected by a thin rigid rod to a hinge in a wind tunnel, as in Fig. P7.112. Air at 20°C and 1 atm flows and levitates the sphere. (a) Plot the angle θ versus sphere diameter d in the range 1 cm ≤ d ≤ 15 cm. (b) Comment on the feasibility of this configuration. Neglect rod drag and weight.

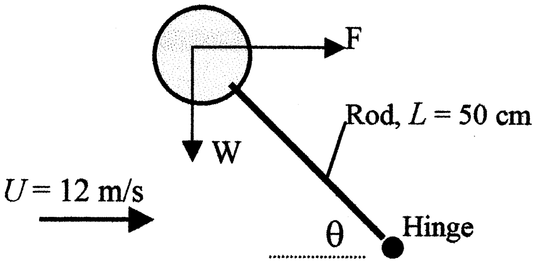


Fig. P7.112

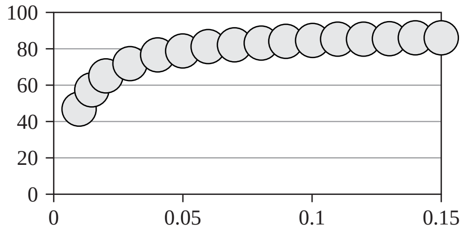
Solution: For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. If rod drag is neglected and $L \gg d$, the balance of moments around the hinge gives:

$$\sum M_{hinge} = 0 = FL\sin\theta - WL\cos\theta, \quad or \quad \tan\theta = \frac{W}{F} = \frac{\rho_w g(\pi/6)d^3}{(\rho_a/2)C_D V^2(\pi/4)d^2}$$

$$\textit{Input the data:}\ \tan\theta = \frac{(0.65)(998)(9.81)(\pi/6)d^3}{(1.2/2)C_D(12)^2(\pi/4)d^2} = 49.1\frac{d}{C_D}\ \textit{with d in meters.}$$

We find C_D from $Re_d = \rho Vd/\mu$ = (1.2)(12)d/(0.000018) = **8E5*d*** (with d in meters). For d = 1 cm, Re_d = 8000, Fig. 7.16b, C_D = 0.5, tanθ = 0.982, θ = 44.5°. At the other extreme, for d = 15 cm, Re_d = 120000, Fig. 7.16b, C_D = 0.5, tanθ = 14.73, θ = 86.1°.

(a) A complete plot is shown at right. (b) This is a ridiculous device for either velocity or diameter.



Problem 7.112: Angles vs. Diameter

P7.113 An auto has m = 1000 kg and a drag-area C_DA = 0.7 m^2, plus constant 70-N rolling resistance. The car coasts without brakes at 90 km/h climbing a hill of 10 percent grade (5.71°). How far up the hill will the car come to a stop?

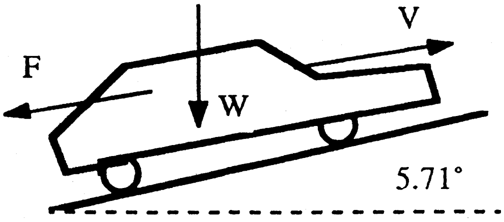


Fig. P7.113

Solution: For sea-level air, take $\rho = 1.225\ kg/m^3$ and $\mu = 1.78E{-}5$ kg/m·s. If x denotes *uphill,* the equation of motion is

$$m\frac{dV}{dt} = -W\sin\theta - F_{rolling} - C_DA\frac{\rho}{2}V^2, \quad \text{separate the variables and integrate:}$$

$$V = V_f\tan\left[\tan^{-1}\left(\frac{V_o}{V_f}\right) - t\frac{W\sin\theta + F_r}{mV_f}\right], \quad \text{where } V_f = \sqrt{\frac{W\sin\theta + F_r}{C_DA\rho/2}}$$

For the particular data of this problem, we evaluate

$$V_f = \sqrt{\frac{9810\sin 5.71° + 70}{0.7(1.225/2)}} \approx 49.4\frac{m}{s}, \quad \frac{W\sin\theta + F_r}{mV_f} = \frac{9810\sin 5.71° + 70}{1000(49.4)} \approx 0.0212$$

$$\text{also } \tan^{-1}\left(\frac{25}{49.4}\right) = 0.469 \text{ radians. So, finally,} \quad V \approx 49.4\ \tan[0.469 - 0.0212t]$$

The car stops at V = 0, or t_{final} = 0.469/0.0212 ≈ **22.1** s. The distance to stop is computed by the same formula as in Prob. 7.98:

$$\Delta x_{max} = \frac{m}{\rho C_DA}\ln\left[1 + \left(\frac{V_o}{V_f}\right)^2\right] = \frac{1000}{1.225(0.7)}\ln\left[1 + \left(\frac{25}{49.4}\right)^2\right] \approx \mathbf{266\ m} \quad \textit{Ans.}$$

P7.114 The deep submergence vehicle ALVIN, in the Chap. 2 opener photo, is 23 ft long and 8.5 ft wide. It weighs about 36,000 lbf in air and ascends (descends) in the seawater due to about 360 lbf of positive (negative) buoyancy. Noting that the leading face of the ship is quite different for ascent and descent, (*a*) estimate the velocity for each direction, in meters per minute. (*b*) How long does it take to ascend from its maximum depth of 4500 m?

Solution: Well, nothing in Table 7.3 looks much like the top or the bottom of ALVIN. Let's just estimate. When ascending, the leading face of ALVIN is cluttered and ugly and approximates a blunt body, so let's guess $C_{D,\text{ascent}} \approx 1.0$. When descending, the approaching water sees a smoother, if still blunt, shape, so let's guess $C_{D,\text{descent}} \approx 0.7$. ALVIN is almost rectangular in outline either way, so we take the area to be $bL \approx$ (23ft)(8.5ft) = 195.5 ft^2.

For seawater, $\rho = 1025\ \text{kg/m}^3 \approx 2.0\ \text{slug/ft}^3$. Then

$$\text{Descent}:\ \ F = 360\,lbf = C_{D,descent}\frac{\rho}{2}V_d^2 A \approx (1.0)\left(\frac{2.0\,slug/ft^3}{2}\right)V_d^2\,(195.5\,ft^2)$$

$$\text{Solve for}\ \ V_{descent} \approx 1.36\frac{ft}{s} = 0.41\frac{m}{s} \approx \mathbf{25}\ \frac{m}{min}\qquad Ans.(a-1)$$

$$\text{Ascent}:\ \ F = 360\,lbf = C_{D,ascent}\frac{\rho}{2}V_a^2 A \approx (0.7)\left(\frac{2.0\,slug/ft^3}{2}\right)V_a^2\,(195.5\,ft^2)$$

$$\text{Solve for}\ \ V_{ascent} \approx 1.62\frac{ft}{s} = 0.49\frac{m}{s} \approx \mathbf{30}\ \frac{m}{min}\qquad Ans.(a-2)$$

According to Mark Spear of the Woods Hole ALVIN Project, these estimates are about right. Students, however, might have quite different estimates of the (unknown) drag coefficients.

(*b*) At 25 m/min, rising from 4500 meters takes (4500 m)/(25 m/min) = 180 min = **3 hours**!

Mark Spear told the writer he takes along a book to read during ascent and descent.

P7.115 The Cessna *Citation* executive jet weighs 67 kN and has a wing area of 32 m^2. It cruises at 10 km standard altitude with a lift coefficient of 0.21 and a drag coefficient of 0.015. Estimate (a) the cruise speed in mi/h; and (b) the horsepower required to maintain cruise velocity.

Solution: At 10 km standard altitude (Table A-6) the air density is 0.4125 kg/m^3. (a) The cruise speed is found by setting lift equal to weight:

$$Lift = 67000\ \text{N} = C_L\frac{\rho}{2}V^2 A_{wing} = 0.21\left(\frac{0.4125\ \text{kg/m}^3}{2}\right)V^2(32\ \text{m}^2),$$

$$\text{Solve}\quad V = 220\ \frac{\text{m}}{\text{s}} = \mathbf{492}\ \frac{\textbf{mi}}{\textbf{h}}\qquad Ans.\ \text{(a)}$$

(b) With speed known, the power is found from the drag:

$$Power = F_{drag}V = \left(C_D\frac{\rho}{2}V^2A\right)V = \left\{0.015\left(\frac{0.4125}{2}\right)(220)^2(32)\right\}(220)$$

$$= 1.05\text{ MW} = \mathbf{1410\ hp} \quad \textit{Ans.}\text{ (b)}$$

P7.116 An airplane weighs 180 kN and has a wing area of 160 m^2 and a mean chord of 4 m. The airfoil properties are given by Fig. 7.25. If the plane is designed to land at $V_o = 1.2V_{stall}$, using a split flap set at 60°, (a) What is the proper landing speed in mi/h? (b) What power is required for takeoff at the same speed?

Solution: For air at sea level, $\rho \approx 1.225$ kg/m^3. From Fig. 7.24 with the flap, $C_{L,max} \approx 1.75$ at $\alpha \approx 6°$. Compute the stall velocity:

$$V_{stall} = \sqrt{\frac{2W}{\rho C_{L,\max}A_p}} = \sqrt{\frac{2(180000\text{ N})}{(1.225\text{ kg/m}^3)(1.75)(160\text{ m}^2)}} = 32.4\ \frac{\text{m}}{\text{s}}$$

$$\text{Then}\quad V_{landing} = 1.2V_{stall} = 38.9\ \frac{m}{s} = \mathbf{87}\ mi/h \quad \textit{Ans.}\text{ (a)}$$

$$C_L = \frac{C_{L,\max}}{(V_{land}/V_{stall})^2} = \frac{1.75}{(1.2)^2} = 1.22$$

For take-off at the same speed of 38.9 m/s, we need a drag estimate. From Fig. 7.25 ***with*** a split flap, $C_{D\infty} \approx 0.04$. We don't have a theory for induced drag with a split flap, so we just go along with the usual finite wing theory, Eq. (7.71). The aspect ratio is b/c = (40 m)/(4 m) =10.

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR} = 0.04 + \frac{(1.22)^2}{\pi(10)} = 0.087,$$

$$F_{drag} = (0.087)\left(\frac{1.225}{2}\right)(38.9)^2(160) = 12900\text{ N}$$

Power required = FV = (12900 N)(38.9 m/s) = 501000 W = **672 hp** *Ans.* (b)

P7.117 The Transition® auto-car in Fig. 7.30 has a weight of 1200 lbf, a wingspan of 27.5 ft, and a wing area of 150 ft^2, with a symmetrical airfoil, $C_{D\infty} = 0.02$. Assume that the fuselage and tail section have a drag-area comparable to the Toyota *Prius* [21], $C_DA \approx 6.24$ ft^2. If the pusher propeller provides a thrust of 250 lbf, how fast, in mi/h, can this car-plane fly at an altitude of 8200 ft?

Solution: From Table A.6, at 8200 ft = 2500 m, air density is 0.957 kg/m^3 = 0.00186 slug/ft^3. The wing has an aspect ratio AR = (27.5 ft)^2/(150 ft^2) = 5.04. The wing lift and drag coefficients are

$$C_L = \frac{2W}{\rho V^2 A_p} = \frac{2(1200\,lbf)}{(0.00186 slug/ft^3)V^2(150)} = \frac{8600}{V^2}\;, \quad \text{with } V \text{ in } \frac{ft}{s}$$

$$C_{D,wing} = C_{D\infty} + \frac{C_L^2}{\pi\,\text{AR}} = 0.020 + \frac{(8600/V^2)^2}{\pi(5.04)} = 0.020 + \frac{4.67\text{E}6}{V^4}$$

The total drag, including the fuselage, equals the propeller thrust:

$$F = T = 250\,lbf = (C_D A_{fuselage} + C_{D,wing}\,A_p)\frac{\rho}{2}V^2$$

$$= [6.24 + (0.020 + \frac{4.67\text{E}6}{V^4})(150)](\frac{0.00186}{2})V^2,$$

$$or: \quad 250\,lbf = (0.00580 + 0.00279 + \frac{6.51\text{E}5}{V^4})V^2$$

The term involving (6.51E5/V^4) is small. Iterate to V_{autocar} = 162 ft/s = **110** mi/h *Ans.*

***P7.118** Suppose the airplane of Prob. 7.116 is now fitted with all the best high-lift devices of Fig. 7.28. (a) What is its minimum stall speed in mi/h? (b) Estimate the stopping distance if the plane lands at $V_o = 1.25V_{\text{stall}}$ with constant $C_L = 3.0$ and $C_D = 0.2$ and the braking force is 20% of the weight on the wheels.

Solution: For air at sea level, $\rho = 1.225$ kg/m^3. From Fig. 7.28 read $C_{L,\text{highest}} \approx 4.0$.

$$\text{(a) Then} \quad V_{stall} = \sqrt{\frac{2\,W}{\rho C_{L,\max} A_p}} = \sqrt{\frac{2(180000\text{ N})}{(1.225\text{ kg/m}^3)(4.0)(160\text{ m}^2)}} = 21.4\ \frac{\text{m}}{\text{s}}$$

$$\text{Thus} \quad V_{land} = 1.25V_{stall} = 26.8\ \frac{\text{m}}{\text{s}} \approx \mathbf{60}\ \frac{\textbf{mi}}{\textbf{h}} \quad \textit{Ans. (a)}$$

(b) With constant lift and drag coefficients, we can set up and solve the equation of motion:

$$\sum F_x = m\frac{dV}{dt} = -F_{drag} - F_{brake} = -C_D\left(\frac{\rho}{2}\right)V^2 A_p - 0.2(Weight - Lift)$$

$$\text{or:}\quad \left(\frac{180000}{9.81}\right)\frac{dV}{dt} = -0.2\left(\frac{1.225}{2}\right)V^2(160) - 0.2\left[180000 - 3.0\left(\frac{1.225}{2}\right)V^2(160)\right]$$

$$\text{Clean this up:}\quad \frac{dV}{dt} = +0.00214V^2 - 1.962$$

We could integrate this twice and calculate $V = 0$ (stopping) at $t = 21.5$ s and $\mathbf{S_{max} = 360}$ **m**. Or, since we are looking for distance, we could convert $dV/dt = (1/2)d(V^2)/ds$ to obtain

$$\frac{dV}{dt} = \frac{1}{2}\frac{dV^2}{ds} = 0.00214V^2 - 1.962,\quad \text{or:}\quad 2\int_0^{S_{\max}} ds = \int_{V_o^2}^{0} \frac{d(V^2)}{0.00214V^2 - 1.962}$$

$$\text{Solution:}\quad S_{\max} = \frac{1}{2(0.00214)}\ln\left[\frac{1.962}{1.962 - 0.00214(26.8)^2}\right] \approx \mathbf{360\ m} \quad \textit{Ans.}\ \text{(b)}$$

P7.119 A transport plane has a mass of 45,000 kg, a wing area of 160 m^2, and an aspect ratio of 7. Assume all lift and drag due to the wing alone, with $C_{D\infty} = 0.020$ and $C_{L,\max} = 1.5$. If the aircraft flies at 9,000 m standard altitude, make a plot of drag (in N) versus speed (from stall to 240 m/s) and determine the optimum cruise velocity (minimum drag per unit speed).

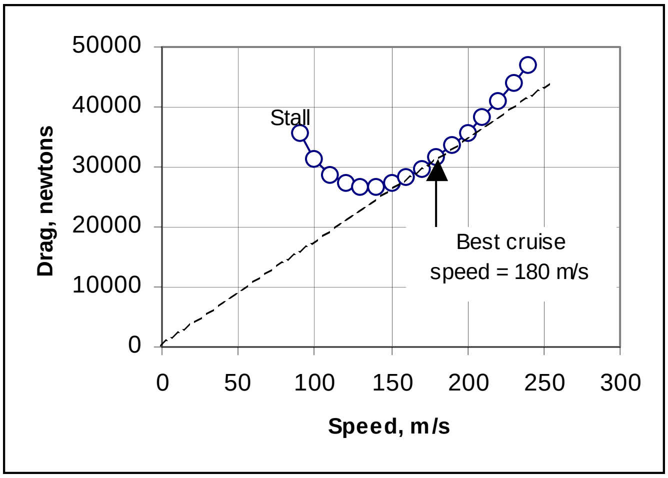
Solution: From Table A.6, at 9000 m, $\rho = 0.4661$ kg/m^3. First compute the stall velocity:

$$V_{stall} = \sqrt{\frac{2W}{\rho A_p C_{L,\max}}} = \sqrt{\frac{2(45000)(9.81\,N)}{0.4661(160)(1.5)}} = 89\ m/s$$

Go up from there with your drag-vs-speed plot. For each speed, compute C_L, C_D, Drag:

$$Given\ V,\quad C_L = \frac{2W}{\rho V^2 A_p};\quad C_D = C_{D\infty} + \frac{C_L^2}{\pi AR};\ Drag = C_D\frac{\rho}{2}V^2 A_p$$

Example: V = 100 m/s, $C_L = 1.187$, $C_D = 0.084$, Drag = 31,300 N. The plot is:



We see that any speed between 150 and 200 m/s is efficient. The actual minimum is 176 m/s. NOTE: Cruise speed varies with altitude and would be much lower at, say, sea-level.

P7.120 Show that, if Eqs. (7.70) and (7.71) are valid, the maximum lift-to-drag ratio occurs when $C_D = 2C_{D\infty}$. What are $(L/D)_{max}$ and α for a symmetric wing when AR = 5.0 and $C_{D\infty} = 0.009$?

Solution: According to our lift and induced-drag approximations, Eqs. (7.70) and (7.71), the lift-to-drag ratio is

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D\infty} + C_L^2/(\pi AR)}; \quad \text{Differentiate:} \quad \frac{d}{dC_L}\left(\frac{L}{D}\right) = 0 \quad \text{if} \quad \mathbf{C_D = 2C_{D\infty}} \quad \textit{Ans.}$$

For our numerical example, compute, at maximum L/D,

$$AR = 5, \quad C_L = \sqrt{C_{D\infty}\pi AR} = \sqrt{(0.009)\pi(5)} \approx 0.376, \quad C_D = 2C_{D\infty} = 0.018$$

$$\text{Therefore,} \quad L/D\big|_{max} = 0.376/0.018 \approx \mathbf{21} \quad \textit{Ans.}$$

Also, $C_L = 0.376 = 2\pi \sin\alpha/[1 + 2/AR] = 2\pi \sin\alpha/[1 + 2/5]$, solve $\alpha \approx \mathbf{4.8°}$ *Ans.*

P7.121 In gliding (unpowered) flight, lift and drag are in equilibrium with the weight. Show, that, with no wind, the craft sinks at an angle $\tan\theta \approx$ drag/lift. For a sailplane with $m = 200$ kg, wing area = 12 m^2, AR = 12, with an NACA 0009 airfoil, estimate (a) stall speed, (b) minimum gliding angle; (c) the maximum distance it can glide in still air at $z = 1200$ m.

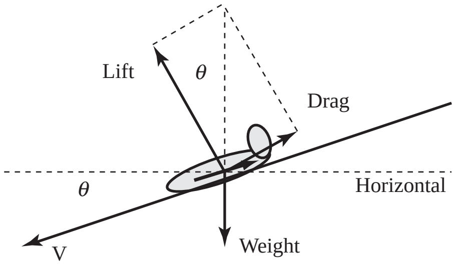


Fig. P7.121

Solution: By the geometry of the figure, with no thrust, wind, or acceleration,

$$W = \frac{L}{\cos\theta} = \frac{D}{\sin\theta}, \quad \text{or:} \quad \boldsymbol{\tan\theta_{glide} = \frac{D}{L}} \quad \textit{Ans.}$$

The NACA 0009 airfoil is shown in Fig. 7.25, with $C_{D\infty} \approx 0.006$. From Table A-6, at z = 1200 m, $\rho \approx 1.09$ kg/m^3. Then, as in part (b) of Prob. 7.120 above,

$$\text{at max}\frac{L}{D}, \quad C_L = \sqrt{C_{D\infty}\pi AR} = \sqrt{0.006\pi(12)} = 0.476, \quad \frac{L}{D}\Big|_{max} = \frac{0.476}{2(0.006)} \approx 39.6$$

$$\text{Thus} \quad \tan\theta_{min} = 1/39.6 \quad \text{or} \quad \boldsymbol{\theta_{min} \approx 1.45°} \quad \textit{Ans.}\text{ (b)}$$

Meanwhile, from Fig. 7.25, $C_{L,max} \approx 1.3$, so the stall speed at 1200 m altitude is

$$V_{stall} = \sqrt{\frac{2\,W}{\rho C_{L,max} A}} = \sqrt{\frac{2(200)(9.81)}{1.09(1.3)(12)}} \approx \mathbf{15.2\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

With θ_{min} = 1.45° and z = 1200 m, the craft can glide 1200/tan(1.45°) ≈ **47 km** *Ans.* (c).

***P7.122** A boat of mass 2500 kg has two hydrofoils, each of chord 30 cm and span 1.5 meters, with $C_{L,max} = 1.2$ and $C_{D\infty} = 0.08$. Its engine can deliver 130 kW to the water. For seawater at 20°C, estimate (a) the minimum speed for which the foils support the boat, and (b) the maximum speed attainable.

Solution: For seawater at 20°C, take ρ = 1025 kg/m^3 and μ = 0.00107 kg/m·s. With two foils, total planform area is 2(0.3 m)(1.5 m) = 0.9 m^2. Thus the stall speed is

$$\mathbf{V_{min}} = \sqrt{\frac{2W}{\rho C_{L,max}A}} = \sqrt{\frac{2(2500)(9.81)}{1025(1.2)(0.9)}} \approx \mathbf{6.66\ \frac{m}{s} \approx (13\ knots)} \quad \textit{Ans.}\text{ (a)}$$

Given AR = 1.5/0.3 = 5.0. At any speed during lifting operation ($V > V_{min}$), the lift and drag coefficients, from Eqs. (7.70) and (7.71), are

$$C_L = \frac{2W}{\rho AV^2} = \frac{2(2500)(9.81)}{1025(0.9)V^2} = \frac{53.2}{V^2};$$

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR} = 0.08 + \frac{(53.2/V^2)^2}{\pi(5.0)} = 0.08 + \frac{180.0}{V^4}$$

$$\text{Power} = DV = \left(0.08 + \frac{180}{V^4}\right)\left(\frac{1025}{2}\right)V^2(0.9)V = 130\ \text{hp} \times 745.7 = 96900\ \text{W}$$

$$\text{Clean up and rearrange:} \quad V^4 - 2627V + 2250 = 0,$$

$$\text{Solve} \quad \mathbf{V_{max} \approx 13.5\ \frac{m}{s} \approx 26\ kn} \quad \textit{Ans.}\text{ (b)}$$

Three other roots: 2 *imaginary* and V_4 = 0.86 m/s (impossible, below stall)

P7.123 In prewar days there was a controversy, perhaps apocryphal, about whether the bumblebee has a legitimate aerodynamic right to fly. The average bumblebee (*Bombus terrestris*) weighs 0.88 g, with a wing span of 1.73 cm and a wing area of 1.26 cm^2. It can indeed fly at 10 m/s. Using fixed-wing theory, what is the lift coefficient of the bee at this speed? Is this reasonable for typical airfoils?

Solution: Assume sea-level air, ρ = 1.225 kg/m^3. Assume that the bee's wing is a low-aspect-ratio airfoil and use Eqs. (7.68) and (7.72):

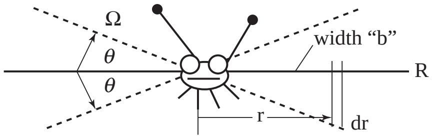
$$AR = \frac{b^2}{A_p} = \frac{(1.73\ \text{cm})^2}{1.26\ \text{cm}^2} = \mathit{2.38}; \quad C_L = \frac{2W}{\rho V^2 A_p} = \frac{2(0.88\text{E–}3)(9.81)}{1.225(10)^2(1.26\text{E–}4)} \approx \mathbf{1.12}$$

This looks unreasonable, $C_L \to C_{L,max}$ and the bee could not fly *slower* than 10 m/s.

Even if this high lift coefficient were possible, the angle of attack would be unrealistic:

$$C_L = 1.12 \overset{?}{=} \frac{2\pi \sin\alpha}{1 + 2/AR}, \quad \text{with } AR = 2.38, \quad \text{solve for } \alpha \approx \mathbf{19°} \quad (\text{too high}, > \alpha_{stall})$$

P7.124 The bumblebee can hover at zero speed by flapping its wings. Using the data of Prob. 7.123, devise a theory for flapping wings where the downstroke approximates a short flat plate normal to the flow (Table 7.3)



and the upstroke is feathered at nearly zero drag. How many flaps per second of such a model wing are needed to support the bee’s weight? (Actual measurements of bees show a flapping rate of 194 Hz.)

Solution: Any “theory” one comes up with might be crude. As shown in the figure, let the wings flap sinusoidally, between ±θ_o, that is, $\theta = \theta_o \cos \Omega t$. Let the upstroke be feathered (zero force), and let the downstroke be strong enough to create a total upward force of 0.75 W on each wing—to compensate for zero lift during upstroke. Assume a short flat plate (Table 7.3), $C_D \approx 1.2$. Then, on each strip *dr* of wing, the elemental drag force is

$$dF = \frac{\rho}{2} C_D V^2 b\, dr, \quad \text{where } V = r\frac{d\theta}{dt} = r\Omega\theta_o \sin(\Omega t)$$

$$F = \int_0^R \frac{\rho}{2} C_D (r\,\Omega\theta_o \sin\Omega t)^2 b\, dr = \frac{\rho}{6} C_D \Omega^2 \theta_o^2 b R^3 [\sin^2 \Omega t]_{avg} \approx 0.75\text{ W}$$

$$\text{Assume full flapping:} \quad \theta_o = \frac{\pi}{2} \quad \text{and} \quad [\sin^2 \Omega t]_{avg} \approx \frac{1}{2}$$

$$\text{Evaluate} \quad F = 0.75(0.00088)(9.81) \approx \frac{1.225}{12}(1.2)\Omega^2\left(\frac{\pi}{2}\right)^2(0.00728)(0.00865)^3$$

$$\text{Solve for} \quad \Omega \approx 2132 \text{ rad/s} \div 2\pi \approx \mathbf{340\ rev/s} \quad \textit{Ans.}$$

This is about 75% higher than the measured value $\Omega_{bee} \approx 194$ Hz, but it’s a crude theory!

P7.125 In 2001, a commercial aircraft lost all power while flying at 33,000 ft over the open Atlantic Ocean, about 60 miles from the Azores Islands. The pilots, with admirable skill, put the plane into a shallow glide and successfully landed in the Azores. Assume that the airplane satisfies Eqs. (7.70) and (7.71), with AR = 7, $C_{d\infty} = 0.02$, and a symmetric airfoil. Estimate its optimum glide distance with a mathematically perfect pilot.

Solution: From Problem P7.120, the maximum lift-to-drag ratio occurs when $C_d = 2C_{d\infty} = 2(0.02) = 0.04$ in the present case. Accordingly, for maximum *L/D* ratio, the lift coefficient is $C_L = [C_{d\infty}\pi \text{AR}]^{1/2} = [0.02\pi(7)]^{1/2} = 0.663$. From Prob. 7.121, the glide angle of an unpowered aircraft is such that $\tan\theta$ = drag/lift = C_d/C_L. Thus, the pilots' optimum glide is:

$$\tan\theta_{\min} = \frac{Drag}{Lift}\bigg|_{\mathbf{min}} = \frac{0.04}{0.663} = \frac{1}{16.6} = \frac{33000\text{ ft}}{Glide\ distance},$$

$$\text{or}\quad \boldsymbol{Glide} = 547000\text{ ft} \approx \mathbf{104\ miles}\quad \textit{Ans.}$$

P7.126 Using the data for the Transition® auto-car from Prob. P7.117, and a maximum lift coefficient of 1.3, estimate the distance for the vehicle to take off at a speed of $1.2V_{\text{stall}}$. Note that we have to add the car-body drag to the wing drag.

Solution: For sea-level conditions, take $\rho = 0.00237$ slug/ft^3. Recall the data: $C_{D\infty} = 0.02$, $A_p = 150$ ft^2, $W = 1200$ lbf, $b = 27.5$ ft, AR $= b^2/A_p = 5.04$, $C_DA \approx 6.24$ ft^2 for the fuselage, and thrust $T = 250$ lbf. Follow the steps in Example 7.9, adding in car-body drag.

$$V_{stall} = \sqrt{\frac{2W}{C_{L,\max}\rho A_p}} = \sqrt{\frac{2(1200)}{(1.3)(0.00237)(150)}} = 72.1\ ft/s$$

$$V_{take-off} = V_o = 1.2V_{stall} = 1.2(72.1) = 86.5\ ft/s$$

$$C_{D,wing} = C_{D\infty} + \frac{C_L^2}{\pi\, AR} = 0.020 + \frac{C_L^2}{\pi(5.04)} = 0.02 + 0.0632C_L^2$$

Find the take-off lift coefficient, the total drag, and the parameter *k* at take-off:

$$C_{Lo} = \frac{2W}{\rho V_o^2 A_p} = \frac{2(1200)}{(0.00237)(86.5)^2(150)} = 0.902$$

$$C_{Do} = 0.02 + 0.0632C_{Lo}^2 = 0.02 + 0.0632(0.902)^2 = 0.0714$$

$$k = \frac{\rho}{2}(C_{Do}A_p + C_D A_{car}) = (\frac{0.00237}{2})[0.0714(150)+6.24] = 0.0201\ slug/ft$$

$$D_o = kV_o^2 = 0.0201(86.5)^2 = 150\ lbf$$

Finally, use the take-off distance S_o formula derived in Example 7.9:

$$S_o = \frac{m}{2k}\ln(\frac{T}{T-D_o}) = \frac{(1200/32.2)}{2(0.0201)}\ln(\frac{250}{250-150}) = (927)(0.916) = \mathbf{850}\ ft \quad Ans.$$

The manufacturer, with better data, states that the Transition® auto-car will take off and clear a 50-ft obstacle within a distance of 1700 ft.

P7.127 The Chap. 7 opener photo of Yves Rossy gives the following data: thrust = 200 lbf, altitude = 8200 ft, and wing span = 8 ft. Further assume a wing area of 12 ft^2, total weight of 280 lbf, $C_{D\infty}$ = 0.08 for the wing, and a drag area of 1.7 ft^2 for Rocket Man. Estimate the maximum velocity possible for this condition, in mi/h.

Solution: From Table A.6, at 8200 ft = 2500 m, air density is 0.957 kg/m^3 = 0.00186 slug/ft^3. The wing has an aspect ratio AR = b^2/A_p = (8 ft)^2/(12 ft^2) = 5.33. The wing lift and drag coefficients are

$$C_L = \frac{2W}{\rho V^2 A_p} = \frac{2(280\,lbf)}{(0.00186\,slug/ft^3)V^2(12)} = \frac{25090}{V^2}\ , \quad \text{with } V \text{ in } \frac{ft}{s}$$

$$C_{D,wing} = C_{D\infty} + \frac{C_L^2}{\pi\,\text{AR}} = 0.080 + \frac{(25090/V^2)^2}{\pi(5.33)} = 0.080 + \frac{3.76E7}{V^4}$$

The total drag, including the fuselage, equals the rocket thrust:

$$F = T = 200\,lbf = (C_D A_{Rossy} + C_{D,wing}\,A_p)\frac{\rho}{2}V^2$$

$$200 = [1.7 + (0.080 + \frac{3.76\text{E}7}{V^4})(12)](\frac{0.00186}{2})V^2,$$

$$or: \quad 200\,lbf = (0.00158 + 0.000893 + \frac{4.19\text{E}5}{V^4})V^2$$

The term involving (4.19E5/V^4) is very small. Iterate to $V_{\max} = 281$ ft/s = **192** mi/h *Ans*. Observers of Rocket Man in the Alps that day recorded $V_{\max} \approx 186$ mi/h.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE7.1 A smooth 12-cm-diameter sphere is immersed in a stream of 20°C water moving at 6 m/s. The appropriate Reynolds number of this sphere is

(a) 2.3E5 **(b) 7.2E5** (c) 2.3E6 (d) 7.2E6 (e) 7.2E7

FE7.2 If, in Prob. FE7.1, the drag coefficient based on frontal area is 0.5, what is the drag force on the sphere?

(a) 17 N (b) 51 N **(c) 102 N** (d) 130 N (e) 203 N

FE7.3 If, in Prob. FE7.1, the drag coefficient based on frontal area is 0.5, at what terminal velocity will an aluminum sphere (SG = 2.7) fall in still water?

(a) 2.3 m/s (b) 2.9 m/s (c) 4.6 m/s (d) 6.5 m/s (e) 8.2 m/s

FE7.4 For flow of sea-level standard air at 4 m/s parallel to a thin flat plate, estimate the boundary-layer thickness at *x* = 60 cm from the leading edge:

(a) 1.0 mm (b) 2.6 mm (c) 5.3 mm **(d) 7.5 mm** (e) 20.2 mm

FE7.5 In Prob. FE7.4, for the same flow conditions, what is the wall shear stress at *x* = 60 cm from the leading edge?

(a) 0.053 Pa (b) 0.11 Pa **(c) 0.016 Pa** (d) 0.32 Pa (e) 0.64 Pa

FE7.6 Wind at 20°C and 1 atm blows at 75 km/h past a flagpole 18 m high and 20 cm in diameter. The drag coefficient based upon frontal area is 1.15. Estimate the wind-induced bending moment at the base of the pole.

(a) 9.7 kN·m (b) 15.2 kN·m (c) 19.4 kN·m (d) 30.5 kN·m (e) 61.0 kN·m

FE7.7 Consider wind at 20°C and 1 atm blowing past a chimney 30 m high and 80 cm in diameter. If the chimney may fracture at a base bending moment of 486 kN·m, and its drag coefficient based upon frontal area is 0.5, what is the approximate maximum allowable wind velocity to avoid fracture?

(a) 50 mi/h (b) 75 mi/h (c) 100 mi/h (d) 125 mi/h **(e) 150 mi/h**

FE7.8 A dust particle of density 2600 kg/m^3, small enough to satisfy Stokes drag law, settles at 1.5 mm/s in air at 20°C and 1 atm. What is its approximate diameter?

(a) 1.8 μm (b) 2.9 μm **(c) 4.4 μm** (d) 16.8 μm (e) 234 μm

FE7.9 An airplane has a mass of 19,550 kg, a wing span of 20 m, and an average wind chord of 3 m. When flying in air of density 0.5 kg/m^3, its engines provide a thrust of 12 kN against an overall drag coefficient of 0.025. What is its approximate velocity?

(a) 250 mi/h (b) 300 mi/h (c) 350 mi/h **(d) 400 mi/h** e) 450 mi/h

FE7.10 For the flight conditions of the airplane in Prob. FE7.9 above, what is its approximate lift coefficient?

(a) 0.1 (b) 0.2 (c) 0.3 **(d) 0.4** (e) 0.5

COMPREHENSIVE PROBLEMS

C7.1 Jane wants to estimate the drag coefficient of herself on her bicycle. She measures the projected frontal area to be 0.40 m^2 and the rolling resistance to be 0.80 N·s/m. Jane coasts down a hill with a constant 4° slope. The bike mass is 15 kg, Jane's mass is 80 kg. She reaches a terminal speed of 14 m/s down the hill. Estimate the aerodynamic drag coefficient C$_D$ of the rider and bicycle combination.

Solution: For air take $\rho \approx 1.2$ kg/m^3. Let x be down the hill. Then a force balance is

$$\sum F_x = 0 = mg\sin\phi - F_{drag} - F_{rolling},$$

$$\text{where}\quad F_{drag} = C_D\frac{\rho}{2}V^2 A,\quad F_{rolling} = C_{RR}V$$

Solve for, and evaluate, the drag coefficient:

$$C_D = \frac{mg\sin\phi - C_{RR}V}{(1/2)\rho V^2 A} = \frac{95(9.81)\sin 4° - 0.8(14)}{(1/2)(1.2)(14)^2(0.4)} \approx \mathbf{1.14}\quad \textit{Ans.}$$

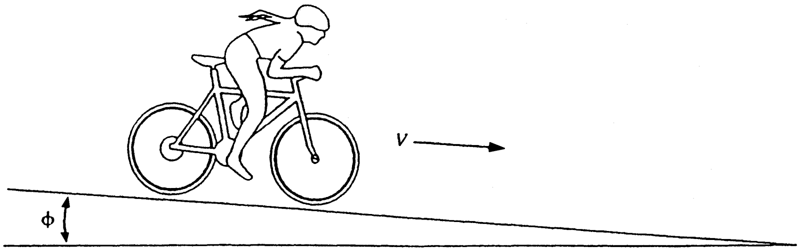


Fig. C7.1

C7.2 Air at 20°C and 1 atm flows at V_{avg} = 5 m/s between long, smooth parallel heat-exchanger plates 10 cm apart, as shown below. It is proposed to add a number of widely spaced 1-cm-long thin 'interrupter' plates to increase the heat transfer, as shown. Although the channel flow is turbulent, the boundary layer over the interrupter plates is laminar. Assume all plates are 1 m wide into the paper. Find (a) the pressure drop in Pa/m without the small plates present. Then find (b) the number of small plates, per meter of channel length, which will cause the overall pressure drop to be 10 Pa/m.

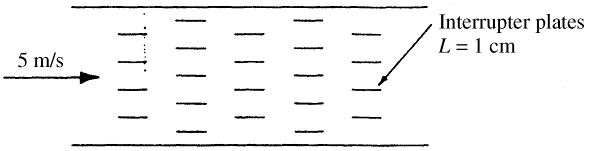


Fig. C7.2

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. (a) For wide plates, the hydraulic diameter is $D_h = 2h = 20$ cm. The Reynolds number, friction factor, and pressure drop for the bare channel (no small plates) is:

$$Re_{Dh} = \frac{\rho V_{avg} D_h}{\mu} = \frac{(1.2)(5.0)(0.2)}{1.8E{-}5} = 66,700 \ \ (turbulent)$$

$$f_{Moody,smooth} \approx 0.0196$$

$$\Delta p_{bare} = f\,\frac{L}{D_h}\frac{\rho}{2}V_{avg}^2 = (0.0196)\left(\frac{1.0\text{ m}}{0.2\text{ m}}\right)\left(\frac{1.2}{2}\right)(5.0)^2 = \mathbf{1.47}\ \frac{\mathbf{Pa}}{\mathbf{m}} \quad \textit{Ans.}\text{ (a)}$$

Each small plate (neglecting the wake effect if the plates are in line with each other) has a *laminar* Reynolds number:

$$Re_L = \frac{\rho V_{avg} L_{plate}}{\mu} = \frac{(1.2)(5.0)(0.01)}{1.8E{-}5} = \mathbf{3333} < 5E5, \quad \therefore\ laminar$$

$$C_{D,laminar} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{3333}} \approx 0.0230$$

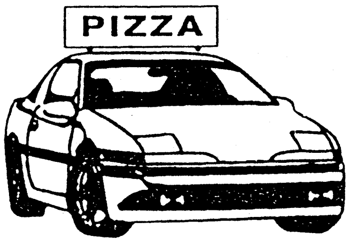
$$F_{1plate} = C_D\frac{\rho}{2}V_{avg}^2 A_{2sides} = (0.0230)\left(\frac{1.2}{2}\right)(5.0)^2(2\times 0.01\times 1) = 0.0069\ \frac{\text{N}}{\text{plate}}$$

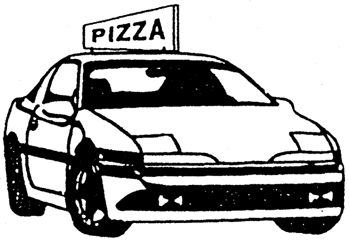
Each plate force must be supported by the channel walls. The effective pressure drop will be the bare wall pressure drop (assumed unchanged) plus the sum of the interrupter-plate forces divided by the channel cross-section area, which is given by (h × 1 m) = 0.1 m^2. The extra pressure drop provided by the plates, for this problem, is (10.0 – 1.47) = **8.53** Pa/m. Therefore we need

$$No.\,of\ plates = \frac{\Delta p_{needed}}{(F/A)_{1plate}} = \frac{8.53\text{ Pa/m}}{(0.0069\text{ N/plate})/(0.1\text{ m}^2)} \approx \mathbf{124\ plates} \quad \textit{Ans.}\text{ (b)}$$

This is the number of small interrupter plates *needed for each meter of channel length* to build up the pressure drop to 10.0 Pa/m.

C7.3 A new pizza store needs a delivery car with a sign attached. The sign is 1.5 ft high and 5 ft long. The boss wants to mount the sign normal to the car's motion. His employee, a student of fluid mechanics, suggests mounting it parallel to the motion. (a) Calculate the drag on the *sign alone* at 40 mi/h (58.7 ft/s) in both orientations. (b) The car has a rolling resistance of 40 lbf, a drag coefficient of 0.4, and a frontal area of 40 ft^2. Calculate the total drag of the car-sign combination at 40 mi/h. (c) Include rolling resistance and calculate the horsepower required in both orientations. (d) If the engine delivers 10 hp for 1 hour on a gallon of gasoline, calculate the fuel efficiency in mi/gal in both orientations, at 40 mi/h.





Solution: For air take $\rho = 0.00237$ slug/ft^3. (a) Table 7.3, blunt plate, $C_D \approx 1.2$:

$$F_{normal} = C_D \frac{\rho}{2}V^2 A = 1.2\left(\frac{0.00237}{2}\right)(58.7)^2(1.5 \times 5.0) \approx \mathbf{37\ lbf} \quad \textit{Ans. } (\text{a—}\textit{normal})$$

For parallel orientation, take m = 3.76E–7 slug/ft·s. Use flat-plate theory for Re_L = 0.00237(58.7)(5.0)/(3.76E–7) = 1.85E6 = transitional—use Eq. (7.49a):

$$C_D = \frac{0.031}{\text{Re}_L^{1/7}} - \frac{1440}{\text{Re}_L} = \frac{0.031}{(1.85\,\text{E}6)^{1/7}} - \frac{1440}{1.85\,\text{E}6} \approx 0.00317$$

$$F_{parallel} = 0.00317\left(\frac{0.00237}{2}\right)(58.7)^2(1.5 \times 5 \times 2\ \textit{sides}) \approx \mathbf{0.19\ lbf} \quad \textit{Ans. } (\text{a—}\textit{parallel})$$

(b) Add on the drag of the car:

$$F_{car} = C_{D,car}\frac{\rho}{2}V^2 A_{car} = 0.4\left(\frac{0.00237}{2}\right)(58.7)^2(40) \approx 65.3 \text{ lbf}$$

(1) *sign* ⊥: *Total Drag* = 65.3 + 36.7 ≈ **102 lbf** *Ans.* (b—*normal*)

(2) *sign* //: *Total Drag* = 65.3 + 0.2 ≈ **65.5 lbf** *Ans.* (b—*parallel*)

(c) Horsepower required = total force times velocity (include rolling resistance):
(1) $P_\perp = FV = (102 + 40)(58.7) = 8330$ ft·lbs/s ÷ 550 ≈ **15.1 hp** *Ans.* (c—*normal*)
(2) $P_{//} = (65.5 + 40)(58.7) = 6190$ ft·lbf/s ÷ 550 ≈ **11.3 hp** *Ans.* (b—*parallel*)

(d) Fuel efficiency:

$$(1)\ \ mpg_\perp = \left(40\ \frac{\text{mi}}{\text{h}}\right)\left(10\ \frac{\text{hp·h}}{\text{gal}}\right)\left(\frac{1}{15.1\text{ hp}}\right) \approx \mathbf{26.5\ \frac{mi}{gal}} \quad \textit{Ans. } (\text{d—}\textit{normal})$$

$$(2)\ \ mpg_{//} = \left(40\ \frac{\text{mi}}{\text{h}}\right)\left(10\ \frac{\text{hp·h}}{\text{gal}}\right)\left(\frac{1}{11.3\text{ hp}}\right) \approx \mathbf{35.4\ \frac{mi}{gal}} \quad \textit{Ans. } (\text{d—}\textit{parallel})$$

We see that the student is correct, there are fine 25% savings with the sign parallel.

C7.4 Consider a simple pendulum with an unusual bob shape: a cup of diameter D whose axis is in the plane of oscillation. Neglect the mass and drag of the rod L. (a) Set up the differential equation for θ(t) and (b) non-dimensionalize this equation. (c) Determine the natural frequency for $\theta \ll 1$. (d) For L = 1 m, D = 1 cm, m = 50 g, and air at 20°C and 1 atm, and θ(0) = 30°, find (numerically) the time required for the oscillation amplitude to drop to 1°.

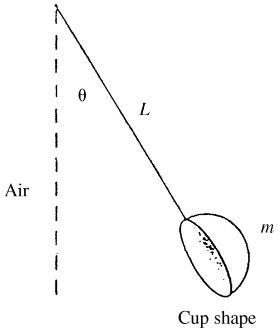


Fig. C7.4

Solution: (a) Let L_{eq} = L + D/2 be the effective length of the pendulum. Sum forces in the direction of the motion of the bob and rearrange into the basic 2nd-order equation:

$$\sum F_{tangential} = -mg\sin\theta - C_D\frac{\rho}{2}V_t^2\frac{\pi}{4}D^2 = \mathrm{m}\frac{dV_t}{dt}, \quad \text{where } V_t = L_{eq}\frac{d\theta}{dt}$$

$$\text{Rearrange:} \quad \boldsymbol{\ddot{\theta} + K\dot{\theta}^2 + \frac{g}{L_{eq}}\sin\theta = 0}, \quad \text{where } K = \frac{C_D\rho L_{eq}\pi D^2}{8\text{ m}} \quad \textit{Ans.}\text{ (a)}$$

Note that $C_D \approx$ **0.4** when moving to the right and about **1.4** moving to the left (Table 7.3).

(b) Now θ is already dimensionless, so define dimensionless time $\tau = t(g/L_{eq})^{1/2}$ and substitute into the differential equation above. We obtain the dimensionless result

$$\boldsymbol{\frac{d^2\theta}{d\tau^2} + K\left(\frac{d\theta}{d\tau}\right)^2 + \theta = 0} \quad \textit{Ans.}\text{ (b)}$$

Thus the only dimensionless parameter is ***K*** from part (a) above.
(c) For $\theta \ll 1$, the term involving K is neglected, and $\sin\theta \approx \theta$ itself. We obtain

$$\ddot{\theta} + \omega_n^2\theta \approx 0, \quad \text{where } \boldsymbol{\omega_n = \sqrt{\frac{g}{L_{eq}}}} \quad \textit{Ans.}\text{ (c)}$$

Thus the natural frequency is $(g/L_{eq})^{1/2}$ just as for the simple drag-free pendulum. Recall that L_{eq} = L + D/2. Note again that K has a different value when moving to the right ($C_D \approx 0.4$) or to the left ($C_D \approx 1.4$).

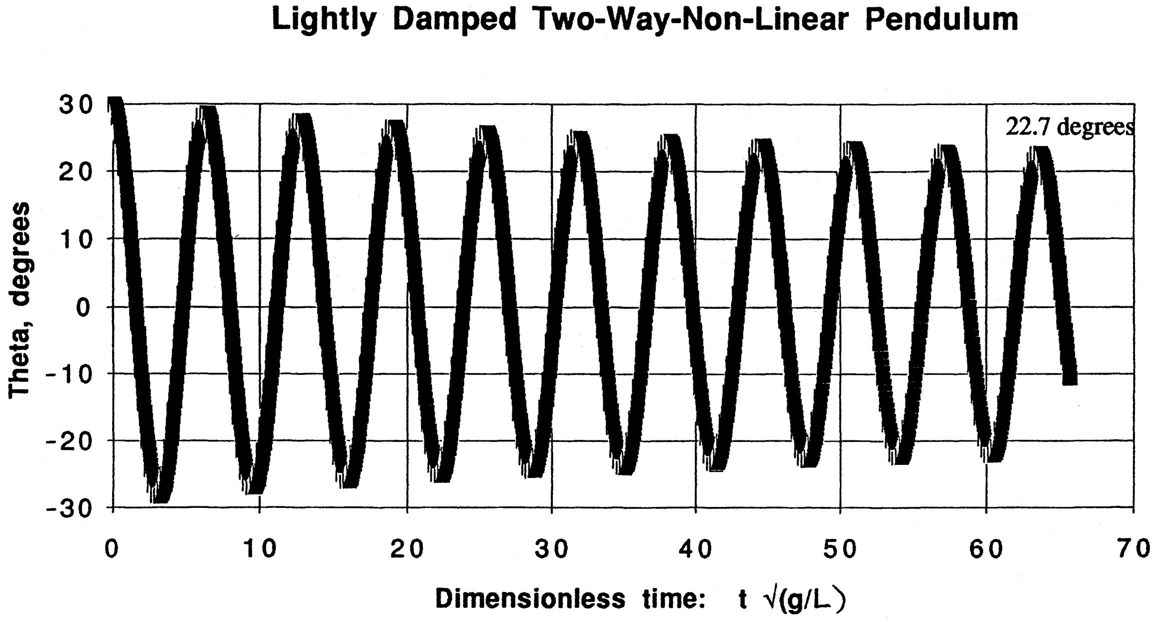
(d) For the given data, ρ_{air} = 1.2 kg/m^3, L_{eq} = L + D/2 = 1.05 m, and the parameter K is

$$K = \frac{C_D(1.2)(1.05)\pi(0.1)^2}{8(0.050)} = 0.099C_D = 0.0396 \ (\textit{moving to the right})$$

$$= 0.1385 \ (\textit{moving to the left})$$

The differential equation from part (b) is then solved for $\theta(0) = 30° = \pi/6$ radians. The natural frequency is $(9.81/1.05)^{1/2} = 3.06$ rad/s, with a dimensionless period of 2π. Integrate numerically, with Runge-Kutta or MatLab or Excel or whatever, until $\theta = 1° = \pi/180$ radians. The time-series results are shown in the figure below.

We see that the pendulum is very *lightly damped*—drag forces are only about 1/50th of the weight of the bob. After ten cycles, the amplitude has only dropped to 22.7°—we will never get down to 1° in the lifetime of my computer. The dimensionless period is 6.36, or only 1% greater than the simple drag-free theoretical value of 2π.



Chapter 8 • Potential Flow and Computational Fluid Dynamics

8.1 Prove that the streamlines $\psi(\mathrm{r}, \theta)$ in polar coordinates, from Eq. (8.10), are orthogonal to the potential lines $\phi(\mathrm{r}, \theta)$.

Solution: The streamline slope is represented by

$$\left.\frac{\mathrm{dr}}{\mathrm{r\,d}\theta}\right|_{\text{streamline}} = \frac{\mathrm{v_r}}{\mathrm{v}_\theta} = \frac{\partial\phi/\partial\mathrm{r}}{(1/\mathrm{r})(\partial\phi/\partial\theta)} = \frac{\mathbf{-1}}{\left(\dfrac{\mathrm{dr}}{\mathrm{r\,d}\theta}\right)_{\text{potential line}}}$$

Since the ψ – slope = $\mathbf{-1}$/(ϕ – slope), the two sets of lines are **orthogonal**. *Ans.*

8.2 The steady plane flow in the figure has the polar velocity components $\mathrm{v}_\theta = \Omega\mathrm{r}$ and $\mathrm{v_r} = 0$. Determine the circulation Γ around the path shown.

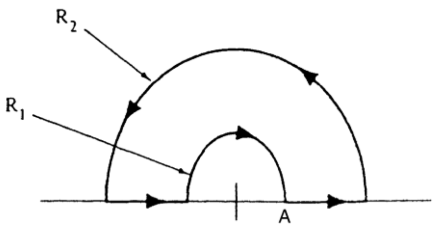


Fig. P8.2

Solution: Start at the inside right corner, point A, and go around the complete path:

$$\Gamma = \oint \boldsymbol{V}\cdot \boldsymbol{ds} = 0(R_2 - R_1) + \Omega R_2(\pi R_2) + 0(R_1 - R_2) + \Omega R_1(-\pi R_1)$$

$$\text{or:}\quad \boldsymbol{\Gamma = \pi\Omega\left(R_2^2 - R_1^2\right)} \quad \textit{Ans.}$$

8.3 Using cartesian coordinates, show that each velocity component (u, v, w) of a potential flow satisfies Laplace's equation separately if $\nabla^2\phi = 0$.

Solution: This is true because the order of integration may be changed in each case:

$$\text{Example:}\quad \nabla^2 u = \nabla^2\left(\frac{\partial\phi}{\partial \mathrm{x}}\right) = \frac{\partial}{\partial \mathrm{x}}(\nabla^2\phi) = \frac{\partial}{\partial \mathrm{x}}(0) = 0 \quad \textit{Ans.}$$

8.4 Is the function *1/r* a legitimate velocity potential in plane polar coordinates? If so, what is the associated stream function $\psi(\mathrm{r}, \theta)$?

Solution: Evaluation of the laplacian of (*1/r*) shows that it is *not* legitimate:

$$\nabla^2\left(\frac{1}{r}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right]=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(-\frac{1}{r^2}\right)\right]=\frac{\mathbf{1}}{\mathbf{r^3}}\neq \mathbf{0} \quad \textbf{Illegitimate} \quad \textit{Ans.}$$

P8.5 A proposed harmonic function $F(x, y, z)$ is given by

$$F = 2x^2 + y^3 - 4xz + f(y)$$

(*a*) If possible, find a function $f(y)$ for which the Laplacian of F is zero. If you do indeed solve part (*a*), can your final function F serve as (*b*) a velocity potential, or (*c*) a stream function?

Solution: Evaluate $\nabla^2 F$ and see if we can find a suitable $f(y)$ to make it zero:

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 4 + 6y + f''(y)$$

This equals zero if $f''(y) = -4-6y$, or: $f(y) = -2y^2 - y^3 + \text{const}$ *Ans.*(*a*)

$$\text{Or}: \ \nabla^2 F = 0 \ \ if \quad \mathbf{F = 2x^2 - 2y^2 - 4xz + constant}$$

Solving for $f(y)$ eliminated y^3, which is not a harmonic function.

(*b*) Since $\nabla^2 F = 0$, **it can indeed be a velocity potential**, although the writer does not think it is a realistic flow pattern. (*c*) Since F is three-dimensional, **it cannot be a stream function**.

8.6 Given the plane polar coordinate velocity potential $\phi = Br^2\cos(2\theta)$, where B is a constant. (*a*) Show that a stream function also exists. (*b*) Find the algebraic form of $\psi(r, \theta)$. (*c*) Find any stagnation points in this flow field.

Solution: (*a*) First find the velocities from ϕ and then check continuity:

$$v_r = \frac{\partial\phi}{\partial r} = 2Br\cos(2\theta); \quad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -2Br\sin(2\theta); \quad \text{Then check}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)=\frac{1}{r}(4Br\cos 2\theta)+\frac{1}{r}(-4Br\cos 2\theta)=4B\cos 2\theta-4B\cos 2\theta=0$$

Yes, continuity is satisfied, so ψ exists. Ans.(*a*)

(b) Find ψ from its definition:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 2Br\cos(2\theta); \quad v_\theta = -\frac{\partial\psi}{\partial r} = -2Br\sin(2\theta)$$

$$\text{Solve } \psi = Br^2\sin(2\theta) \qquad Ans.(b)$$

(*c*) By checking to see where v_r and $v_\theta = 0$ from part (*a*), we find that the only stagnation point is **at the origin of coordinates**, $r = 0$. These functions define *plane stagnation flow*, Fig. 8.19*b*.

8.7 Consider a flow with constant density and viscosity. If the flow possesses a velocity potential as defined by Eq. (8.1), show that it exactly satisfies the full Navier-Stokes equation (4.38). If this is so, why do we back away from the full Navier-Stokes equation in solving potential flows?

Solution: If $\mathbf{V} = \nabla\phi$, the full Navier-Stokes equation is satisfied identically:

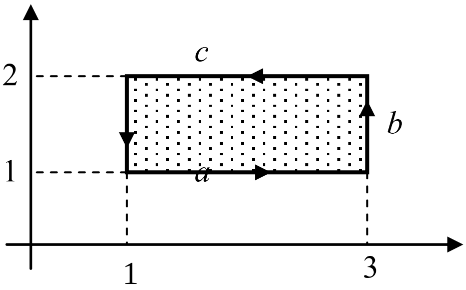
$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{V} \quad \text{becomes}$$

$$\rho\left[\nabla\left(\frac{\partial\phi}{\partial t}\right) + \nabla\left(\frac{V^2}{2}\right)\right] = -\nabla p - \nabla(\rho g z) + \mu\nabla(\nabla^2\phi), \text{ where the last term is } \textbf{zero}.$$

The viscous (final) term drops out identically for potential flow, and what remains is

$$\frac{\partial\phi}{\partial t} + \frac{V^2}{2} + \frac{p}{\rho} + gz = \text{constant} \quad (\textbf{Bernoulli's equation})$$

The Bernoulli relation is an exact solution of Navier-Stokes for potential flow. We don't exactly "back away," we need also to solve $\nabla^2\phi = 0$ in order to find the velocity potential.



P8.8 For the velocity distribution $u = -By$, $v = +Bx$, $w = 0$, evaluate the circulation Γ about the rectangular closed curve defined by (x, y) = (1,1), (3,1), (3,2), and (1,2). Interpret your result, especially vis-à-vis the velocity potential.

Solution: Given that $\Gamma = \int \mathbf{V}\cdot\mathbf{ds}$ around the curve, divide the rectangle into (a, b, c, d) pieces as shown.

$$\Gamma = \int_a u\,ds + \int_b v\,ds + \int_c u\,ds + \int_d v\,ds = (-B)(2) + (3B)(1) + (-2B)(2) + (-B)(1) = \mathbf{+4B} \quad \textit{Ans.}$$

The flow is *rotational*. Check $|\text{curl}\mathbf{V}| = 2B$ = constant, so $\Gamma = (2B)A_{\text{region}} = (2B)(2) = 4B$.

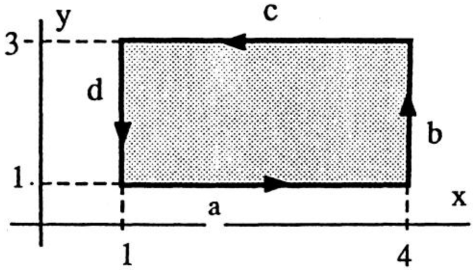


Fig. P8.9

8.9 Consider the two-dimensional flow u = –Ax, v = +Ay, where A is a constant. Evaluate the circulation Γ around the rectangular closed curve defined by (x, y) = (1, 1), (4, 1), (4, 3), and (1, 3). Interpret your result especially *vis-a-vis* the velocity potential.

Solution: Given $\Gamma = \int \mathbf{V}\cdot\mathbf{ds}$ around the curve, divide the rectangle into (a, b, c, d) pieces as shown:

$$\Gamma = \int_a u\,dx + \int_b v\,dy + \int_c u\,dx + \int_d v\,dy$$

$$= \int_1^4 (-Ax)\,dx + \int_1^3 Ay\,dy + \int_1^4 Ax\,dx + \int_1^3 (-Ay)\,dy = \mathbf{0} \quad \textit{Ans.}$$

The circulation is zero because the flow is **irrotational**: curl $\mathbf{V} \equiv \mathbf{0}$, $\boldsymbol{\Gamma} = \int \mathbf{d}\boldsymbol{\phi} \equiv \mathbf{0}$.

8.10 A two-dimensional Rankine half-body, 8 cm thick, is placed in a water tunnel at 20°C. The water pressure far up-stream along the body centerline is 105 kPa. (*a*) What is the nose radius of the half-body? (*b*) At what tunnel flow velocity will cavitation bubbles begin to form on the surface of the body?

Solution: (*a*) The nose radius is the distance *a* in Fig. 8.6, the Rankine half-body:

$$a = \frac{thickness}{2\pi} = \frac{8cm}{2\pi} = 1.27cm \qquad Ans.(a)$$

(*b*) At 20°C the vapor pressure of water is 2337 Pa. Maximum velocity occurs, as shown, on the upper surface at $\theta \approx 63°$, where $z \approx 2.04a \approx 2.6$ cm and $V \approx 1.26U_\infty$. Write the Bernoulli equation between upstream and V, assuming the surface pressure is just vaporizing at the minimum-pressure point:

$$\frac{p_\infty}{\rho g} + \frac{U_\infty^2}{2g} + z_\infty = \frac{p_{vap}}{\rho g} + \frac{(1.26U_\infty)^2}{2g} + z_{surf}, \text{ or :}$$

$$\frac{105000}{9790} + \frac{U_\infty^2}{2(9.81)} + 0 = \frac{2337}{9790} + \frac{(1.26U_\infty)^2}{2(9.81)} + 0.026, \quad Solve\ U_\infty = 18.7\frac{m}{s} \quad Ans.(b)$$

8.11 A power-plant discharges cooling water through the manifold in Fig. P8.11, which is 55 cm in diameter and 8 m high and is perforated with 25,000 holes 1 cm in diameter. Does this manifold simulate a line source? If so, what is the equivalent source strength *m*?

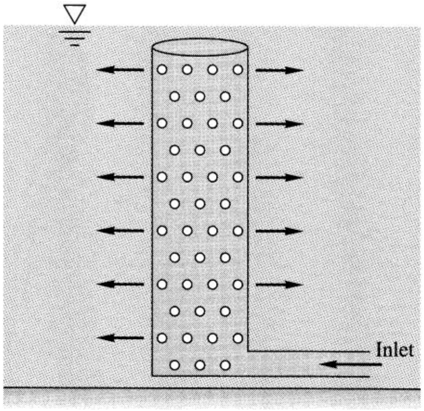


Fig. P8.11

Solution: With that many small holes, equally distributed and presumably with equal flow rates, the manifold **does indeed** simulate a line source of strength

$$\mathrm{m} = \frac{\mathbf{Q}}{\mathbf{2\pi b}}, \quad \text{where } \mathrm{b} = 8 \text{ m and } \mathrm{Q} = \sum_{i=1}^{25000} \mathrm{Q}_{1\,\mathrm{hole}} \quad Ans.$$

8.12 Consider the flow due to a vortex of strength K at the origin. Evaluate the circulation from Eq. (8.15) about the clockwise path from (a, 0) to (2a, 0) to (2a, $3\pi/2$) to (a, $3\pi/2$) and back to (a, 0). Interpret your result.

Solution: Break the path up into (1, 2, 3, 4) as shown. Then

$$\Gamma = \int_{\text{path}} \mathbf{V}\cdot\mathbf{ds}$$

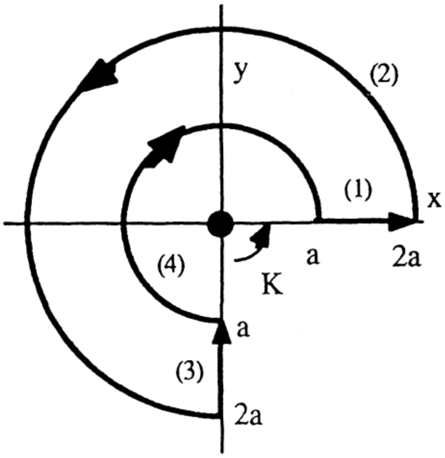


Fig. P8.12

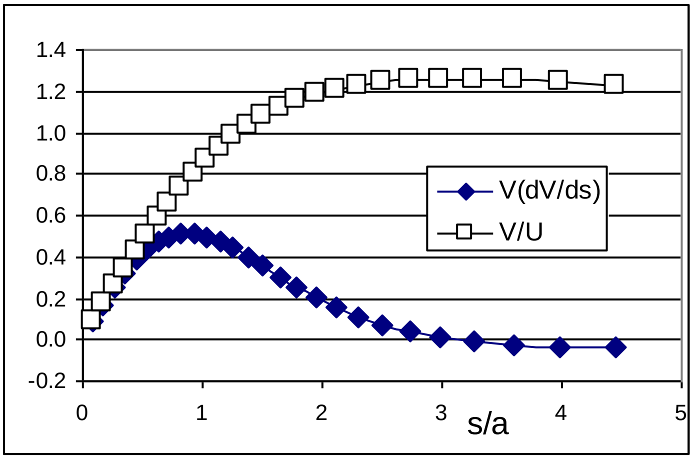
$$\Gamma = \int_{(1)} u\,ds + \int_{(2)} v_\theta\,ds + \int_{(3)} v\,ds + \int_{(4)} v_\theta\,ds$$

$$= 0 + \int_{(2)} \frac{K}{2a} 2a\,d\theta + 0 + \int_{(4)} \frac{K}{a}(-a\,d\theta) = K\left(\frac{3\pi}{2}\right) + K\left(-\frac{3\pi}{2}\right) = 0 \quad \textit{Ans.}$$

There is zero circulation **about all closed paths which do not enclose the origin**.

P8.13 Starting at the stagnation point in Fig. 8.6, the fluid *acceleration* along the half-body surface rises to a maximum and eventually drops off to zero far downstream. (*a*) Does this maximum occur at the point in Fig. 8.6 where $U_{max} = 1.26U$? (*b*) If not, does the maximum acceleration occur before or after that point? Explain.

Solution: Since the flow is steady, the fluid acceleration along the half-body surface is convective, $dU/dt = U(dU/ds)$, where s is along the surface. (*a*) At the point of maximum velocity in Fig. 8.6, $dU/ds = 0$, hence $dU/dt = 0$, so answer (*a*) is **No**. (*b*) A plot of $U(x)$ along the surface is shown in Fig. 8.9*b*, and we see that the slope dU/ds is rather small (and negative) downstream of the maximum-velocity point. Therefore maximum acceleration occurs ***before* the point of maximum velocity**, at about $s/a \approx 0.9$. The actual value of maximum acceleration (not asked) is approximately $(V/U)[d(V/U/)d(s/a)] \approx 0.51$. A graph (not requested) is as follows:



Fig, P8.13

8.14 A tornado may be modeled as the circulating flow shown in Fig. P8.14, with $\upsilon_r = \upsilon_z = 0$ and $\upsilon_\theta(r)$ such that

$$\upsilon_\theta = \begin{cases} \omega r & r \le R \\ \dfrac{\omega R^2}{r} & r > R \end{cases}$$

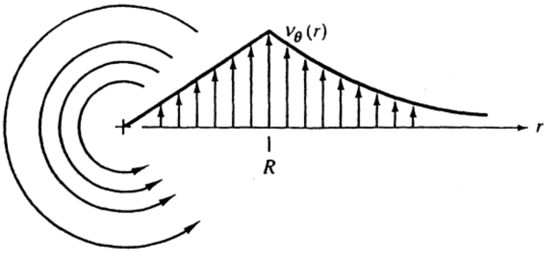


Fig. P8.14

Determine whether this flow pattern is irrotational in either the inner or outer region. Using the r-momentum equation (D.5) of App. D, determine the pressure distribution $p(r)$ in the tornado, assuming $p = p_\infty$ as $r \to \infty$. Find the location and magnitude of the lowest pressure.

Solution: The inner region is solid-body **rotation**, the outer region is **irrotational**:

$$\text{Inner region:}\quad \Omega_z = \frac{1}{r}\frac{d}{dr}(r v_\theta) = \frac{1}{r}\frac{d}{dr}(r\omega r) = \mathbf{2\omega = constant \neq 0} \quad \textit{Ans.}\text{ (inner)}$$

$$\text{Outer region:}\quad \Omega_z = \frac{1}{r}\frac{d}{dr}(\omega R^2/r) = \mathbf{0} \quad \text{(irrotational)} \quad \textit{Ans.}\text{ (outer)}$$

The pressure is found by integrating the r-momentum equation (D-5) in the Appendix:

$$\frac{dp}{dr} = \rho v_\theta^2/r, \quad \text{or:}\quad p_{\text{outer}} = \int \frac{\rho}{r}\left(\frac{\omega \text{R}^2}{r}\right)^2 dr = -\rho\omega^2 R^4/2r^2 + \text{constant}$$

$$\text{when}\quad r = \infty,\ p = p_\infty, \quad \text{hence}\quad \boldsymbol{p_{\text{outer}} = p_\infty - \rho\omega^2 R^4/(2r^2)} \quad \textit{Ans.}\text{ (outer)}$$

$$\text{At the match point, } r = R,\quad p_{\text{outer}} = p_{\text{inner}} = p_\infty - \rho\omega^2 R^2/2$$

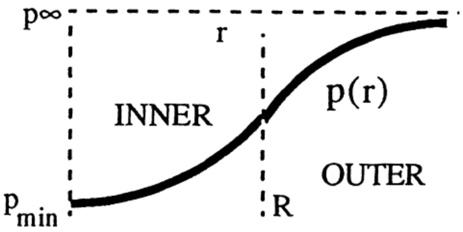
In the inner region, we integrate the radial pressure gradient and match at $r = R$:

$$p_{\text{inner}} = \int \frac{\rho}{r}(\omega r)^2 dr = \rho\omega^2 r^2/2 + \text{constant}, \quad \text{match to}\ \ p(R) = p_\infty - \rho\omega^2 R^2/2$$

$$\text{finally,}\quad \boldsymbol{p_{\text{inner}} = p_\infty - \rho\omega^2 R^2 + \rho\omega^2 r^2/2} \quad \textit{Ans.}\text{ (inner)}$$

The minimum pressure occurs at the origin, $r = 0$:

$$p_{\min} = \boldsymbol{p_\infty - \rho\omega^2 R^2} \quad \textit{Ans.} \text{ (min)}$$



8.15 A *category-3* hurricane on the Saffir-Simpson scale (www.encyclopedia.com) has a maximum velocity of 130 mi/h. Let the match-point radius be R = 18 km (see Fig. P8.14). Assuming sea-level standard conditions at large r, (a) find the minimum pressure; (b) find the pressure at the match-point; and (c) show that both minimum and match-point pressures are independent of R.

Solution: Convert 130 mi/h = 58.1 m/s = ωR. Let ρ = 1.22 kg/m^3. (a) From Prob. 8.14,

$$p_{\min} = p_\infty - \rho(\omega R)^2 = 101350\,\text{Pa} - (1.22\text{ kg/m}^3)(58.1\text{ m/s})^2 = \mathbf{97200\ Pa} \quad \textit{Ans.} \text{ (a)}$$

(b) Again from Prob. 8.14, the match pressure only drops half as low as the minimum pressure:

$$p_{match} = p_\infty - \frac{\rho}{2}(\omega R)^2 = 101350 - \frac{1.22}{2}(58.1)^2 = \mathbf{99300\ Pa} \quad \textit{Ans.} \text{ (b)}$$

(c) We see from above that both $p_{\min}$ and p_{match} have R in their formulas, but only in conjunction with ω. That is, these pressures depend only upon $V_{\max}$, wherever it occurs.

8.16 Air flows at 1.2 m/s along a flat wall when it meets a jet of air issuing from a slot at A. The jet volume flow is 0.4 m^3/s per m of width into the paper. If the jet is approximated as a line source, (*a*) locate the stagnation point S. (*b*) How far vertically will the jet flow extend?

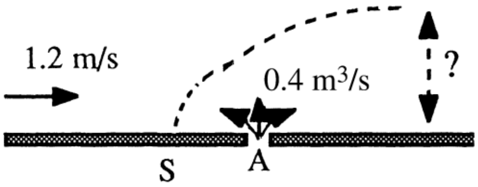


Fig. P8.16

Solution: This is only *half* a source. The equivalent source strength is $m = 2(0.4\text{m}^3/\text{s}) / [2\pi(1\text{m})] = 0.127\ \text{m}^2/\text{s}$. Then, as in Fig. 8.9*a*, the stagnation point S is at $a = m/U = (0.127\ \text{m}^2/\text{s}) / (1.2\ \text{m/s})$ = **0.106 m to the left of A**. *Ans*. (*a*) The effective half-body, shown as a dashed line in Fig. P8.16, extends above the wall to a distance equal to $\pi a = \pi(0.106)$ = **0.333 m**. *Ans*.(*b*)

8.17 Find the position (*x*, *y*) on the upper surface of the half-body in Fig. 8.5a for which the local velocity equals the uniform stream velocity. What should the pressure be at this point?

Solution: The surface velocity and surface contour are given by Eq. (8.18):

$$V^2 = U_\infty^2\left(1+\frac{a^2}{r^2}+\frac{2a}{r}\cos\theta\right) \quad \text{along the surface} \quad \frac{r}{a}=\frac{\pi-\theta}{\sin\theta}$$

If $V = U_\infty$, then $a^2/r^2 = -2a\cos\theta/r$, or $\cos\theta = -a/(2r)$. Combine with the surface profile above, and we obtain an equation for θ alone: $\boldsymbol{\tan\theta = -2(\pi - \theta)}$. The final solution is:

$$\theta = 113.2°;\quad r/a = 1.268;\quad \mathbf{x/a = -0.500};\quad \mathbf{y/a = 1.166} \quad \textit{Ans.}$$

Since the velocity equals U_∞ at this point, the surface pressure $\boldsymbol{p = p_\infty}$. *Ans.*

8.18 Plot the streamlines and potential lines of the flow due to a line source of strength *m* at (a, 0) plus a line source 3*m* at (–a, 0). What is the flow pattern viewed from afar?

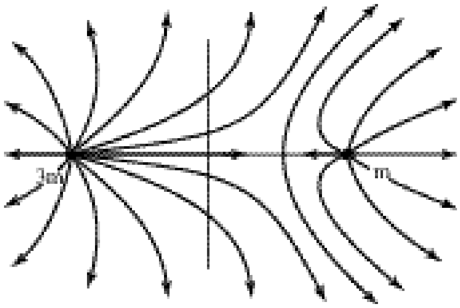
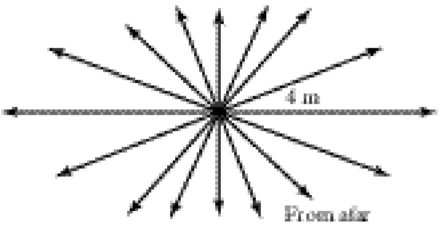


Fig. P8.18

Solution: The pattern viewed close-up is shown above. The pattern viewed from afar is at right and represents a single source of strength **4*m***. *Ans.*



8.19 Plot the streamlines and potential lines of the flow due to a line source of strength 3*m* at (a, 0) plus a line sink of strength –*m* at (–a, 0). What is the pattern viewed from afar?

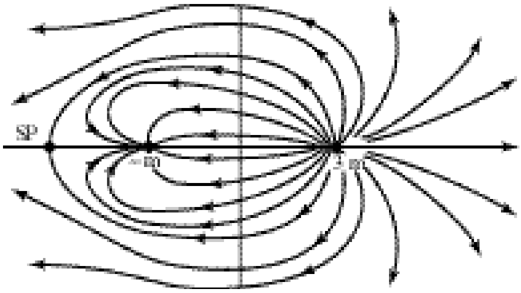
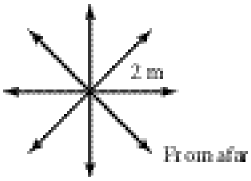


Fig. P8.19

Solution: The pattern viewed close-up is shown at upper right—there is a stagnation point to the left of the sink, at (x, y) = (–2a, 0). The pattern viewed from afar is at right and represents a single source of strength +**2*m***. *Ans.*



8.20 Plot the streamlines of the flow due to a line vortex of strength +*K* at (0, +a) plus a line vortex of strength –*K* at (0, –a). What is the pattern viewed from afar?

Solution: The pattern viewed close-up is shown at right (see Fig. 8.17*b* of the text). The pattern viewed from afar represents **little or nothing**, since the two vortices cancel strengths and cause no flow at ∞. *Ans.*

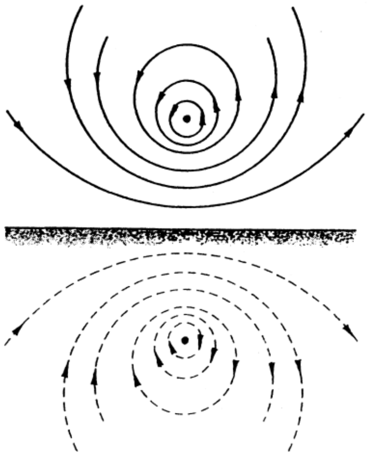
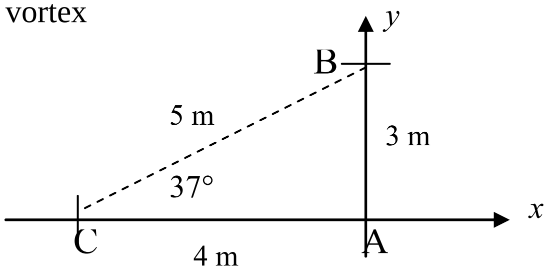


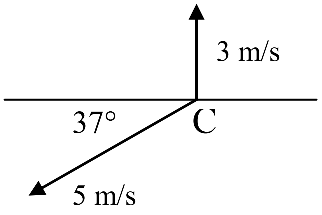
Fig. P8.20

P8.21 At point A in Fig. P8.21 is a clockwise line vortex of strength $K = 12\ \text{m}^2/\text{s}$. At point B is a line source of strength $m = 25\ \text{m}^2/\text{s}$. Determine the resultant velocity induced by these two at point C .

Fig. P8.21



Solution: The vortex induces a velocity $K/r_{AC} = (12\text{m}^2/\text{s})/4\text{m} = 3$ m/s vertically at point C. The source induces a velocity $m/r_{BC} = (25\text{m}^2/\text{s})/5\text{m} = 5$ m/s down to the left at a 37° angle. Sum vertical and horizontal velocities from the sketch at right:



$$\sum u = (3m/s)\cos 90^o - (5m/s)\cos 37^o = 0 - 5(0.8) = \mathbf{-4}\ m/s \quad \textit{Ans.}$$
$$\sum v = (3m/s)\sin 90^o - (5m/s)\sin 37^o = 3 - 5(0.6) = \mathbf{0}\ m/s \quad \textit{Ans.}$$

The resultant induced velocity is 4 m/s to the left of point C.

8.22 Consider inviscid stagnation flow, $\psi = Kxy$ (see Fig. 8.19*b*), Superimposed with a source at the origin of strength *m*. Plot the resulting streamlines in the upper half plane, using the length scale $a = (m/K)^{1/2}$. Give a physical explanation of the flow pattern.

Solution: The sum of stagnation flow plus a line source at the origin is

$$\psi = Kxy + m\theta; \quad \text{Define} \quad x^* = x/a, \quad y^* = y/a, \quad a = \sqrt{m/K}$$
$$\textit{Then plot}\ \frac{\psi}{m} = x^* y^* + \theta, \quad \textit{where} \quad \theta = \tan^{-1}\left(\frac{y^*}{x^*}\right)$$

The plot is below, using MATLAB, and represents **stagnation flow toward a bump of height *a***.

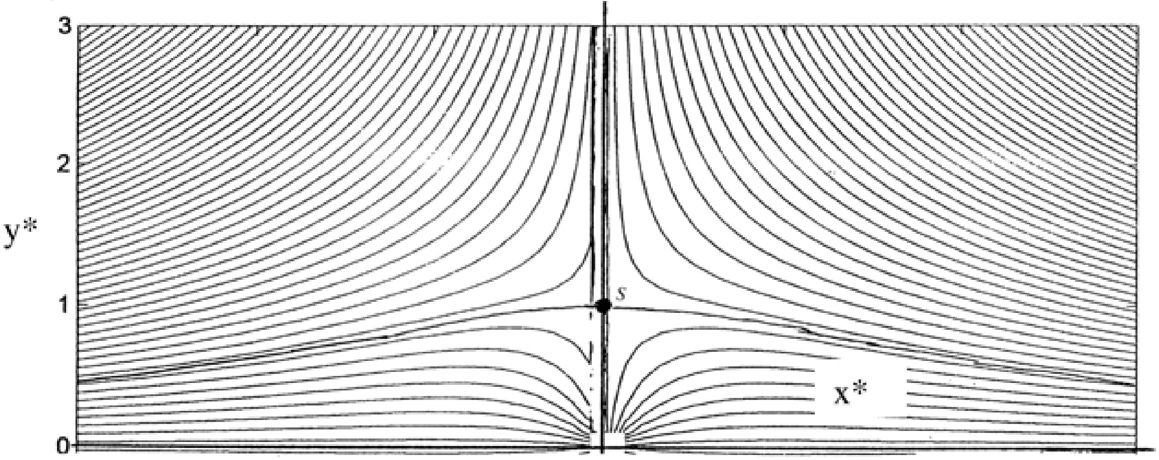


Fig. P8.22

8.23 Find the resultant velocity vector induced at point A in Fig. P8.23 due to the combination of uniform stream, vortex, and line source.

Solution: The velocities caused by each term—stream, vortex, and sink—are shown below. They have to be added together vectorially to give the final result:

$$\mathbf{V = 11.3\ \frac{m}{s}}\ \text{ at } \theta = \mathbf{44.2°}\ \angle \quad \textit{Ans.}$$

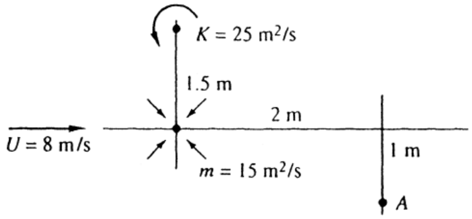
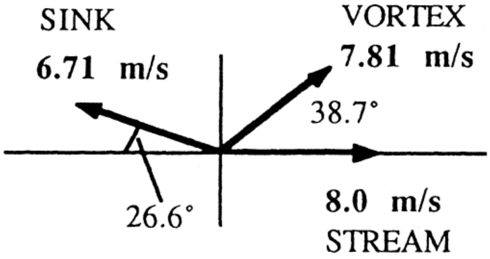


Fig. P8.23



8.24 Line sources of equal strength $m = Ua$, where U is a reference velocity, are placed at $(x, y) = (0, a)$ and $(0, -a)$. Sketch the stream and potential lines in the upper half plane. Is $y = 0$ a "wall"? If so, sketch the pressure coefficient

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2}$$

along the wall, where p_0 is the pressure at (0, 0). Find the minimum pressure point and indicate where flow separation might occur in the boundary layer.

Solution: This problem is an "image" flow and is sketched in Fig. 8.17a of the text. Clearly $y = 0$ is a "wall" where

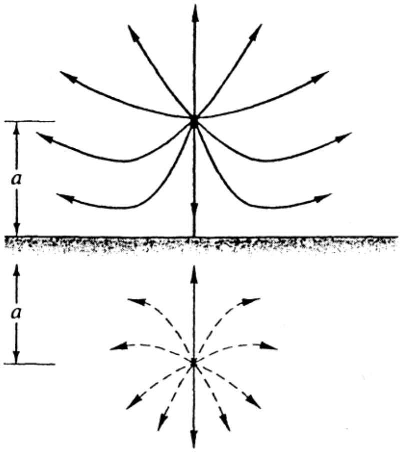
$$u = 2u_s = \frac{2Ua}{\sqrt{x^2 + a^2}} \cdot \frac{x}{\sqrt{x^2 + a^2}}$$
$$= 2Ua/(x^2 + a^2)$$

From Bernoulli, $p + \rho u^2/2 = p_0$,

$$C_p = \frac{p - p_o}{(1/2)\rho U^2} = -\frac{u^2}{U^2}$$

$$= -\left[\frac{\mathbf{2x/a}}{\mathbf{1 + (x/a)^2}}\right]^2 \quad \textit{Ans.}$$

The minimum pressure coefficient is $C_{p,min} = -1.0$ at x = a, as shown in the figure. Beyond this point, pressure increases (*adverse* gradient) and separation is possible.



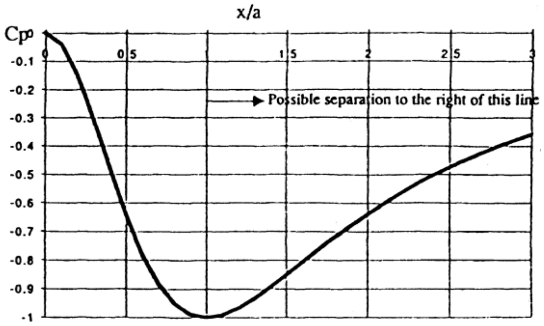


Fig. P8.24

8.25 Let the vortex/sink flow of Eq. (4.134) simulate a tornado as in Fig. P8.25. Suppose that the circulation about the tornado is $\Gamma = 8500\ m^2/s$ and that the pressure at $r = 40$ m is 2200 Pa less than the far-field pressure. Assuming inviscid flow at sea-level density, estimate (a) the appropriate sink strength $-m$, (b) the pressure at $r = 15$ m, and (c) the angle β at which the streamlines cross the circle at $r = 40$ m (see Fig. P8.25).

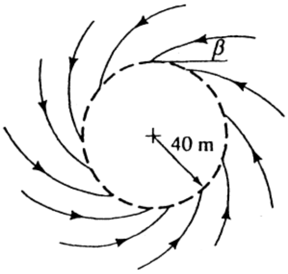


Fig. P8.25

Solution: The given circulation yields the circumferential velocity at $r = 40$ m:

$$v_\theta = \frac{\Gamma}{2\pi r} = \frac{8500}{2\pi(40)} \approx 33.8\ \frac{m}{s}$$

Assuming sea-level density $\rho = 1.225\ \mathrm{kg/m^3}$, we use Bernoulli to find the radial velocity:

$$p_\infty + \frac{\rho}{2}(0)^2 = (p_\infty - \Delta p) + \frac{\rho}{2}\left(v_\theta^2 + v_r^2\right) = p_\infty - 2200 + \frac{1.225}{2}\left[(33.8)^2 + v_r^2\right]$$

$$\text{Solve for}\quad v_r \approx 49.5\ \frac{\mathrm{m}}{\mathrm{s}} = \frac{m}{r} = \frac{m}{40},\quad \therefore\ \boldsymbol{m \approx 1980\ \frac{m^2}{s}} \quad \textit{Ans.}\ \text{(a)}$$

With circumferential and radial (inward) velocity known, the streamline angle β is

$$\beta = \tan^{-1}\left(\frac{v_r}{v_\theta}\right) = \tan^{-1}\left(\frac{49.5}{33.8}\right) \approx \mathbf{55.6°} \quad \textit{Ans.}\ \text{(c)}$$

(b) At $r = 15$ m, compute $v_r = m/r = 1980/15 \approx 132$ m/s (unrealistically high) and $v_\theta = \Gamma/2\pi r = 8500/[2\pi(15)] \approx 90$ m/s (high again, there is probably a viscous core here). Then we use Bernoulli again to compute the pressure at $r = 15$ m:

$$p + \frac{1.225}{2}[(132)^2 + (90)^2] = p_\infty, \quad \text{or} \quad p \approx p_\infty - \mathbf{15700\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

If we assume sea-level pressure of 101 kPa at ∞, then $p_{\text{absolute}} = 101 - 16 \approx$ **85 kPa**.

8.26 A coastal power plant takes in cooling water through a vertical perforated manifold, as in Fig. P8.26. The total volume flow intake is 110 m³/s. Currents of 25 cm/s flow past the manifold, as shown. Estimate (a) how far downstream and (b) how far normal to the paper the effects of the intake are felt in the ambient 8-m-deep waters.

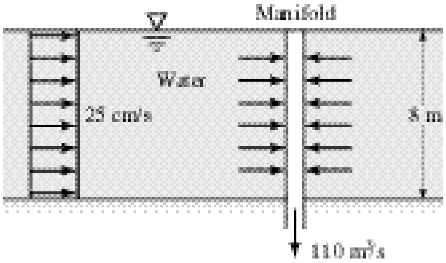


Fig. P8.26 (identical to 5/e Fig. P4.72)

Solution: The sink strength *m* leads to the desired dimensions. The distance downstream from the sink is *a* and the distance normal to the paper is *pa* (see Fig. 8.6):

$$m = \frac{Q}{2\pi\, b} = \frac{110 m^3/s}{2\pi(8m)} = 2.19\frac{m^2}{s}$$

$$a = \frac{m}{U_\infty} = \frac{2.19 m^2/s}{0.25 m/s} = 8.75\,m \qquad Ans.(a)$$

$$Width = \pi\, a = \pi(8.75m) = 27.5\,m \text{ on each side} \qquad Ans.(b)$$

8.27 Water at 20°C flows past a half-body as shown in Fig. P8.27. Measured pressures at points A and B are 160 kPa and 90 kPa, respectively, with uncertainties of 3 kPa each. Estimate the stream velocity and its uncertainty.

Solution: Since Eq. (8.18) is for the upper surface, use it by noting that $V_C = V_B$ in the figure:

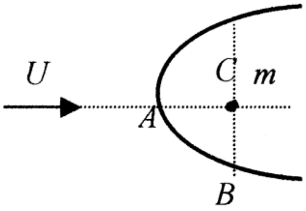


Fig. P8.27

$$\frac{r_C}{a} = \frac{\pi - \pi/2}{\sin(\pi/2)} = \frac{\pi}{2}, \quad V_C^2 = V_B^2 = U_\infty^2\left[1 + \left(\frac{2}{\pi}\right)^2 + \frac{2}{(\pi/2)}\cos(\pi/2)\right] = 1.405U_\infty^2$$

$$\text{Bernoulli:} \quad p_A + \frac{\rho}{2}V_A^2 = 160000 + 0 = p_B + \frac{\rho}{2}V_B^2 = 90000 + \frac{998}{2}\left(1.405U_\infty^2\right)$$

Solve for $\mathbf{U_\infty \approx 10.0\ m/s}$ *Ans.*

The uncertainty in ($p_A - p_B$) is as high as 6000 Pa, hence the uncertainty in U_∞ is **±0.4 m/s**. *Ans.*

8.28 Sources of equal strength *m* are placed at the four symmetric positions (a, a), (–a, a), (a, –a), and (–a, –a). Sketch the streamline and potential-line patterns. Do any plane "walls" appear?

Solution: This is a double-image flow and creates two walls, as shown. Each quadrant has the same pattern: a source in a "corner." *Ans.*

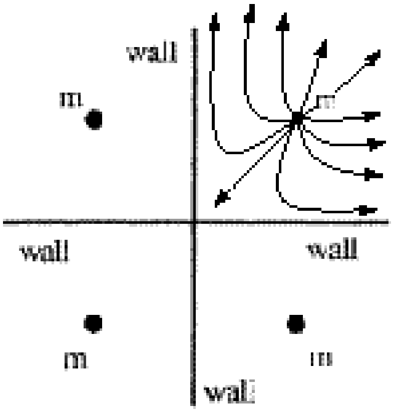


Fig. P8.28

8.29 A uniform water stream, U_∞ = 20 m/s and ρ = 998 kg/m^3, combines with a source at the origin to form a half-body. At (x, y) = (0, 1.2 m), the pressure is 12.5 kPa less than p_∞. (a) Is this point outside the body? Estimate (b) the appropriate source strength *m* and (c) the pressure at the nose of the body.

Solution: We know, from Fig. 8.5 and Eq. 8.18, the point on the half-body surface just above "*m*" is at y = πa/2, as shown, where a = *m*/U. The Bernoulli equation allows us to compute the necessary source strength *m* from the pressure at (x, y) = (0, 1.2 m):

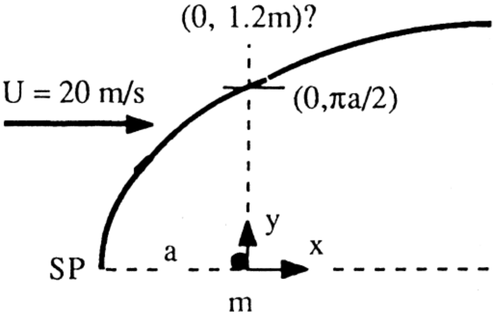


Fig. P8.29

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_\infty - 12500 + \frac{998}{2}\left[(20)^2 + \left(\frac{m}{1.2}\right)^2\right]$$

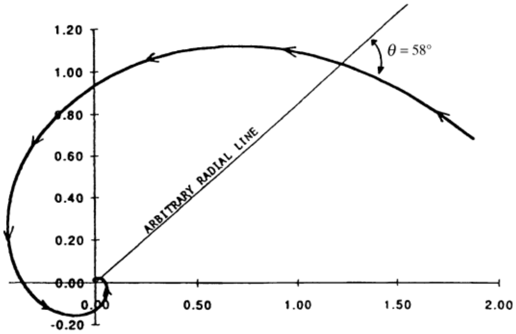
$$\text{Solve for}\quad \boldsymbol{m \approx 6.0\ \frac{m^2}{s}}\quad \textit{Ans.}\ \text{(b)}\quad \text{while}\ \ a = \frac{m}{U} = \frac{6.0}{20} = 0.3\ \text{m}$$

The body surface is thus at $y = \pi a/2 =$ **0.47 m** above m. Thus the point in question, y = 1.2 m above *m*, is **outside the body**. *Ans.* (a)

At the nose SP of the body, (x, y) = (–a, 0), the velocity is zero, hence we predict

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_{nose} + \frac{\rho}{2}(0)^2, \quad \text{or} \quad \mathbf{p_{nose} \approx p_\infty + 200\ kPa} \quad \textit{Ans. } (c)$$

8.30 A tornado is simulated by a line sink $m = -1000$ m²/s plus a line vortex $K = 1600$ m²/s. Find the angle between any streamline and a radial line, and show that it is independent of both r and θ. If this tornado forms in sea-level standard air, at what radius will the local pressure be equivalent to 29 inHg?



Solution: For sea-level air take $p = 101350$ Pa and $\rho = 1.2255$ kg/m³. The combined stream function of this flow is

$$\psi = -K\ln r - m\theta, \quad \textit{with} \quad \tan\theta = \left|\frac{v_\theta}{v_r}\right| = \frac{K/r}{m/r} = \frac{K}{m} = \frac{1600}{1000} = 1.6$$

$$\text{The desired angle is } \theta_{streamline} = \tan^{-1}(1.6) = \mathbf{58^o} \qquad \textit{Ans.}$$

(*b*) Convert 29 in Hg = 97900 Pa. Bernoulli's equation from far away ($U_\infty = 0$) gives

$$p_\infty + \frac{\rho}{2}U_\infty^2 = 101350 + 0 = p_{local} + \frac{\rho}{2}V^2 = 97900 + \frac{1.2255}{2}V^2, \text{ solve}$$

$$V = 74.9 \ \ m/s = \frac{\sqrt{(1600)^2 + (1000)^2}}{r} = \frac{1887 m^2/s}{r} \quad \text{Solve} \quad r = \mathbf{25}m \qquad \textit{Ans.}(b)$$

8.31 A Rankine half-body is formed as shown in Fig. P8.31. For the conditions shown, compute (a) the source strength *m* in m^2/s; (b) the distance *a*; (c) the distance *h*; and (d) the total velocity at point A.

Solution: The vertical distance above the origin is a known multiple of *m* and *a*:

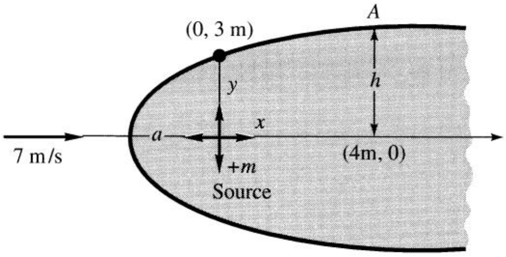


Fig. P8.31

$$y_{x=0} = 3\text{ m} = \frac{\pi m}{2U} = \frac{\pi m}{2(7)} = \frac{\pi a}{2},$$

$$\text{or}\quad \boldsymbol{m \approx 13.4\ \frac{m^2}{s}} \quad\text{and}\quad \boldsymbol{a \approx 1.91\text{ m}} \quad \textit{Ans.}\text{ (a, b)}$$

The distance *h* is found from the equation for the body streamline:

$$\text{At}\quad x = 4\text{ m},\quad r_{\text{body}} = \frac{m(\pi-\theta)}{U\sin\theta} = \frac{13.4(\pi-\theta)}{7\sin\theta} = \frac{4.0}{\cos\theta},\quad \text{solve for}\ \theta \approx 47.8^\circ$$

$$\text{Then}\quad r_A = 4.0/\cos(47.8^\circ) = 5.95\text{ m}\quad\text{and}\quad \mathbf{h} = r\sin\theta \approx \mathbf{4.41\ m}\quad \textit{Ans.}\text{ (c)}$$

The resultant velocity at point A is then computed from Eq. (8.18):

$$V_A = U\left(1+\frac{a^2}{r^2}+\frac{2a}{r}\cos\theta_A\right)^{1/2} = 7\left[1+\left(\frac{1.91}{5.95}\right)^2+2\left(\frac{1.91}{5.95}\right)\cos 47.8^\circ\right]^{1/2} \approx \mathbf{8.7\ \frac{m}{s}}\quad \textit{Ans.}\text{ (d)}$$

P8.32 A source and sink of equal strength, $m = 25\ \text{m}^2/\text{s}$, are near a wall, as in Fig. P8.32. Find the resultant velocity induced by this pair at point A on the wall.

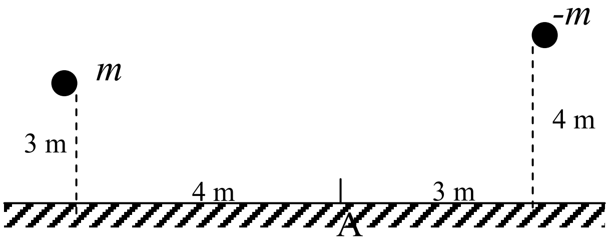
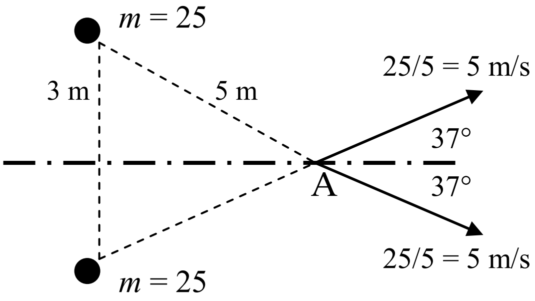
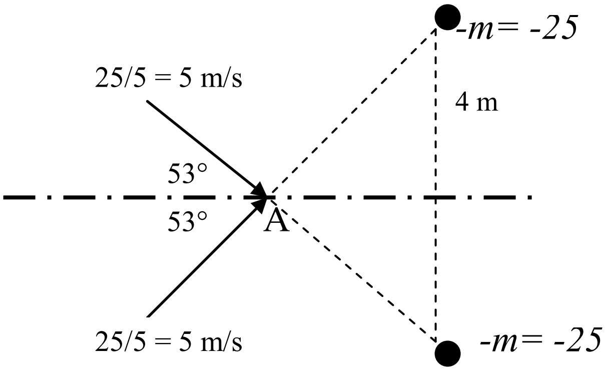


Fig. P8.32

Solution: To create the "wall", we need a mirror image source-sink pair below the wall. Then we can sketch the source pair and the sink pair separately, as follows:





You can see by symmetry that the new velocity normal to the wall is **zero**. Add up all the velocities to the right along the wall:

$$\sum u \;=\; 2(5\,m/s)\cos(37^{o}) \;+\; 2(5\,m/s)\cos(53^{o}) = 10(0.8) + 10(0.6) = 8 + 6 = \mathbf{14}\frac{m}{s}\ \textit{Ans.}$$

8.33 Sketch the streamlines, especially the body shape, due to equal line sources m at (0, +a) and (0, –a) plus a uniform stream U_∞ = ma.

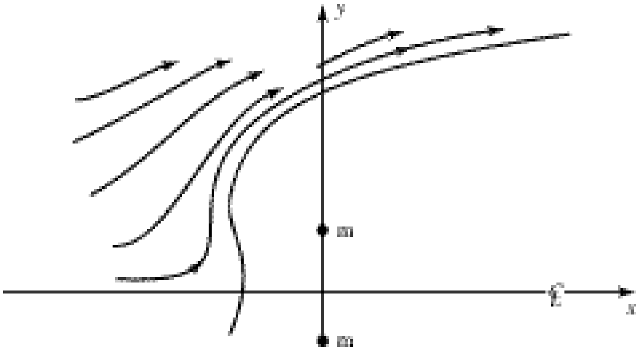


Fig. P8.33

Solution: As shown, a half-body shape is formed which has a dimple at the nose. The stagnation point, for this special case U_∞ = ma, is at x = –a. The half-body shape varies with the parameter (U_∞a/m).

8.34 Consider three equally spaced line sources m placed at (x, y) = (+a, 0), (0, 0), and (−a, 0). Sketch the resulting streamlines and note any stagnation points. What would the pattern look like from afar?

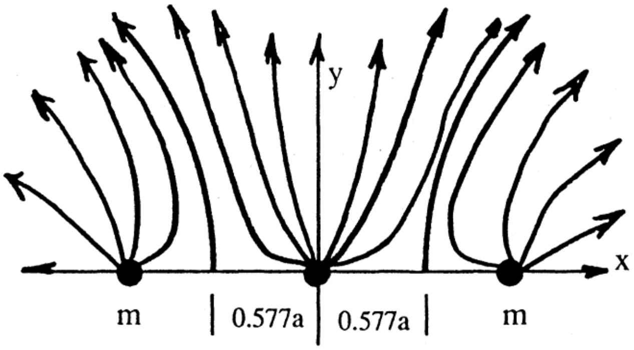


Fig. P8.34

Solution: The pattern (symmetrical about the x-axis) is shown above. There are two stagnation points, at x = ±a/√3 = **±0.577a**. Viewed from afar, the pattern would look like a single source of strength **3 m**.

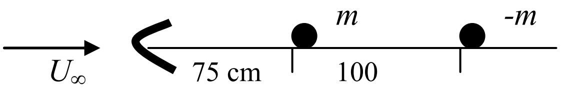
P8.35 A uniform stream, U_∞ = 4 m/s, approaches a Rankine oval as in Fig. 8.13, with a = 50 cm. (*a*) Find the strength m of the source-sink pair, in m^2/s, which will cause the total length of the oval to be 250 cm. (*b*) What is the maximum width of this oval?

Solution: (*a*) The straightforward, but unsatisfying, way to find length is to simply use Eq. (8.35) for the half-length *L* of the oval. We are given *a* = 50 cm and *L* = 250/2 = 125 cm. Thus

$$\frac{L}{a} = \left(1+\frac{2m}{U_\infty a}\right)^{1/2} = \frac{125\,cm}{50\,cm} = 2.5\ ,\ \text{hence}\ \frac{2m}{U_\infty a} = (2.5)^2 - 1 = 5.25\ ,$$

$$\text{or}:\ m = 2.625 U_\infty a = (2.625)(4\,m/s)(0.5\,m) = \mathbf{5.25}\ m^2/s \qquad Ans.(a)$$

More satisfying would be to make the sketch at right to show the front stagnation point, where



$$\sum u_{stagnation} = 0 = 4 - \frac{m}{0.75} + \frac{m}{1.75}\ ;\quad \text{Solve}\ \ m = \mathbf{5.25}\ m^2/s \qquad Ans.(a)$$

(*b*) Once we know 2*m*/(U_∞*a*) = 5.25, Eq. (8.35) gives the width (2*h*) of the oval:

$$\frac{h}{a} = \cot\left[\frac{h/a}{2m/(U_\infty a)}\right] = \cot\left[\frac{h/a}{5.25}\right],\quad \text{Iteration or EES}:\ \frac{h}{a} \approx 2.22$$

Hence the oval **width** = 2*h* = 2(2.22*a*) = 2(2.22)(0.5m) = 2.22 m = **222** cm *Ans.*(*b*)

8.36 When a line source-sink pair with *m* = 2 m^2/s is combined with a uniform stream, it forms a Rankine oval whose minimum dimension is 40 cm, as shown. If *a* = 15 cm, what are the stream velocity and the maximum velocity? What is the length?

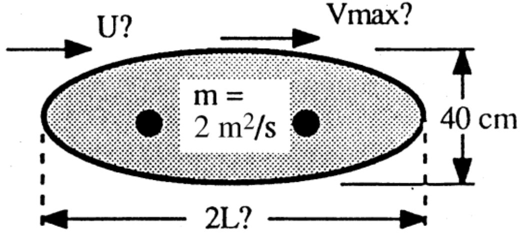


Fig. P8.36

Solution: We know h/a = 20/15, so from Eq. (8.30) we may determine the stream velocity:

$$\frac{h}{a}=\frac{20}{15}=\cot\left[\frac{h/a}{2m/(U_\infty a)}\right]=\cot\left[\frac{20/15}{2(2)/(0.15U_\infty)}\right],\quad \text{solve for } \mathbf{U_\infty \approx 12.9\ \frac{m}{s}} \quad \textit{Ans.}$$

$$\text{Then}\quad \frac{m}{U_\infty a}=\frac{2}{12.9(0.15)}=1.036,\quad \frac{L}{a}=[1+2(1.036)]^{1/2}=1.75,\quad \mathbf{2L \approx 53\ cm} \quad \textit{Ans.}$$

$$\text{Finally,}\quad \frac{V_{max}}{U_\infty}=1+\frac{2m/(U_\infty a)}{1+(h/a)^2}=1+\frac{2(1.036)}{1+(20/15)^2}=1.75,\quad \mathbf{V_{max} \approx 22.5\ \frac{m}{s}} \quad \textit{Ans.}$$

8.37 A Rankine oval 2 m long and 1 m high is immersed in a stream U_∞ = 10 m/s, as in Fig. P8.37. Estimate (a) the velocity at point A and (b) the location of point B where a particle approaching the stagnation point achieves its maximum deceleration.

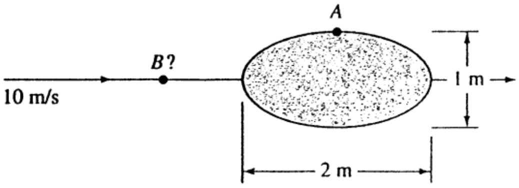


Fig. P8.37

Solution: (a) With L/h = 2.0, we may evaluate Eq. (8.30) to find the source-sink strength:

$$\frac{h}{a}=\cot\left[\frac{h/a}{2m/(U_\infty a)}\right] \quad \text{and} \quad \frac{L}{a}=\left(1+\frac{2m}{U_\infty a}\right)^{1/2}$$

$$\text{converges to}\quad \frac{L}{h}=2.0 \quad \text{if} \quad \frac{m}{U_\infty a}=\mathbf{0.3178}$$

$$\text{Meanwhile,}\quad \frac{h}{a}=0.6395 \ \text{and}\ \frac{L}{a}=1.2789 \quad \text{thus}\ a=\frac{1\ \text{meter}}{1.2789}\approx \mathbf{0.782\ m}$$

$$\text{Also compute}\quad V_{max}/U_\infty=1.451,\quad \text{hence}\ V_{max}=1.451(10)\approx \mathbf{14.5\ m/s}. \quad \textit{Ans.}\ \text{(a)}$$

(b) Along the x-axis, at any x ≤ –L, the velocity toward the body nose has the form

$$u=U_\infty+\frac{m}{a+x}+\frac{m}{a-x},\quad \text{where}\ m\approx 0.3178U_\infty a$$

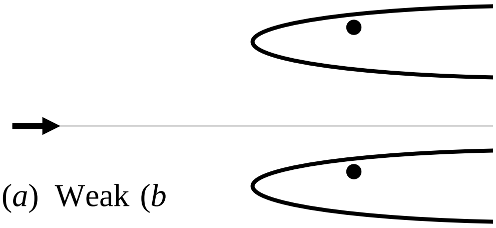
$$\text{Then}\quad \frac{du}{dt}=u\frac{\partial u}{\partial x}=\left[U_\infty+\frac{m}{a+x}+\frac{m}{a-x}\right](-m)\left[\frac{1}{(a+x)^2}-\frac{1}{(a-x)^2}\right]$$

For this value of m, the maximum deceleration occurs at $\mathbf{x = -1.41a}$ *Ans.*

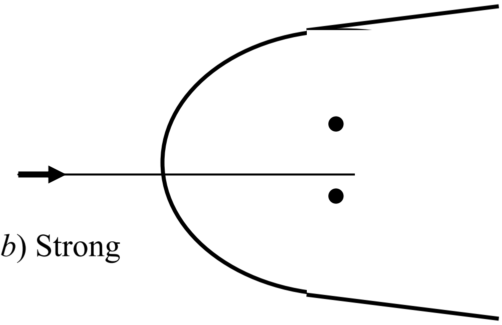
This is quite near the nose (which is at x = –1.28a). The numerical value of the maximum deceleration is $\mathbf{(du/dt)_{max} \approx -0.655U_\infty^2/a}$.

P8.38 Consider potential flow of a uniform stream in the *x* direction plus two equal sources, one at (*x, y*) = (0, +a) and the other at (*x, y*) = (0, -a). Sketch your ideas of the body contours that would arise if the sources were (*a*) very weak; and (*b*) very strong.

Solution: Weak sources ($m \ll U_\infty a$) create two approximate half-bodies, strong sources ($m \gg U_\infty a$) create one big approximate half-body. *Ans.*



(*a*) Weak (*b*



(*b*) Strong

(Sorry for the poor sketching. Au)

8.39 For the Rankine oval of Fig. P8.37, if the fluid is water at 20°C and the pressure far upstream along the body centerline is 115 kPa, determine the freestream velocity U_∞ for which the fluid will cavitate at point A.

Solution: For water at 20°C take $\rho = 998$ kg/m^3 and $p_{vap} = 2337$ Pa. If you have solved Prob. P8.37, then you already know that, for the thickness ratio h/L = ½, $m/U_\infty a = 0.32$, $u_{max}/U_\infty = 1.453$. Otherwise you have to work that out here. Then Bernoulli's equation yields the correct velocity:

$$\frac{p_\infty}{\rho g} + \frac{U_\infty^2}{2g} + z_\infty = \frac{115000}{9790} + \frac{U_\infty^2}{2(9.81)} + 0 = \frac{p_{vap}}{\rho g} + \frac{(1.453U_\infty)^2}{2g} + z_A = \frac{2337}{9790} + \frac{(1.453U_\infty)^2}{2(9.81)} + 0.5$$

$$\textit{Solve for} \qquad U_\infty = \mathbf{13.95\,\frac{m}{s}} \qquad \textit{Ans.}$$

8.40 Modify the Rankine oval in Fig. P8.37 so that the stream velocity and body length are the same but the thickness is unknown (not 1 m). The fluid is water at 30°C and the pressure far upstream along the body centerline is 108 kPa. Find the body thickness for which cavitation will occur at point A.

Solution: For water at 30°C, take ρ = 996 kg/m^3 and p_{vap} = 4242 Pa. Bernoulli's equation between far upstream and point A yields

$$\frac{p_\infty}{\rho g}+\frac{U_\infty^2}{2g}+z_\infty=\frac{p_{vap}}{\rho g}+\frac{u_{\max}^2}{2g}+z_A, \quad \text{or:}$$

$$\frac{108,000}{996(9.81)}+\frac{10^2}{2(9.81)}+0=\frac{4242}{996(9.81)}+\frac{[K(10)]^2}{2(9.81)}+h$$

The unknowns are the ratio K = $u_{\max}/U_\infty$ and the thickness (2h), which depend upon the parameter ($m/U_\infty a$), which we will have to find by iteration. Trying out the answers to Prob. P8.37, $m/U_\infty a$ = 0.32, $u_{\max}$ = 14.5 m/s, and h = 0.5 m, we find p_A = 48,000 Pa, so that body is too thin, not enough pressure drop. We can iterate, or use EES, and the final solution is:

$$\frac{m}{U_\infty a}=0.891; \quad h=0.732m; \quad u_{\max}=17.15\frac{m}{s}, \quad \text{Thickness}=2h=1.465m \quad \textit{Ans.}$$

8.41 A Kelvin oval is formed by a line-vortex pair with K = 9 m^2/s, a = 1 m, and U = 10 m/s. What are the height, width, and shoulder velocity of this oval?

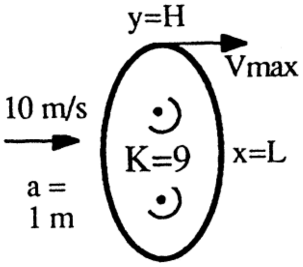


Fig. P8.41

Solution: With reference to Fig. 8.12 and Eq. (8.41), the oval is described by

$$\psi=0,\ x=0,\ y=H: \quad UH=\frac{K}{2}\ln\left[\frac{(H+a)^2}{(H-a)^2}\right], \quad \text{with } \frac{K}{Ua}=\frac{9}{10(1)}=\mathbf{0.9}$$

Solve by iteration for H/a ≈ 1.48, or **2H** = oval height ≈ **2.96 m** *Ans.*

$$\text{Similarly,}\quad \frac{L}{a}=\left(\frac{2K}{Ua}-1\right)^{1/2}=[2(0.9)-1]^{1/2}=0.894, \quad \mathbf{2L}=\text{width}\approx\mathbf{1.79\ m} \quad \textit{Ans.}$$

$$\text{Finally,}\quad V_{\max}=U+\frac{K}{H-a}-\frac{K}{H+a}=10+\frac{9}{0.48}-\frac{9}{2.48}\approx\mathbf{25.1\ \frac{m}{s}} \quad \textit{Ans.}$$

P8.42 The vertical keel of a sailboat approximates a Rankine oval 125 cm long and 30 cm thick. The boat sails in seawater in standard atmosphere at 14 knots, parallel to the keel. At a section 2 m below the surface, estimate the lowest pressure on the surface of the keel.

Solution: Assume standard sea level pressure of 101,350 Pa. From Table A.3, the density of seawater is approximately 1025 kg/m^3. At 2 meters depth, we would have

$$p_\infty = p_{atm} + \rho g \Delta z = 101350 + (1025)(9.81)(2.0) = 101350 + 20110 \approx 121,500\ Pa$$

The length-to-width ratio of the keel is 125/30 = 4.17, which the writer intended to lie exactly in Table 8.1 and thus avoid extra computation. The table shows $u_{max}/U_\infty = 1.187$ for that condition. Convert U_∞ = 14 kn x 0.51444 = 7.20 m/s. Then u_{max} = 1.187(7.20) = 8.55 m/s, where the lowest pressure will occur. Use Bernoulli's equation at a constant (hence cancelled) depth:

$$p_\infty + \frac{\rho}{2}U_\infty^2 = 121,500 + \frac{1025}{2}(7.20)^2 = p_{lowest} + \frac{\rho}{2}u_{\max}^2 = p_{lowest} + \frac{1025}{2}(8.55)^2$$

$$\text{Solve}\quad p_{lowest} = 121,500 + 26,600 - 37,500 = 110,600\,Pa \approx \mathbf{111}\ kPa \quad Ans.$$

8.43 Consider water at 20°C flowing past a 1-m-diameter cylinder. What doublet strength in m^2/s is required to simulate this flow? If the stream pressure is 200 kPa, use inviscid theory to estimate the surface pressure at (a) 180°; (b) 135°; and (c) 90°.

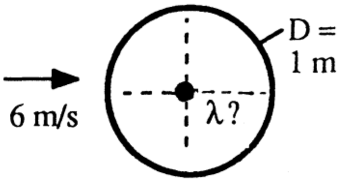


Fig. P8.43

Solution: For water at 20°C, take ρ = 998 kg/m^3. The required doublet strength is

$$\lambda = U_\infty a^2 = (6.0\ \text{m/s})(0.5\ \text{m})^2 \approx \mathbf{1.5}\ \frac{\mathbf{m^3}}{\mathbf{s}} \quad Ans.$$

The surface pressures are computed from Bernoulli's equation, with $V_{surface} = 2U_\infty \sin\theta$:

$$p_s + \frac{\rho}{2}(2U_\infty \sin\theta)^2 = p_\infty + \frac{\rho}{2}U_\infty^2, \quad \text{or:} \quad p_s = 200000 + \frac{998}{2}(6)^2(1 - 4\sin^2\theta)$$

(a) at 180°, ps ≈ **218000 Pa**; (b) at 135°, **182000 Pa**; (c) at 90°, **146000 Pa** *Ans.* (a, b, c)

8.44 Suppose that circulation is added to the cylinder flow of Prob. 8.43 sufficient to place the stagnation points at $\theta = 35°$ and 145°. What is the required vortex strength K in m^2/s? Compute the resulting pressure and surface velocity at (a) the stagnation points, and (b) the upper and lower shoulders. What will be the lift per meter of cylinder width?

Solution: Recall that Prob. 8.43 was for water at 20°C flowing at 6 m/s past a 1-m-diameter cylinder, with p_∞ = 200 kPa. From Eq. (8.35),

$$\sin\theta_{stag} = \sin(35°) = \frac{K}{2U_\infty a} = \frac{K}{2(6\text{ m/s})(0.5\text{ m})}, \quad \text{or:} \quad K = 3.44\ m^2/s \quad \textit{Ans.}$$

(a) At the stagnation points, velocity is zero and pressure equals stagnation pressure:

$$p_{stag} = p_\infty + \frac{\rho}{2}U_\infty^2 = 200{,}000\ Pa + \frac{998\text{ kg/m}^3}{2}(6\text{ m/s})^2 = \mathbf{218{,}000\ Pa} \quad \textit{Ans.}\text{ (a)}$$

(b) At any point on the surface, from Eq. (8.37),

$$p_{stag} = 218000 = p_{surf} + \frac{\rho}{2}\left(-2U_\infty \sin\theta + \frac{K}{a}\right)^2 = p_{surf} + \frac{998}{2}\left[-2(6)\sin\theta + \frac{3.44}{0.5}\right]^2$$

At the upper shoulder, $\theta = 90°$,

$$p = 218000 - \frac{998}{2}(-5.12)^2 \approx \mathbf{204{,}900\ Pa} \quad \textit{Ans.}\text{ (b—upper)}$$

At the lower shoulder, $\theta = 270°$,

$$p = 218000 - \frac{998}{2}(-18.88)^2 \approx \mathbf{40{,}100\ Pa} \quad \textit{Ans.}\text{ (b—lower)}$$

8.45 If circulation K is added to the cylinder flow in Prob. 8.43, (a) for what value of K will the flow begin to cavitate at the surface? (b) Where on the surface will cavitation begin? (c) For this condition, where will the stagnation points lie?

Solution: Recall that Prob. 8.43 was for water at 20°C flowing at 6 m/s past a 1-m-diameter cylinder, with p_∞ = 200 kPa. From Table A.5, p_{vap} = 2337 Pa. (b) Cavitation will occur at the lowest pressure point, which is **at the bottom shoulder (θ = 270°)** in Fig. 8.10. *Ans.* (b)

(a) Use Bernoulli's equation to estimate the velocity at $\theta = 270°$ if the pressure there is $p$vap:

$$p_o = 218000\text{ Pa} = p_{vap} + \frac{\rho}{2}V_{surf}^2 = 2337\text{ Pa} + \frac{998\text{ kg/m}^3}{2}\left[(-2)(6\text{ m/s})\sin(270°) + \frac{K}{0.5\text{ m}}\right]^2$$

$$\text{Solve for:}\quad \mathbf{K_{cavitation} \approx 4.39\ m^2/s} \quad \textit{Ans.}\text{ (a)}$$

(c) The locations of the two stagnation points are given by Eq. (8.35):

$$\sin\theta_{stag} = \frac{K}{2U_\infty a} = \frac{4.39\text{ m}^2/\text{s}}{2(6\text{ m/s})(0.5\text{ m})} = 0.732, \quad \theta_{stag} = \mathbf{47°}\ \textit{and}\ \mathbf{133°} \quad \textit{Ans.}\text{ (c)}$$

8.46 A cylinder is formed by bolting two semicylindrical channels together on the inside, as shown in Fig. P8.46. There are 10 bolts per meter of width on each side, and the inside pressure is 50 kPa (gage). Using potential theory for the outside pressure, compute the tension force in each bolt if the fluid outside is sea-level air.

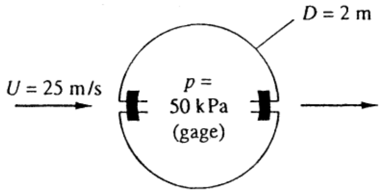


Fig. P8.46

Solution: For sea-level air take $\rho = 1.225$ kg/m^3. Use Bernoulli to find surface pressure:

$$p_\infty + \frac{\rho}{2}U_\infty^2 = 0 + \frac{1.225}{2}(25)^2 = p_s + \frac{1.225}{2}(2U_\infty\sin\theta)^2, \quad \text{or:}\quad p_s = 383 - 1531\sin^2\theta$$

$$\text{compute}\quad F_{down} = 2\int_0^{\pi/2} p\sin\theta\, ba\, d\theta = 2\int_0^{\pi/2} (383 - 1531\sin^2\theta)\sin\theta(1\text{ m})(1\text{ m})\,d\theta = -1276\ \frac{N}{m}$$

This is small potatoes compared to the force due to *inside* pressure:

$$F_{up} = 2p_{inside}ab = 2(50000)(1)(1) = 100000\ \frac{N}{m}$$

$$\text{Total force per meter} = 100000 - (-1276) = 101276 \div 20\text{ bolts} \approx \mathbf{5060\ \frac{N}{bolt}} \quad \textit{Ans.}$$

8.47 A circular cylinder is fitted with two pressure sensors, to measure pressure at "a" (180°) and "b" (105°), as shown. The intent is to use this cylinder as a stream velocimeter. Using inviscid theory, derive a formula for calculating U_∞ from p_a, p_b, ρ, and radius *a*.

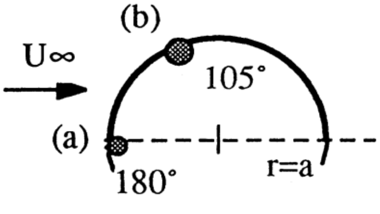


Fig. P8.47

Solution: We relate the pressures to surface velocities from Bernoulli's equation:

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_a + \frac{\rho}{2}(0)^2 = p_b + \frac{\rho}{2}(2U_\infty \sin 105°)^2, \quad \text{or:} \quad \mathbf{U_\infty = \frac{1}{\sin 105°}\sqrt{\frac{p_a - p_b}{2\rho}}} \quad \textit{Ans.}$$

This is not a bad idea for a velocimeter, except that (1) it should be calibrated; and (2) it must be carefully aligned so that sensor "a" exactly faces the oncoming stream.

8.48 Wind at U_∞ and p_∞ flows past a Quonset hut which is a half-cylinder of radius a and length L (Fig. P8.48). The internal pressure is p_i. Using inviscid theory, derive an expression for the upward force on the hut due to the difference between p_i and p_s.

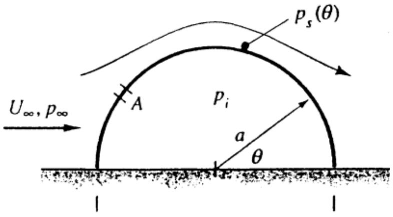


Fig. P8.48

Solution: The analysis is similar to Prob. 8.46 on the previous page. If p_o is the stagnation pressure at the nose ($\theta = 180°$), the surface pressure distribution is

$$p_s = p_o - \frac{\rho}{2}U_s^2 = p_o - \frac{\rho}{2}(2U_\infty \sin\theta)^2 = p_o - 2\rho U_\infty^2 \sin^2\theta$$

Then the net upward force on the half-cylinder is found by integration:

$$F_{up} = \int_0^\pi (p_i - p_s)\sin\theta\, ab\, d\theta = \int_0^\pi (p_i - p_o + 2\rho U_\infty^2 \sin^2\theta)\sin\theta\, ab\, d\theta,$$

or: $\mathbf{F_{up} = (p_i - p_o)2ab + \frac{8}{3}\rho U_\infty^2\, ab}$ *Ans.* $\left(\text{where } p_o = p_\infty + \frac{\rho}{2}U_\infty^2\right)$

8.49 In strong winds, the force in Prob. 8.48 above can be quite large. Suppose that a hole is introduced in the hut roof at point A (see Fig. P8.48) to make p_i equal to the surface pressure p_A. At what angle θ should hole A be placed to make the net force zero?

Solution: Set F = 0 in Prob. 8.48 and find the proper pressure from Bernoulli:

$$F_{up} = 0 \text{ if } \quad p_i = p_o - \frac{4}{3}\rho U_\infty^2, \quad \text{but also } \; p_i = p_A = p_o - \frac{\rho}{2}(2U_\infty \sin\theta_A)^2$$

$$\text{Solve for } \quad \sin\theta_A = \sqrt{2/3} = 0.817 \quad \text{or} \quad \theta_A \approx \mathbf{125°} \quad \textit{Ans.}$$

(or **55°** = poor position on rear of body)

8.50 It is desired to simulate flow past a ridge or "bump" by using a streamline *above* the flow over a cylinder, as shown in Fig. P8.50. The bump is to be *a*/2 high, as shown. What is the proper elevation *h* of this streamline? What is U_{max} on the bump compared to U_∞?

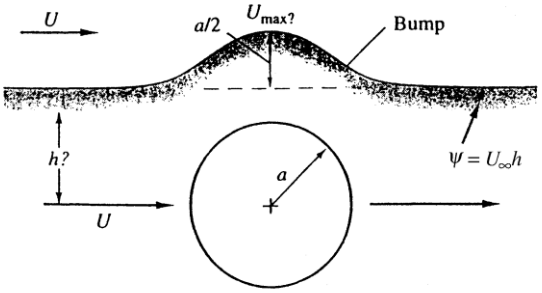


Fig. P8.50

Solution: Apply the equation of the streamline (Eq. 8.32) to θ=180° and also 90°:

$$\psi = U_\infty \sin\theta\left(r - \frac{a^2}{r}\right) \quad \text{at } \theta = 180° \text{ (the freestream) gives } \psi = U_\infty h$$

$$\text{Then, at } \theta = 90°, \quad r = h + \frac{a}{2}, \quad \psi = U_\infty h = U_\infty \sin 90°\left(h + \frac{a}{2} - \frac{a^2}{h + a/2}\right)$$

$$\text{Solve for} \quad \mathbf{h = \frac{3}{2}a} \quad \textit{Ans.} \quad \text{(corresponds to } r = 2a\text{)}$$

The velocity at the hump (r = 2a, θ = 90°) then follows from Eq. (8.33):

$$U_{\max} = U_\infty \sin 90° \left[1+\frac{a^2}{(2a)^2}\right] \quad \text{or} \quad \mathbf{U_{max} = \frac{5}{4}U_\infty} \quad \textit{Ans.}$$

P8.51 A hole is placed in the front of a cylinder to measure the stream velocity of sea level fresh water. The measured pressure at the hole is 2840 lbf/ft^2. If the hole is misaligned by 12° from the stream, and misinterpreted as stagnation pressure, what is the error in velocity?

Solution: Take sea level pressure as 2117 lbf/ft^2 and ρ_{water} = 1.936 slug/ft^3. If we think the velocity at the holes is zero (stagnation), then Bernoulli's equation predicts

$$p_o \approx 2840\,\frac{lbf}{ft^2} \approx p_\infty + \frac{\rho}{2}V^2 = 2117 + \frac{1.936}{2}V^2 \;; \quad Solve\;\; V \approx 27.3\,\frac{ft}{s}$$

In fact, though, there is a *velocity* at the hole, equal by theory to $2U_\infty \sin^2(12°)$. So the proper Bernoulli equation at the hole is

$$p_\infty + \frac{\rho}{2}U_\infty^2 = 2117 + \frac{1.936}{2}U_\infty^2 = p_{hole} + \frac{1.936}{2}V_{hole}^2 = 2840 + \frac{1.936}{2}[2U_\infty \sin(12^o)]^2$$

$$\text{Solve for} \quad U_\infty = \Big[\frac{2}{1.936}\frac{(2840-2117)}{(1-4\sin^2 12^o)}\Big]^{1/2} \approx 30.1\,\frac{ft}{s}$$

Thus the misaligned estimate of 27.3 ft/s is low by about **9.2 %**. *Ans.*

If we solve this problem analytically, for an angle β of misalignment, we would find that the error in velocity is a function only of β, not density or stream pressure:

$$\frac{V_{hole}}{U_\infty} = \sqrt{1-4\sin^2\beta} \qquad \text{(always a low estimate)}$$

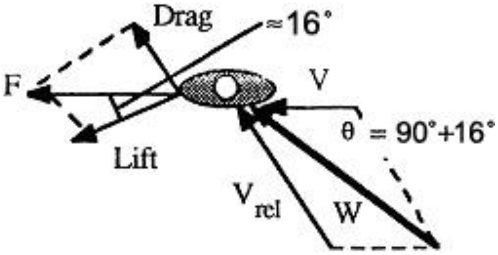
8.52 The Flettner-rotor sailboat in Fig. E8.2 has a water drag coefficient of 0.006 based on a wetted area of 45 ft^2. If the rotor spins at 220 rev/min, find the maximum boat speed that can be achieved in 15 mi/h winds. Find the optimum wind angle.

Solution: Recall that the rotor has a diameter of 2.5 ft and is 10 ft high. Standard air density is 0.00238 slug/ft^3. As in Ex. 8.2, estimate $C_L \approx 2.5$ and $C_D \approx 0.7$. If the boat speed is V and the wind is W, the relative velocity V_{rel} is shown in the figure at right. Thrust = drag:

$$F = \left(C_L^2 + C_D^2\right)^{1/2}\frac{\rho}{2}V_{rel}^2 DL$$

$$= \text{Boat drag} = C_{d,\text{boat}}\frac{\rho_{\text{water}}}{2}V^2 A_{\text{wetted}},$$

Fig. E8.2



$$\text{or:}\quad [(2.5)^2 + (0.7)^2]^{1/2}\left(\frac{0.00238}{2}\right)V_{rel}^2(2.5)(10) = (0.006)\left(\frac{1.99}{2}\right)V^2(45),$$

$$\text{or:}\quad V_{rel} = 1.865V$$

Convert W = 15 mi/h = 22 ft/s. As shown in the figure, the angle between the wind lift and wind drag is $\tan^{-1}(C_D/C_L) = \tan^{-1}(0.7/2.5) \approx 16°$. Then, by geometry, the angle θ between the relative wind and the boat speed (see figure above) is $\theta = 180 - 74 = 106°$. The law of cosines, applied to the wind-vector triangle above, then determines the boat speed:

$$W^2 = V^2 + V_{rel}^2 - 2VV_{rel}\cos\theta,$$

$$\text{or:}\quad (22)^2 = V^2 + (1.865V)^2 - 2V(1.865V)\cos(106°)$$

$$\text{Solve for}\quad V_{boat} \approx 10.4\ \frac{ft}{s}\quad \textit{Ans.}$$

For this "optimum" condition (which directs the resultant wind force along the keel or path of the boat), the angle β between the wind and the boat direction (see figure on the previous page) is

$$\frac{\sin\beta}{1.865(10.4)} = \frac{\sin(110°)}{22},\quad \text{or:}\quad \beta \approx 58° \quad \textit{Ans.}$$

8.53 Modify Prob. P8.52 as follows. For the same sailboat data, find the wind velocity, in mi/h, which will drive the boat at an optimum speed of **8 kn** parallel to its keel.

Solution: Convert 8 knots = 13.5 ft/s. Again estimate $C_L \approx 2.5$ and $C_D \approx 0.7$. The geometry is the same as in Prob. P8.52, hence V_{rel} still equals $1.865V = 25.2$ ft/s. The law of cosines still holds for the velocity diagram in the figure:

$$W^2 = V^2 + V_{rel}^2 - 2VV_{rel}\cos\theta = (13.5)^2 + (25.2)^2 - 2(13.5)(25.2)\cos(106°) = 1005\ \text{ft}^2/\text{s}^2$$

$$\text{Solve for}\quad W_{wind} = 31.7\ \text{ft/s} = 21.6\ \text{mi/h}\quad \textit{Ans.}$$

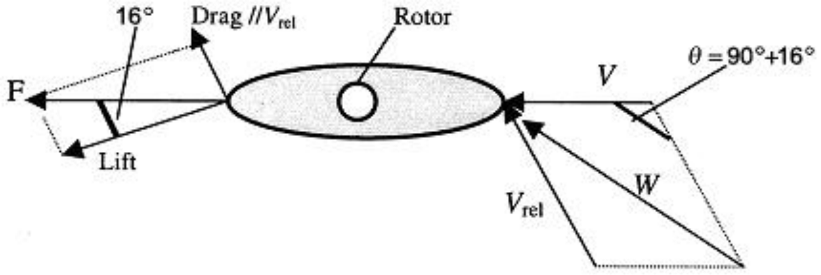


Fig. P8.52

8.54 The original Flettner rotor ship was approximately 100 ft long, displaced 800 tons, and had a wetted area of 3500 ft^2. As sketched in Fig. P8.54, it had two rotors 50 ft high and 9 ft in diameter rotating at 750 r/min, which is far outside the range of Fig. 8.11. The measured lift and drag coefficients for each rotor were about 10 and 4, respectively. If the ship is moored and subjected to a crosswind of 25 ft/s, as in Fig. P8.54, what will the wind force parallel and normal to the ship centerline be? Estimate the power required to drive the rotors.

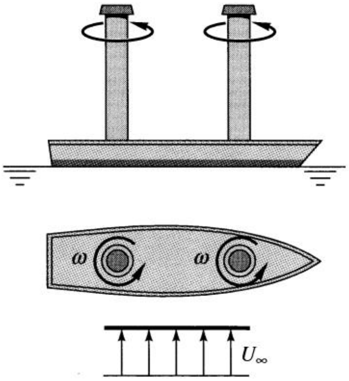
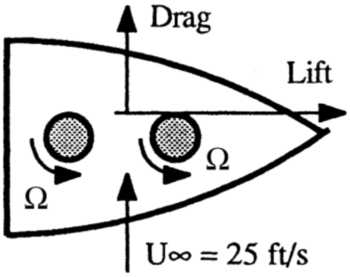


Fig. P8.54

Solution: For sea-level air take ρ = 0.00238 slug/ft^3 and μ = 3.71E–7 slug/ft·s. Then compute the forces:



$$\text{Lift} = C_L \frac{\rho}{2} U_\infty^2 DL = 10\left(\frac{0.00238}{2}\right)(25)^2(9\text{ ft})(50\text{ ft}) \times (2\,\text{rotors}) \approx 6700\ lbf\ \text{(parallel)} \quad \textit{Ans.}$$

$$\text{Drag} = (4/10)\text{Lift} \approx 2700\ lbf\ \text{(normal)} \quad \textit{Ans.}$$

$$\text{Drag} = (4/10)\text{Lift} \approx 2700\ lbf\ \text{(normal)} \quad \textit{Ans.}$$

We don't have any *formulas* in the book for the (viscous) torque of a rotating cylinder (you could find results in refs. 1 and 2 of Chap. 7). As a good approximation, assume the cylinder simulates a flat plate of length $2\pi R = 2\pi(4.5) = 28.3$ ft. Then the shear stress is:

$$\tau_w = C_f \frac{\rho}{2}U^2 \approx \frac{0.027}{Re_L^{1/7}}\frac{\rho}{2}(\Omega R)^2, \quad \Omega R = 750\left(\frac{2\pi}{60}\right)(4.5) = 353\ \frac{ft}{s}, \quad \text{and}$$

$$Re_L = \frac{0.00238(353)(28.3)}{3.71E{-}7} \approx 6.42E7, \quad \text{whence } \tau_w \approx \mathbf{0.308}\ \frac{lbf}{ft^2}$$

$$\text{Then} \quad \text{Torque} = \tau w(\pi D\, L)R = 0.308\pi(9)(50)(4.5) \approx 1958\ ft{\cdot}lbf$$

$$\text{Total power} = T\Omega = (1958)\left(750\frac{2\pi}{60}\right) \times (2\ \text{rotors}) \div 550\ \frac{hp}{ft{\cdot}lbf/s} \approx \mathbf{560\ hp} \quad \textit{Ans.}$$

8.55 Assume that the Flettner rotor ship of Fig. P8.54 has a water-resistance coefficient of 0.005. How fast will the ship sail in seawater at 20°C in a 20 ft/s wind if the keel aligns itself with the resultant force on the rotors? [This problem involves relative velocities.]

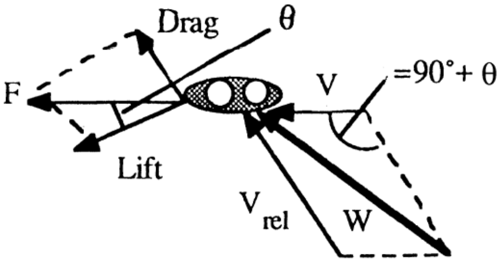


Fig. P8.55

Solution: For air, take $\rho = 0.00238$ slug/ft^3. For seawater, take $\rho = 1.99$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. Recall D = 9 ft, L = 50 ft, 2 rotors at 750 rev/min, $C_L \approx 10.0$, $C_D \approx 4.0$. In the sketch above, the drag and lift combine along the ship's keel. Then

$$\theta = \tan^{-1}\left(\frac{4}{10}\right) \approx 21.8°, \quad \text{so angle between } \mathbf{V} \text{ and } \mathbf{V_{rel}} = 90 + \theta \approx \mathbf{111.8°}$$

$$\text{Thrust} \quad F = [(10)^2 + (4)^2]^{1/2}\left(\frac{0.00238}{2}\right)V_{rel}^2(9)(50)(2\ \text{rotors})$$

$$= \text{Drag} = C_d(\rho/2)V^2 A_{wetted} = (0.005)(1.99/2)(3500)V^2, \quad \text{solve } \mathbf{V_{rel} \approx 1.23V}$$

$$\text{Law of cosines:} \quad W^2 = V^2 + V_{rel}^2 - 2VV_{rel}\cos(\theta + 90°),$$

$$\text{or:} \quad (20)^2 = V^2 + (1.23V)^2 - 2V(1.23V)\cos(111.8°), \quad \text{solve for } \mathbf{V_{ship} \approx 10.8\ \frac{ft}{s}} \quad \textit{Ans.}$$

8.56 A proposed freestream velocimeter would use a cylinder with pressure taps at $\theta = 180°$ and at 150°. The pressure difference would be a measure of stream velocity U_∞.

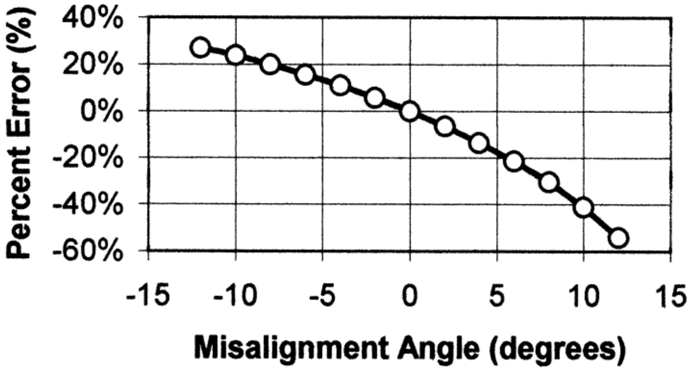
However, the cylinder must be aligned so that one tap exactly faces the freestream. Let the misalignment angle be δ, that is, the two taps are at $(180° + \delta)$ and $(150° + \delta)$. Make a plot of the percent error in velocity measurement in the range $-20° < \delta < +20°$ and comment on the idea.

Solution: Recall from Eq. (8.34) that the surface velocity on the cylinder equals $2U_\infty \sin\theta$. Apply Bernoulli's equation at both points, 180° and 150°, to solve for stream velocity:

$$p_1 + \frac{\rho}{2}[2U_\infty \sin(180° + \delta)]^2 = p_2 + \frac{\rho}{2}[2U_\infty \sin(150° + \delta)]^2,$$

$$\text{or:}\quad U_\infty = \frac{\sqrt{\Delta p/2\rho}}{\sqrt{\sin^2(150° + \delta) - \sin^2(180° + \delta)}}$$

The error is zero when $\delta = 0°$. Thus we can plot the percent error versus δ. When $\delta = 0°$, the denominator above equals 0.5. When $\delta = 5°$, the denominator equals 0.413, giving an error on the low side of $(0.413/0.5) - 1 = -17\%$! The plot below shows that this is a very poor idea for a velocimeter, since even a small misalignment causes a large error.



Problem 8.56

8.57 In principle, it is possible to use rotating cylinders as aircraft wings. Consider a cylinder 30 cm in diameter, rotating at 2400 rev/min. It is to lift a 55-kN airplane flying at 100 m/s. What should the cylinder length be? How much power is required to maintain this speed? Neglect end effects on the rotating wing.

Solution: Assume sea-level air, $\rho = 1.23\ \text{kg/m}^3$. Use Fig. 8.11 for lift and drag:

$$\frac{a\omega}{U_\infty} = \frac{(0.15)[2400(2\pi/60)]}{100} \approx 0.38. \quad \text{Fig. 8.11: Read } C_L \approx 1.8,\ C_D \approx 1.1$$

$$\text{Then}\quad \text{Lift} = 55000\text{ N} = C_L\frac{\rho}{2}U_\infty^2 DL = (1.8)\left(\frac{1.23}{2}\right)(100)^2(0.3)L,$$

$$\text{solve}\quad \mathbf{L \approx 17\ m}\quad \textit{Ans.}$$

$$\text{Drag} = C_D\frac{\rho}{2}U_\infty^2 DL = (1.1)\left(\frac{1.23}{2}\right)(100)^2(0.3)(17) \approx 33600\text{ N}$$

$$\text{Power required} = FU = (33600)(100) \approx \mathbf{3.4\ MW!}\quad \textit{Ans.}$$

The power requirements are ridiculously high. This airplane has way too much drag.

8.58 Plot the streamlines due to a line sink (–*m*) at the origin, plus line sources (+*m*) at (*a*, 0) and (4*a*, 0). *Hint*: A cylinder of radius 2*a* appears.

Solution: The overall stream function is

$$\psi = m\tan^{-1}\left(\frac{y-4a}{x}\right) + m\tan^{-1}\left(\frac{y-a}{x}\right) - m\tan^{-1}(y/x)$$

The cylinder shape, of radius 2*a*, is the streamline $\psi = -\pi/2$. *Ans.*

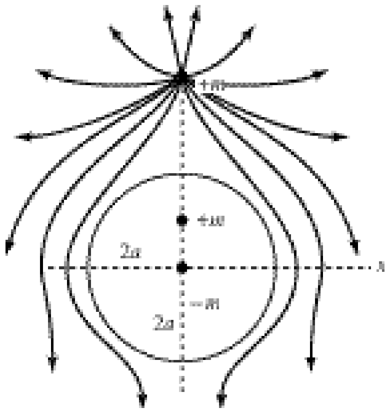


Fig. P8.58

P8.59 The Transition® car-plane in Fig. 7.30 lifts a maximum of 1320 lbf at sea level. Suppose a 1-ft-diameter rotating cylinder, 27.5 ft long, replaces the wing, satisfying Fig. 8.15. (*a*) What rotation rate, in r/min, would lift the car-plane at a take-off speed of 60 mi/h? (*b*) Estimate the drag of the cylinder at this rotation rate. Neglect end effects.

Solution: For sea-level air, take $\rho = 0.00238$ slug/ft^3. The radius $a = d/2 = 0.5$ ft. Convert the speed 60 mi/h = 88 ft/s. (*a*) We have enough information to evaluate the lift coefficient of the cylinder, defined by Eq. (8.44):

$$C_L = \frac{2(lift)}{\rho V^2(2ba)} = \frac{2(1320\,lbf)}{(0.00238)(88)^2[2(27.5)(0.5)]} = 5.21$$

From Fig. 8.15, the writer finds this lift coefficient at about $(a\omega)/U_\infty \approx 2.5$. Thus

$$\frac{a\omega}{U_\infty} = 2.5 = \frac{(0.5\,ft)\omega}{(88\;ft/s)}\;,\quad \text{solve}\quad \omega = 440\frac{rad}{s}\times\frac{60}{2\pi} \approx \mathbf{4200}\,\frac{r}{min} \quad Ans.(a)$$

(*b*) Go back to Fig. 8.15 at the same $(a\omega)/U_\infty \approx 2.5$, using the *right-side* scale for drag, and read $C_D \approx 0.5$. Then the drag is

$$Drag = C_D\frac{\rho}{2}V^2(2ba) = (0.5)(\frac{0.00238}{2})(88)^2[2(27.5)(0.5)] \approx \mathbf{127}\;lbf \quad Ans.(b)$$

This cylinder-plane might fly! Its drag is relatively small. The actual car-plane in Fig. 7.30 has a propeller that develops a thrust on the order of 250 lbf. However, the Reynolds number in Fig. 8.15 is only 3800, while this car-plane cylinder has $Re_D \approx$ 550,000. The drag will vary.

8.60 One of the corner-flow patterns of Fig. 8.18 is given by the cartesian stream function $\psi = A(3yx^2 - y^3)$. Which one? Can this correspondence be proven from Eq. (8.49)?

Solution: This ψ is Fig. 8.18a, **flow in a 60° corner**. [Its velocity potential was given earlier Eq. (8.49) of the text.] The trigonometric form (Eq. 8.49 for n = **3**) is

$$\psi = Ar^3\sin(3\theta),\quad \text{but}\quad \sin(3\theta) \equiv 3\sin\theta\cos^2\theta - \sin^3\theta.$$

Introducing $y = r\sin\theta$ and $x = r\cos\theta$, we obtain $\psi = \mathbf{A(3yx^2 - y^3)}$ *Ans.*

8.61 Plot the streamlines of Eq. (8.53) in the upper right quadrant for $n = 4$. How does the velocity increase with x outward along the x axis from the origin? For what corner angle and value of n would this increase be linear in x? For what corner angle and n would the increase be as x^5?

Solution: For $n = 4$ pute

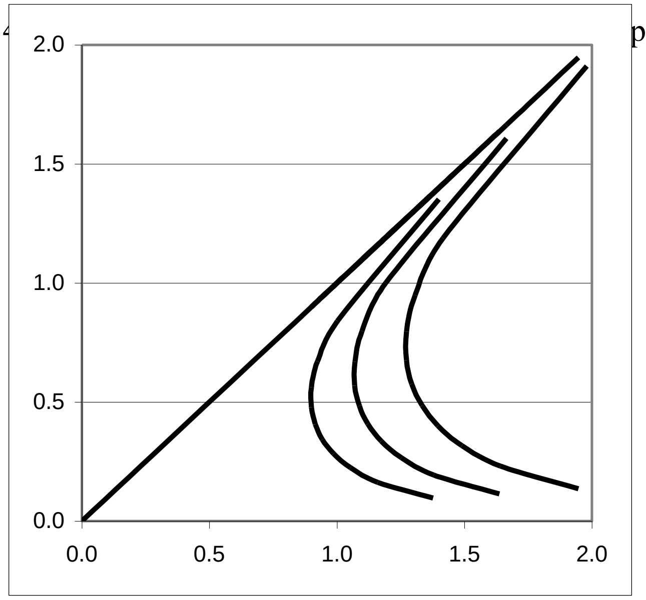


Fig. P8.61

$$n = 4: \quad \psi = Ar^4 \sin(4\theta), \quad v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 4Ar^3 \cos(4\theta)$$

$$\text{Along the } x\text{-axis,} \quad \theta = 0,\ r = x,\ v_r = u = (\text{const})\boldsymbol{x^3} \quad \textit{Ans. (a)}$$

In general, for any n, the flow along the x-axis is $u = (\text{const})x^{n-1}$. Thus u is linear in x for $n = \mathbf{2}$ (a 90° corner). *Ans*. (b). And $u = Cx^5$ if $n = \mathbf{6}$ (a 30° corner). *Ans*. (c)

8.62 Combine stagnation flow, Fig. 8.14b, with a source at the origin:

$$f(z) = Az^2 + m\ln(z)$$

Plot the streamlines for $m = AL^2$, where L is a length scale. Interpret.

Solution: **The imaginary part of this complex potential is the stream function:**

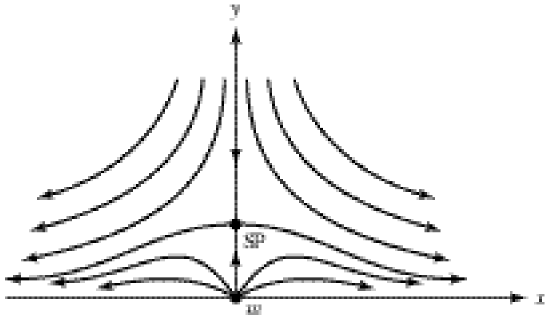


Fig. P8.62

$$\psi = 2Axy + m\tan^{-1}\left(\frac{y}{x}\right), \quad \text{with } m = AL^2$$

The streamlines are shown on the figure above. The source pushes the oncoming stagnation flow away from the vicinity of the origin. There is a stagnation point above the source, at (x, y) = (**0, L/√2**). Thus we have "stagnation flow near a bump." *Ans.*

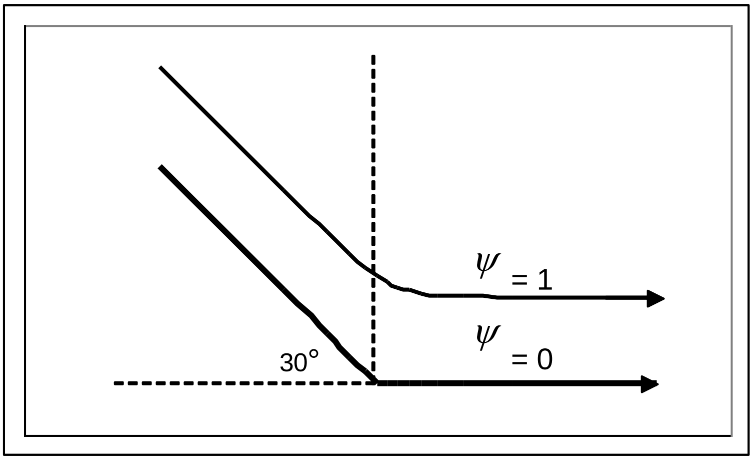
8.63 The superposition in Prob. 8.62 above leads to stagnation flow near a curved *bump*, in contrast to the flat wall of Fig. 8.15*b*. Determine the maximum height *H* of the bump as a function of the constants *A* and *m*.

Solution: The bump crest is a stagnation point:

$$v_{\text{bump crest}} = -2AH + \frac{m}{H} = 0 \quad \text{whence } \boldsymbol{H_{\text{bump}} = \sqrt{\frac{m}{2A}}} \quad \textit{Ans.}$$

P8.64 Consider the polar-coordinate stream function $\psi = B\,r^{1.2}\sin(1.2\,\theta)$, with B equal, for convenience, to 1.0 $\text{ft}^{0.8}/\text{s}$. (*a*) Plot the streamline $\psi = 0$ in the upper half plane. (*b*) Plot the streamline $\psi = 1.0$ and interpret the flow pattern. (*c*) Find the locus of points above $\psi = 0$ for which the resultant velocity = 1.2 ft/s.

Solution: (*a*) We find that $\psi = 0$ along the two radial lines $\theta = 0°$ and $\theta = 150°$. (*b*) We can plot the line $\psi = 1.0 = (1.0)\,r^{1.2}\sin(1.2\,\theta)$ by, for example, setting θ equal to a range of values and solving for r, whence $x = r\cos\theta$ and $y = r\sin\theta$. The plot is as follows:



We interpret this pattern as potential flow around a 150° corner. *Ans.*(*a, b*)

(*c*) Evaluate the velocity components and find the resultant:

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 1.2\,B r^{1.2}\cos(1.2\theta) \;\; ; \;\; v_\theta = -\frac{\partial \psi}{\partial r} = -1.2\,B r^{1.2}\sin(1.2\theta)$$

$$V = \sqrt{v_r^2 + v_\theta^2} = 1.2\,B r^{0.2} = 1.2(1.0\frac{ft^{0.8}}{s})r^{0.2} = 1.2\frac{ft}{s} \text{ if } \mathbf{r = 1.0\ ft} \quad \textit{Ans.}(c)$$

The velocity equals 1.2 ft/s **on a circle of unit radius about the corner** (the origin).

8.65 Potential flow past a wedge of half-angle θ leads to an important application of laminar-boundary-layer theory called the *Falkner-Skan flows* [Ref. 15 of Chap. 8, pp. 242–247]. Let x denote distance along the wedge wall, as in Fig. P8.65, and let $\theta = 10°$. Use Eq. (8.49) to find the variation of surface velocity $U(x)$ along the wall. Is the pressure gradient adverse or favorable?

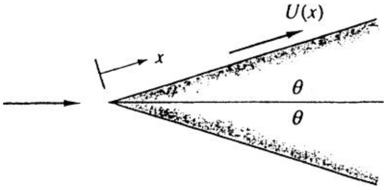


Fig. P8.65

Solution: As discussed above, all wedge flows are "corner flows" and have a velocity along the wall of the form $u = (\text{const})x^{n-1}$, where n = π/(turning angle). In this case, the turning angle is $\beta = (\pi - \theta)$, where $\theta = 10° = \pi/18$. Hence the proper value of n here is:

$$n = \frac{\pi}{\beta} = \frac{\pi}{\pi - \pi/18} = \frac{18}{17}, \quad \text{hence } U = Cx^{n-1} \mathbf{= Cx^{1/17}} \;\; \textbf{(favorable gradient)} \quad \textit{Ans.}$$

8.66 The inviscid velocity along the wedge in Prob. 8.65 has the form $U(x) = Cx^m$, where $m = n - 1$ and n is the exponent in Eq. (8.49). Show that, for any C and n, computation of the laminar boundary-layer by Thwaites' method, Eqs. (7.53) and (7.54), leads to a unique value of the Thwaites parameter λ. Thus wedge flows are called *similar* [Ref. 15 of Chap. 8, p. 244].

Solution: The momentum thickness is computed by Eq. (7.54), assuming $\theta_0 = 0$:

$$\theta^2 = \frac{0.45\nu}{U^6}\int_0^x U^5\,dx = \frac{0.45\nu}{C^6x^{6m}}\int_0^x C^5x^{5m}\,dx = \frac{0.45\nu\ x^{1-m}}{C(5m+1)}$$ Then use Eq. (7.53):

$$\lambda = \frac{\theta^2}{\nu}\frac{dU}{dx} = \left(\frac{0.45x^{1-m}}{C(5m+1)}\right)(mCx^{m-1}) = \mathbf{\frac{0.45m}{5m+1}}$$ (independent of C) *Ans.*

8.67 Investigate the complex potential function $f(z) = U_\infty(z + a^2/z)$, where a is a constant, and interpret the flow pattern.

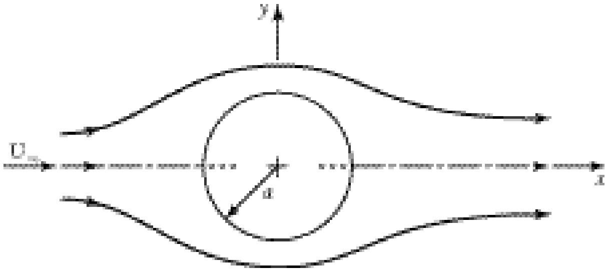


Fig. P8.67

Solution: This represents flow past a **circular cylinder** of radius a, with stream function and velocity potential identical to the expressions in Eqs. (8.31) and (8.32) with $K = 0$. [There is no circulation.]

8.68 Investigate the complex potential function $f(z) = U_\infty z + m\ln[(z + a)/(z - a)]$, where m and a are constants, and interpret the flow pattern.

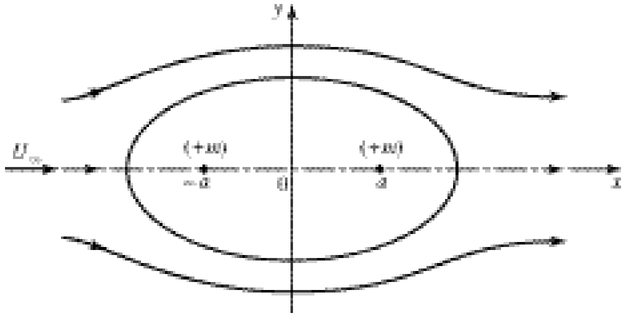
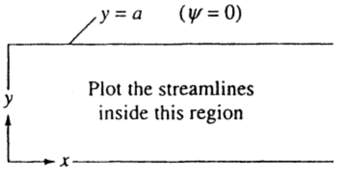


Fig. P8.68

Solution: This represents flow past a **Rankine oval**, with stream function identical to that given by Eq. (8.29).

8.69 Investigate the complex potential function $f(z) = A\cosh(\pi z/a)$, where a is a constant, and plot the streamlines inside the region shown in Fig. P8.69. What hyphenated French word might describe this flow pattern?

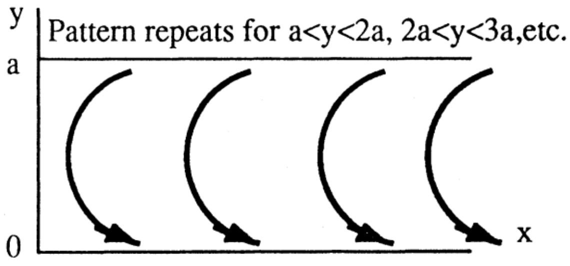
Fig. P8.69



Solution: This potential splits into

$$\psi = A\sinh(\pi x/a)\sin(\pi y/a)$$
$$\phi = A\cosh(\pi x/a)\cos(\pi y/a)$$

and represents flow in a "cul-de-sac" or blind alley.



8.70 Show that the complex potential $f(z) = U_\infty[z + (a/4)\coth(\pi z/a)]$ represents flow past an oval shape placed midway between two parallel walls $y = \pm a/2$. What is a practical application?

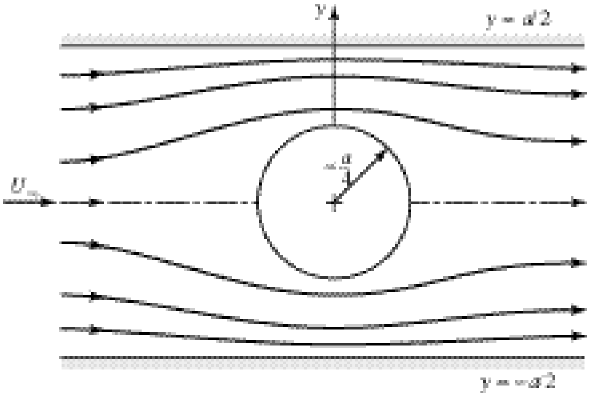


Fig. P8.70

Solution: The stream function of this flow is

$$\psi = U_\infty\left[y - \frac{(a/4)\sin(2\pi y/a)}{\cosh(2\pi x/a) - \cos(2\pi y/a)}\right]$$

The streamlines are shown in the figure. The body shape, trapped between y = ±a/2, is nearly a cylinder, with width a/2 and height 0.51a. A nice application is the estimate of wall “blockage” effects when a body (say, in a wind tunnel) is trapped between walls.

8.71 Figure P8.71 shows the streamlines and potential lines of flow over a thin-plate weir as computed by the complex potential method. Compare qualitatively with Fig. 10.16*a*. State the proper boundary conditions at all boundaries. The velocity potential has equally spaced values. Why do the flow-net “squares” become smaller in the overflow jet?

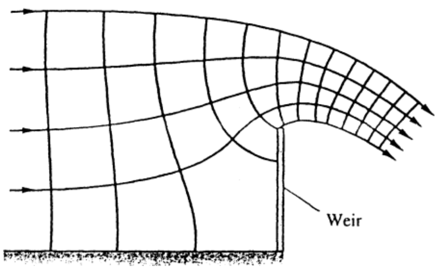
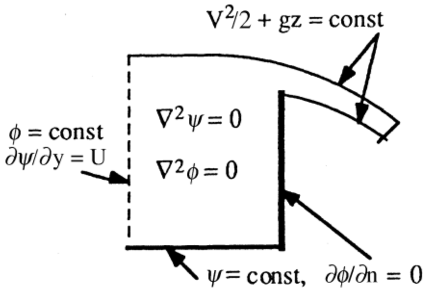


Fig. P8.71

Solution: Solve Laplace's equation for either ψ or ϕ (or both), find the velocities $u = \partial\phi/\partial x$, $v = \partial\phi/\partial y$, force the (constant) pressure to match Bernoulli's equation on the free surfaces (whose shape is *a priori* unknown). The squares become smaller in the overfall jet because the velocity is increasing.


$V^2/2 + gz = \text{const}$
$\phi = \text{const}$
$\partial\psi/\partial y = U$
$\nabla^2\psi = 0$
$\nabla^2\phi = 0$
$\psi = \text{const}$, $\partial\phi/\partial n = 0$

8.72 Use the method of images to construct the flow pattern for a source +*m* near two walls, as in Fig. P8.72.

Sketch the velocity distribution along the lower wall (y = 0). Is there any danger of flow separation along this wall?

Solution: This pattern is the same as that of Prob. 8.28. It is created by placing **four** identical sources at (x, y) = (±a, ±a), as shown. Along the wall ($x \geq 0$, $y = 0$), the velocity first increases from 0 to a maximum at x = a. Then the velocity *decreases* for x > a, which is an *adverse* pressure gradient—**separation may occur**. *Ans.*

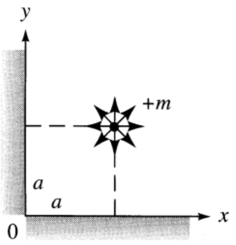
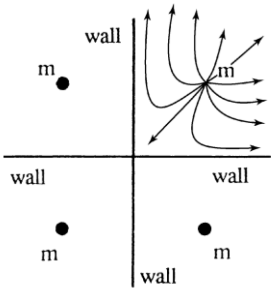


Fig. P8.72



8.73 Set up an image system to compute the flow of a source at *unequal* distances from *two* walls, as shown in Fig. P8.73. Find the point of maximum velocity on the y-axis.

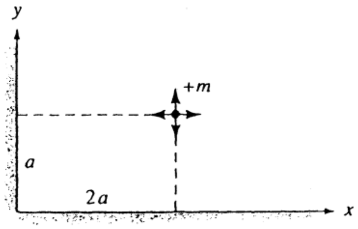
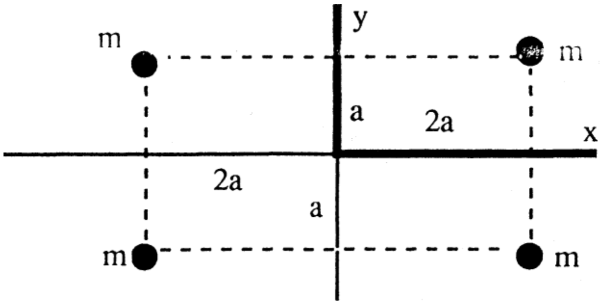


Fig. P8.73

Solution: Similar to Prob. 8.72 on the previous page, we place identical sources (+m) at the symmetric (but non-square) positions (x, y) = (±2a, ±a) as shown below. The induced velocity along the wall (x > 0, y = 0) has the form

$$U = \frac{2m(x+2a)}{(x+2a)^2 + a^2} + \frac{2m(x-2a)}{(x-2a)^2 + a^2}$$



This velocity has a maximum (to the *right*) at **x ≈ 2.93a**, **U ≈ 1.387 m/a**. *Ans.*

8.74 A positive line vortex *K* is trapped in a corner, as in Fig. P8.74. Compute the total induced velocity at point B, (x, y) = (2a, a), and compare with the induced velocity when no walls are present.

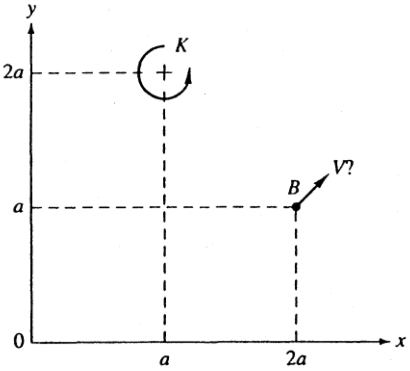
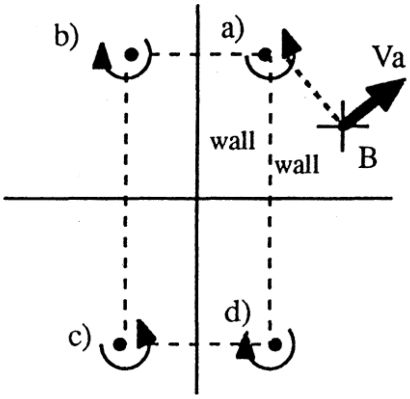


Fig. P8.74

Solution: The two walls are created by placing vortices, as shown at right, at (x, y) = (±a, ±2a). With only one vortex (#a), the induced velocity **V**a would be

$$\mathbf{V}_a = \frac{K}{2a}\mathbf{i} + \frac{K}{2a}\mathbf{j}, \quad \text{or} \quad \frac{K}{a\sqrt{2}} \text{ at } 45° \,□$$

as shown at right. With the walls, however, we have to add this vectorially to the velocities induced by vortices b, c, and d.



$$\text{With walls:} \quad \mathbf{V} = \sum \mathbf{V}_{a,b,c,d} = \frac{K}{a}\left(\frac{1}{2} - \frac{1}{10} - \frac{1}{6} + \frac{3}{10}\right)\mathbf{i} + \frac{K}{a}\left(\frac{1}{2} - \frac{3}{10} + \frac{1}{6} - \frac{1}{10}\right)\mathbf{j},$$

$$\text{or:} \quad \mathbf{V}_B = 0.533\frac{K}{a}\mathbf{i} + 0.267\frac{K}{a}\mathbf{j} = \frac{8K}{15a}\mathbf{i} + \frac{4K}{15a}\mathbf{j} \quad \textit{Ans.}$$

The presence of the walls thus causes a significant change in the magnitude and direction of the induced velocity at point B.

8.75 Using the four-source image pattern needed to construct the flow near a corner shown in Fig. P8.72, find the value of the source strength *m* which will induce a wall velocity of 4.0 m/s at the point (*x*, *y*) = (*a*, 0) just below the source shown, if *a* = 50 cm.

Solution: The flow pattern is formed by four equal sources *m* in the 4 quadrants, as in the figure at right. The sources above and below the point A(*a*, 0) cancel each other at A, so the velocity at A is caused only by the two left sources. The velocity at A is the sum of the two horizontal components from these 2 sources:

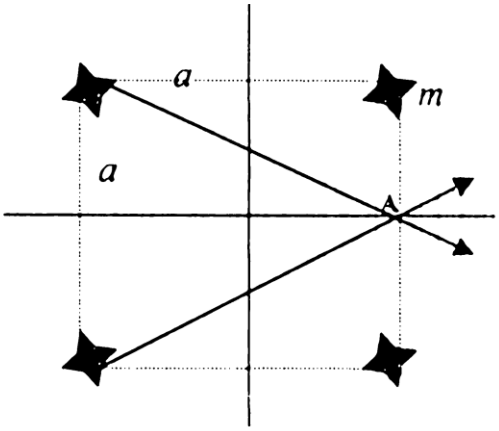


Fig. P8.75

$$V_A = 2\frac{m}{\sqrt{a^2+(2a)^2}}\frac{2a}{\sqrt{a^2+(2a)^2}} = \frac{4ma}{5a^2} = \frac{4m}{5(0.5m)} = 4\ \frac{\text{m}}{\text{s}} \quad \text{if } \boldsymbol{m = 2.5\ \frac{m^2}{s}} \qquad \textit{Ans.}$$

8.76 Use the method of images to approximate the flow past a cylinder at distance 4*a* from the wall, as in Fig. P8.76. To illustrate the effect of the wall, compute the velocities at points A, B, C, and D, comparing with a cylinder flow in an infinite expanse of fluid (without walls).

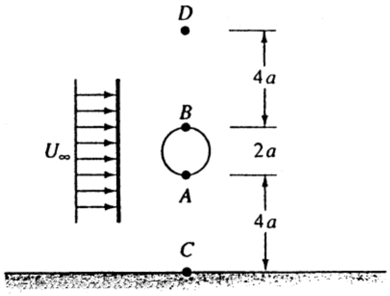


Fig. P8.76

Solution: Let doublet #1 be above the wall, as shown, and let image doublet #2 be below the wall, at (x, y) = (0, −5*a*). Then, at any point on the y-axis, the total velocity is

$$V_{x=0} = -v_\theta\big|_{90^\circ} = U_\infty[1+(a/r_1)^2+(a/r_2)^2]$$

Since the images are 10*a* apart, the cylinders are only slightly out-of-round and the velocities at A, B, C, D may be tabulated as follows:

Point:	**A**	**B**	**C**	**D**
$r1$:	a	a	$5a$	$5a$
$r2$:	$9a$	$11a$	$5a$	$15a$
V walls:	$2.012U_\infty$	$2.008U_\infty$	$1.080U_\infty$	$1.044U_\infty$
V no walls:	$2.0U_\infty$	$2.0U_\infty$	$1.04U_\infty$	$1.04U_\infty$

The presence of the walls causes only a slight change in the velocity pattern.

8.77 Discuss how the flow pattern of Prob. 8.58 might be interpreted to be an *image*-system construction for circular walls. Why are there two images instead of one?

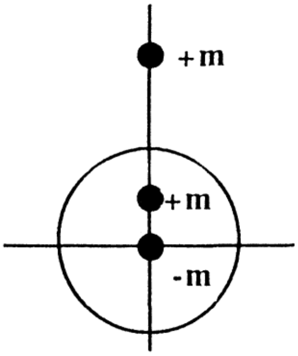


Fig. P8.77

Solution: The missing "image sink" in this problem is at y = +∞ so is not shown. If the source is placed at y = *a* and the image source at y = *b*, the radius of the cylinder will be $R=\sqrt{(\mathbf{ab})}$. For further details about this type of imaging, see Chap. 8, Ref. 3, p. 230.

8.78 Indicate the system of images needed to construct the flow of a uniform stream past a Rankine half-body centered between parallel walls, as in Fig. P8.78. For the particular dimensions shown, estimate the position ℓ of the nose of the resulting half-body.

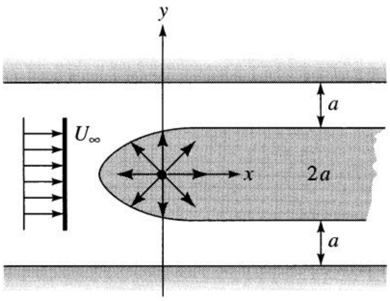
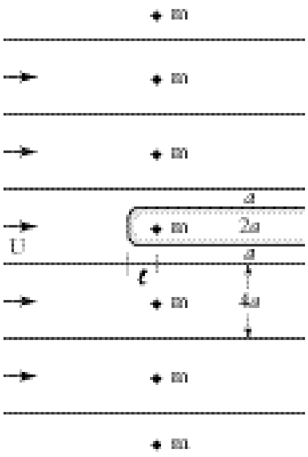


Fig. P8.78

Solution: A body between *two* walls is created by an infinite array of sources, as shown at right. The source strength *m* fits the half-body size "2a":

$$Q = U(2a) = 2\pi m, \quad \text{or:} \quad \mathbf{m = Ua/\pi}$$



The distance ℓ to the nose denotes the stagnation point, where

$$U = \frac{m}{L}\left[1 + \frac{2}{1+(4a/L)^2} + \frac{2}{1+(8a/L)^2} + \cdots\right]$$

where m = Ua/π, as shown. We solve this series summation for $\boldsymbol{\ell \approx 0.325a}$. *Ans.*

8.79 Indicate the system of images needed to simulate the flow of a line source placed unsymmetrically between two parallel walls, as in Fig. P8.79. Compute the velocity on the lower wall at x = +*a*. How many images are needed to establish this velocity to within ±1%?

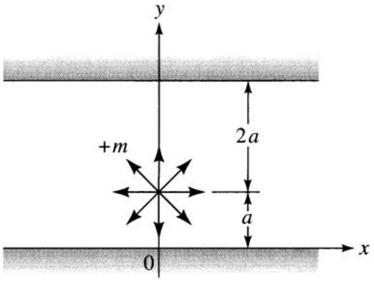
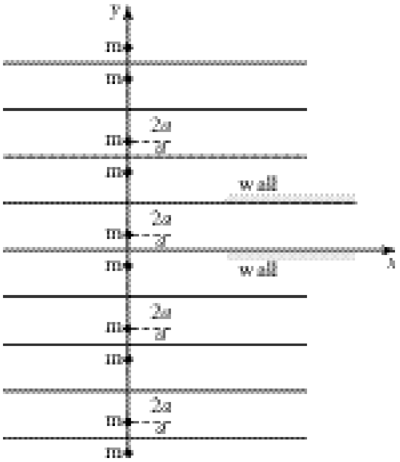


Fig. P8.79

Solution: To form a wall at y = 0 and also at y = 3a, with the source located at (0, a), one needs an infinite number of **pairs** of sources, as shown. The velocity at (a, 0) is an



infinite sum:

$$u(a,\,0)=\frac{2m}{a}\left[\frac{1}{1^2+1}+\frac{1}{5^2+1}+\frac{1}{7^2+1}+\frac{1}{11^2+1}+\frac{1}{13^2+1}+\cdots\right]$$

Accuracy within 1% is reached after 18 terms: **u(a, 0) ≈ 1.18 m/a**. *Ans.*

8.80 The beautiful expression for lift of a two-dimensional airfoil, Eq. (8.69), arose from applying the *Joukowski transformation*, $\zeta = z + a^2/z$, where $z = x + iy$ and $\zeta = \eta + i\beta$. The constant *a* is a length scale. The theory transforms a certain circle in the *z* plane into an airfoil in the ζ plane. Taking *a* = 1 unit for convenience, show that (a) a circle with center at the origin and radius >1 will become an *ellipse* in the ζ plane, and (b) a circle with center at $x = -\varepsilon << -1, y = 0,$ and radius $(1+\varepsilon)$ will become an *airfoil* shape in the ζ plane. *Hint*: Excel is excellent for solving this problem.

Solution: Introduce z = x + iy into the transformation and find real and imaginary parts:

$$\zeta = (x+iy)+\frac{1}{x+iy}\left(\frac{x-iy}{x-iy}\right)=x\left(1+\frac{1}{x^2+y^2}\right)+iy\left(1-\frac{1}{x^2+y^2}\right)=\eta+i\beta$$

Thus η and β are simple functions of *x* and *y*, as shown. Thus, if the circle in the z plane has radius C > 1, the coordinates in the ζ plane will be

$$\eta = x\left(1+\frac{1}{C^2}\right)\qquad \beta = y\left(1-\frac{1}{C^2}\right)$$

The circle in the z plane will transform into an *ellipse* in the ζ plane of major axis $(1 + 1/C^2)$ and minor axis $(1 - 1/C^2)$. This is shown on the next page for C = 1.1. If the circle center is at $(-\varepsilon, 0)$ and radius $C = 1+\varepsilon$, an *airfoil* will form, because a sharp (trailing) edge will form on the right and a fat (elliptical) leading edge will form on the left. This is also shown below for $\varepsilon = 0.1$.

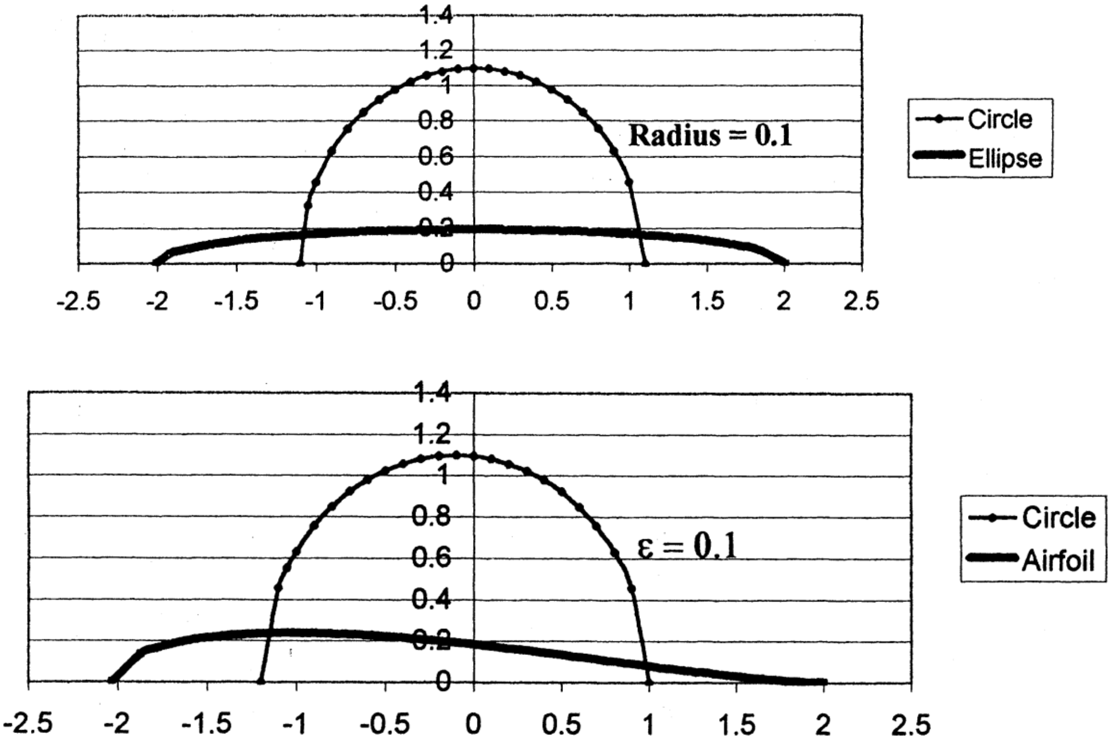


Fig. P8.80

***8.81** Given an airplane of weight W, wing area A, aspect ratio AR, and flying at an altitude where the density is ρ. Assume all drag and lift is due to the wing, which has an infinite-span drag coefficient $C_{D\infty}$. Further assume sufficient thrust to balance whatever drag is calculated. (*a*) Find an algebraic expression for the *best cruise velocity* V_b, which occurs when the ratio of drag to speed is a minimum. (*b*) Apply your formula to the data in Prob. P7.119, for which a laborious graphing procedure gave an answer $V_b \approx 180$ m/s.

Solution: The drag force, for a finite aspect ratio, is given by

$$D = (C_{D\infty} + \frac{C_L^2}{\pi\ \mathrm{AR}})\frac{\rho}{2}V^2 A \quad \text{and} \quad C_L = \frac{2W}{\rho V^2 A}$$

Combine these: $$\frac{D}{V} = \frac{\rho}{2}A(C_{D\infty}V + \frac{\mathrm{K}}{V^3}) \quad \text{where } \mathrm{K} = \frac{4W^2}{\rho^2 A^2 \pi\ \mathrm{AR}}$$

(*a*) Differentiate this to find the minimum of (*D*/*V*), or best cruise velocity:

$$\frac{d(D/V)}{dV}=0 \text{ when } V=V_b=\left(\frac{3\mathrm{K}}{C_{D\infty}}\right)^{1/4}=\left(\frac{12W^2}{\rho^2A^2C_{D\infty}\pi\mathrm{AR}}\right)^{1/4} \qquad Ans.(a)$$

(*b*) In Prob. P7.119, the data were m = 45000 kg, A = 160 m^2, AR = 7, $C_{D\infty}$ = 0.020, and the density (at standard 9000-m altitude) was 0.4661 kg/m^3. Calculate W = 441,450 N and apply your new formula:

$$V_b=\left(\frac{12\ \ W^2}{\rho^2A^2C_{D\infty}\pi\mathrm{AR}}\right)^{1/4}=\left[\frac{12(441450N)^2}{(0.4661kg/m^3)^2(160m^2)^2(0.02)\pi(7)}\right]^{1/4}=176\frac{m}{s} \quad Ans.(b)$$

8.82 The ultralight plane *Gossamer Condor* in 1977 was the first to complete the Kremer Prize figure-eight course solely under human power. Its wingspan was 29 m, with Cav = 2.3 m and a total mass of 95 kg. Its drag coefficient was approximately 0.05. The pilot was able to deliver 1/4 horsepower to propel the plane. Assuming two-dimensional flow at sea level, estimate (a) the cruise speed attained, (b) the lift coefficient; and (c) the horsepower required to achieve a speed of 15 knots.

Solution: For sea-level air, take ρ = 1.225 kg/m^3. With CD known, we may compute V:

$$Power=F_{drag}V=\frac{1}{4}\mathrm{hp}(745.7)=186\ \mathrm{W}=\left[C_D\frac{\rho}{2}V^2bC\right]V$$

$$=\ 0.05\left(\frac{1.225}{2}\right)V^2(29)(2.3)V=186;$$

$$\text{Solve}\quad V^3=91.3 \quad or \quad \mathbf{V=4.5\ \frac{m}{s}} \quad Ans.\text{ (a)}$$

Then, with V known, we may compute the lift coefficient from the known weight:

$$C_L=\frac{weight}{(\rho/2)V^2bC}=\frac{95(9.81)\ \mathrm{N}}{(1.225/2)(4.5)^2(29)(2.3)}=\mathbf{1.13} \quad Ans.\text{ (b)}$$

Finally, compute the power if V = 15 knots = 7.72 m/s:

$$P=FV=\left(C_D\frac{\rho}{2}V^2bC\right)V=0.05\left(\frac{1.225}{2}\right)(7.72)^2(29)(2.3)(7.72)$$

$$=940\ \mathrm{W}=\mathbf{1.26\ hp} \quad Ans.\text{ (c)}$$

P8.83 The Boeing 747-8 aircraft has a gross weight of 975,000 lbf, wing area of 5500 ft^2, wingspan of 224.75 ft, and C_{Do} = 0.022. When cruising at 35,000 ft standard altitude, the four engines each provide 13,500 lbf of thrust. Assuming that all lift and drag are due to the wing, estimate the cruise velocity, in mi/h.

Solution: From Table A.6, at 35,000 ft = 10,668 m, ρ = 0.3793 kg/m^3 = 0.000736 slug/ft^3. Calculate the wing aspect ratio: AR = b^2/A_p = $(224.75)^2/(5500)$ = 9.18. We know the aircraft weight (= lift) and the available thrust (= drag). Thrust is 4(13,500) = 54,000 lbf. Therefore

$$C_L = \frac{2(weight)}{\rho V^2 A_p} = \frac{2(975,000)}{(0.000736)V^2(5500)} = \frac{481,700}{V^2}$$

$$C_D = C_{Do} + \frac{C_L^2}{\pi AR} = 0.022 + \frac{(481700/V^2)^2}{\pi(9.18)} = \frac{2(drag)}{\rho V^2 A_p} = \frac{2(48,000)}{(0.000736)V^2(5500)}$$

The unknown is velocity V, awkwardly in both equations. We can either iterate or use EES. Actually, the equations are *extremely sensitive* – a large change in V, say, from 700 to 900 ft/s, only causes a slight change in aircraft drag. So guess V around 550 mi/h, which is what we know about the average speed of the Boeing 747.

The final result is V = 806 ft/s = **550 mi/h**. *Ans.*

8.84 Reference 12 contains inviscid theory calculations for the surface velocity distributions V(*x*) over an airfoil, where *x* is the chordwise coordinate. A typical result for small angle of attack is shown below. Use these data, plus Bernoulli's equation, to estimate (a) the lift coefficient; and (b) the angle of attack if the airfoil is symmetric.

x/c	V/U_∞ (upper)	V/U_∞ (lower)
0.0	0.0	0.0
0.025	0.97	0.82
0.05	1.23	0.98
0.1	1.28	1.05
0.2	1.29	1.13
0.3	1.29	1.16
0.4	1.24	1.16
0.6	1.14	1.08
0.8	0.99	0.95
1.0	0.82	0.82

Solution: From Bernoulli's equation, the surface pressures may be computed, whence the lift coefficient then follows from an integral of the pressure difference:

Surface: $$p = p_\infty + \frac{\rho}{2}\left(U_\infty^2 - V^2\right),\quad L = \int_0^C (p_{\text{lower}} - p_{\text{upper}})b\,dx = \int_0^C \frac{\rho}{2}\left(V^2_{\text{upper}} - V^2_{\text{lower}}\right)b\,dx$$

Non-dimensionalize: $$C_L = \frac{L}{(\rho/2)U_\infty^2 bC} = \int_0^1 \left[(V/U)^2_{\text{upp}} - (V/U)^2_{\text{low}}\right] d\left(\frac{x}{C}\right)$$

Thus the lift coefficient is an integral of the difference in $(V/U)^2$ on the airfoil. Such a plot is shown below. The area between the curves is approximately

$$\int_0^1 \Delta(V/U)^2 \approx C_L \approx \mathbf{0.21} \quad \textit{Ans.}\text{ (a)};\quad \alpha = \sin^{-1}\left(\frac{0.21}{2\pi}\right) \approx \mathbf{1.9^\circ} \quad \textit{Ans.}\text{ (b)}$$

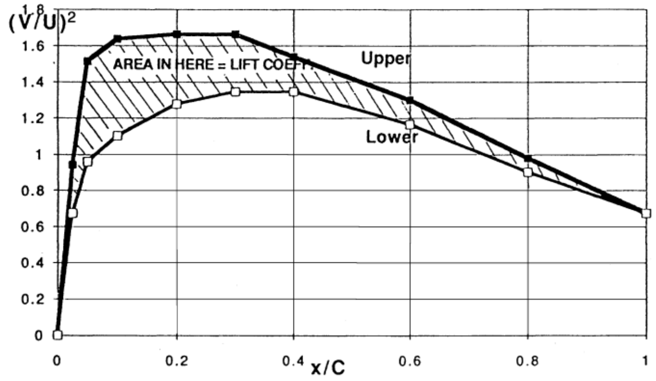


Fig. P8.84

8.85 A wing of 2 percent camber, 5-in chord, and 30-in span is tested at a certain angle of attack in a wind tunnel with sea-level standard air at 200 ft/s and is found to have lift of 30 lbf and drag of 1.5 lbf. Estimate from wing theory (a) the angle of attack, (b) the minimum drag of the wing and the angle of attack at which it occurs, and (c) the maximum lift-to-drag ratio.

Solution: For sea-level air take $\rho = 0.00238$ slug/ft^3. Establish the lift coefficient first:

$$C_L = \frac{2L}{\rho V^2 bC} = \frac{2(30\text{ lbf})}{0.00238(200)^2(30/12)(5/12)} = 0.605 = \frac{2\pi \sin[\alpha + \tan^{-1}(0.04)]}{1 + 2/(6.0)}$$

$$\text{Solve for} \quad \alpha \approx \mathbf{5.1^\circ} \quad \textit{Ans.}\ \text{(a)}$$

Find the induced drag and thence the minimum drag (when lift = zero):

$$C_D = \frac{C_L}{20} = 0.0303 = C_{D\infty} + \frac{C_L^2}{\pi AR} = C_{D\infty} + \frac{(0.605)^2}{\pi(6.0)}, \quad \text{solve for } C_{D\infty} = 0.0108$$

Then $D_{min} = C_{D\infty}\frac{\rho}{2}V^2 bC = 0.0108\left(\frac{0.00238}{2}\right)(200)^2\left(\frac{30}{12}\right)\left(\frac{5}{12}\right) \approx \mathbf{0.54\ lbf}$ *Ans.* (b)

Finally, the maximum L/D ratio occurs when $C_D = 2C_{D\infty}$, or:

$$C_L\left(\text{at max } \frac{L}{D}\right) = \sqrt{\pi AR C_{D\infty}} = \sqrt{\pi(6)(0.0108)} = 0.451,$$

$$\text{whence} \quad (L/D)_{max} = \frac{0.451}{2(0.0108)} \approx \mathbf{21} \quad \textit{Ans.}\ \text{(c)}$$

8.86 An airplane has a mass of 20,000 kg and flies at 175 m/s at 5000-m standard altitude. Its rectangular wing has a 3-m chord and a symmetric airfoil at 2.5° angle of attack. Estimate (a) the wing span; (b) the aspect ratio; and (c) the induced drag.

Solution: For air at 5000-m altitude, take $\rho = 0.736$ kg/m^3. We know W, find b:

$$L = W = 20000(9.81) = C_L\frac{\rho}{2}V^2 bC = \frac{2\pi \sin 2.5^\circ}{1 + 2(3/b)}\left(\frac{0.736}{2}\right)(175)^2 b(3.0)$$

$$\text{Rearrange to} \quad b^2 - 21.2b - 127 = 0, \quad \text{or} \quad b \approx \mathbf{26.1\ m} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Aspect ratio} \quad AR = b/C = 26.1/3.0 \approx \mathbf{8.7} \quad \textit{Ans.}\ \text{(b)}$$

With aspect ratio known, we can solve for lift and induced-drag coefficients:

$$\mathrm{C_L} = \frac{2\pi\sin 2.5^\circ}{1+2/8.7} = 0.223, \quad \mathrm{C_{Di}} = \frac{\mathrm{C_L^2}}{\pi\mathrm{AR}} = \frac{(0.223)^2}{\pi(8.7)} = 0.00182$$

Induced drag = (0.00182)(0.736/2)(175)^2(26.1)(3) ≈ **1600 N** *Ans*. (c)

8.87 A freshwater boat of mass 400 kg is supported by a rectangular hydrofoil of aspect ratio 8, 2% camber, and 12% thickness. If the boat travels at 7 m/s and α = 2.5°, estimate (a) the chord length; (b) the power required if $C_{D\infty}$ = 0.01, and (c) the top speed if the boat is refitted with an engine which delivers 20 hp to the water.

Solution: For fresh water take ρ = 998 kg/m^3. (a) Use Eq. (8.82) to estimate the chord length:

$$Lift = C_L\frac{\rho}{2}V^2bC = \frac{2\pi\sin\left[\dfrac{2.5}{57.3}+2(0.02)\right]}{1+2/8}\left(\frac{998\ \text{kg/m}^3}{2}\right)(7\ \text{m/s})^2(8C)C,$$

$$= (400\ \text{kg})(9.81\ \text{m/s}^2)$$

$$\text{Solve for}\quad C^2 = 0.0478\ \text{m}^2 \quad or \quad \mathbf{C \approx 0.219\ m} \quad Ans.\ \text{(a)}$$

(b) At 7 m/s, the power required depends upon the drag, with C_L = 0.420 from part (a):

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi(AR)} = 0.01 + \frac{(0.420)^2}{\pi(8)} = 0.0170$$

$$F = C_D\frac{\rho}{2}V^2bC = (0.0170)\left(\frac{998\ \text{kg/m}^3}{2}\right)(7\ \text{m/s})^2(8\times 0.219\ \text{m})(0.219\ \text{m}) = 159\ \text{N}$$

$$Power = FV = (159\ \text{N})(7\ \text{m/s}) = 1120\ \text{W} = \mathbf{1.5\ hp} \quad Ans.\ \text{(b)}$$

(c) Set up the power equation again with velocity unknown:

$$Power = FV = \left[\left(C_{D\infty} + \frac{C_L^2}{\pi(AR)}\right)\left(\frac{\rho}{2}\right)V^2bC\right]V, \quad C_L = \frac{2(Weight)}{\rho V^2bC} = \frac{2\pi\sin(\alpha-\alpha_{ZL})}{1+2/(AR)}$$

Enter the data: $C_{D\infty}$ = 0.01, C = 0.219 m, b = 1.75 m, AR = 8, Weight = 3924 N, Power = 20 hp = 14914 W, α_{ZL} = −2.29°, ρ = 998 kg/m^3. Iterate (or use EES) to find the results:

$$\mathrm{C_L} = 0.0526,\ \mathrm{C_D} = 0.0101,\ \alpha = -1.69^\circ,\ \mathbf{V_{max} = 19.8\ m/s \approx 44\ mi/h} \quad Ans.\ \text{(c)}$$

P8.88 The Transition® car-plane in Fig. 7.30 has a gross weight of 1320 lbf, a wingspan of 27.5 ft, and a wing area of 150 ft^2. Assume that the wing uses an NACA 2412 airfoil. Estimate the angle of attack of this car-plane when cruising at 115 mi/h at a standard altitude of 8200 ft.

Solution: At 8200 ft = 2500 m, take $\rho_{air} = 0.957\ kg/m^3 = 0.00186\ slug/ft^3$. The aspect ratio is AR = $b^2/A_p = (27.5)^2/150 = 5.04$. Convert V = 115 mi/h = 169 ft/s. The lift coefficient is

$$C_L = \frac{2W}{\rho V^2 A_p} = \frac{2(1320)}{(0.00186)(169)^2(150)} = 0.331$$

From Table 8.3 for an NACA 2412 airfoil, α_{ZL} = - 2.1°. Use Eq. (8.82) to find α:

$$C_L = 0.331 = 2\pi\sin(\alpha - \alpha_{ZL})/[1+2/AR] = 2\pi\sin(\alpha + 2.1^o)/[1+2/5.04]\ ;\ \text{Solve}\ \alpha \approx \mathbf{2.1^o}\quad \text{Ans.}$$

8.89 The Beechcraft T-34C airplane has a gross weight of 5500 lbf, a wing area of 60 ft^2, and cruises at 322 mi/h at 10000 feet standard altitude. It is driven by a propeller which delivers 300 hp to the air. Assume that the airfoil is the NACA 2412 section from Figs. 8.24 and 8.25 and neglect all drag except the wing. What is the appropriate aspect ratio for this wing?

Solution: At 10000 ft = 3048 m, $\rho \approx 0.00176\ slug/ft^3$. Convert 322 mi/h = 472 ft/s. From the weight and power we can compute the lift and drag coefficients:

$$C_L = \frac{2W}{\rho V^2 A} = \frac{2(5500)}{0.00176(472)^2(60)} = 0.468 = \frac{C_{L\infty}}{1+2/AR}$$

$$\text{Power} = 300\ \text{hp} \times 550 = 165000\ \frac{\text{ft·lbf}}{\text{s}}$$

$$= C_D\left(\frac{0.00176}{2}\right)(472)^2(60)(472), \quad \text{or} \quad C_D \approx 0.0297$$

With C_L & C_D known at AR = ∞ from Figs. 8.24 and 8.25, we can compute AR. From Table 8.3, α_{ZL} for the 2412 airfoil is – 2.1°. The unknowns are α and AR:

$$C_{L\infty} \approx 6.1\sin(\alpha + 2.1^{o}) \; ; \quad C_{D\infty} \approx 0.007$$

$$0.468 = \frac{C_{L\infty}}{1+2/AR}; \quad C_D = 0.0297 = C_{D\infty} + \frac{(0.468)^2}{\pi AR}$$

By iteration, the solution converges to $\alpha \approx$ **5.2°** and **AR ≈ 3.1**. *Ans.*

8.90 NASA is developing a swing-wing airplane called the Bird of Prey [37]. As shown in Fig. P8.90, the wings pivot like a pocketknife blade: forward (a), straight (b), or backward (c). Discuss a possible advantage for each of these wing positions. If you can't think of any, read the article [37] and report to the class.

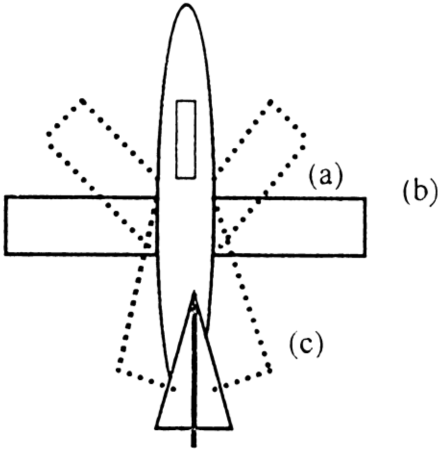


Fig. P8.90

Solution: Each configuration has a different advantage: (a) highly maneuverable but unstable, needs computer control; (b) maximum lift at low speeds, best for landing and take-off; (c) maximum speed possible with wings swept back.

8.91 If $\phi(r, \theta)$ in axisymmetric flow is defined by Eq. (8.85) and the coordinates are given in Fig. 8.29, determine what partial differential equation is satisfied by ϕ.

Solution: The velocities are related to ϕ by Eq. (8.87), and direct substitution gives

$$\frac{\partial}{\partial r}(r^2 v_r \sin\theta) + \frac{\partial}{\partial \theta}(r v_\theta \sin\theta) = 0, \quad \text{with } v_r = \frac{\partial \phi}{\partial r} \quad \text{and} \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

Thus the PDE for ϕ is: $\mathbf{\sin\theta \frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{\partial \phi}{\partial \theta}\sin\theta\right) = 0}$ *Ans.*

This linear but complicated PDE is *not* Laplace's Equation in axisymmetric coordinates.

8.92 A point source with volume flow Q = 30 m^3/s is immersed in a uniform stream of speed 4 m/s. A Rankine half-body of revolution results. Compute (a) the distance from the source to the stagnation point; and (b) the two points (r, θ) on the body surface where the local velocity equals 4.5 m/s. [See Fig. 8.31]

Solution: The properties of the Rankine half-body follow from Eqs. (8.89) and (8.94):

$$m = \frac{Q}{4\pi} = \frac{30}{4\pi} = \mathbf{2.39}\ \frac{m^3}{s}, \quad \text{hence } \mathbf{a} = \sqrt{\frac{m}{U}} = \sqrt{\frac{2.39}{4}} \approx \mathbf{0.77\ m} \quad \textit{Ans.}\text{ (a)}$$

[It's a *big* half-body, to be sure.] Some iterative computation is needed to find the body shape and the velocities along the body surface:

Surface, $\psi = -Ua^2$: $r = a\dfrac{\sqrt{2(1+\cos\theta)}}{\sin\theta} = a\csc\left(\dfrac{\theta}{2}\right)$, with velocity components

$$v_r = U\cos\theta + m/r^2 \quad \text{and} \quad v_\theta = -U\sin\theta$$

A brief tabulation of surface velocities reveals *two* solutions, both for $\theta < 90°$:

θ:	40°	50°	**50.6°**	60°	70°	80°	**88.1°**	90°	100°
V, m/s:	4.37	4.49	**4.50**	4.58	4.62	4.59	**4.50**	4.47	4.27

As in Fig. 8.26, V rises to a peak of 4.62 m/s (1.155U) at 70.5°, passing through 4.5 m/s at (r, θ) ≈ (**1.808 m, 50.6**°) and (**1.111 m, 88.1**°). *Ans.*

8.93 The Rankine body of revolution of Fig. 8.26 could simulate the shape of a pitot-static tube (Fig. 6.30). According to inviscid theory, how far downstream from the nose should the static-pressure holes be placed so that the local surface velocity is within ±0.5% of U? Compare your answer with the recommendation x ≈ 8D in Fig. 6.30.

Solution: We search iteratively along the surface until we find **V = 1.005U**:

$$\text{Along} \quad r/a = \csc(\theta/2), \quad v_r = U\cos\theta + \frac{m}{r^2}, \quad v_\theta = -U\sin\theta, \quad V = \sqrt{v_r^2 + v_\theta^2}, \quad m = Ua^2$$

The solution is found at $\theta \approx \mathbf{8.15°}$, $x \approx 13.93a$, $\mathbf{\Delta x/D} = 14.93a/4a \approx \mathbf{3.73}$. *Ans.*
[Further downstream, at $\Delta x = 8D$, we find that $V \approx 1.001U$, or within 0.1%.]

8.94 Determine whether the Stokes streamlines from Eq. (8.86) are everywhere orthogonal to the Stokes potential lines from Eq. (8.87), as is the case for cartesian and plan polar coordinates.

Solution: Compare the ratio of velocity components for lines of constant ψ:

$$\left.\frac{dr}{r\,d\theta}\right|_{\text{streamline}} = \frac{v_r}{v_\theta} = \frac{-(1/r^2\sin\theta)(\partial\psi/\partial\theta)}{(1/r\sin\theta)(\partial\psi/\partial r)} = \frac{\partial\phi/\partial r}{(1/r)(\partial\phi/\partial\theta)},$$

$$\text{or:}\quad \frac{-(1/r)(\partial\psi/\partial\theta)}{(\partial\psi/\partial r)} = -1\Big/\left[\frac{-(1/r)(\partial\phi/\partial\theta)}{\partial\phi/\partial r}\right],\quad \text{thus } [\text{slope}]_{\psi\text{ line}} = \frac{-1}{[\text{slope}]_{\phi\text{ line}}}$$

They **are** orthogonal. *Ans.*

8.95 Show that the axisymmetric potential flow formed by a point source $+m$ at $(-a, 0)$, a point sink-$(-m)$ at $(+a, 0)$, and a stream U in the x direction becomes a Rankine body of revolution as in Fig. P8.95. Find analytic expressions for the length 2L and diameter 2R of the body.

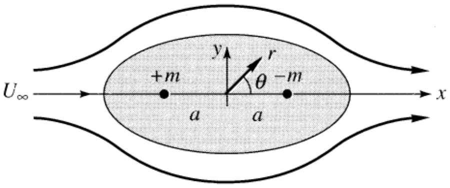


Fig. P8.95

Solution: The stream function for this three-part superposition is given below, and the body shape (the *Rankine ovoid*) is given by $\psi = 0$.

$$\psi = -\frac{U}{2}r^2\sin^2\theta + m(\cos\theta_2 - \cos\theta_1),\quad \text{where “2” and “1” are from the source/sink.}$$

$$\text{Stagnation points at } r = L,\ \theta = 0,\ \pi,\quad \text{or}\quad U + \frac{m}{(L+a)^2} - \frac{m}{(L-a)^2} = 0$$

$$\text{Solve for}\quad \mathbf{[(L/a)^2 - 1]^2 = \frac{4m}{Ua^2}(L/a)}\quad \textit{Ans.}$$

Similarly, the maximum radius R of the ovoid occurs at $\psi = 0$, $\theta = \pm 90°$:

$$0 = -\frac{U}{2}R^2 + 2m\cos\theta_2,\quad \text{where } \cos\theta_2 = \frac{a}{\sqrt{a^2+R^2}},$$

$$\text{or:}\quad \mathbf{(R/a)^2\sqrt{1+(R/a)^2} = \frac{4m}{Ua^2}}\quad \textit{Ans.}$$

Some numerical values of length and diameter are as follows:

m/Ua^2:	0.01	0.1	1.0	10.0	100.0
L/a:	1.100	1.313	1.947	3.607	7.458
R/a:	0.198	0.587	1.492	3.372	7.348
L/R:	5.553	2.236	1.305	1.070	1.015

As m/Ua^2 increases, the ovoid approaches a large spherical shape, L/R ≈ 1.0.

8.96 Consider inviscid flow along the streamline approaching the front stagnation point of a sphere, as in Fig. 8.32. Find (*a*) the maximum fluid deceleration along this streamline, and (*b*) its position.

Solution: Along the stagnation streamline, the flow is purely radial, hence $v_\theta = 0$. Thus

$$V_{toward\ stagnation} = -v_r\,|_{\theta=\pi} = -U_\infty \cos(\pi)(1-\frac{a^3}{r^3}) = U_\infty(1-\frac{a^3}{r^3})$$

$$\frac{dV}{dr} = -3U_\infty\frac{a^3}{r^4};\quad \text{Thus deceleration} = V\frac{dV}{dr} = 3U_\infty^2\frac{a^3}{r^4}(1-\frac{a^3}{r^3}) = \text{maximum when}$$

$$\frac{r}{a} = (7/4)^{1/3} = 1.205 \quad Ans.(b) \text{ and } (V\frac{dV}{dr})_{\max} = 0.6097\frac{U_\infty^2}{a} \quad Ans.(a)$$

8.97 The Rankine body of revolution in Fig. P8.97 is 60 cm long and 30 cm in diameter. When it is immersed in the low-pressure water tunnel as shown, cavitation may appear at point *A*. Compute the stream velocity *U*, neglecting surface wave formation, for which cavitation occurs.

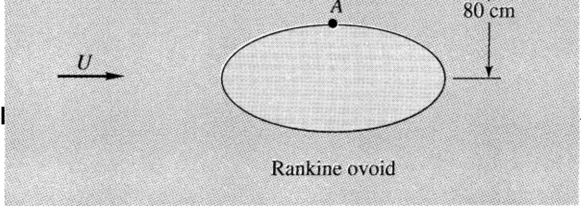


Fig. P8.97

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and pv = 2337 Pa. For an ovoid of ratio L/R = 60/30 = **2.0**, we may interpolate in the Table of Prob. 8.95 to find

$$\frac{m}{Ua^2} = 0.1430, \quad \frac{L}{a} = 1.3735, \quad \text{hence } a = \frac{30}{1.3735} = 21.84 \text{ cm}, \quad R = 0.687a = 15.0 \text{ cm},$$

$$m = 0.143Ua^2 = 0.00682U, \quad U_{max} = U + \frac{2ma}{r_2^3} = U + \frac{2(0.00682U)(0.2184)}{[(.2184)^2 + (.15)^2]^{3/2}} \approx \mathbf{1.16U}$$

Then Bernoulli's equation allows us to compute U when point A reaches vapor pressure:

$$p_\infty + \frac{\rho}{2}U^2 + \rho g z_\infty \approx p_A + \frac{\rho}{2}V_A^2 + \rho g z_A$$

$$\text{where} \quad V_A = 1.16U \quad \text{and} \quad p_\infty = p_{atm} + \rho g(z_{surf} - z_\infty)$$

$$40000 + 9790(0.8) + \frac{998}{2}U^2 + 0 = 2337 + \frac{998}{2}(1.16U)^2 + 9790(0.15)$$

$$\text{Solve for cavitation speed} \quad \mathbf{U \approx 16\ \frac{m}{s}} \quad \textit{Ans.}$$

8.98 We have studied the point source (sink) and the line source (sink) of infinite depth into the paper. Does it make any sense to define a finite-length line sink (source) as in Fig. P8.98? If so, how would you establish the mathematical properties of such a finite line sink? When combined with a uniform stream and a point source of equivalent strength as in Fig. P8.98, should a closed-body shape be formed? Make a guess and sketch some of these possible shapes for various values of the dimensionless parameter $m/(U_\infty L^2)$.

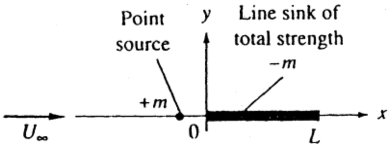
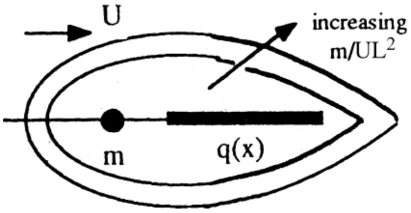


Fig. P8.98

Solution: Yes, the "sheet" sink makes good sense and will create a body with a sharper trailing edge. If q(x) is the local sink strength, then $m = \int q(x)\,dx$, and the body shape is a teardrop which becomes fatter with increasing m/UL^2.



8.99 Consider air flowing past a hemisphere resting on a flat surface, as in Fig. P8.99. If the internal pressure is *pi*, find an expression for the pressure force on the hemisphere. By analogy with Prob. 8.49 at what point *A* on the hemisphere should a hole be cut so that the pressure force will be zero according to inviscid theory?

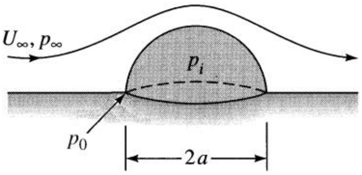


Fig. P8.99

Solution: Recall from Eq. (8.100) that the velocity along the sphere surface is

$$V_s = \frac{3}{2}U_\infty \sin\theta \quad \text{and} \quad F_{up} = \int_0^{\pi/2} (p_i - p_s)\, 2\pi a \sin\theta\, a\, d\theta \cos\theta, \quad \text{where}$$

$$p_s(\text{Bernoulli}) = p_o - \frac{\rho}{2}\left(\frac{3}{2}U_\infty \sin\theta\right)^2, \quad \text{work out } F = \pi a^2(p_i - p_o) + \frac{9\pi}{16}\rho U_\infty^2 a^2$$

This force is zero if we put a hole at point **A** ($\theta = \theta$A) such that

$$p_A = p_o - \frac{9}{16}\rho U_\infty^2 = p_o - \frac{\rho}{2}V_A^2, \quad \text{or} \quad V_A = \sqrt{\frac{9}{8}}U_\infty = 1.061U_\infty \overset{?}{=} \frac{3}{2}U_\infty \sin\theta_A$$

Solve for **θA ≈ 45°** or **135°** *Ans.*

8.100 A 1-m-diameter sphere is being towed at speed *V* in fresh water at 20°C as shown in Fig. P8.100. Assuming inviscid theory with an undistorted free surface, estimate the speed *V* in m/s at which cavitation will first appear on the sphere surface. Where will cavitation appear? For this condition, what will be the pressure at point *A* on the sphere which is 45° up from the direction of travel?

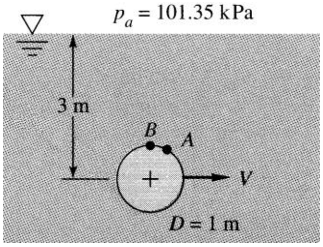


Fig. P8.100

Solution: For water at 20°C, take ρ = 998 kg/m^3 and p_v = 2337 Pa. Cavitation will occur at the lowest-pressure point, which is point **B** on the *top* of the cylinder, at $\theta = 90°$:

$$p_\infty + \frac{\rho}{2}U_\infty^2 + \rho g z_\infty = p_B + \frac{\rho}{2}V_B^2 + \rho g z_B, \quad \text{where } p_\infty = p_{atm} + \rho g(z_{surface} - z_\infty)$$

$$\text{Thus}\quad [101350 + 9790(3)] + \frac{998}{2}U_\infty^2 + 0 = 2337 + \frac{998}{2}(1.5U_\infty)^2 + 9790(0.5)$$

Solve for $\mathbf{U_\infty \approx 14.1\ \frac{m}{s}}$ *Ans.* to cause cavitation at point B ($\theta = 90°$)

With the stream velocity known, we may now solve for the pressure at point A ($\theta = 45°$):

$$p_A + \frac{998}{2}\left[\frac{3}{2}(14.1)\sin 45°\right]^2 + 9790(0.5\sin 45°) = 101350 + 9790(3) + \frac{998}{2}(14.1)^2$$

Solve for $\mathbf{p_A \approx 115000\ Pa}$ *Ans.*

8.101 Consider a steel sphere (SG = 7.85) of diameter 2 cm, dropped from rest in water at 20°C. Assume a constant drag coefficient C_D = 0.47. Accounting for the sphere's hydrodynamic mass, estimate (a) its terminal velocity; and (b) the time to reach 99% of terminal velocity. Compare these to the results when hydrodynamic mass is neglected, $V_{terminal} \approx 1.95$ m/s and $t_{99\%} \approx 0.605$ s, and discuss.

Solution: For water take $\rho = 998$ kg/m^3. Add hydrodynamic mass to the differential equation:

$$(m+m_h)\frac{dV}{dt} = W_{net} - C_D\frac{\rho}{2}V^2A, \quad A = \frac{\pi}{4}D^2 \quad \text{and} \quad W_{net} = (\rho_{steel} - \rho_{water})g\frac{\pi}{6}D^3$$

Separate the variables and integrate: $V = \sqrt{\dfrac{2W_{net}}{C_D\rho A}}\tanh\left(t\sqrt{\dfrac{W_{net}C_D\rho A}{2(m+m_h)^2}}\right)$

(a) The terminal velocity is the coefficient of the *tanh* function in the previous equation:

$$V_f = \sqrt{\frac{2W_{net}}{C_D\rho A}} = \sqrt{\frac{2(7834-998)(9.81)(\pi/6)(0.02)^3}{(0.47)(998)(\pi/4)(0.02)^2}}$$

$$= \mathbf{1.95\ \frac{m}{s}} \quad \text{(same as when } m_h = 0\text{)} \quad \textit{Ans. (a)}$$

(b) Noting that tanh(2.647) = 0.99, we find the time to approach 99% of *V*f to be

$$t\sqrt{\frac{W_{net}C_D\rho A}{2(m+m_h)^2}} = 2.647$$

$$= t\sqrt{\frac{(7834-998)(9.81)(\pi/6)(0.02)^3(0.47)(998)(\pi/4)(0.02)^2}{2[(7834+998/2)(\pi/6)(0.02)^3]^2}} = 4.122t$$

Solve for $t = 2.647/4.122 = \mathbf{0.642\ s}$ (6% more than when $m_h = 0$) *Ans.* (b)

8.102 A golf ball weighs 0.102 lbf and has a diameter of 1.7 in. A professional golfer strikes the ball at an initial velocity of 250 ft/s, an upward angle of 20°, and a backspin (front of the ball rotating upward). Assume that the lift coefficient on the ball (based on frontal area) follows Fig. P7.108. If the ground is level and drag is neglected, make a simple analysis to predict the impact point (a) without spin and (b) with backspin of 7500 r/min.

Solution: For sea-level air, take $\rho = 0.00238$ slug/ft^3. (a) If we neglect drag and spin, we just use classical particle physics to predict the distance travelled:

$$V = V_{oz} - gt = 0 \quad \text{when} \quad t = \frac{V_o\sin\theta_o}{g}, \quad x = V_{ox}(2t) = \frac{V_o^2}{g}2\sin\theta_o\cos\theta_o$$

$$\text{Substitute} \quad \Delta x_{impact} = \frac{(250)^2}{32.2}2\sin 20°\cos 20° \approx \mathbf{1250\ ft} \quad \textit{Ans. (a)}$$

For part (b) we have to estimate the lift of the spinning ball, using Fig. P7.108:

$$\omega = 7500\left(\frac{2\pi}{60}\right) = 785\ \frac{rad}{s}, \quad \frac{\omega R}{U} = \frac{785(1.7/24)}{250} \approx 0.22\text{: Read } C_L \approx 0.02$$

Estimate average lift $\mathrm{L} \approx \mathrm{C_L}(\rho/2)\mathrm{V}^2\pi\mathrm{R}^2$

$$= (0.02)\left(\frac{0.00238}{2}\right)(250)^2\pi\left(\frac{1.7}{24}\right)^2 \approx 0.023 \text{ lbf}$$

Write the equations of motion in the z and x directions and assume average values:

$$\mathrm{m}\ddot{\mathrm{x}} = -\mathrm{L}\sin\theta, \text{ with } \theta_{\mathrm{avg}} \approx 10°, \quad \ddot{\mathrm{x}}_{\mathrm{avg}} \approx -\frac{0.023}{0.102/32.2}\sin 10° \approx -1.3\ \frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$\text{Then } \mathrm{x} \approx \mathrm{V_{ox}}\mathrm{t} - \frac{1}{2}\ddot{\mathrm{x}}_{\mathrm{avg}}\mathrm{t}^2, \quad \text{where } \mathrm{t} = 2 \times (\text{time to reach the peak})$$

$$\mathrm{m}\ddot{\mathrm{z}} = \mathrm{L}\cos\theta - \mathrm{W}, \quad \text{or} \quad \ddot{\mathrm{z}}_{\mathrm{avg}} \approx \frac{0.023}{0.102/32.2}\cos 10° - 32.2 \approx -25\ \frac{\mathrm{ft}}{\mathrm{s}^2}$$

Using these admittedly crude estimates, the travel distance to impact is estimated:

$$\mathrm{t_{peak}} = \frac{\mathrm{V_{oz}}}{\mathrm{a_z}} = \frac{250\sin 20°}{25} \approx 3.4 \text{ s}, \quad \mathrm{t_{impact}} = 2\mathrm{t_{peak}} \approx 6.8 \text{ s},$$

$$\Delta\mathrm{x_{impact}} = \mathrm{V_{ox}}\mathrm{t} - \frac{1}{2}\mathrm{a_x}\mathrm{t}^2 = 250\cos 20°(6.8) - \frac{1.3}{2}(6.8)^2 \approx \mathbf{1570\ ft} \quad \textit{Ans.}\text{ (b)}$$

These are 400-yard to 500-yard drives, on the fly! It would be nice, at least when teeing off, to have zero viscous drag on the golfball.

8.103 Consider inviscid flow past a sphere, as in Fig. 8.32. Find (*a*) the point on the front surface where the fluid acceleration $a_{\max}$ is maximum; and (*b*) the magnitude of $a_{\max}$. (*c*) If the stream velocity is 1 m/s, find the sphere diameter for which $a_{\max}$ is ten times the acceleration of gravity. Comment.

Solution: Along the sphere surface, the flow is purely tangential, hence $v_r = 0$. Thus

$$V_{along\ surface} = -v_\theta|_{r=a} = 1.5U_\infty\sin\left(\frac{x}{a}\right), \quad \text{where } x \text{ is along the surface}$$

$$\frac{dV}{dx} = \frac{1.5U_\infty}{a}\cos\left(\frac{x}{a}\right); \quad \text{Thus accel} = V\frac{dV}{dx} = \frac{9U_\infty^2}{4a}\sin\left(\frac{x}{a}\right)\cos\left(\frac{x}{a}\right) = \text{maximum when}$$

$$\frac{x}{a} = \frac{\pi}{4} \text{ or } 45^\circ \text{ from stagnation point} \quad \textit{Ans.}(b) \quad \text{and} \quad \left(V\frac{dV}{dx}\right)_{\max} = \frac{9}{8}\frac{U_\infty^2}{a} \quad \textit{Ans.}(a)$$

(*c*) From *Ans*.(*a*) above, if U_∞ = 1 m/s and the maximum acceleration is ten g's, then

$$a_{\max} = 10(9.81\frac{m}{s^2}) = \frac{9U_\infty^2}{8a} = \frac{9(1m/s)^2}{8a}, \quad \text{Solve} \quad a = 0.0115m \quad \textit{Ans.}(c)$$

Comment: This is a fairly large sphere, 23-mm diameter. It shows that fluid flow past small bodies can cause large accelerations, even thousands of g's.

8.104 Consider a cylinder of radius *a* moving at speed U_∞ through a still fluid, as in Fig. P8.104. Plot the streamlines relative to the cylinder by modifying Eq. (8.32) to give the relative flow with K = 0. Integrate to find the total relative kinetic energy, and verify the hydrodynamic mass of a cylinder from Eq. (8.104).

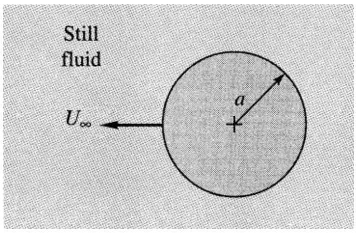


Fig. P8.104

Solution: For this two-dimensional polar-coordinate system, a differential **mass** is:

$$dm = \rho b r\, dr\, d\theta, \quad \text{where } b = \text{width into the paper}$$

$$\text{Subtract off the stream U to get } v_r|_{rel} = -U\cos\theta(a^2/r^2),\ v_\theta|_{rel} = -U\sin\theta(a^2/r^2)$$

That is, the velocities "relative" to the cylinder are, in fact, the velocities induced by the doublet. Now introduce the element kinetic energy into Eq. (8.102) and integrate:

$$KE = \int_{fluid} \frac{1}{2} dm\, V_{rel}^2 = \int_0^{2\pi}\int_a^{\infty} \frac{1}{2}(\rho b r\, dr\, d\theta)[\{U\cos\theta a^2/r^2\}^2 + \{U\sin\theta a^2/r^2\}^2] = \frac{\boldsymbol{\pi}}{\mathbf{2}}\boldsymbol{\rho}\mathbf{Ua^2b}$$

$$\text{Then, by definition,} \quad m_{hydro} = \frac{KE}{U^2/2} = \boldsymbol{\pi\rho}\mathbf{a^2b} = \text{cylinder displaced mass} \quad \textit{Ans.}$$

P8.105 A 22-cm-diameter solid aluminum sphere (SG = 2.7) is accelerating at 12 m/s^2 in water at 20°C. (*a*) According to potential theory, what is the hydrodynamic mass of the sphere? (*b*) Estimate the force being applied to the sphere at this instant.

Solution: For water at 20°C, take $\rho = 998\ \text{kg/m}^3$. The sphere radius is $a = 11$ cm. (*a*) Equation (8.103) gives the hydrodynamic mass, which is proportional to the *water* density:

$$m_h = \frac{2}{3}\rho_{water}\,\pi a^3 = \frac{2}{3}(998\frac{kg}{m^3})\pi(0.11m)^3 = \mathbf{2.78}\ kg \qquad Ans.(a)$$

(*b*) The sphere mass is $(\pi/6)\rho_{\text{sphere}}(D^3) = (\pi/6)(2.7 \times 998)(0.22)^3 = 15.02$ kg. Add this to the hydrodynamic mass, with the given acceleration, and use Eq. (8.101) to compute the applied force:

$$\sum F = (m_{sphere} + m_h)\frac{dV}{dt} = (15.02 + 2.78kg)(12\frac{m}{s^2}) = \mathbf{214}\ N \qquad Ans.(b)$$

8.106 Laplace's equation in polar coordinates, Eq. (8.11), is complicated by the variable radius *r*. Consider the finite-difference mesh in Fig. P8.106, with nodes (i, j) at equally spaced $\Delta\theta$ and Δr. Derive a finite-difference model for Eq. (8.11) similar to our cartesian expression in Eq. (8.109).

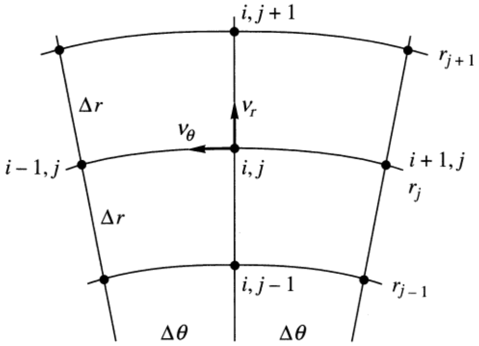


Fig. P8.106

Solution: We are asked to model

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = 0$$

There are two possibilities, depending upon whether you split up the first term. I suggest

$$\frac{1}{r_{ij}\,\Delta r}\left[\left(r_{ij} + \frac{\Delta r}{2}\right)\left(\frac{\psi_{i,j+1} - \psi_{i,j}}{\Delta r}\right) - \left(r_{ij} - \frac{\Delta r}{2}\right)\left(\frac{\psi_{i,j} - \psi_{i,j-1}}{\Delta r}\right)\right] + \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{r_{ij}^2(\Delta\theta)^2} \approx 0$$

Clean up: $\mathbf{(2+2\zeta)\psi_{i,j} \approx \psi_{i+1,j} + \psi_{i-1,j} + \zeta(1+\eta)\psi_{i,\,j+1} + \zeta(1-\eta)\psi_{i,j-1}}$ *Ans.*

where $\zeta = (r_{ij}\,\Delta\theta/\Delta r)^2$ and $\eta = \Delta r/(2r_{ij})$

8.107 Set up the numerical problem of Fig. 8.30 for an expansion angle of 30°. A new grid system and non-square mesh may be needed. Give the proper nodal equation and boundary conditions. If possible, program this 30° expansion and solve on a digital computer.

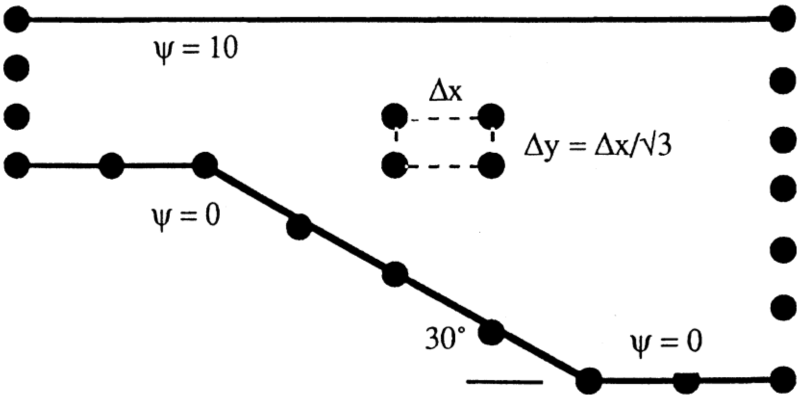


Fig. P8.107

Solution: Assuming the *same* 2:1 expansion, from U(in) = 10 m/s to U(out) = 5 m/s, we need a non-square mesh to make nodes fall along the slanted line at a 30° slope. The mesh size, as shown, should be Δy = Δx/√3. The scale-ratio β from Equation (8.108) equals $(\Delta x/\Delta y)^2 = \mathbf{3.0}$, and the model for each node is

$$2(1+\beta)\psi_{i,j} = \psi_{i-1,j} + \psi_{i+1,j} + \beta(\psi_{i,j-1} + \psi_{i,j+1}), \quad \text{where } \beta = 3.0 \quad \textit{Ans.}$$

The stream function would retain the same boundary values as in Fig. 8.30: $\psi = \mathbf{0}$ along the lower wall, $\psi = \mathbf{10}$ along the upper wall, and linear variation, between 0 and 10, along the inlet and exit planes. [See Fig. 8.31 also for boundary values.]

If we keep the same **vertical** nodal spacing in Fig. 8.30, Δy = **20** cm, then we need a horizontal spacing Δx = **34.64** cm, and the total length of the duct—from i = 1 to i = 16—will be 519.6 cm, whereas in Fig. 8.30 this total length is only 300 cm.

The numerical results will not be given here.

8.108 Consider two-dimensional potential flow into a step contraction as in Fig. P8.108. The inlet velocity U_1 = 7 m/s, and the outlet velocity U_2 is uniform. The nodes (i, j) are labelled in the figure. Set up the complete finite-difference algebraic relation for all nodes. Solve, if possible, on a digital computer and plot the streamlines.

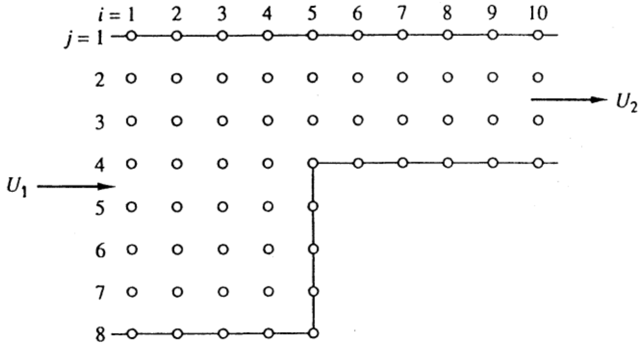


Fig. P8.108

Solution: By continuity, $U_2 = U_1(7/3) = 16.33$ m/s. For a square mesh, the standard Laplace model, Eq. (8.109), holds. For simplicity, assume unit mesh widths $\Delta x = \Delta y = 1$.

$$\text{Solve} \quad \psi_{i,j} = \frac{1}{4}(\psi_{i,j+1} + \psi_{i,j-1} + \psi_{i+1,j} + \psi_{i-1,j})$$

with $\psi = 0$ on the lower wall and $\psi = 49$ on the upper wall.

The writer's numerical solution is tabulated below.

i =	1	2	3	4	5	6	7	8	9	10
j =1, ψ=	49.00	49.00	49.00	49.00	49.00	49.00	49.00	49.00	49.00	49.00
j = 2	42.00	40.47	38.73	36.66	34.54	33.46	32.98	32.79	32.79	26.67
j = 3	35.00	32.17	28.79	24.38	19.06	17.30	16.69	16.46	16.38	16.33
j = 4	28.00	24.40	19.89	13.00	0.00	0.00	0.00	0.00	0.00	0.00
j = 5	21.00	17.53	13.37	7.73	0.00					
j = 6	14.00	11.36	8.32	4.55	0.00					
j = 7	7.00	5.59	4.02	2.14	0.00					
j = 8	0.00	0.00	0.00	0.00	0.00					

8.109 Consider inviscid potential flow through a two-dimensional 90° bend with a contraction, as in Fig. P8.109. Assume uniform flow at the entrance and exit. Make a finite-difference computer model analysis for small grid size (at least 150 nodes), determine the dimensionless pressure distribution along the walls, and sketch the streamlines. [You may use either square or rectangular grids.]

Solution: This problem is "digital computer enrichment" and will not be presented here.

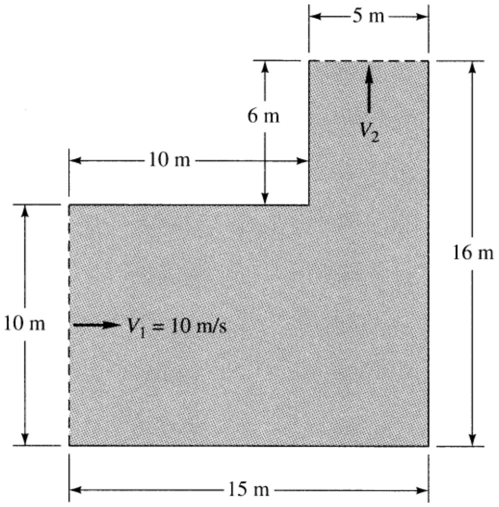


Fig. P8.109

8.110 For fully developed laminar incompressible flow through a straight noncircular duct, as in Sec. 6.8, the Navier-Stokes Equation (4.38) reduce to

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{\mu}\frac{dp}{dx}=\text{const}<0$$

where (y, z) is the plane of the duct cross section and x is along the duct axis. Gravity is neglected. Using a nonsquare rectangular grid (Δx, Δy), develop a finite-difference model for this equation, and indicate how it may be applied to solve for flow in a rectangular duct of side lengths a and b.

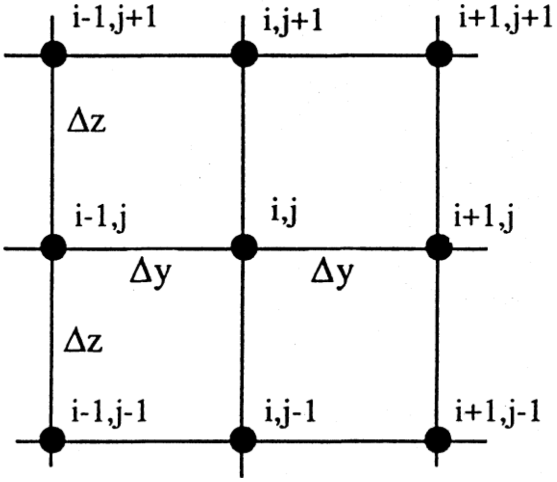


Fig. P8.110

Solution: An appropriate square grid is shown above. The finite-difference model is

$$\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}+\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta z)^2}\approx\frac{1}{\mu}\frac{dp}{dx},\quad \text{or,}\quad \text{if } \Delta y=\Delta z,$$

$$\mathbf{u_{i,j}=\frac{1}{4}\left[u_{i,j+1}+u_{i,j-1}+u_{i+1,j}+u_{i-1,j}-\frac{(\Delta y)^2}{\mu}\frac{dp}{dx}\right]}\quad \textit{Ans.}$$

This is "Poisson's equation," it looks like the Laplace model plus the constant "source" term involving the mesh size (Δy) and the pressure gradient and viscosity.

8.111 Solve Prob. 8.110 numerically for a rectangular duct of side length *b* by 2*b*, using at least 100 nodal points. Evaluate the volume flow rate and the friction factor, and compare with the results in Table 6.4:

$$Q\approx 0.1143\frac{b^4}{\mu}\left(-\frac{dp}{dx}\right)\qquad f\,\mathrm{Re}_{D_h}\approx 62.19$$

where $Dh = 4A/P = 4b/3$ for this case. Comment on the possible truncation errors of your model.

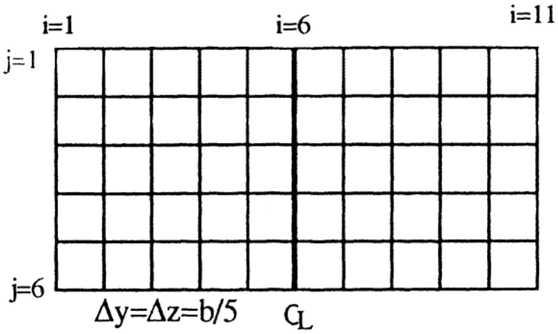


Fig. P8.111

Solution: A typical square mesh is shown in the figure above. It is appropriate to nondimensionalize the velocity and thus get the following dimensionless model:

$$\mathbf{V}=\frac{u}{(b^2/\mu)(-dp/dx)};\ \text{Then}\ V_{ij}=\frac{1}{4}\left[V_{i,j+1}+V_{i,j-1}+V_{i+1,j}+V_{i-1,j}+\left(\frac{\Delta y}{b}\right)^2\right],\ \text{with } \Delta y=\Delta z$$

The boundary conditions are: No-slip along all the outer surfaces: V = 0 along i = 1, i =11, j = 1, and j = 6. The internal values V_{ij} are then computed by iteration and sweeping over the interior field. Some computed results for this mesh, Δy/b = 0.2, are as follows:

i =		1	2	3	4	5	6 (centerline)
j = 1,	V =	0.000	0.000	0.000	0.000	0.000	0.000
j = 2,	V =	0.000	0.038	0.058	0.067	0.072	0.073
j = 3,	V =	0.000	0.054	0.084	0.100	0.107	0.109
j = 4,	V =	0.000	0.054	0.084	0.100	0.107	0.109
j = 5,	V =	0.000	0.038	0.058	0.067	0.072	0.073
j = 6,	V =	0.000	0.000	0.000	0.000	0.000	0.000

The solution is doubly symmetric because of the rectangular shape. [This mesh is too **coarse**, it only has 27 interior points.] After these dimensionless velocities are computed, the volume flow rate is computed by integration:

$$Q = \int_0^b \int_0^{2b} u\,dy\,dz = \frac{b^4}{\mu}\left(-\frac{dp}{dx}\right)\int_0^1 \int_0^2 V\frac{dy}{b}\frac{dz}{b} = \text{constant}\,\frac{b^4}{\mu}\left(-\frac{dp}{dx}\right)$$

The double integral was evaluated numerically by summing over all the mesh squares. Two mesh sizes were investigated by the writer, with good results as follows:

$$\Delta y/b = 0.2{:}\quad Q/(b^4/\mu)(-dp/dx) = \text{“constant”} \approx 0.1063\,(7\%\,\text{off})\quad (27\,\text{grid nodes})$$

$$= \mathbf{0.1}\qquad\qquad \text{“constant”} \approx \mathbf{0.1123}\,(2\%\,\text{off})\quad (171\,\text{nodes})\quad \textit{Ans.}$$

The accuracy is good, and the numerical model is very simple to program.

8.112 In CFD textbooks [5, 23-27], one often replaces the left-hand side of Eq. (8.119*b*) and (8.119*c*), respectively, with the following two expressions:

$$\text{Replace}\quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\quad \textit{by}\quad \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(vu);$$

$$\text{Replace}\quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\quad \textit{by}\quad \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2)$$

Are these equivalent expressions, or are they merely simplified approximations? Either way, why might these forms be better for finite-difference purposes?

Solution: These expressions are indeed *equivalent* because of the 2-D incompressible continuity equation. In the first example,

$$\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) \equiv 2u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = \left[u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right] + u\cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}$$

and similarly for the second example. They are more convenient numerically because, being non-linear terms, they are easier to model as *the difference in products* rather than the *product of differences*.

8.113 Repeat Example 8.7 using the *implicit* method of Eq. (8.118). Take Δt = 0.2 s and Δy = 0.01 m, which ensures that an explicit model would *diverge*. Compare your accuracy with Example 8.7.

Solution: Recall that SAE 30 oil (υ = 3.25E–4 m^2/s) was at rest at (t = 0) when the wall suddenly began moving at U = 1 m/s. Find the oil velocity at (y, t) = (3 cm, 1 s). This time σ = (3.25E–4)(0.2)/$(0.01)^2$ = **0.65** > 0.5, therefore an implicit method is required. We set up a grid with 11 nodes, going out to y = 0.1 m (N = 11), and use Eq. (8.118) to sweep all nodes for each time step. Stop at t = 1 s (j = 6):

$$u_n^{j+1} \approx \frac{u_n^j + 0.65\left(u_{n-1}^{j+1} + u_{n+1}^{j+1}\right)}{1 + 2(0.65)}$$

The results are shown in the table below.

j	Time	u1	u2	u3	u4	U5	u6	u7	u8	u9	u10	u11
1	0.0	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.2	1.000	0.310	0.096	0.030	0.009	0.003	0.001	0.000	0.000	0.000	0.000
3	0.4	1.000	0.473	0.197	0.077	0.029	0.010	0.004	0.001	0.000	0.000	0.000
4	0.6	1.000	0.568	0.283	0.129	0.055	0.023	0.009	0.003	0.001	0.000	0.000
5	0.8	1.000	0.629	0.351	0.180	0.086	0.039	0.017	0.007	0.003	0.001	0.000
6	1.0	1.000	0.671	0.406	**0.226**	0.117	0.058	0.027	0.012	0.005	0.002	0.000

This time the computed value of u4 (y = 0.03 m) at j = 6 (t = 1 s) is **0.226**, or about **6% lower** than the exact value of *0.241*. This accuracy is comparable to the explicit method of Example 8.7 and uses twice the time step.

***8.114** The following problem is not solved in this Manual. It requires Boundary-Element-Code software. If your institution has such software (see, e.g., the programming in Ref. 7), this advanced exercise is quite instructive about potential flow about airfoils.

If your institution has an online potential-flow boundary-element computer code, consider flow past a symmetric airfoil, as in Fig. P8.114. The basic shape of an NACA symmetric airfoil is defined by the function [12]

$$\frac{2y}{t_{\max}} \approx 1.4845\zeta^{1/2} - 0.63\zeta - 1.758\zeta^2 + 1.4215\zeta^3 - 0.5075\zeta^4$$

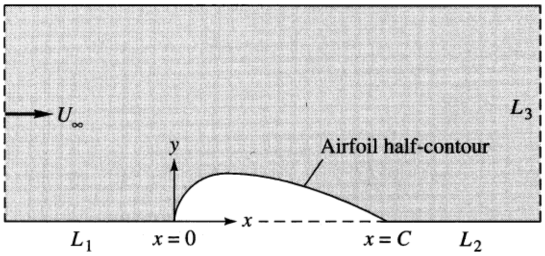


Fig. P8.114

where $\zeta = x/C$ and the maximum thickness t_{max} occurs at $\zeta = 0.3$. Use this shape as part of the lower boundary for zero angle of attack. Let the thickness be fairly large, say, t_{max} = 0.12, 0.15, or 0.18. Choose a generous number of nodes (≥60), and calculate and plot the velocity distribution V/U_∞ along the airfoil surface. Compare with the theoretical results in Ref. 12 for NACA 0012, 0015, or 0018 airfoils. If time permits, investigate the effect of the boundary lengths L_1, L_2, and L_3, which can initially be set equal to the chord length C.

8.115 Use the explicit method of Eq. (8.115) to solve Problem 4.85 *numerically* for SAE 30 oil ($\nu = 3.25\text{E}{-4}\ \text{m}^2/\text{s}$) with U_o = 1 m/s and $\omega = M$ rad/s, where M is the number of letters in your surname. (The writer will solve it for M = 5.) When steady oscillation is reached, plot the oil velocity versus time at y = 2 cm.

Solution: Recall that Prob. 4.85 specified an oscillating wall, $u_{wall} = U_o\sin(\omega t)$. One would have to experiment to find that the "edge" of the shear layer, that is, where the wall no longer influences the ambient still fluid, is about $y \approx 7$ cm. For reasonable accuracy, we could choose Δy = 0.5 cm, that is, 0.005 m, so that N = 15 is the outer "edge." For explicit calculation, we require

$$\sigma = \nu\Delta t/\Delta y^2 = (3.25\text{E}{-4})\Delta t/(0.005)^2 < 0.5, \quad \text{or} \quad \Delta t < 0.038\ \text{s}.$$

We choose Δt = 0.0333 s, $\boldsymbol{\sigma = 0.433}$, with 1 cycle covering about 37 time steps. Use Eq. (8.115):

$$u_n^{j+1} \approx 0.433\left(u_{n-1}^j + u_{n+1}^j\right) + 0.133u_n^j \quad \textit{for } 2 \le n \le 14 \quad \textit{and} \quad u_1 = 1.0\sin(5t),\ u_{15} = 0$$

Apply this algorithm to all the internal nodes (2 < n < 14) for many (200) time steps, up to about t = 6 sec. The results for y = 2 cm, n = 5, are shown in the plot below. The amplitude has dropped to 0.18 m/s with a phase lag of 20° [Ref. 15 of Chap. 8, p. 139].

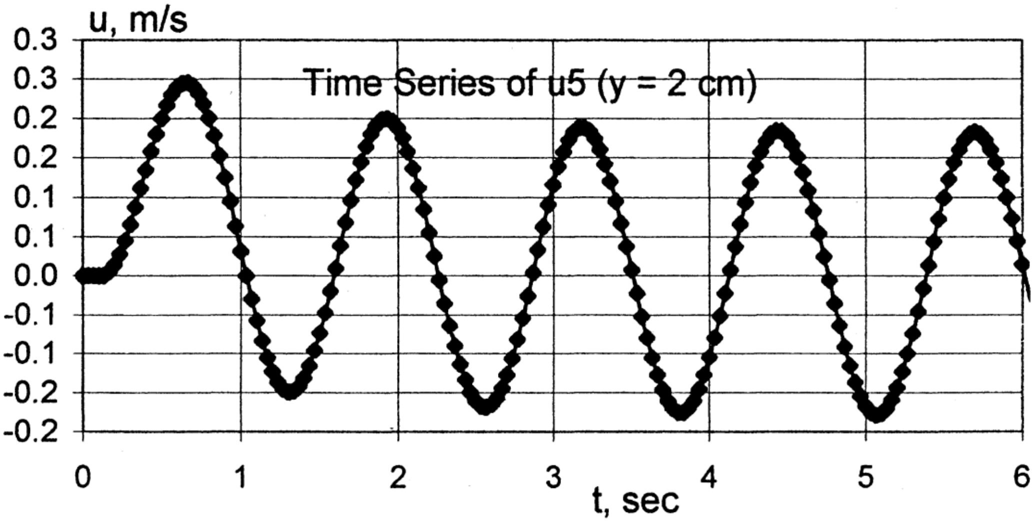


Fig. P8.115

COMPREHENSIVE PROBLEMS

C8.1 Did you know you can solve iterative CFD problems on an Excel spreadsheet? Successive relaxation of Laplace's equation is easy, since each nodal value is the average of its 4 neighbors. Calculate irrotational potential flow through a contraction as shown in the figure. To avoid "circular reference," set the Tools/Options/Calculate menu to "iteration." For full credit, attach a printout of your solution with ψ at each node.

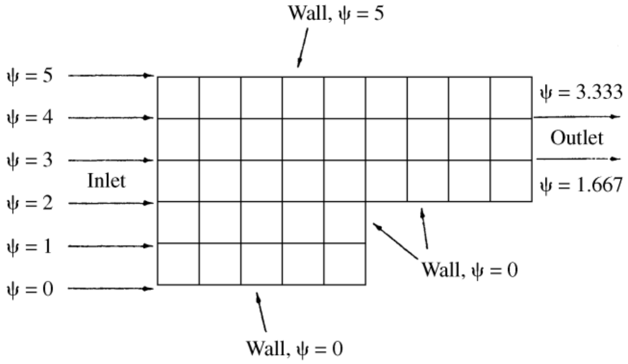


Fig. C8.1

Solution: Do exactly what the figure shows: Set bottom nodes at $\psi = 0$, top nodes at $\psi = 5$, left nodes at $\psi = 0$, 1, 2, 3, 4, 5 and right nodes at $\psi = 0$, 1.667, 3.333, and 5. Iterate with Eq. (8.115) for at least 100 iterations. Any initial guesses will do—the author chose 2.0 at all interior nodes. The final converged nodal values of stream function are shown in the table below.

i, j	1	2	3	4	5	6	7	8	9	10
1	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
2	4.000	3.948	3.880	3.780	3.639	3.478	3.394	3.357	3.342	3.333
3	3.000	2.911	2.792	2.602	2.298	1.879	1.742	1.694	1.675	1.667
4	2.000	1.906	1.773	1.539	1.070	0.000	0.000	0.000	0.000	0.000
5	1.000	0.941	0.856	0.710	0.445	0.000				
6	0.000	0.000	0.000	0.000	0.000	0.000				

C8.2 Use an *explicit* method, similar to but not identical to Eq. (8.115), to solve the case of SAE 30 oil starting from rest near a fixed wall. Far from the wall, the oil accelerates linearly, that is, $u_\infty = u_N = at$, where $a = 9\ \text{m/s}^2$. At t = 1 s, determine (a) the oil velocity at y = 1 cm; and (b) the instantaneous boundary-layer thickness (where $u \approx 0.99u_\infty$).

Hint: There is a non-zero pressure gradient in the outer (shear-free) stream, n = N, which must be included in Eq. (8.114).

Solution: To account for the stream acceleration as $\partial^2 u/\partial t^2 = 0$, we add a term:

$$\rho\frac{\partial u}{\partial t} = \rho a + \mu\frac{\partial^2 u}{\partial y^2}, \quad \text{which changes the model of Eq. (8.115) to}$$

$$u_n^{j+1} \approx a\Delta t + \sigma\left(u_{n-1}^j + u_{n+1}^j\right) + (1-2\sigma)u_n^j$$

The added term **aΔt** keeps the outer stream accelerating linearly. For SAE 30 oil, ν = 3.25E–4 m^2/s. As in Prob. 8.115 of this Manual, choose N = 15, Δy = 0.005 m, Δt = 0.0333 s, σ **= 0.433**, and let u1 = 0 and uN = u15 = *at* = 9t. All the inner nodes, 2 < n <14, are computed by the explicit relation just above. After 30 time-steps, t = 1 sec, the tabulated velocities below show that the velocity at *y* = 1 cm (n = 3) is u3 ≈ **4.41** m/s, and the position "δ" where u = 0.99u$_\infty$ = 8.91 m/s is at approximately **0.053** meters. *Ans*. These results are in good agreement with the known exact analytical solution for this flow.

j	Time	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11	u12	u13	u14	u15
31	**1.00**	0.000	2.496	**4.405**	5.832	6.871	7.607	8.114	8.453	8.673	8.811	**8.895**	8.944	8.972	8.988	9

C8.3 Model potential flow through the upper-half of the symmetric diffuser shown below. The expansion angle is θ = 18.5°. Use a *non-square* mesh and calculate and plot (a) the velocity distribution; and (b) the pressure coefficient along the centerline (the bottom boundary).

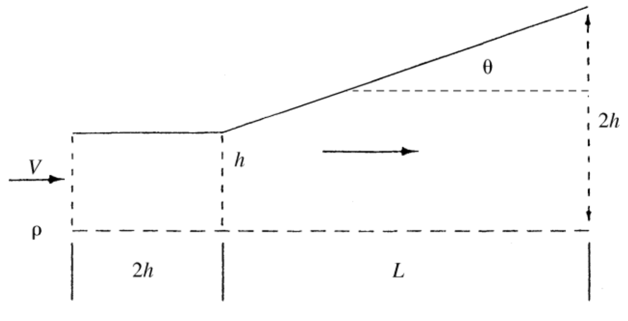


Fig. C8.3

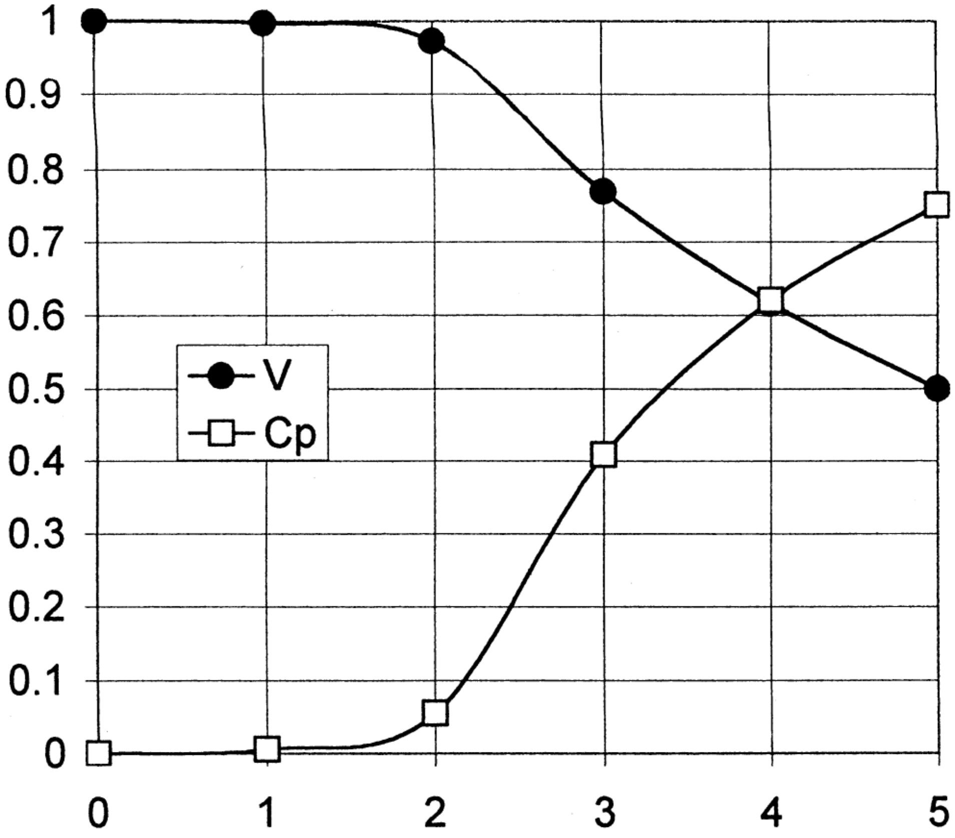
Solution: The tangent of 18.5° is 0.334, so $L \approx 3h$ and 3:1 rectangles are appropriate. If we make them *h* long and *h*/3 high, then *i* = 1 to 6 and *j* = 1 to 7. The model is given by Eq. (8.108) with $\beta = (3/1)^2 = 9$. That is,

$$2(1+9)\psi_{i,j} \approx \psi_{i-1,j} + \psi_{i+1,j} + 9(\psi_{i,j-1} + \psi_{i,j+1})$$

to be iterated over the internal nodes. For convenience, take $\psi_{top} = 10{,}000$ and $\psi_{bottom} = 0$. The iterated nodal solutions are as follows:

i, j	1	2	3	4	5	6
1						10000
2					10000	8333
3				10000	8083	6667
4	10000	10000	10000	7604	6111	5000
5	6667	6657	6546	5107	4097	3333
6	3333	3324	3240	2563	2055	1667
7	0	0	0	0	0	0

The velocities are found by taking differences: $u \approx \Delta\psi/\Delta y$ along the centerline. A plot is then made, as shown below, of velocity along the centerline (j = 7). The pressure coefficient is defined by $C_p = (p - p_{entrance})/[(1/2)\rho V^2_{entrance}]$. These are also plotted on the graph, along the centerline, using Bernoulli's equation.



Velocity and Pressure Coefficient Distribution along the Centerline of the Diffuser in Problem C8.3, Assuming unit Velocity at the Entrance.

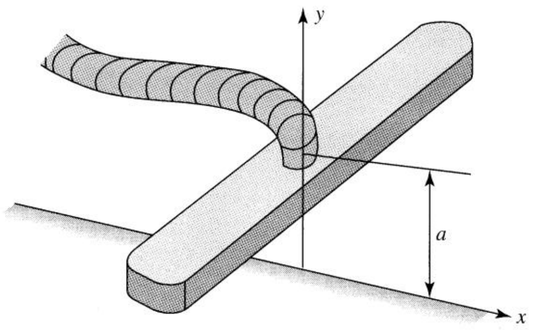
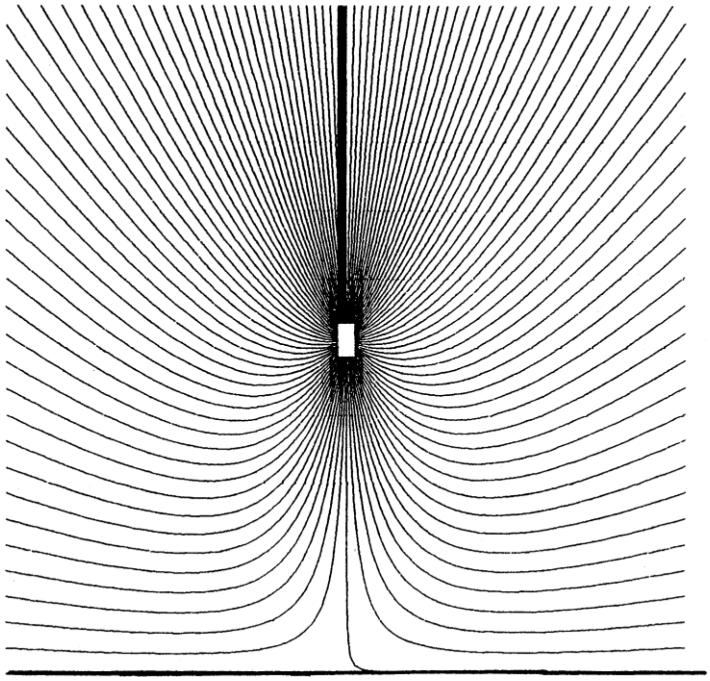


Fig. C8.4

C8.4 Use potential flow to approximate the flow of air being sucked into a vacuum cleaner through a 2-D slit attachment, as in the figure. Model the flow as a line sink of strength (–m), with its axis in the *z*-direction at height *a* above the floor. (a) Sketch the streamlines and locate any stagnation points. (b) Find the velocity V(x) along the floor in terms of *a* and *m*. (c) Define a velocity scale *U* = *m/a* and plot the pressure coefficient $C_p = (p - p_\infty)/[(1/2)\rho U^2]$ along the floor. (d) Find where C_p is a minimum—the vacuum cleaner should be most effective here. (e) Where did you *expect* the cleaner to be most effective, at x = 0 or elsewhere? (Experiment with dust later.)

Solution: (a) The "floor" is created by a sink at (0, +a) and an *image* sink at (0, –a), exactly like Fig. 8.17a of the text. There is one stagnation point, at the *origin*. The streamlines are shown below in a plot constructed from a MATLAB contour. *Ans*. (a)



(b) At any point *x* along the wall, the velocity *V* is the sum of image flows:

$$V = 2\upsilon_{r,sink}\cos\theta = \frac{2m}{r}\frac{x}{r} = \frac{\mathbf{2}\boldsymbol{mx}}{\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{a}^{\mathbf{2}}} = V_{along\ wall} \quad \textit{Ans.}\ \text{(b)}$$

(c) Use the Bernoulli equation to calculate pressure coefficient along the wall:

$$C_p = \frac{p - p_\infty}{(1/2)\rho U^2} = \frac{(1/2)\rho(U_\infty^2 - V^2)}{(1/2)\rho U^2} = -\frac{V^2}{U^2};\ C_{p,wall} = -\frac{\mathbf{4}\boldsymbol{x}^{\mathbf{2}}\boldsymbol{a}^{\mathbf{2}}}{(\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{a}^{\mathbf{2}})^{\mathbf{2}}} \quad \textit{Ans.}\ \text{(c)}$$

(d) The minimum wall-pressure coefficient is found by differentiation:

$$\frac{dC_p}{dx} = 0 = \frac{d}{dx}\left[\frac{-4x^2a^2}{(x^2 + a^2)^2}\right] \text{ occurs at } x^2 = a^2, \quad \textit{or}: \quad \boldsymbol{x} = \pm\boldsymbol{a} \quad \textit{Ans.}\ \text{(d)}$$

(e) Unexpected result! But experiments *do* show best cleaning at about $x \approx \pm a$.

C8.5 Consider three-dimensional, incompressible, irrotational flow. Use two methods to prove that the viscous term in the Navier-Stokes equation is zero: (a) using vector notation; and (b) expanding out the scalar terms using irrotationality.

Solution: (a) For irrotational flow, $\nabla \times \mathbf{V} = 0$, and $\mathbf{V} = \nabla\phi$, so the viscous term may be rewritten in terms of ϕ and then we get Laplace's equation:

$$\mu\nabla^2 V = \mu\nabla^2(\nabla\phi) = \mu\nabla(\cancel{\nabla^2\phi}) \equiv 0 \text{ from Laplace's equation.} \quad \textit{Ans.}\ \text{(a)}$$

(b) Expansion illustration: write out the *x*-term of $\nabla^2\mathbf{V}$, using irrotationality:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \mathbf{w}}{\partial \mathbf{x}}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} + \frac{\partial w}{\partial z}\right) \equiv 0 \quad \textit{Ans.}\ \text{(b)}$$

Similarly, $\nabla^2\upsilon = \nabla^2 w = 0$. The viscous term always vanishes for irrotational flow.

C8.6 Reconsider the lift-drag data for the NACA 4412 airfoil from Prob. 8.83. (a) Again draw the polar lift-drag plot and compare qualitatively with Fig. 7.26. (b) Find the maximum value of the lift-to-drag ratio. (c) Demonstrate a straight-line construction on the polar plot which will immediately yield the maximum *L*/*D* in (b). (d) If an aircraft could use this two-dimensional wing in actual flight (no induced drag) and had a perfect pilot, estimate how far (in miles) this aircraft could glide to a sea-level runway if it lost power at 25,000 ft altitude.

Solution: (a) Simply calculate $C_L(\alpha)$ and $C_D(\alpha)$ and plot them versus each other, as shown below:

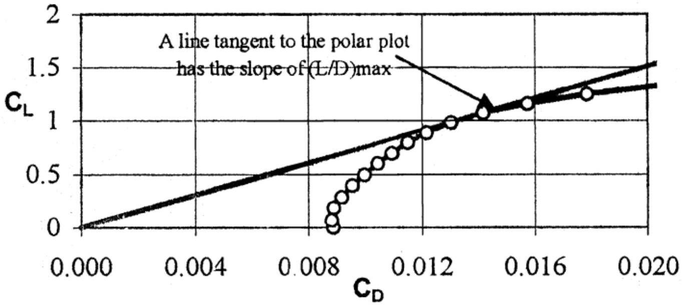


Fig. C8.6

(b, c) By calculating the ratio *L*/*D*, we could find a maximum value of **76** at $\alpha = 9°$. *Ans*. (b) This can be found graphically by **drawing a tangent from the origin** to the polar plot. *Ans*. (c)

(d) If the pilot could glide down at a constant angle of attack of 9°, the airplane could coast to a maximum distance of (76)(25000 ft)/(5280 ft/mi) = **360 miles**. *Ans*. (d)

C8.7 Find a formula for the stream function for flow of a doublet of strength λ at a distance a from a wall, as in Fig. C8.7. (a) Sketch the streamlines. (b) Are there any stagnation points? (c) Find the maximum velocity along the wall and its position.

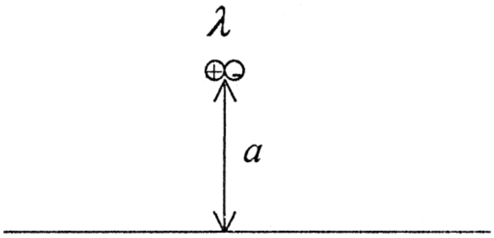
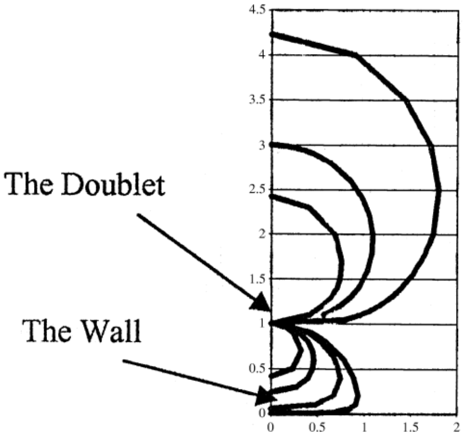


Fig. C8.7

Solution: Use an image doublet of the same strength and orientation at the $(x, y) = (0, -a)$. The stream function for this combined flow will form a "wall" at $y = 0$ between the two doublets:

$$\psi = -\frac{\lambda(y+a)}{x^2+(y+a)^2} - \frac{\lambda(y-a)}{x^2+(y-a)^2}$$

(a) The streamlines are shown on the next page for one quadrant of the doubly-symmetric flow field. They are fairly circular, like Fig. 8.8, above the doublet, but they flatten near the wall.



Problem C8.7

(b) There are **no stagnation points** in this flow field. *Ans*. (b)

(c) The velocity along the wall ($y = 0$) is found by differentiating the stream function:

$$u_{wall} = \frac{\partial\psi}{\partial y}\Big|_{y=0} = -\frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2} - \frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2}$$

The maximum velocity occurs at $x = 0$, that is, right between the two doublets:s

$$u_{w,\max} = \frac{2\lambda}{a^2} \quad \textit{Ans.}\ \text{(c)}$$

Chapter 9 • Compressible Flow

9.1 An ideal gas flows adiabatically through a duct. At section 1, p_1 = 140 kPa, T_1 = 260°C, and V_1 = 75 m/s. Farther downstream, p_2 = 30 kPa and T_2 = 207°C. Calculate V_2 in m/s and $s_2 - s_1$ in J/(kg·K) if the gas is (a) air, k = 1.4, and (b) argon, k = 1.67.

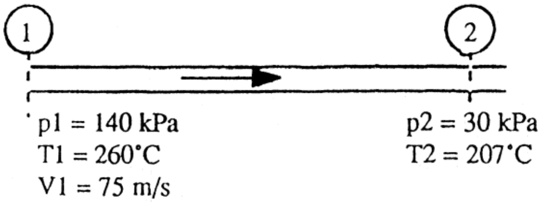


Fig. P9.1

Solution: (a) For air, take k = 1.40, R = 287 J/kg·K, and c_p = 1005 J/kg·K. The adiabatic steady-flow energy equation (9.23) is used to compute the downstream velocity:

$$c_pT+\frac{1}{2}V^2=\text{constant}=1005(260)+\frac{1}{2}(75)^2=1005(207)+\frac{1}{2}V_2^2 \quad \text{or} \quad \mathbf{V_2 \approx 335\ \frac{m}{s}} \quad \textit{Ans.}$$

$$\text{Meanwhile,}\ \ s_2-s_1=c_p\ln(T_2/T_1)-R\ln(p_2/p_1)=1005\ln\left(\frac{207+273}{260+273}\right)-287\ln\left(\frac{30}{140}\right),$$

$$\text{or}\quad s2-s1=-105+442\approx \mathbf{337\ J/kg\cdot K} \quad \textit{Ans.}\ \text{(a)}$$

(b) For argon, take k = 1.67, R = 208 J/kg·K, and c_p = 518 J/kg·K. Repeat part (a):

$$c_pT+\frac{1}{2}V^2=518(260)+\frac{1}{2}(75)^2=518(207)+\frac{1}{2}V_2^2,\quad \text{solve}\ \ \mathbf{V_2=246\ \frac{m}{s}} \quad \textit{Ans.}$$

$$s_2-s_1=518\ln\left(\frac{207+273}{260+273}\right)-208\ln\left(\frac{30}{140}\right)=-54+320\approx \mathbf{266\ J/kg\cdot K} \quad \textit{Ans.}\ \text{(b)}$$

9.2 Solve Prob. 9.1 if the gas is steam. Use two approaches: (a) an ideal gas from Table A.4; and (b) real steam from EES or the steam tables [15].

Solution: For steam, take $k = 1.33$, $R = 461$ J/kg·K, and $c_p = 1858$ J/kg·K. Then

$$c_pT+\frac{1}{2}V^2 = 1858(260)+\frac{1}{2}(75)^2 = 1858(207)+\frac{1}{2}V_2^2,\quad \text{solve } \mathbf{V_2 \approx 450\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$s_2 - s_1 = 1858\ln\left(\frac{207+273}{260+273}\right) - 461\ln\left(\frac{30}{140}\right) = -195+710 \approx \mathbf{515\ J/kg\cdot K} \quad \textit{Ans.}\text{ (a)}$$

(b) For real steam, we look up each enthalpy and entropy in EES or the Steam Tables:

$$\text{at 140 kPa and 260°C,}\quad \text{read } h_1 = 2.993\text{E}6\ \frac{J}{kg};$$

$$\text{at 30 kPa and 207°C,}\quad h_2 = 2.892\text{E}6\ \frac{J}{kg}$$

$$\text{Then}\quad h+\frac{1}{2}V^2 = 2.993\text{E}6+\frac{1}{2}(75)^2 = 2.892\text{E}6+\frac{1}{2}V_2^2,\quad \text{solve } V_2 \approx 443\ \frac{m}{s} \quad \textit{Ans.}\text{ (b)}$$

$$\text{at 140 kPa and 260°C,}\quad \text{read } s_1 = 7915\frac{J}{kg\cdot K},\quad \text{at 30 kPa and 207°C,}\quad s_2 = 8427\ \frac{J}{kg\cdot K}$$

$$\text{Thus}\quad s_2 - s_1 = 8427 - 7915 \approx \mathbf{512\ J/kg\cdot K} \quad \textit{Ans.}\text{ (b)}$$

These are within ±1.5% of the ideal gas estimates (a). Steam is nearly ideal in this range.

9.3 If 8 kg of oxygen in a *closed tank* at 200°C and 300 kPa is heated until the pressure rises to 400 kPa, calculate (a) the new temperature; (b) the total heat transfer; and (c) the change in entropy.

Solution: For oxygen, take $k = 1.40$, $R = 260$ J/kg·K, and $c_v = 650$ J/kg·K. Then

$$\rho_1 = \rho_2,\quad \therefore\ T_2 = T_1(p_2/p_1) = (200+273)\left(\frac{400}{300}\right) = 631\text{ K} \approx \mathbf{358°C} \quad \textit{Ans.}\text{ (a)}$$

$$Q = mc_v\Delta T = (8)(650)(358-200) \approx \mathbf{8.2E5\ J} \quad \textit{Ans.}\text{ (b)}$$

$$s_2 - s_1 = mc_v\ln(T_2/T_1) = (8)(650)\ln\left(\frac{358+273}{200+273}\right) \approx \mathbf{1500\ \frac{J}{K}} \quad \textit{Ans.}\text{ (c)}$$

P9.4 Consider steady adiabatic airflow in a duct. At section B, the pressure is 154 kPa and the density is 1.137 kg/m^3. At section D, the pressure is 28.2 kPa and the temperature is –19°C. (*a*) Find the entropy change, if any. (*b*) Which way is the air flowing?

Solution: Convert T_D = -19+273 = 254 K. We need the temperature at section B:

$$T_B = \frac{p_B}{R\rho_B} = \frac{154,000\,Pa}{(287)(1.137\,kg/m^3)} = 472\,K$$

$$\text{Then } s_B - s_D = c_p \ln(\frac{T_B}{T_D}) - R\ln(\frac{p_B}{p_D}) = (1005)\ln(\frac{472}{254}) - (287)\ln(\frac{154,000}{28,200})$$

$$\text{or: } s_B - s_D = 623 - 487 \approx \mathbf{+133}\ \frac{J}{kg-K} \quad Ans.(a)$$

The entropy is higher at B. Therefore the (adiabatic) **flow is from D to B**. *Ans.*(*b*)

9.5 Steam enters a nozzle at 377°C, 1.6 MPa, and a steady speed of 200 m/s and accelerates isentropically until it exits at saturation conditions. Estimate the exit velocity and temperature.

Solution: At saturation conditions, steam is *not ideal*. Use EES or the Steam Tables:

At 377°C and 1.6 MPa, read h_1 = 3.205E6 J/kg and s_1 = 7153 J/kg·K

At *saturation* for $s_1 = s_2$ = 7153, read p_2 = 185 kPa,

T_2 = 118°C, and h_2 = 2.527E6 J/kg

$$\text{Then } h + \frac{1}{2}V^2 = 3.205E6 + \frac{1}{2}(200)^2 = 2.527E6 + \frac{1}{2}V_2^2, \text{ solve } \mathbf{V_2 \approx 1180\ \frac{m}{s}} \quad Ans.$$

This exit flow is **supersonic**, with a Mach number exceeding 2.0. We are assuming with this calculation that a (supersonic) shock wave does not form.

P9.6 Use EES, other software, or the Gas Tables, to estimate c_p and c_v, their ratio, and their difference, for carbon dioxide at 800K and 100 kPa. Compare with estimates similar to Eqs. (9.4).

Solution: The writer used EES, for example, c_p = CP(CarbonDioxide,T=800,P=100) and obtained the following results at 800K and 100 kPa:

c_p = **1169** J/kg-K ; c_v = **980** J/kg-K ; $k = c_p/c_v$ = **1.19** ; $c_p - c_v$ = **189** J/kg-K (=R)

The difference is still equal to the gas constant R, but the specific heats are about 50% higher than would be estimated from Table A.4, which states (at room temperature), that k = 1.30 and R = 189 J/kg-K:

$$c_p\,|_{CO2} \approx \frac{kR}{k-1} = \frac{1.3(189)}{1.3-1} = 819\,\frac{J}{kg-K}\;;\quad c_v\,|_{CO2} \approx \frac{R}{k-1} = \frac{189}{1.3-1} = 630\,\frac{J}{kg-K}$$

So we give up a little accuracy by assuming constant specific heats if temperature changes are large.

P9.7 Air flows through a variable-area duct. At section 1, A_1 = 20 cm^2, p_1 = 300 kPa, ρ_1 = 1.75 kg/m^3, and V_1 = 122.5 m/s. At section 2, the area is exactly the same, but the density is much lower: ρ_2 = 0.266 kg/m^3, and T_2 = 281 K. There is no transfer of work or heat. Assume one-dimensional steady flow. (*a*) How can you reconcile these differences? (*b*) Find the mass flow at section 2. Calculate (*c*) V_2, (*d*) p_2, and (*e*) $s_2 - s_1$. *Hint*: This problem requires the continuity equation.

Solution: Part (*a*) is too confusing, let's try (*b, c, d, e*) first. (*b*) The mass flow must be constant:

$$\dot{m}_1 = \dot{m}_2 = \rho_1 A_1 V_1 = (1.75\frac{kg}{m^3})(0.0020\,m^2)(122.5\frac{m}{s}) = 0.0429\,\frac{kg}{s}\quad Ans.(b)$$

$$\text{Then } V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{0.0429\,kg/s}{(0.266\,kg/m^3)(0.002\,m^2)} = 806\,\frac{m}{s}\quad Ans.(c)$$

That's pretty fast! Check $a_2 = (kRT_2)^{1/2} = [1.4(287)(281)]^{1/2}$ = 336 m/s. Hence the Mach number at section 2 is Ma$_2 = V_2/a_2$ = 806/336 = 2.40. The flow at section 2 is *supersonic*! (*d*) The pressure at section 2 is easy, since the density and temperature are given:

$$p_2 = \rho_2 R T_2 = (0.266\,kg/m^3)(287\,m^2/s^2-K)(281K) = 21{,}450\ Pa \quad Ans(d)$$

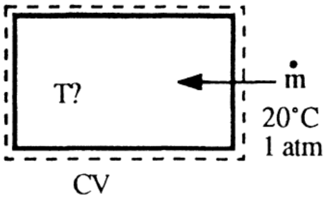
$$\text{Similarly},\ T_1 = \frac{p_1}{R\rho_1} = \frac{(300000\ Pa)}{(287\,m^2/s_2-K)(1.75\,kg/m^3)} = 597\,K$$

(*e*) Finally, with pressures and temperatures known, the entropy change follows from Eq. (9.8):

$$s_2 - s_1 = c_p \ln(\frac{T_2}{T_1}) - R\ln(\frac{p_2}{p_1}) = 1005\ln(\frac{281}{597}) - 287\ln(\frac{21450}{300000}) = -757+757 \approx 0\ \frac{J}{kg-K} \quad Ans.(e)$$

Ahah! Now I get it. (*a*) The flow is **isentropic**. *Ans.*(*a*) The stagnation properties, T_o = 605 K, p_o = 319 kPa, and ρ_o = 1.805 kg/m^3 are constant in the flow from section 1 to section 2.

9.8 Atmospheric air at 20°C enters and fills an insulated tank which is initially evacuated. Using a control-volume analysis from Eq. (3.63), compute the tank air temperature when it is full.



Solution: The energy equation during filling of the adiabatic tank is

$$\frac{dQ}{dt} + \frac{dW_{shaft}}{dt} = 0 + 0 = \frac{dE_{CV}}{dt} - h_{atm}\dot{m}_{entering}, \quad \text{or, after filling,}$$

$$E_{CV,final} - E_{CV,initial} = h_{atm}m_{entered}, \quad \text{or:} \quad mc_vT_{tank} = mc_pT_{atm}$$

$$\text{Thus} \quad \mathbf{T_{tank}} = (c_p/c_v)T_{atm} = (1.4)(20+273) \approx \mathbf{410\,K = 137°C} \quad Ans.$$

9.9 Liquid hydrogen and oxygen are burned in a combustion chamber and fed through a rocket nozzle which exhausts at 1600 m/s and exit pressure equal to ambient pressure of 54 kPa. The nozzle exit diameter is 45 cm, and the jet exit density is 0.15 kg/m^3. If the exhaust gas has a molecular weight of 18, estimate (a) the exit gas temperature; (b) the mass flow; and (c) the thrust generated by the rocket.

Solution: (a) From Eq. (9.3), estimate Rgas and hence the gas exit temperature:

$$R_{gas} = \frac{\Lambda}{M} = \frac{8314}{18} = 462\ \frac{J}{kg\cdot K}, \quad \text{hence } T_{exit} = \frac{p}{R\rho} = \frac{54000}{462(0.15)} \approx \mathbf{779\ K} \quad Ans.\ (a)$$

(b) The mass flow follows from the exit velocity:

$$\dot{m} = \rho AV = \left(0.15\ \frac{\text{kg}}{\text{m}^3}\right)\frac{\pi}{4}(0.45m)^2(1600m/s) \approx 38\ \frac{kg}{s} \quad \textit{Ans.}\text{ (b)}$$

(c) The thrust was derived in Problem 3.68. When $p_{exit} = p_{ambient}$, we obtain

$$Thrust = \rho_e A_e V_e^2 = \dot{m}V_e = 38(1600) \approx \mathbf{61{,}100\ N} \quad \textit{Ans.}\text{ (c)}$$

9.10 A certain aircraft flies at the same Mach number regardless of its altitude. Compared to its speed at 12000-m Standard Altitude, it flies 127 km/h faster at sea level. Determine its Mach number.

Solution: At sea level, T_1 = 288.16 K. At 12000 m standard, T_2 = 216.66 K. Then

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340.3\ \frac{m}{s};\quad a_2 = \sqrt{kRT_2} = 295.0\ \frac{m}{s}$$

$$\text{Then } \Delta V_{plane} = Ma(a_2 - a_1) = Ma(340.3 - 295.0) = Ma(45.3) = [127\text{ km/h}] = 35.27\text{ m/s}$$

$$\text{Solve for}\quad \mathbf{Ma} = \frac{35.27}{45.3} \approx 0.78 \quad \textit{Ans.}$$

9.11 At 300°C and 1 atm, estimate the speed of sound of (a) nitrogen; (b) hydrogen; (c) helium; (d) steam; and (e) uranium hexafluoride $^{238}UF_6$ ($k \approx 1.06$).

Solution: The gas constants are listed in Appendix Table A.4 for all but uranium gas (e):

(a) nitrogen: k = 1.40, R = 297, T = 300 + 273 = 573 K:

$$a = \sqrt{kRT} = \sqrt{1.40(297)(573)} \approx \mathbf{488\ m/s} \quad \textit{Ans.}\text{ (a)}$$

(b) hydrogen: k = 1.41, R = 4124, $a = \sqrt{1.41(4124)(573)} \approx \mathbf{1825\ m/s}$ *Ans.* (b)

(c) helium: k = 1.66, R = 2077: $a = \sqrt{1.66(2077)(573)} \approx \mathbf{1406\ m/s}$ *Ans.* (c)

(d) steam: k = 1.33, R = 461: $a = \sqrt{1.33(461)(573)} \approx \mathbf{593\ m/s}$ *Ans.* (d)

[NOTE: The EES “soundspeed” function would predict a_{steam} = 586 m/s.]

(e) For uranium hexafluoride, we need only to compute *R* from the molecular weight:

(e) $^{238}UF_6$: $\mathrm{M} = 238 + 6(19) = 352, \quad \therefore\ \mathrm{R} = \dfrac{8314}{352} \approx 23.62\ \mathrm{m^2/s^2 \cdot K}$

then $\mathrm{a} = \sqrt{1.06(23.62)(573)} \approx \mathbf{120\ m/s}$ *Ans.* (e)

9.12 Assume that water follows Eq. (1.19) with *n* ≈ 7 and *B* ≈ 3000. Compute the bulk modulus (in kPa) and the speed of sound (in m/s) at (a) 1 atm; and (b) 1100 atm (the deepest part of the ocean). (c) Compute the speed of sound at 20°C and 9000 atm and compare with the measured value of 2650 m/s (A. H. Smith and A. W. Lawson, *J. Chem. Phys.*, vol. 22, 1954, p. 351).

Solution: We may compute these values by differentiating Eq. (1.19) with $k \approx 1.0$:

$$\frac{p}{p_a} = (B+1)(\rho/\rho_a)^n - B; \quad \text{Bulk modulus } K = \rho\frac{dp}{d\rho} = n(B+1)p_a(\rho/\rho_a)^n, \quad a = \sqrt{K/\rho}$$

We may then substitute numbers for water, with p_a = 101350 Pa and ρ_a = 998 kg/m^3:

(a) at 1 atm: $K_{water} = 7(3001)(101350)(1)^7 \approx$ **2.129E9 Pa** (21007 atm) *Ans.* (a)

speed of sound $a_{water} = \sqrt{K/\rho} = \sqrt{2.129E9/998} \approx \mathbf{1460\ m/s}$ *Ans.* (a)

(b) at 1100 atm: $\rho = 998\left(\dfrac{1100+3000}{3001}\right)^{1/7} = 998(1.0456) \approx 1044\ \mathrm{kg/m^3}$

$K = K_{atm}(1.0456)^7 = (2.129E9)(1.3665) = \mathbf{2.91E9\ Pa}$ (28700 atm) *Ans.* (b)

$a = \sqrt{K/\rho} = \sqrt{2.91E9/1044} \approx \mathbf{1670\ m/s}$ *Ans.* (b)

(c) at 9000 atm: $\rho = 998\left(\dfrac{9000+3000}{3001}\right)^{1/7} = 1217\ \dfrac{kg}{m^3}; \quad K = K_a\left(\dfrac{1217}{998}\right)^7,$

or: $K = 8.51E9\ \mathrm{Pa}, \quad a = \sqrt{K/\rho} = \sqrt{8.51E9/1217} \approx \mathbf{2645\ m/s}$ (within 0.2%) *Ans.* (c)

P9.13 Consider steam at 500 K and 200 kPa. Estimate its speed of sound by three different methods: (*a*) using the handy new EES thermophysical function SOUNDSPEED(Steam, p = p_1,T = T_1); (*b*) assuming an ideal gas from Table B.4; or (*c*) using finite differences for isentropic densities between 210 kPa and 190 kPa.

Solution: (*a*) Enter EES and use the new function, setting units to kPa and degrees Kelvin:

$$a_{EES} = \text{SOUNDSPEED}(Steam, p=200, T=500) = \mathbf{547}\ \frac{m}{s} \qquad Ans.(a)$$

(*b*) Ideal gas approximation: From Table B.4 for H_2O, *k* = 1.33 and *R* = 461 m^2/s^2-K:

$$a_{ideal\ gas} = \sqrt{k\,RT} = \sqrt{1.33(461)(500)} \approx \mathbf{554}\ \frac{m}{s} \qquad Ans.(b)$$

This is 1.3% higher than EES, not bad. In this region, a better *k* would 1.30, not 1.33.

(*c*) Using finite differences of density and pressure at the same entropy as the given state:

$$\text{Entropy level:} \quad s_o = \text{ENTROPY}(Steam, p=200, T=500) = 7.6168\, kJ/kg-K$$

$$\text{At } p_2 = 210\,kPa, \text{ compute } \rho_2 = DENSITY(Steam, p=210, s=s_o) = 0.9073\ kg/m^3$$

$$\text{At } p_1 = 190\,kPa, \text{ compute } \rho_1 = DENSITY(Steam, p=190, s=s_o) = 0.8404\ kg/m^3$$

$$\text{Finite differences: } a^2 \approx \frac{\Delta p}{\Delta \rho}\Big|_s = \frac{210000-190000}{0.9073-0.8404} = \frac{20000}{0.0669} = 298950\frac{m^2}{s^2}$$

$$\text{Finally,} \quad a_{differences} \approx \sqrt{298950} \approx \mathbf{547}\ \frac{m}{s} \qquad Ans.(c)$$

Part (*c*) is the same as the EES result, so maybe that’s how the new function works?

P9.14 At 1 atm and 20°C, the density of methyl alcohol is 49.4 lbm/ft^3. At 300 atm, its density increases to 50.9 lbm/ft^3. Use this data to estimate the speed of sound. Comment on the possible uncertainty of this estimate.

Solution: For convenience, convert the density data to SI units: 49.4 lbm/ft^3 = 790.7 kg/m^3, and 50.9 lbm/ft^3 = 814.7 kg/m^3. Then use finite differences to approximate the formula:

$$a_{methanol} \approx \sqrt{\frac{\Delta p}{\Delta \rho}} = \sqrt{\frac{(300-1)(101350\,Pa)}{(814.7-790.7)kg/m^3}} = \sqrt{\frac{30303650}{24.0}} \approx \mathbf{1124}\ \frac{m}{s} \quad Ans.$$

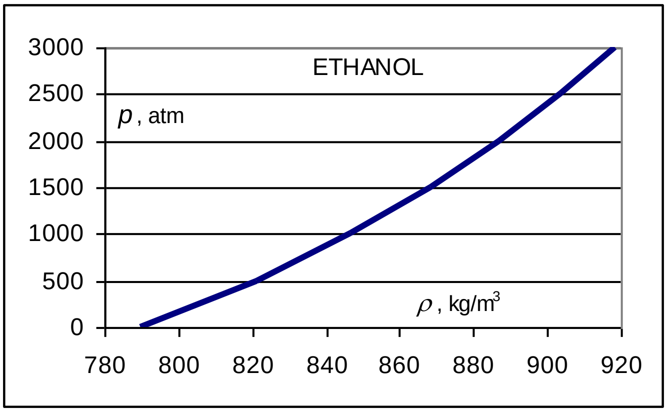
Since it relies on a small difference between two large densities, the density measurement must be very accurate. For example, a 1% error in density might cause a 50% error in speed of sound.

P9.15 The pressure-density relation for ethanol is approximated by Eq. (1.19) with B = 1600 and n = 7. Use this relation to estimate the speed of sound of ethanol at a pressure of 2000 atmospheres.

Solution: Recall that Eq. (1.19) is a curve-fit equation of state for liquids:

$$\frac{p}{p_o} \approx (B+1)\left(\frac{\rho}{\rho_o}\right)^n - B \qquad (1.19)$$

It looks like this, with ρ_o = 790 kg/m^3 from Table A.3. At 2000 atm, $\rho \approx$ 887 kg/m^3.



We see that the slope (or speed of sound squared) increases with pressure. Differentiate:

$$\frac{dp}{d\rho} = a^2 = \frac{p_o}{\rho_o} n(B+1)\left(\frac{\rho}{\rho_o}\right)^{n-1}$$

$$Ethanol:\ a = \sqrt{\frac{101350\,Pa}{790\,kg/m^3}(7)(1600+1)\left(\frac{887\,kg/m^3}{790\,kg/m^3}\right)^6} \approx 1700\,\frac{m}{s} \quad Ans.$$

At 1 atm, the speed of sound of ethanol is about 1200 m/s.

9.16 A weak pressure wave (sound wave) Δp propagates through still air. Discuss the type of reflected pulse which occurs, and the boundary conditions which must be satisfied, when the wave strikes normal to, and is reflected from, (a) a solid wall; and (b) a free liquid surface.

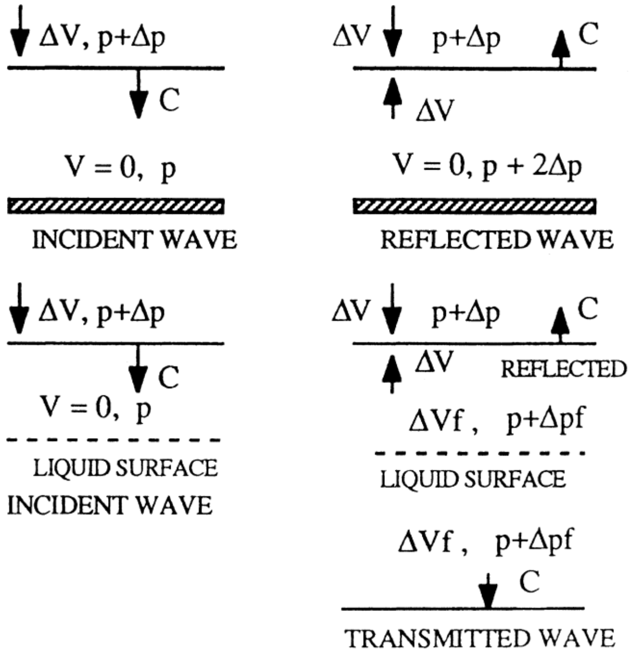


Fig. P9.16

Solution: (a) When reflecting from a solid wall, the velocity to the wall must be zero, so the wall pressure rises to $p + 2\Delta p$ to create a compression wave which cancels out the oncoming particle motion ΔV.

(b) When a compression wave strikes a liquid surface, it reflects and transmits to keep the particle velocity ΔV_f and the pressure $p + \Delta p_f$ the same across the liquid interface:

$$\Delta V_f = \frac{2\rho C\,\Delta V}{\rho C + \rho_{liq}C_{liq}}; \qquad \Delta p_f = \frac{2\rho_{liq}C_{liq}\,\Delta p}{\rho C + \rho_{liq}C_{liq}} \quad \textit{Ans.}\ \text{(b)}$$

If $\rho_{liq}C_{liq} \geq\geq \rho C$ of air, then $\Delta V_f \approx 0$ and $\Delta p_f \approx 2\Delta p$, which is case (a) above.

9.17 A submarine at a depth of 800 m sends a sonar signal and receives the reflected wave back from a similar submerged object in 15 s. Using Prob. 9.12 as a guide, estimate the distance to the other object.

Solution: It probably makes little difference, but estimate a at a depth of 800 m:

$$\text{at 800 m,}\quad p = 101350 + 1025(9.81)(800) = 8.15E6\ \text{Pa} = 80.4\ \text{atm}$$

$$p/p_a = 80.4 = 3001(\rho/1025)^7 - 3000,\quad \text{solve } \rho \approx 1029\ \text{kg/m}^3$$

$$a = \sqrt{n(B+1)p_a(\rho/\rho_a)^7/\rho} = \sqrt{7(3001)(101350)(1029/1025)^7/1029} \approx 1457\ \text{m/s}$$

Hardly worth the trouble: One-way distance ≈ $a\Delta t/2 = 1457(15/2) \approx$ **10900 m**. *Ans.*

9.18 Race cars at the Indianapolis Speedway average speeds of 185 mi/h. After determining the altitude of Indianapolis, find the Mach number of these cars and estimate whether compressibility might affect their aerodynamics.

Solution: Rush to the Almanac and find that Indianapolis is at 220 m altitude, for which Table A.6 predicts that the standard speed of sound is 339.4 m/s = 759 mi/h. Thus the Mach number is

$$Ma_{racer} = V/a = 185\ \text{mph}/759\ \text{mph} = \mathbf{0.24}\quad \textit{Ans.}$$

This is less than 0.3, so the Indianapolis Speedway need not worry about compressibility.

P9.19 In 1976, the SR-71A *Blackbird*, flying at 20 km standard altitude, set the jet-powered aircraft speed record of 3326 km/h. (*a*) Estimate the temperature, in °C, at its front stagnation point. (*b*) At what Mach number would it have a front stagnation-point temperature of 500°C?

Solution: At 20 km altitude, from Table A.6, T = 216.66K and a = 295.1 m/s. Convert the velocity from 3326 km/h to (3316)(1000)/(3600) = 924 m/s. Then Ma = V/a = 924/295.1 = **3.13**. Compute

$$T_o = T(1+0.2Ma^2) = (216.66)[1+0.2(3.13)^2] = 641K = 368\ ^oC \quad \textit{Ans.(a)}$$

(*b*) To have a front stagnation temperature of 500°C = 773 K, we could calculate

$$T_o = 773K = (216.66)[1+0.2Ma^2]\ ,\quad \textit{solve for}\ \ Ma = 3.58 \qquad \textit{Ans.(b)}$$

The SR-71A couldn't fly that fast because of structural and heat transfer limitations.

P9.20 Air flows isentropically in a channel. Properties at section 1 are V_1 = 250 m/s, T_1 = 330 K, and p_1 = 80 kPa. At section 2 downstream, the temperature has dropped to 0°C. Find (*a*) the pressure, (*b*) velocity, and (*c*) Mach number at section 2.

Solution: Assume k = 1.4 and, of course, convert T_2 = 0°C = 273 K. (*b*) The adiabatic energy equation will yield the new velocity:

$$T_1 + \frac{V_1^2}{2c_p} = T_o = 330 + \frac{(250)^2}{2(1005)} = 361K = T_2 + \frac{V_2^2}{2c_p} = 273 + \frac{V_2^2}{2(1005)}$$

$$Solve\ \ for\ \ V_2 = \mathbf{421}\ \frac{m}{s} \qquad Ans.(b)$$

(*a*) To calculate p_2, we could go through the stagnation pressure (which is 110 kPa) or we could simply use the ideal gas temperature ratio, Eq. (9.9):

$$\frac{p_2}{p_1} = (\frac{T_2}{T_1})^{k/(k-1)} = \frac{p_2}{80} = (\frac{273}{330})^{3.5} = 0.515, \quad \text{or:} \quad p_2 = \mathbf{41}\ kPa \qquad Ans.(a)$$

We have velocity and temperature at section 2, so we can easily calculate the Mach number:

$$Ma_2 = \frac{V_2}{a_2} = \frac{V_2}{\sqrt{kRT_2}} = \frac{421}{\sqrt{1.4(287)(273)}} = \frac{421 m/s}{331 m/s} = \mathbf{1.27} \qquad Ans.(c)$$

9.21 CO_2 expands isentropically through a duct from p_1 = 125 kPa and T_1 = 100°C to p_2 = 80 kPa and V_2 = 325 m/s. Compute (a) T_2; (b) Ma_2; (c) T_o; (d) p_o; (e) V_1; and (f) Ma_1.

Solution: For CO2, from Table A.4, take $k = 1.30$ and $R = 189$ J/kg·K. Compute the specific heat: $c_p = kR/(k - 1) = 1.3(189)/(1.3 - 1) = 819$ J/kg·K. The results follow in sequence:

(a) $T_2 = T_1(p_2/p_1)^{(k-1)/k} = (373\text{ K})(80/125)^{(1.3-1)/1.3} = \mathbf{336\ K}$ *Ans.* (a)

(b) $a_2 = \sqrt{kRT_2} = \sqrt{(1.3)(189)(336)} = 288\text{ m/s},\ Ma_2 = V_2/a_2 = 325/288 = \mathbf{1.13}$ *Ans.* (b)

(c) $T_{o1} = T_{o2} = T_2\left(1+\frac{k-1}{2}Ma_2^2\right) = (336)\left[1+\frac{0.3}{2}(1.13)^2\right] = \mathbf{401\ K}$ *Ans.* (c)

(d) $p_{o1} = p_{o2} = p_2\left(1+\frac{k-1}{2}Ma_2^2\right)^{1.3/(1.3-1)} = (80)\left[1+\frac{0.3}{2}(1.13)^2\right]^{1.3/0.3} = \mathbf{171\ kPa}$ *Ans.* (d)

(e) $T_{o1} = 401\ K = T_1 + \frac{V_1^2}{2c_p} = 373 + \frac{V_1^2}{2(819)}$, Solve for $\mathbf{V_1 = 214\ m/s}$ *Ans.* (e)

(f) $a_1 = \sqrt{kRT_1} = \sqrt{(1.3)(189)(373)} = 303\text{ m/s},\quad Ma_1 = V_1/a_1 = 214/303 = \mathbf{0.71}$ *Ans.* (f)

9.22 Given the pitot stagnation temperature and pressure and the static-pressure measurements in Fig. P9.22, estimate the air velocity *V*, assuming (a) incompressible flow and (b) compressible flow.

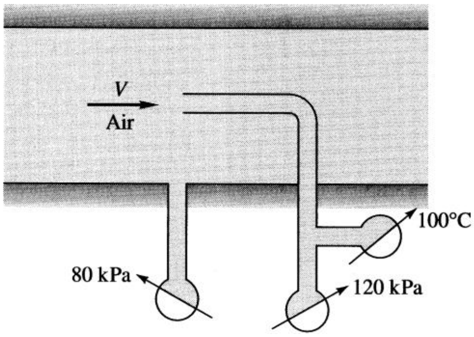


Fig. P9.22

Solution: Given p = 80 kPa, po = 120 kPa, and T = 100°C = 373 K. Then

$$\rho_o = \frac{p_o}{RT_o} = \frac{120000}{287(373)} = 1.12\text{ kg/m}^3$$

(a) 'Incompressible':

$$\rho = \rho_o,\ V \approx \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2(120000-80000)}{1.12}} \approx \mathbf{267}\ \frac{\mathbf{m}}{\mathbf{s}}\ (7\%\text{ low})\quad \textit{Ans. (a)}$$

(b) Compressible: $T = T_o(p/p_o)^{(k-1)/k} = 373(80/120)^{0.4/1.4} = 332$ K. Then $T_o = 373\text{ K} = T + V^2/2c_p = 332 + V^2/[2(1005)]$, solve for $\mathbf{V = 286\ m/s}$. *Ans.* (b)

P9.23 A gas, assumed ideal, flows isentropically from point 1, where the velocity is negligible, the pressure is 200 kPa, and the temperature is 300°C, to point 2, where the pressure is 40 kPa. What is the Mach number Ma_2 if the gas is (*a*) air; (*b*) argon; or (*c*) CH_4? (*d*) Can you tell, without calculating, which gas will be the coldest at point 2?

Solution: This is a standard exercise in using the isentropic-flow formulas. The term "negligible velocity" is code for *stagnation conditions*, hence $p_o = 200$ kPa and $T_o = 300°C = 573$ K. Work it out for the three different gases, using the ideal-gas isentropic-flow formulas:

$$\frac{p_o}{p} = (1+\frac{k-1}{2}Ma^2)^{k/(k-1)} \quad ; \quad \frac{T_o}{T} = (1+\frac{k-1}{2}Ma^2)$$

For $p_o/p = 200/40 = 5.0$ and $T_o = 573$ K, we obtain

(*a*) Air, $k = 1.40$; $\mathrm{Ma}_2 = 1.709$; $T_2 = 362$ K *Ans.*(*a*)

(*b*) Argon, $k = 1.67$; $\mathrm{Ma}_2 = 1.646$; $T_2 = 301$ K *Ans.*(*b*)

(*c*) CH_4, $k = 1.32$; $\mathrm{Ma}_2 = 1.727$; $T_2 = 388$ K *Ans.*(*c*)

(*d*) The **argon** cools off the most *Ans.*(*d*). But, since both c_p and V vary with k, the writer was not smart enough to divine which was coolest without calculating the temperatures.

9.24 For low-speed (nearly incompressible) gas flow, the stagnation pressure can be computed from Bernoulli's equation

$$p_0 = p + \frac{1}{2}\rho V^2$$

(a) For higher subsonic speeds, show that the isentropic relation (9.28*a*) can be expanded in a power series as follows:

$$p_0 \approx p + \frac{1}{2}\rho V^2\left(1+\frac{1}{4}\mathrm{Ma}^2+\frac{2-k}{24}\mathrm{Ma}^4+\cdots\right)$$

(b) Suppose that a pitot-static tube in air measures the pressure difference $p0 - p$ and uses the Bernoulli relation, with stagnation density, to estimate the gas velocity. At what Mach number will the error be 4 percent?

Solution: Expand the isentropic formula into a binomial series:

$$\frac{p_o}{p} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{k/(k-1)} = 1+\frac{k}{k-1}\frac{k-1}{2}\mathrm{Ma}^2 + \frac{k}{k-1}\frac{1}{2}\left(\frac{k}{k-1}-1\right)\left(\frac{k-1}{2}\mathrm{Ma}^2\right)^2 + \cdots$$

$$= 1+\frac{k}{2}\mathrm{Ma}^2 + \frac{k}{8}\mathrm{Ma}^4 + \frac{k(2-k)}{48}\mathrm{Ma}^6 + \cdots$$

Use the ideal gas identity $(1/2)\rho V^2 \equiv (1/2)kp(\mathrm{Ma}^2)$ to obtain

$$\frac{p_o - p}{(1/2)\rho V^2} = \mathbf{1+\frac{1}{4}Ma^2+\frac{2-k}{24}Ma^4+\cdots} \quad \textit{Ans.}$$

The error in the incompressible formula, $2\Delta p/\rho_o V^2$, is 4% when

$$\frac{V}{\sqrt{2(p_o - p)/\rho}} = \sqrt{\frac{\rho_o/\rho}{1+(1/4)\mathrm{Ma}^2+[(2-k)/24]\mathrm{Ma}^4}} = 1.04,$$

$$\text{where} \quad \frac{\rho_o}{\rho} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{1/(k-1)}$$

For k = 1.4, solve this for 4% error at **Ma ≈ 0.576** *Ans.*

9.25 If it is known that the air velocity in the duct is 750 ft/s, use that mercury manometer measurement in Fig. P9.25 to estimate the static pressure in the duct, in psia.

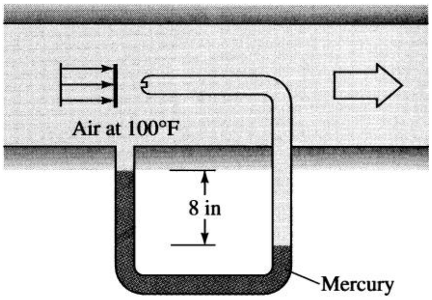


Fig. P9.25

Solution: Estimate the air specific weight in the manometer to be, say, 0.07 lbf/ft³. Then

$$p_o - p\big|_{\text{measured}} = (\rho g_{\text{mercury}} - \rho g_{\text{air}})h = (846 - 0.07)\left(\frac{8}{12}\ \text{ft}\right) \approx 564\ \text{lbf/ft}^2$$

$$\text{Given} \quad T = 100°\text{F} = 560°\text{R}, \quad a = \sqrt{kRT} = \sqrt{1.4(1717)(560)} \approx 1160\ \text{ft/s}$$

Then Ma = V/a = 750/1160 ≈ 0.646

$$\text{Finally,} \quad \frac{p_o - p}{p} = [1+0.2(0.646)^2]^{3.5} - 1 = 1.324 - 1 = 0.324 = \frac{564}{p}$$

Solve for $p_{\text{static}} \approx 1739$ psf ≈ **12.1 lbf/in²** (abs) *Ans.*

9.26 Show that for isentropic flow of a perfect gas if a pitot-static probe measures p_0, p, and T_0, the gas velocity can be calculated from

$$V^2 = 2c_pT_0\left[1-\left(\frac{p}{p_0}\right)^{(k-1)/k}\right]$$

What would be a source of error if a shock wave were formed in front of the probe?

Solution: Assuming isentropic flow past the probe,

$$\mathrm{T} = \mathrm{T_o(p/p_o)^{(k-1)/k}} = \mathrm{T_o} - \frac{\mathrm{V}^2}{2\mathrm{c_p}}, \quad \text{solve} \quad \mathbf{V^2 = 2c_pT_o\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \quad \textit{Ans.}$$

If there is a *shock wave* formed in front of the probe, this formula will yield the air velocity inside the shock wave, because the probe measures p_{o2} *inside* the shock. The stagnation pressure in the outer stream is *greater,* as is the velocity outside the shock.

P9.27 A pitot tube, mounted on an airplane flying at 8000 m standard altitude, reads a stagnation pressure of 57 kPa. Estimate (*a*) the velocity in mi/h, and (*b*) the Mach number.

Solution: We assume that the static pressure is the standard atmosphere pressure at 8000 m, which from Table B.6 is 35,581 Pa. Then the isentropic pressure formula will yield the Mach number:

$$\frac{p_o}{p} = \frac{57000}{35581} = 1.602 = (1+0.2Ma^2)^{3.5}, \quad Solve \;\; Ma = \mathbf{0.85} \quad Ans.(b)$$

Gratifyingly, the speed of sound at 8000 m is given right in Table B.6: a = 308 m/s. Then

$$V = (Ma)(a) = (0.85)(308\,m/s) = 262\,m/s = \mathbf{586}\,mi/h \qquad Ans.(a)$$

9.28 A large vacuum tank, held at 60 kPa absolute, sucks sea-level standard air through a converging nozzle of throat diameter 3 cm. Estimate (a) the mass flow rate; and (b) the Mach number at the throat.

Solution: For sea-level air take To = 288 K, ρo = 1.225 kg/m^3, and po = 101350 Pa. The pressure ratio is given, and we can assume isentropic flow with k = 1.4:

$$\frac{p_e}{p_o} = \frac{60000}{101350} = \left(1 + 0.2Ma_e^2\right)^{-3.5}, \quad \text{solve } \mathbf{Ma_e \approx 0.899} \quad \textit{Ans.}\ \text{(b)}$$

We can then solve for exit temperature, density, and velocity, finally mass flow:

$$\rho_e = \rho_o[1 + 0.2(0.899)^2]^{-2.5} \approx 0.842\ \frac{\text{kg}}{\text{m}^3}, \quad T_e = \frac{p_e}{R\rho_e} = \frac{60000}{287(0.842)} \approx 248\ \text{K}$$

$$V_e = Ma_e a_e = 0.899[1.4(287)(248)]^{1/2} \approx 284\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,} \quad \dot{m} = \rho_e A_e V_e = (0.842)\frac{\pi}{4}(0.03)^2(284) \approx \mathbf{0.169\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(a)}$$

9.29 Steam from a large tank, where T = 400°C and p = 1 MPa, expands isentropically through a small nozzle until, at a section of 2-cm diameter, the pressure is 500 kPa. Using the Steam Tables, estimate (a) the temperature; (b) the velocity; and (c) the mass flow at this section. Is the flow subsonic?

Solution: "Large tank" is code for stagnation values, thus To = 400°C and po = 1 MPa. This problem involves dogwork in the tables and well illustrates why we use the ideal-gas law so readily. Using $k \approx 1.33$ for steam, we find the flow is slightly supersonic:

$$\text{Ideal-gas simplification:} \quad \frac{p_o}{p} = \frac{1000}{500} = 2.0 \approx \left[1 + \left(\frac{1.33-1}{2}\right)\text{Ma}^2\right]^{\frac{1.33}{0.33}},$$

$$\text{Solve} \quad \mathbf{Ma} \approx 1.07 \qquad \text{Ans.(a)}$$

That was quick. Instead, for more accuracy, use EES, assuming constant entropy:

$$\text{At To} = 400°\text{C} \quad \text{and po} = 1\ \text{MPa}, \quad \text{read } s_o \approx 7465\ \frac{\text{J}}{\text{kg·K}} \quad \text{and } h_o \approx 3.264\text{E}6\ \frac{\text{J}}{\text{kg}}$$

Then, at p = 0.5 MPa, assuming s = so, read T ≈ 302°C ≈ **575 K** *Ans.* (a)

Also read h ≈ 3.067E6 J/kg and $\rho \approx 1.908$ kg/m^3.

With h and ho known, the velocity follows from the adiabatic energy equation:

$$h + V^2/2 = h_o, \quad \text{or} \quad 3.067E6 + V^2/2 = 3.264E6\ \frac{J}{kg}\left(\text{or } \frac{m^2}{s^2}\right),$$

$$\text{Solve} \quad V \approx 627\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\text{ (b)}$$

The speed of sound is now included in EES, thanks, EES!

$$a \;=\; \text{SOUNDSPEED}(steam, p = 500, s = 7.465) \;=\; 583\ \frac{m}{s}$$

$$\text{Then} \quad Ma = V/a = \frac{627}{583} \approx 1.075 \quad \textit{Ans.}\text{ (a)} \quad \text{(slightly supersonic)}$$

Finally, the mass flow is computed from the density and velocity:

$$\dot{m} \;=\; \rho A V \;=\; (1.908 kg/m^3)(\pi/4)(0.02m)^2(627 m/s) \;\approx\; \mathbf{0.376}\ kg/s \qquad \textit{Ans.}(c)$$

We could have done nearly as well (±2%) by simply assuming an ideal gas with $k \approx 1.33$.

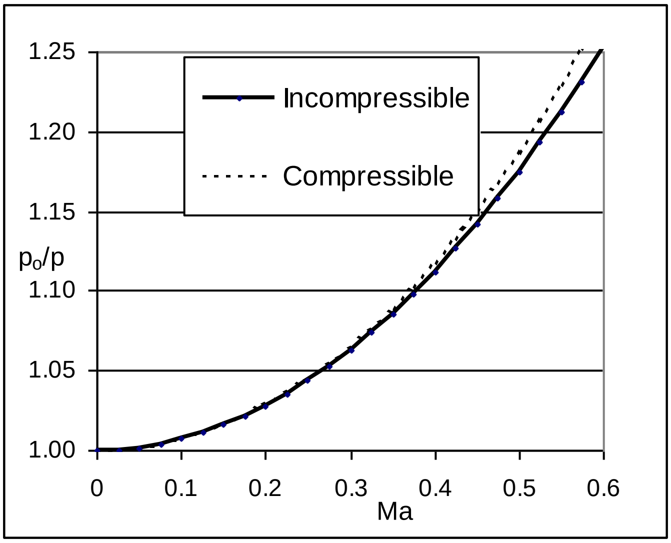
P9.30 When does the incompressible-flow assumption begin to fail for pressures? Construct a graph of p_o/p for incompressible flow of air as compared to Eq. (9.28*a*). Neglect gravity. Plot both versus Mach number for $0 \le Ma \le 0.6$ and decide for yourself where the deviation is too great.

Solution: The Bernoulli incompressible equation can be converted to Mach number form:

$$p_o|_{incompressible} = p + \frac{\rho}{2}V^2, \text{ or: } \frac{p_o}{p} = 1 + \frac{\rho V^2}{2p} = 1 + \frac{kV^2}{2kRT} = 1 + \frac{k}{2}\frac{V^2}{a^2} = 1 + \frac{k}{2}Ma^2$$

$$\text{Compare with} \quad \frac{p_o}{p}|_{compressible} = [1 + \frac{(k-1)}{2}Ma^2]^{k/(k-1)}$$

The two formulas are compared in the chart below. The difference becomes visible (but less than 1%) at Ma = 0.3 (the usual criterion) but is still small (<2%) at Ma = 0.5.



$k = 1.40$

9.31 Air flows adiabatically through a duct. At one section, V1 = 400 ft/s, T1 = 200°F, and p1 = 35 psia, while farther downstream V2 = 1100 ft/s and p2 = 18 psia. Compute (a) Ma2; (b) Umax; and (c) po2/po1.

Solution: (a) Begin by computing the stagnation temperature, which is constant (adiabatic):

$$T_{o1} = T_{o2} = T_1 + \frac{V^2}{2c_p} = (200+460) + \frac{(400)^2}{2(6010)} = 673°R = T_2 + \frac{V^2}{2c_p}$$

$$\text{Then}\quad T_2 = 673 - \frac{(1100)^2}{2(6010)} = 573°R,$$

$$Ma_2 = \frac{V_2}{a_2} = \frac{1100}{\sqrt{1.4(1717)(573)}} = \frac{1100}{1173} \approx \mathbf{0.938} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\quad U_{max} = \sqrt{2c_pT_o} = \sqrt{2(6010)(673)} \approx \mathbf{2840\ ft/s} \quad \textit{Ans.}\text{ (b)}$$

$$\text{(c)}\quad \text{We need } Ma_1 = V_1/a_1 = 400/\sqrt{1.4(1717)(200+460)} = 400/1260 \approx 0.318$$

$$\text{then}\quad p_{o1} = p_1\left(1+0.2Ma_1^2\right)^{3.5} = 1.072p_1 = 37.53\text{ psia}$$

$$\text{and}\quad p_{o2} = p_2\left(1+0.2Ma_2^2\right)^{3.5} = 1.763p_2 = 31.74\text{ psia},\quad \therefore\ \frac{p_{o2}}{p_{o1}} = \frac{31.74}{37.53} \approx \mathbf{0.846} \quad \textit{Ans.}\text{ (c)}$$

9.32 The large compressed-air tank in Fig. P9.32 exhausts from a nozzle at an exit velocity of 235 m/s. The mercury manometer reads h = 30 cm. Assuming isentropic flow, compute the pressure (a) in the tank and (b) in the atmosphere. (c) What is the exit Mach number?

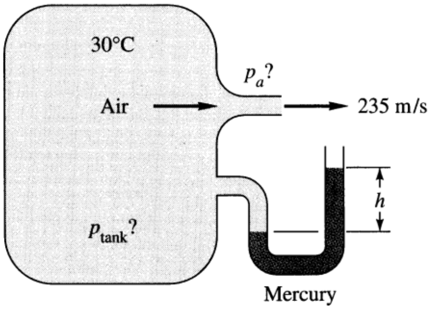


Fig. P9.32

Solution: The tank temperature = To = 30°C = 303 K. Then the exit jet temperature is

$$T_e = T_o - \frac{V_e^2}{2c_p} = 303 - \frac{(235)^2}{2(1005)} = 276\text{ K}, \quad \therefore\ Ma_e = \frac{235}{\sqrt{1.4(287)(276)}} \approx \mathbf{0.706} \quad \textit{Ans.}\text{ (c)}$$

$$\text{Then}\quad \frac{p_{tank}}{p_e} = \left(1+0.2Ma_e^2\right)^{3.5} = \mathbf{1.395} \quad \text{and} \quad p_{tank} - p_e = (\rho_{mercury} - \rho_{tank})gh$$

$$\text{Guess}\quad \rho_{tank} \approx 1.6\text{ kg/m}^3, \quad \therefore\ p_o - p_e \approx (13550-1.6)(9.81)(0.30) \approx \mathbf{39900\ Pa}$$

Solve the above two simultaneously for $p_e \approx$ **101 kPa** and $p_{tank} \approx$ **140.8 kPa** *Ans.* (a, b)

9.33 Air flows isentropically from a reservoir, where p = 300 kPa and T = 500 K, to section 1 in a duct, where A1 = 0.2 m^2 and V1 = 550 m/s. Compute (a) Ma1; (b) T1; (c) p1; (d) $\dot{m}$; and (e) A*. Is the flow choked?

Solution: Use the energy equation to calculate T1 and then get the Mach number:

$$T_1 = T_o - \frac{V_1^2}{2c_p} = 500 - \frac{(550)^2}{2(1005)} = \mathbf{350\ K} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Then}\quad a_1 = \sqrt{1.4(287)(350)} = 375\text{ m/s}, \quad Ma_1 = V_1/a_1 = \frac{550}{375} \approx \mathbf{1.47} \quad \textit{Ans.}\text{ (a)}$$

The flow **must be choked** in order to produce supersonic flow in the duct. *Answer.*

$$p_1 = p_o\Big/\left(1+0.2\ Ma_1^2\right)^{3.5} = 300/[1+0.2(1.47)^2]^{3.5} \approx \mathbf{86\ kPa} \quad \textit{Ans.}\text{ (c)}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{86000}{287(350)} \approx 0.854\ \frac{kg}{m^3}, \quad \therefore\ \dot{m} = \rho AV = (0.854)(0.2)(550) \approx \mathbf{94\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(d)}$$

$$\text{Finally,}\quad \frac{A}{A^*} = \frac{1}{Ma}\frac{(1+0.2Ma^2)^3}{1.728} = 1.155 \quad \text{if}\ Ma = 1.47,$$

$$\therefore\ A^* = \frac{0.2}{1.155} \approx \mathbf{0.173\ m^2} \quad \textit{Ans.}\ \text{(e)}$$

P9.34 Carbon dioxide, in a large tank at 100°C and 151 kPa, exhausts to a 1-atm environment through a converging nozzle whose throat area is 5 cm^2. Using isentropic ideal-gas theory, calculate (*a*) the exit temperature, and (*b*) the mass flow.

Solution: For CO_2, from Table A.4, take $R = 189\ m^2/s^2\text{-}K$ and $k = 1.30$. Find the exit Mach number from the exit pressure ratio (guessing that the exit flow is subsonic, so $p_{exit} = p_{atm}$):

$$\frac{p_o}{p} = \frac{151000Pa}{101350Pa} = [1+(\frac{k-1}{2})Ma_e^2]^{k/(k-1)} = [1+0.15Ma_e^2]^{1.3/0.3}\ ,\ \text{solve}\quad Ma_e = 0.802$$

$$\text{Then}\ T_e = \frac{T_o}{[1+(\frac{k-1}{2})Ma_e^2]} = \frac{373K}{[1+0.15*(0.802)^2]} = 340K = 67^oC \qquad Ans.(a)$$

To compute the mass flow, either find V_e and ρ_e or use the mass-flow function, Eq. (9.47).

$$\rho_o = \frac{151000}{(189)(373)} = 2.14\frac{kg}{m^3}\ ;\ \rho_e = \frac{2.14}{[1+0.15(0.802)^2]^{1/0.3}} = 1.58\frac{kg}{m^3}$$

$$V_e = Ma_e\, a_e = Ma_e\sqrt{kRT_e} = (0.802)\sqrt{1.3(189)(340)} = 232\, m/s$$

$$Finally,\quad \dot{m} = \rho_e A_e V_e = (1.58)(0.0005m^2)(232) = 0.183\ kg/s \quad Ans.(b)$$

With pressures known, an alternate method would use the mass-flow function, Eq. (9.47):

$$\frac{\dot{m}\sqrt{RT_o}}{A\,p_o} = \frac{\dot{m}\sqrt{(189)(373)}}{(0.0005)(151000)} = \sqrt{\frac{2k}{k-1}(\frac{p}{p_o})^{2/k}[1-(\frac{p}{p_o})^{(k-1)/k}]} =$$

$$= \sqrt{\frac{2(1.3)}{0.3}(\frac{101350}{151000})^{2/1.3}[1-(\frac{101350}{151000})^{0.3/1.3}]}\ ,\ \text{Solve}\quad \dot{m} = 0.183\frac{kg}{s}\quad Ans.(b)$$

The mass-flow function is a bit messy, but it gives the mass flow *directly*.

9.35 Helium, at To = 400 K, enters a nozzle isentropically. At section 1, where A1 = 0.1 m^2, a pitot-static arrangement (see Fig. P9.25) measures stagnation pressure of 150 kPa and static pressure of 123 kPa. Estimate (a) Ma1; (b) mass flow; (c) T1; and (d) A*.

Solution: For helium, from Table A.4, take $k = 1.66$ and $R = 2077$ J/kg·K. (a) The local pressure ratio is given, hence we can estimate the Mach number:

$$\frac{p_o}{p_1} = \frac{150}{123} = \left[1+\frac{1.66-1}{2}Ma_1^2\right]^{1.66/(1.66-1)},\quad \text{solve for}\ \mathbf{Ma_1 \approx 0.50}\quad \textit{Ans.}\ \text{(a)}$$

Use this Mach number to estimate local temperature, density, velocity, and mass flow:

$$T_1 = \frac{T_o}{1+(k-1)Ma_1^2/2} = \frac{400}{1+0.33(0.50)^2} \approx \mathbf{370\ K}\quad \textit{Ans.}\ \text{(c)}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{123000}{2077(370)} \approx 0.160\ \frac{\text{kg}}{\text{m}^3}$$

$$V_1 = Ma_1 a_1 = 0.50[1.66(2077)(370)]^{1/2} \approx \mathbf{565}\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,}\quad \dot{m} = \rho_1 A_1 V_1 = (0.160)(0.1)(565) \approx \mathbf{9.03}\ \frac{\mathbf{kg}}{\mathbf{s}}\quad \textit{Ans.}\,\text{(b)}$$

Finally, A* can be computed from Eq. (9.44), using $k = 1.66$:

$$\frac{A_1}{A^*} = \frac{1}{Ma_1}\left[\frac{1+0.33\,Ma_1^2}{(1.66+1)/2}\right]^{(1/2)(2.66)/(0.66)} \approx 1.323,\quad \mathbf{A^* \approx 0.0756\ m^2}\quad \textit{Ans.}\,\text{(d)}$$

9.36 An air tank of volume 1.5 m^3 is at 800 kPa and 20°C when it begins exhausting through a converging nozzle to sea-level conditions. The throat area is 0.75 cm^2. Estimate (a) the initial mass flow; (b) the time to blow down to 500 kPa; and (c) the time when the nozzle ceases being choked.

Solution: For sea level, $p$ambient = 101.35 kPa < 0.528$p$tank, hence the flow is choked until the tank pressure drops to $p$ambient/0.528 = 192 kPa. (a) We obtain

$$\dot{m}_{initial} = \dot{m}_{\max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{800000(0.75E\text{–}4\ \mathrm{m}^2)}{\sqrt{287(293)}} = \mathbf{0.142}\ \frac{\mathrm{kg}}{\mathrm{s}} \quad \textit{Ans.}\,\text{(a)}$$

(b) For a control volume surrounding the tank, a mass balance gives

$$\frac{d}{dt}(\rho_o \upsilon) = \frac{\upsilon}{RT_o}\frac{dp_o}{dt} = -\dot{m} = -0.6847\frac{p_o A^*}{\sqrt{RT_o}}, \quad \text{separate the variables:}$$

$$\frac{p(t)}{p(0)} = \exp\left[-0.6847\frac{A^*\sqrt{RT_o}}{\upsilon}t\right] = \mathbf{e}^{-0.00993t} \quad \text{until p(t) drops to 192 kPa}$$

At 500 kPa, we obtain 500/800 = exp(–0.00993t), or **t ≈ 47 s** *Ans.* (b)

At choking (192 kPa), 192/800 = exp(–0.00993t), or **t ≈ 144 s** *Ans.* (c)

9.37 Make an exact control volume analysis of the blowdown process in Fig. P9.37, assuming an insulated tank with negligible kinetic and potential energy. Assume critical flow at the exit and show that both po and To *decrease* during blowdown. Set up first-order differential equations for po(t) and To(t) and reduce and solve as far as you can.

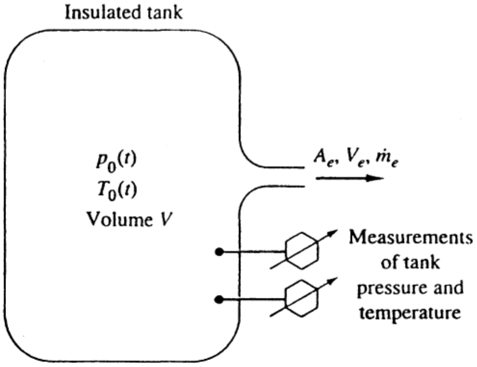


Fig. P9.37

Solution: For a CV around the tank, write the mass and the energy equations:

$$\text{mass:}\quad \frac{d}{dt}(\rho_o \upsilon) = -\dot{m}, \quad \text{or} \quad \frac{d}{dt}\left(\frac{p_o}{RT_o}\upsilon\right) = -B\frac{p_o}{\sqrt{T_o}}, \quad \text{where } B = \frac{0.6847A^*}{\sqrt{R}}$$

$$\text{energy:}\quad \frac{dQ}{dt} + \frac{dW}{dt} = 0 = \frac{d}{dt}\left(\frac{p_o}{RT_o}\upsilon c_v T_o\right) + \dot{m}c_p T_o$$

We may rearrange and combine these to give a single differential equation for To:

$$\frac{dT_o}{dt} = -CT_o^{3/2}, \quad \text{where } C = \frac{0.6847}{\upsilon}(k-1)A^*\sqrt{R}, \quad \text{or} \quad \int \frac{dT_o}{T_o^{3/2}} = -C\int dt$$

$$\text{Integrate:} \quad \mathbf{T_o(t) = \left[\frac{1}{\sqrt{T_o(0)}} + \frac{1}{2}Ct\right]^{-2}} \quad \textit{Ans.}$$

With To(t) known, we could go back and solve the mass relation for po(t), but in fact that is not necessary. We simply use the isentropic-flow assumption:

$$\frac{p_o(t)}{p_o(0)} = \left[\frac{T_o(t)}{T_o(0)}\right]^{k/(k-1)} = \mathbf{\left[1 + \frac{1}{2}CT_o^{1/2}(0)t\right]^{-2k/(k-1)}} \quad \left(C = \frac{0.6847(k-1)A^*\sqrt{R}}{\upsilon}\right) \quad \textit{Ans.}$$

Clearly, tank pressure also decreases with time as the tank blows down.

9.38 Prob. 9.37 makes an ideal senior project or combined laboratory and computer problem, as described in Ref. 30, sec. 8.6. In Bober and Kenyon's lab experiment, the tank had a volume of 0.0352 ft^3 and was initially filled with air at 50 lb/in^2 gage and 72°F. Atmospheric pressure was 14.5 lb/in^2 absolute, and the nozzle exit diameter was 0.05 in. After 2 s of blowdown, the measured tank pressure was 20 lb/in^2 gage and the tank temperature was –5°F. Compare these values with the theoretical analysis of Prob. 9.37.

Solution: Use the formulas derived in Prob. 9.37 above, with the given data:

$$T_o(0) = 72 + 460 = 532°R,$$

$$\text{“C”} = \frac{0.6847(\pi/4)(0.05/12)^2(1.4-1)\sqrt{1717}}{0.0352\ \text{ft}^3} \approx 0.0044\ \frac{1}{\text{s}\cdot°\text{R}^{1/2}}$$

$$\text{Then} \quad T_o(t) \approx T_o(0)\left[1 + \frac{1}{2}(0.0044)\sqrt{532}t\right]^{-2} = 532/[1 + 0.0507t]^2$$

$$\text{Similarly,} \quad p_o = p_o(0)[T_o/T_o(0)]^{k/(k-1)} = (50 + 14.5\ \text{psia})/[1 + 0.0507t]^7$$

Some numerical predictions from these two formulas are as follows:

t, sec:	0	0.5	1.0	1.5	2.0
To, °R:	532.0	506.0	481.9	459.5	**438.6°R**
po, psia:	64.5	54.1	45.6	38.6	**32.8 psia**

At t = 2 sec, the tank temperature is 438.6°R = **–21.4°F**, compared to –5°F measured.

At t = 2 sec, the tank pressure is 32.8 psia = **18.3 psig,** compared to 20 psig measured. The discrepancy is probably due to heat transfer through the tank walls warming the air.

9.39 Consider isentropic flow in a channel of varying area, between sections 1 and 2. Given Ma1 = 2.0, we desire that V2/V1 equal 1.2. Estimate (a) Ma2 and (b) A2/A1. (c) Sketch what this channel looks like, for example, does it converge or diverge? Is there a throat?

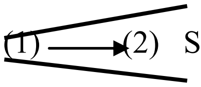
Solution: This is a problem in iteration, ideally suited for EES. Algebraically,

$$\frac{V_2}{V_1}=\frac{Ma_2 a_2}{Ma_1 a_1}=\frac{Ma_2}{Ma_1}\,\frac{\cancel{a_o}\left[1+0.2Ma_2^2\right]^{-1/2}}{\cancel{a_o}\left[1+0.2Ma_1^2\right]^{-1/2}}=1.2,\quad \text{given that } Ma_1=2.0$$

For adiabatic flow, ao is constant and cancels. Introducing Ma1 = 2.0, we have to solve $\mathrm{Ma}_2/[1+0.2\mathrm{Ma}_2^2]^{1/2}\approx 1.789$. By iteration, the solution is: **Ma2 = 2.98** *Ans*. (a)

$$\text{Then}\quad \frac{A_2}{A_1}=\frac{A_2/A^*}{A_1/A^*}=\frac{4.1547}{1.6875}(\text{Table B.1})\approx \mathbf{2.46}\quad \textit{Ans.}\ \text{(b)}$$

There is no throat between, a **supersonic expansion**. *Ans*. (c)



Supersonic

P9.40 Steam, in a tank at 300 kPa and 600 K, discharges isentropically to a low-pressure atmosphere through a converging nozzle with exit area 5 cm^2. (*a*) Using an ideal gas approximation from Table B.4, estimate the mass flow. (*b*) Without actual calculations, indicate how you would use real properties of steam, from EES, to find the mass flow.

Solution: The code words "low-pressure atmosphere" mean that the flow is choked at the exit. For steam, from Table B.4, assume *k* = 1.33 and *R* = 461 m^2/s^2-K. Then we are at maximum mass flow:

$$k=1.33:\ \dot{m}_{\max}=[k^{1/2}(\frac{2}{k+1})^{0.5(k+1)/(k-1)}]\frac{p_o\,A^*}{\sqrt{RT_o}}=$$

$$=[0.6726]\,\frac{(300000\,Pa)(0.0005\,m^2)}{\sqrt{(461\ J/kg-K)(600\,K)}}\approx \mathbf{0.192}\,\frac{kg}{s}\quad \textit{Ans.}(a)$$

(*b*) Using EES for real steam, we don't have these nice power-law formulas, but we can use the energy equation and the continuity equation and the fact that the entropy is constant:

$$\text{Energy: } h + \frac{1}{2}V^2 = h_o \quad ; \quad \text{Continuity: } \dot{m} = \rho A V = \text{constant} \; ; \quad s = s_o = \text{constant}$$

First establish s_o from the given p_o and T_o. Then guess a velocity V, perhaps starting at 300 m/s, compute h from energy, then compute $\rho = \rho(h, s_o)$ from the EES thermophysical functions. This enables us to calculate the (guessed) mass flow = $\rho A V$. Is it a maximum? Probably not. Keep changing V until you reach a maximum mass flow. The final result obtained by the writer is

$$\text{At} \quad V = 569\ m/s \ \text{and} \ \rho = 0.683\,kg/m^3, \ \dot{m} = \dot{m}_{\max} = \mathbf{0.191}\ \frac{kg}{s} \quad Ans.(b)$$

The perfect-gas result is very accurate. Steam is nearly ideal in this superheat region. Meanwhile, you could also use the new EES thermophysical function SOUNDSPEED for this problem, monitoring a for each guessed velocity V and computing the Mach number V/a. you would find that, at V = 559 m/s, the mass flow is maximum, and Ma = 1.000 (choked).

9.41 Air, with a stagnation pressure of 100 kPa, flows through the nozzle in Fig. P9.41, which is 2 m long and has an area variation approximated by

$$A \approx 20 - 20x + 10x^2$$

with A in cm^2 and x in m. It is desired to plot the complete family of isentropic pressures $p(x)$ in this nozzle, for the range of inlet pressures $1 < p(0) < 100$ kPa. Indicate those inlet pressures which are not physically possible and discuss briefly. If your computer has an online graphics routine, plot at least 15 pressure profiles; otherwise just hit the highlights and explain.

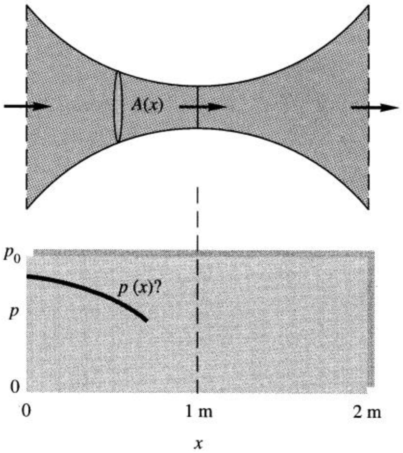


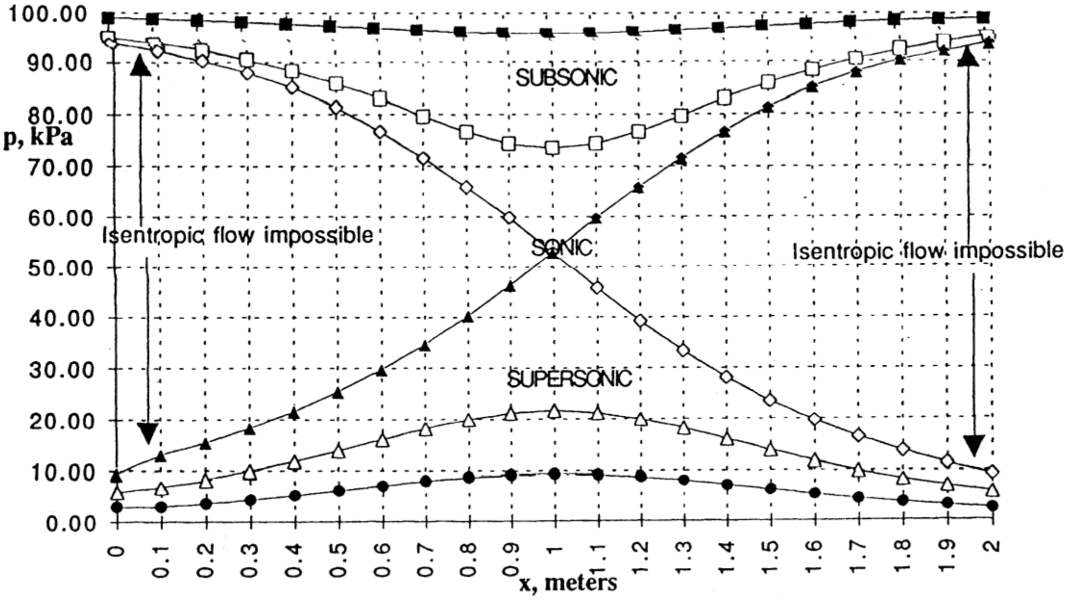
Fig. P9.41

Solution: There is a subsonic entrance region of high pressure and a supersonic entrance region of low pressure, both of which are bounded by a sonic (critical) throat, and both of which have a ratio $A_{x=0}/A^* = 2.0$. From Table B.1 or Eq. (9.44), we find these two conditions to be bounded by

a) subsonic entrance: $A/A^* = 2.0$, $Ma_e \approx 0.306$, $p_e \approx 0.9371p_o \approx$ **93.71 kPa**

b) supersonic entrance: $A/A^* = 2.0$, $Ma_e \approx 2.197$, $p_e \approx 0.09396p_o \approx$ **9.396 kPa**

Thus *no isentropic flow can exist* between entrance pressures $9.396 < p_e < 93.71$ kPa. The complete family of isentropic pressure curves is shown in the graph on the following page. They are **not** easy to find, because we have to convert implicitly from area ratio to Mach number.



9.42 A bicycle tire is filled with air at 169.12 kPa (abs) and 30°C. The valve breaks, and air exhausts into the atmosphere of 100 kPa (abs) and 20°C. The valve exit is 2-mm-diameter and is the smallest area in the system. Assuming one-dimensional isentropic flow, (a) find the initial Mach number, velocity, and temperature at the exit plane. (b) Find the initial mass flow rate. (c) Estimate the exit velocity using the *incompressible Bernoulli equation*. How well does this estimate agree with part (a)?

Solution: (a) Flow is *not* choked, because the pressure ratio is less than 1.89:

$$\frac{p_o}{p} = \frac{169.12}{100} = \left(1 + 0.2Ma_e^2\right)^{3.5}, \quad \text{solve } \mathbf{Ma_e = 0.90}; \quad \text{Read } T_e = 0.8606T_o = \mathbf{261\ K}$$

$$V_e = Ma_e a_e = (0.90)\sqrt{1.4(287)(261)} = 0.90(324) = \mathbf{291\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

(b) Evaluate the exit density at Ma = 0.90 and thence the mass flow:

$$\rho_e = \frac{p_e}{RT_e} = \frac{100000}{287(261)} = 1.335\ \frac{\text{kg}}{\text{m}^3},$$

$$\textit{Then} \quad \dot{m} = \rho_e A_e V_e = (1.335)\frac{\pi}{4}(0.002)^2(291) = \mathbf{0.00122\ \frac{kg}{s}} \quad \textit{Ans. (b)}$$

(c) Assume $\rho = \rho_o = \rho_{tire}$, for how would we know ρ_{exit} if we didn't use compressible-flow theory? Then the incompressible Bernoulli relation predicts

$$\rho_o = \frac{p_o}{RT_o} = \frac{169120}{287(303)} = 1.945\ \frac{\text{kg}}{\text{m}^3}$$

$$V_{e,inc} \approx \sqrt{\frac{2\Delta p}{\rho_o}} = \sqrt{\frac{2(169120-100000)}{1.945}} \approx \mathbf{267}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(c)}$$

This is **8% lower** than the "exact" estimate in part (a).

P9.43 Air flows isentropically through a variable-area duct. At section 1, $A_1 = 20\ \text{cm}^2$, $p_1 = 300$ kPa, $\rho_1 = 1.75\ \text{kg/m}^3$, and $\text{Ma}_1 = 0.25$. At section 2, the area is exactly the same, but the flow is much faster. Compute (*a*) V_2; (*b*) Ma_2; and (*c*) T_2, and (*d*) the mass flow. (*e*) Is there a sonic throat between sections 1 and 2? If so, find its area.

Solution: If the areas are the same but the velocities different, there *must* be a sonic throat in be-tween. (*e*) We can find the throat area A^* right away. For $k = 1.40$, from Eq. (9.45), $\text{Ma}_1 = 0.25$,

$$\frac{A_1}{A^*} = \frac{20cm^2}{A^*} = \frac{1}{Ma_1}\frac{(1+0.2Ma_1{}^2)^3}{1.728} = 2.4027\ ,\ \textit{solve}\quad A^* = A_{throat} = 8.32\ cm^2 \quad \textit{Ans.}(e)$$

(*d*) Compute V_1 and then we can find the mass flow:

$$T_1 = \frac{p_1}{R\rho_1} = \frac{300000}{287(1.75)} = 597\,K\ ;\ V_1 = Ma_1\,a_1 = 0.25\sqrt{1.4(287)(597)} = 123\,m/s$$

$$\textit{Then}\quad \dot{m} = \rho_1 A_1 V_1 = (1.75\frac{kg}{m^3})(0.0020\,m^2)(123\frac{m}{s}) = 0.43\ \frac{kg}{s} \quad \textit{Ans.}(d)$$

$$\textit{We also need}\ \ T_o = T_1(1+0.2Ma_1^2) = (597K)[1+0.2(0.25)^2] = 604K$$

Now go over to section 2 and compute those properties. We have the same area ratio:

$$\frac{A_2}{A^*}\ also\ =\ 2.4027\ ;\quad Table\,B.1:\quad Ma_2\ =\ 2.400\quad Ans.(b)$$

$$T_2 = T_o/(1+0.2Ma_2^2)\ =\ (604\,K)/[1+0.2(2.400)^2]\ =\ 281\,K\qquad Ans.(c)$$

$$V_2\ =\ Ma_2\sqrt{kRT_2}\ =\ (2.400)\sqrt{1.4(287)(281)}\ =\ 806\ \frac{m}{s}\qquad Ans.(a)$$

9.44 In Prob. 3.34 we knew nothing about compressible flow at the time so merely assumed exit conditions $p2$ and $T2$ and computed $V2$ as an application of the continuity equation. Suppose that the throat diameter is 3 in. For the given stagnation conditions in the rocket chamber in Fig. P3.34 and assuming $k = 1.4$ and a molecular weight of 26, compute the actual exit velocity, pressure, and temp-erature according to one-dimensional theory. If pa = 14.7 lbf/in^2 absolute, compute the thrust from the analysis of Prob. 3.68. This thrust is entirely independent of the stagna-tion temperature (check this by changing To to 2000°R if you like). Why?

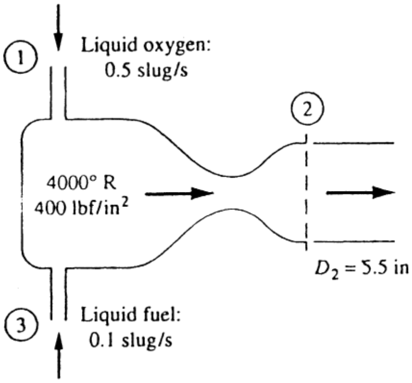


Fig. P3.34

Solution: If **M** = 26, then Rgas = 49720/26 = 1912 ft·lbf/slug°R. Assuming choked flow in the throat (to produce a supersonic exit), the exit area ratio yields the exit Mach number:

$$\frac{A_e}{A^*} = \left(\frac{D_e}{D^*}\right)^2 = \left(\frac{5.5}{3.0}\right)^2 = 3.361,\ \text{whence Eq. 9.45 (for } k = 1.4\text{) predicts Ma}_e \approx \mathbf{2.757}$$

$$\text{Then isentropic }\ p_e = 400/[1+0.2(2.757)^2]^{3.5} \approx \mathbf{15.7\ psia}\quad \textit{Ans.}$$

$$T_e = 4000°\text{R}/[1+0.2(2.757)^2] \approx \mathbf{1587°R}\quad \textit{Ans.}$$

$$\text{Then}\quad V_e = Ma_e\sqrt{kRT_e} = 2.757\sqrt{1.4(1912)(1587)} \approx \mathbf{5680\ ft/s}\quad \textit{Ans.}$$

$$\text{We also need}\quad \rho_e = p_e/RT_e = (15.7\times 144)/[1912(1587)] \approx 0.000747\ \text{slug/ft}^3$$

$$\text{From Prob. 3.68, Thrust}\ \ F = A_e\left[\rho_e V_e^2 + (p_e - p_a)\right],$$

$$\text{or:}\quad F = \frac{\pi}{4}\left(\frac{5.5}{12}\right)^2[0.000747(5680)^2 + (15.7-14.7)\times 144] \approx \mathbf{4000\ lbf}\quad \textit{Ans.}$$

Thrust is independent of T_o because $\rho_e \propto 1/T_o$ and $V_e \propto \sqrt{T_o}$, so T_o cancels out.

P9.45 Air flows isentropically from a large tank through a variable-area nozzle. At section 1, where $A_1 = 12\ \text{cm}^2$, conditions are $p_1 = 20.5$ kPa, $T_1 = 232$ K, and $V_1 = 733$ m/s. Find (*a*) the mass flow; (*b*) Ma_1; (*c*) the throat area, if any; and (*d*) the pressure and temperature in the tank.

Solution: We have enough information at section 1 to find the mass flow:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{20500}{(287)(232)} = 0.308\frac{kg}{m^3}\ ;\ \dot{m} = \rho_1 A_1 V_1 = (0.308)(0.0012)(733) = \mathbf{0.271}\ \frac{kg}{s}\quad Ans.(a)$$

And the information at section 1 also yields the local Mach number:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(232)} = 305\frac{m}{s}\ ;\quad Ma_1 = \frac{V_1}{a_1} = \frac{733\,m/s}{305\,m/s} = \mathbf{2.40}\quad Ans.(b)$$

The local flow is supersonic, so there definitely is a sonic throat:

$$Ma_1 = 2.40:\ \frac{A_1}{A^*} = 2.403\ ,\ A^* = \frac{12\,cm^2}{2.403} \approx \mathbf{5.0}\ cm^2\quad Ans.(c)$$

Finally, the tank (stagnation) conditions are calculated by the isentropic ratios:

$$Ma_1 = 2.40:\ \frac{p_o}{20.5\,kPa} = [1+0.2(2.4)^2]^{3.5} = 14.6\ ,\quad p_o = \mathbf{300}\ kPa$$

$$\frac{T_o}{232\,K} = [1+0.2(2.4)^2] = 2.15,\quad T_o = \mathbf{500}\ K\qquad Ans.(d)$$

P9.46 A one-dimensional isentropic airflow has the following properties at one section where the area is 53 cm^2: p = 12 kPa, ρ = 0.182 kg/m^3, and V = 760 m/s. Determine (a) the throat area; (b) the stagnation temperature; and (c) the mass flow.

Solution: We already have what we need to compute the mass flow:

$$\dot{m} = \rho AV = (0.182\,kg/m^3)(0.0053\,m^2)(760\,m/s) = 0.733\;\;kg/s \qquad Ans.(c)$$

Now we need the Mach number at this section:

$$T = \frac{p}{\rho R} = \frac{12000\;Pa}{(0.182\,kg/m^3)(287\,m^2/s^2K)} = 230\,K$$

$$Ma = \frac{V}{a} = \frac{V}{\sqrt{kRT}} = \frac{760}{\sqrt{1.4(287)(230)}} = \frac{760}{304} = 2.50$$

The flow at this station is *supersonic*; therefore a sonic throat exists. We may now calculate

$$T_o = T(1+0.2Ma^2) = (230K)[1+0.2(2.50)^2] = 518\,K \qquad Ans.(b)$$

$$\frac{A}{A^*} = \frac{(1+0.2Ma^2)^3}{1.728\,Ma} = \frac{[1+0.2(2.50)^2]^3}{1.728(2.50)} = 2.64\;;\; A^* = \frac{0.0053}{2.64} = 0.0020\,m^2 \quad Ans.(a)$$

9.47 In wind-tunnel testing near Mach 1, a small area decrease caused by model blockage can be important. Let the test section area be 1 m^2 and unblocked conditions are Ma = 1.1 and T = 20°C. What model area will first cause the test section to choke? If the model cross-section is 0.004 sq.m., what % change in test-section velocity results?

Solution: First evaluate the unblocked test conditions:

$$\mathrm{T = 293°K,\;\; a = \sqrt{kRT} = \sqrt{1.4(287)(293)} = 343\;\frac{m}{s},\quad \therefore\; V = (1.1)(343) = \mathbf{377}\;\frac{\mathbf{m}}{\mathbf{s}}}$$

$$\text{Also,}\quad \frac{A}{A^*} = \frac{[1+0.2(1.1)^2]^3}{1.728(1.1)} = \underset{\text{(unblocked)}}{1.007925},\quad \text{or}\quad A^* = 1.0/1.007925 \approx \mathbf{0.99214\;m^2}$$

If A is blocked by 0.004 m^2, then A_{new} =1.0 – 0.004 = 0.996 m^2, and now

$$\frac{A_{new}}{A^*} = \frac{0.996}{0.99214} = 1.00389, \quad \text{solve Eq. (9.45) for } \text{Ma(blocked)} \approx \mathbf{1.0696}$$

$$\text{Same } T_o = 364 \text{ K, new } T = 296 \text{ K, new } a = 345 \text{ m/s, new } \mathbf{V} = \text{Ma}(a) \approx \mathbf{369\ \frac{m}{s}} \quad \textit{Ans.}$$

Thus a 0.4% decrease in test section area has caused a 2.1% decrease in test velocity.

9.48 A force F = 1100 N pushes a piston of diameter 12 cm through an insulated cylinder containing air at 20°C, as in Fig. P9.48. The exit diameter is 3 mm, and p_a = 1 atm. Estimate (a) V_e, (b) V_p, and (c) $\dot{m}_e$.

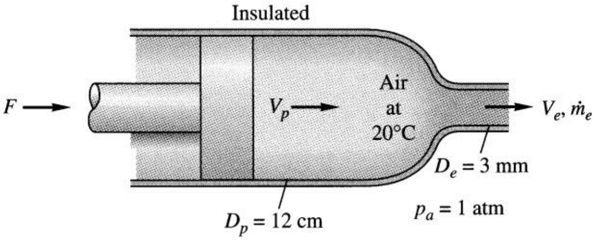


Fig. P9.48

Solution: First find the pressure inside the large cylinder:

$$p_p = \frac{F}{A} + 1\,\text{atm} = \frac{1100}{(\pi/4)(0.12)^2} + 101350 \approx 198600 = 1.96\text{ atm}$$

Since this is greater than (1/0.5283) atm, the small cylinder is ***choked***, and thus

$$V_{exit} = \sqrt{\frac{2k}{k+1}RT_o} = \sqrt{\frac{2(1.4)}{1.4+1}(287)(293)} \approx \mathbf{313\ m/s} \quad \textit{Ans.}\text{ (a)}$$

$$V_{piston} = (\rho_e/\rho_p)(A_e/A_p)V_e = (0.6339)(0.003/0.12)^2(313) = \mathbf{0.124\ m/s} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Finally,} \quad \dot{m} = \dot{m}_{max} = 0.6847\frac{(198600)(\pi/4)(0.003)^2}{\sqrt{287(293)}} \approx \mathbf{0.00331\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (c)}$$

The mass flow increases with F, but the piston velocity and exit velocity are independent of F if the exit flow is choked.

9.49 Consider the venturi nozzle of Fig. 6.40*c*, with $D = 5$ cm and $d = 3$ cm. Air stagnation temperature is 300 K, and the upstream velocity $V1 = 72$ m/s. If the throat pressure is 124 kPa, estimate, with isentropic flow theory, (a) $p1$; (b) $Ma2$; and (c) the mass flow.

Solution: Given one-dimensional isentropic flow of air. The problem looks sticky—sparse, scattered information, implying laborious iteration. But the energy equation yields $V1$ and Ma1:

$$T_o = T_1 + \frac{V_1^2}{2c_p} = 300\,\text{K} = T_1 + \frac{(72\text{ m/s})^2}{2(1005\,\text{J/kg·K})}, \quad \text{solve for } T_1 = 297.4\,\text{K}$$

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{72}{\sqrt{1.4(287)(297.4)}} = \frac{72\,\text{m/s}}{346\,\text{m/s}} = 0.208$$

Area-ratio calculations will then yield A^* and $Ma2$ and then po and $p1$:

$$\frac{A_1}{A^*} = \frac{(\pi/4)(0.05\,\text{m})^2}{A^*} = \frac{\left(1+0.2\,Ma_1^2\right)^3}{1.728\,Ma_1} = \frac{[1+0.2(0.208)^2]^3}{1.728(0.208)} = 2.85,$$

$$\text{Solve} \quad A^* = 0.0006886\text{ m}^2$$

$$\frac{A_2}{A^*} = \frac{(\pi/4)(0.03\text{ m})^2}{0.0006886\text{ m}^2} = \frac{\left(1+0.2Ma_2^2\right)^3}{1.728Ma_2} = 1.027, \quad \text{Solve } \boldsymbol{Ma_2 = 0.831} \quad \textit{Ans.}\text{ (b)}$$

$$p_o = p_2\left(1+0.2\,Ma_2^2\right)^{3.5} = (124\,\text{kPa})[1+0.2(0.831)^2]^{3.5} = 195\,\text{kPa}$$

$$p_1 = p_o\Big/\left(1+0.2\,Ma_1^2\right)^{3.5} = (195\,\text{kPa})/[1+0.2(0.208)^2]^{3.5} = \mathbf{189\ kPa} \quad \textit{Ans.}\text{ (a)}$$

The mass flow follows from any of several formulas. For example:

$$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_1}{RT_1}\right)A_1V_1 = \left[\frac{189000}{287(297.4)}\right]\left(\frac{\pi}{4}\right)(0.05)^2(72) = \mathbf{0.313}\ \frac{\textbf{kg}}{\textbf{s}} \quad \textit{Ans.}\text{ (c)}$$

P9.50 Carbon dioxide is stored in a tank at 100 kPa and 330 K. It discharges to a second tank through a converging nozzle whose exit area is 5 cm^2. What is the initial mass flow rate if the second tank has a pressure of (*a*) 70 kPa, or (*b*) 40 kPa?

Solution: For CO_2, from Table B.4, $R = 189\ m^2/s^2$-K and $k = 1.30$. Calculate the pressure p^* if the flow is choked at the exit:

$$\frac{p_o}{p^*} = [1+\frac{(k-1)}{2}(1.0)^2]^{k/(k-1)} = [1+\frac{(1.3-1)}{2}(1.0)^2]^{1.3/0.3} = 1.8324$$

$$\text{Hence, for this case, } p^* = \frac{100\,kPa}{1.8324} = 54.6\,kPa$$

Thus, for case (*a*), $p_b = 70$, the nozzle is *not* choked. For case (*b*), $p_b = 40$, it *is* choked. (*a*) We could plow through the various exit properties and finally find the mass flow:

$$p_e = 70\,kPa\ ;\ Ma_e = 0.756\ ;\ \rho_e = 1.22\frac{kg}{m^3}\ ;\ a_e = 273\frac{m}{s}\ ;\ V_e = Ma_a\,a_e = 207\frac{m}{s}$$

$$\dot{m} = \rho_e A_e V_e = (1.22)(0.0005)(207) \approx \mathbf{0.126}\ kg/s \quad Ans(a)$$

Or we could do it all in one step with the handy mass flow function, Eq.(9.47), $p/p_o = 0.546$:

$$\frac{\dot{m}\sqrt{RT_o}}{A_e\,p_o} = \sqrt{\frac{2(1.3)}{1.3-1}(0.546)^{2/1.3}[1-(0.546)^{0.3/1.3}]} = 0.629$$

$$\text{Hence}\quad \dot{m} = 0.629\frac{(0.0005)(100,000)}{\sqrt{(189)(330)}} = \mathbf{0.126}\ \frac{kg}{s} \quad Ans.(a)$$

(*b*) For an outside pressure of only 40 kPa < 54.6 kPa, the exit is choked:

$$\dot{m} = \dot{m}_{\max} = [k^{1/2}(\frac{2}{k+1})^{0.5(k+1)/(k-1)}]\frac{p_o\,A^*}{\sqrt{RT_o}} =$$

$$= [0.6673]\frac{(100,000)(0.0005)}{\sqrt{(189)(330)}} = \mathbf{0.134}\ \frac{kg}{s} \quad Ans.(b)$$

P9.51 The scramjet engine of Fig. 9.30 is supersonic throughout. A sketch is shown in Fig. C9.8. Test the following design. The flow enters at Ma = 7 and air properties for 10,000 m altitude. Inlet area is 1 m^2, the minimum area is 0.1 m^2, and the exit area is 0.8 m^2. If there is no combustion, (*a*) will the flow still be supersonic in the throat? Also, determine (*b*) the exit Mach number, (*c*) exit velocity, and (*d*) exit pressure.

Solution: From Table B.6 at 10,000 m, read $p_1 = 26416$ Pa, $T_1 = 223.16$ K, and $\rho_1 = 0.4125\ kg/m^3$. Establish area ratio and stagnation conditions at the inlet, section 1:

$$Ma_1 = 7:\ \frac{A_1}{A^*} = 104.1\ ;\ \ p_{o1} = (26416)[1+0.2(7)^2]^{3.5} = 1.094\text{E}8\ Pa$$

$$T_{o1} = (223.16)[1+0.2(7)^2] = 2410\ K$$

Now, assuming isentropic flow (no combustion), work your way through the area ratios:

$$A_2 = 0.1m^2:\ A_2/A^* = (104.1)(0.1m^2/1.0m^2) = 10.41,\ \text{for which}\ \ Ma_{throat} = \mathbf{3.97}\ \ Ans.(a)$$

$$A_4 = 0.8m^2:\ A_4/A^* = (104.1)(0.8m^2/1m^2) = 83.28,\ \ \text{for which}\ \ Ma_4 = \mathbf{6.655}\ \ \ Ans.(b)$$

$$T_4 = 2410K/[1+0.2(6.655)^2] = 244K,\ V_4 = Ma_a\, a_4 = (6.655)\sqrt{1.4(287)(244)} = \mathbf{2080}\ m/s\ \ Ans.(c)$$

$$p_4 = 1.094\text{E}8\,Pa/[\{1+0.2(6.655)^2\}^{3.5}] = \mathbf{36,400}\ Pa\qquad Ans.(d)$$

9.52 A converging-diverging nozzle exits smoothly to sea-level standard atmosphere. It is supplied by a 40-m^3 tank initially at 800 kPa and 100°C. Assuming isentropic flow, estimate(a) the throat area; and (b) the tank pressure after 10 sec of operation. The exit area is 10 cm^2.

Solution: The phrase "exits smoothly" means that exit pressure = atmospheric pressure, which is 101 kPa. Then the pressure ratio specifies the exit Mach number:

$$\text{p}_\text{o}/\text{p}_\text{exit} = \frac{800}{101} = \left[1+0.2\,\text{Ma}_\text{e}^2\right]^{3.5},\quad \text{solve for}\ \ \mathbf{Ma_{exit} \approx 2.01}$$

$$\text{Thus}\quad \text{A}_\text{e}/\text{A}^* = 1.695\quad \text{and}\quad \text{A}^* = (10\ \text{cm}^2)/1.695 \approx \mathbf{5.9\ cm^2}\quad Ans.\text{(a)}$$

$$\text{Further,}\quad \dot{\text{m}} = \dot{\text{m}}_\text{max} = 0.6847(800000)(0.00059)\big/\sqrt{287(373)} \approx 0.99\ \text{kg/s}$$

The initial mass in the tank is quite large because of large volume and high pressure:

$$\rho_\text{o} = \frac{\text{p}_\text{o}}{\text{RT}_\text{o}} = \frac{800000}{287(373)} \approx 7.47\ \frac{\text{kg}}{\text{m}^3},\quad \text{thus}\ \ \text{m}_{\text{tank},t=0} = \rho\upsilon = (7.47)(40) \approx \mathbf{299\ kg}$$

After 10 sec, blowing down at 0.99 kg/s, we have about 299 − 10 ≈ 289 kg left in the tank. The pressure will drop to about 800(289/299) ≈ **773 kPa**. *Ans.* (b).

9.53 Air flows steadily from a reservoir at 20°C through a nozzle of exit area 20 cm^2 and strikes a vertical plate as in Fig. P9.53. The flow is subsonic throughout. A force of 135 N is required to hold the plate stationary. Compute (a) V_e, (b) Ma$_e$, and (c) p_0 if p_a = 101 kPa.

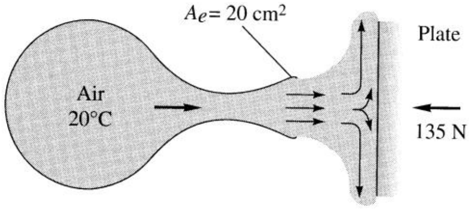


Fig. P9.53

Solution: Assume p_e = 1 atm. For a control volume surrounding the plate, we deduce that

$$F = 135\,\text{N} = \rho_e V_e^2 A_e = kp_e \text{Ma}_e^2 A_e = 1.4(101350)(0.002\ \text{m}^2)\text{Ma}_e^2,$$

$$\text{or}\quad \mathbf{Ma_e \approx 0.69}\quad \textit{Ans.}\ \text{(b)}$$

$$T_e = 293/[1+0.2(0.69)^2] \approx 268\ \text{K},\quad a_e = \sqrt{1.4(287)(268)} = 328\ \frac{\text{m}}{\text{s}},\ \text{thus}$$

$$V_e = a_e \text{Ma}_e = (328)(0.69) \approx \mathbf{226\ m/s}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{Finally,}\quad p_{\text{tank}} = p_o = 101350[1+0.2(0.69)^2]^{3.5} \approx \mathbf{139000\ Pa}\quad \textit{Ans.}\ \text{(c)}$$

P9.54 The airflow in Prob. P9.46 undergoes a normal shock just past the section where data was given. Determine the (*a*) Mach number, (*b*) pressure, and (*c*) velocity just downstream of the shock. [Recall the given data: the area is 53 cm^2, p = 12 kPa, ρ = 0.182 kg/m^3, and V = 760 m/s.]

Solution: In Prob. P9.46 we found that the local Mach number at the section was 2.5.

Table B.2: Ma$_2$ = **0.513** *Ans.*(*a*) ;

p_2/p_1 = 7.125 , hence p_2 = 7.125*12000 = **85500 Pa** *Ans.*(*b*)

V_1/V_2 = ρ_2/ρ_1 = 3.333 hence V_2 = (760)/(3.333) = **228 m/s** *Ans.*(*c*)

The shock results were quite easy – finding the upstream Mach number was the hard part.

9.55 Air, supplied by a reservoir at 450 kPa, flows through a converging-diverging nozzle whose throat area is 12 cm^2. A normal shock stands where $A_1 = 20\ \text{cm}^2$. (a) Compute the pressure just downstream of this shock. Still farther downstream, where $A_3 = 30\ \text{cm}^2$, estimate (b) p_3; (c) A_3*; and (d) Ma_3.

Solution: If a shock forms, the throat must be **choked** (sonic). Use the area ratio at (1):

$$\frac{A_1}{A^*} = \frac{20}{12} = 1.67, \quad \text{or} \quad Ma_1 \approx 1.985, \quad \text{whence } p_1 = \frac{450}{[1+0.2(1.985)^2]^{3.5}} \approx 59\ \text{kPa}$$

$$\text{Then, across the shock,} \quad \frac{p_2}{p_1} = \frac{2.8(1.985)^2 - 0.4}{2.4} = 4.43,$$

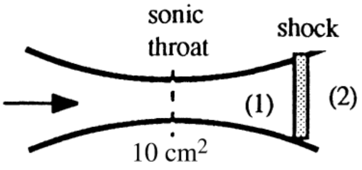
$$p_2 = 4.43(59) \approx \mathbf{261\ kPa} \quad \textit{Ans.}\ (a)$$

$$\text{Across the shock, at } Ma_1 = 1.985, \quad \frac{A_2^*}{A_1^*} = 1.374, \quad A_2^* = 1.374(12) \approx \mathbf{16.5\ cm^2} \quad \textit{Ans.}\ (c)$$

$$\text{At } A_3 = 30\ \text{cm}^2, \ \frac{A_3}{A_2^*} = \frac{30}{16.5} = 1.82, \quad \text{whence subsonic } Ma_3 \approx \mathbf{0.34} \quad \textit{Ans.}\ (d)$$

$$\text{Finally,} \quad p_{o2} = \frac{p_{o1}}{1.374} = 328\ \text{kPa}, \quad p_3 = \frac{328}{[1+0.2(0.34)^2]^{3.5}} \approx \mathbf{303\ kPa} \quad \textit{Ans.}\ (b)$$

9.56 Air from a reservoir at 20°C and 500 kPa flows through a duct and forms a normal shock downstream of a throat of area 10 cm^2. By an odd coincidence it is found that the stagnation pressure downstream of this shock exactly equals the throat pressure. What is the area where the shock wave stands?



Solution: If a shock forms, the throat is **sonic**, $A^* = 10\ \text{cm}^2$. Now

$$p_1^* = 0.5283 p_{o1} = 0.5283(500) \approx \mathbf{264\ kPa} = p_{o2} \quad \text{also}$$

$$\text{Then} \quad \frac{p_{o2}}{p_{o1}} = \frac{264}{500} = 0.5283: \quad \text{Table B.2, read } Ma_1 \approx 2.43$$

$$\text{So} \quad A_1/A_1^* = \frac{[1+0.2(2.43)^2]^{3.0}}{1.728(2.43)} \approx 2.47, \quad \text{or} \quad A_1(\text{at shock}) = 2.47(10) \approx \mathbf{24.7\ cm^2} \quad \textit{Ans.}$$

9.57 Air flows from a tank through a nozzle into the standard atmosphere, as in Fig. P9.57. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the tank pressure; and (b) the mass flow.

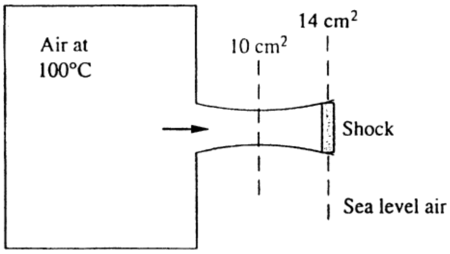


Fig. P9.57

Solution: The throat must be *sonic*, and the area ratio at the shock gives the Mach number:

$$A_1/A^* = \frac{14}{10} = 1.4 = \frac{\left[1+0.2Ma_1^2\right]^3}{1.728Ma_1}, \quad \text{solve } Ma_1 \approx 1.76 \text{ upstream of the shock}$$

$$\text{Then} \quad p_2/p_1|_{shock} = \frac{2.8(1.76)^2 - 0.4}{2.4} \approx 3.46, \quad p_2 = 1 \text{ atm}, \quad p_1 = \frac{101350}{3.46} \approx 29289 \text{ Pa}$$

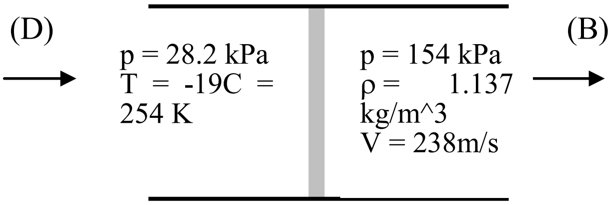
$$\text{Thus} \quad p_{tank} = p_{o1} = 29289[1+0.2(1.76)^2]^{3.5} \approx \mathbf{159100\ Pa} \quad \textit{Ans.}\text{(a)}$$

Given that To = 100°C = 373 K and a critical throat area of 10 cm^2, we obtain

$$\dot{m} = \dot{m}_{max} = 0.6847p_oA^*/\sqrt{RT_o} = 0.6847(159100)(0.001)/\sqrt{287(373)}$$

$$\approx \mathbf{0.333\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (b)}$$

P9.58 The data of Prob. P9.4 represent conditions before and after a normal shock wave. If the velocity at section B is 238 m/s, what are (*a*) the velocity at D; (*b*) the stagnation temperature, (*c*) the stagnation pressure at D, and (*d*) the Mach number at section D?



Solution: Assume *k* = 1.4. From Prob. P9.4, we found that section D is *upstream*. The pressure ratio will give us the upstream Mach number Ma$_D$:

$$\frac{p_B}{p_D} = \frac{154,000Pa}{28,200\,Pa} = 5.46\,; \quad \text{Table B.1 or Eq.(9.54)}: \; Ma_D \approx \mathbf{2.20} \quad \textit{Ans.(d)}$$

With the upstream Mach number known, we can find the stagnation pressure there:

$$p_{0D} = p_B(1+0.2Ma_D^2)^{3.5} = (28.2\,kPa)[1+0.2(2.20)^2]^{3.5} = \mathbf{300}\ kPa \quad Ans.(c)$$

The downstream stagnation pressure (not asked) is much less, 189 kPa. Similarly, given T_D = -19°C = 254 K, we can calculate the stagnation temperature (good for both sides):

$$T_{0D} = T_D(1+0.2Ma_D^2) = (254K)[1+0.2(2.20)^2] = \mathbf{500}\ K = T_{0B} \quad Ans.(b)$$

Finally, we could compute V_D from the ratio across the shock, Eq. (9.58),

$$\frac{V_D}{V_B} = \frac{(k+1)Ma_D^2}{(k-1)Ma_D^2+2} = \frac{2.4(2.20)^2}{(0.4)(2.20)^2+2} = 2.95 = \frac{V_D}{238}, \quad V_D = \mathbf{702}\frac{m}{s} \quad Ans.(a)$$

Alternately, $V_D = Ma_D(kRT_D)^{1/2} = (2.20)[1.4(287)(254)]^{1/2} = (2.20)(319) = 702$ m/s.

9.59 Air, at stagnation conditions of 450 K and 250 kPa, flows through a nozzle. At section 1, where the area = 15 cm^2, there is a normal shock wave. If the mass flow is 0.4 kg/s, estimate (a) the Mach number; and (b) the stagnation pressure just downstream of the shock.

Solution: If there is a shock wave, then the mass flow is maximum:

$$\dot{m}_{max} = 0.4\ \frac{\text{kg}}{\text{s}} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{250000A^*}{\sqrt{287(450)}}, \quad \text{solve } A^* = 0.000840\ \text{m}^2$$

$$\text{Then}\quad \frac{A_1}{A^*} = \frac{0.0015}{0.00084} = 1.786 \quad \text{Table B.1:}\quad \text{Read } Ma_{1,upstream} \approx 2.067$$

$$\text{Finally, from Table B.2,}\quad \text{read } \mathbf{Ma_{1,\,downstream} \approx 0.566} \quad Ans.\text{ (a)}$$

$$\text{Also, Table B.2:}\quad \frac{p_{o2}}{p_{o1}} = 0.690, \quad \mathbf{p_{0,downstream}} = 0.690(250) \approx \mathbf{172\ kPa} \quad Ans.\text{ (b)}$$

9.60 When a pitot tube such as Fig. (6.30) is placed in a supersonic flow, a normal shock will stand in front of the probe. Suppose the probe reads p_O = 190 kPa and p = 150 kPa. If the stagnation temperature is 400 K, estimate the (supersonic) Mach number and velocity upstream of the shock.

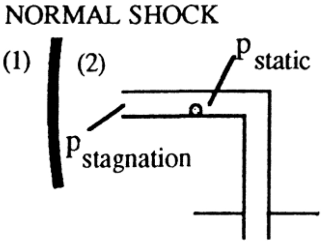


Fig. P9.60

Solution: We can immediately find Ma *inside* the shock:

$$p_{o2}/p_2 = \frac{190}{150} = 1.267 = \left(1+0.2\mathrm{Ma}_2^2\right)^{3.5}, \quad \text{solve } \mathrm{Ma}_2 \approx 0.591$$

$$\text{Then, across the shock,} \quad \mathrm{Ma}_1^2 = \frac{0.4(0.591)^2+2}{2.8(0.591)^2-0.4}, \quad \text{solve } \mathbf{Ma_1 \approx 1.92} \quad \mathit{Ans.}$$

$$T_1 = \frac{400}{[1+0.2(1.92)^2]} = 230\text{ K}, \quad a_1 = \sqrt{1.4(287)(230)} \approx 304\text{ m/s},$$

$$V_1 = \mathrm{Ma}_1 a_1 = (1.92)(304) \approx \mathbf{585\ m/s} \quad \mathit{Ans.}$$

P9.61 Air flows from a large tank, where *T* = 376 K and *p* = 360 kPa, to a design condition where the pressure is 9800 Pa. The mass flow is 0.9 kg/s. However, there is a normal shock in the exit plane just after this condition is reached. Estimate (*a*) the throat area; and, just downstream of the shock, (*b*) the Mach number, (*c*) the temperature, and (*d*) the pressure.

Solution: The low design pressure definitely indicates a supersonic condition:

$$\frac{p}{p_o} = \frac{9800}{360000} = 0.0272 = (1+0.2Ma_1^2)^{-3.5}, \quad solve\ for \quad Ma_1 = 3.00$$

$$Choked: \dot{m} = \dot{m}_{max} = 0.6847\frac{p_oA^*}{\sqrt{RT_o}} = 0.6847\frac{(360000)A^*}{\sqrt{287(376)}}, \quad Solve \quad A^* = 0.0012\,m^2 \quad Ans.(a)$$

Find T_1 in front of the shock and then use the normal shock conditions:

$$T_1 = T_o/(1+0.2Ma_1^2) = (376K)/[1+0.2(3.0)^2] = 134\,K$$

$$\textit{Table B.2, } Ma_1{=}3.0: \quad Ma_2 = 0.475 \quad \textit{Ans.}(b)$$

$$T_2/T_1 = 2.679\,, \quad T_2 = (2.679)(134K) = 360\,K \quad \textit{Ans.}(c)$$

$$p_2/p_1 = 10.333\,, \quad p_2 = (10.333)(9800) = 101{,}300\,Pa \quad \textit{Ans.}(d)$$

The problem worked out so that the external pressure was 1 atm.

9.62 An atomic explosion propagates into still air at 14.7 psia and 520°R. The pressure just inside the shock is 5000 psia. Assuming *k* = 1.4, what are the speed C of the shock and the velocity V just inside the shock?

Solution: The pressure ratio tells us the Mach number of the shock motion:

$$\mathrm{p_2/p_1} = \frac{5000}{14.7} = 340 = \frac{2.8\mathrm{Ma}_1^2 - 0.4}{2.4}, \quad \text{solve for } \mathrm{Ma}_1 \approx \mathbf{17.1}$$

$$\mathrm{a}_1 = \sqrt{1.4(1717)(520)} = 1118 \text{ ft/s}, \quad \therefore\ \mathrm{V}_1 = \mathrm{C} = 17.1(1118) \approx \mathbf{19100\ \frac{ft}{s}} \quad \textit{Ans.}\text{ (a)}$$

We then compute the velocity ratio across the shock and thence the relative motion inside:

$$\mathrm{V_2/V_1} = \frac{0.4(17.1)^2 + 2}{2.4(17.1)^2} = 0.1695, \quad \therefore \quad \mathrm{V}_2 = 0.1695(19100) = 3240 \text{ ft/s}$$

$$\text{Then} \quad \mathrm{V_{inside}} = \mathrm{C} - \mathrm{V}_2 = 19100 - 3240 \approx \mathbf{15900\ ft/s} \quad \textit{Ans.}\text{ (b)}$$

9.63 Sea-level standard air is sucked into a vacuum tank through a nozzle, as in Fig. P9.63. A normal shock stands where the nozzle area is 2 cm^2, as shown. Estimate (a) the pressure in the tank; and (b) the mass flow.

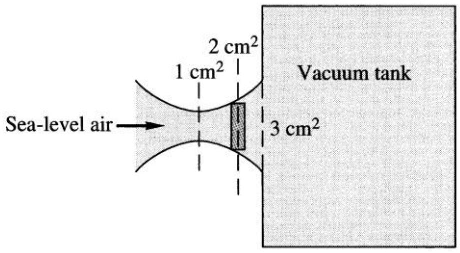


Fig. P9.63

Solution: The flow at the exit section ("3") is *subsonic* (after a shock) therefore must equal the tank pressure. Work our way to 1 and 2 at the shock and thence to 3 in the exit:

$$p_{01} = 101350 \text{ Pa}, \quad A_1/A^* = 2.0, \quad \text{thus } Ma_1 \approx 2.1972, \quad p_1 = \frac{101350}{[1+0.2(2.2)^2]^{3.5}} \approx 9520 \text{ Pa}$$

$$\frac{p_2}{p_1} = \frac{2.8(2.2)^2 - 0.4}{2.4} = 5.47, \quad \therefore\ p_2 = 5.47(9520) \approx 52030 \text{ Pa}$$

$$\text{Also compute} \quad A_2^*/A_1^* \approx 1.59, \quad \text{or} \quad A_2^* = \mathbf{1.59\ cm^2}$$

Also compute p_{o2} = 101350/1.59 = 63800 Pa. Finally compute $A_3/A_2^* = 3/1.59 = 1.89$, read Ma_3 = 0.327, whence $p_3 = 63800/[1 + 0.2(0.327)^2]^{3.5} \approx$ **59200 Pa**. *Ans.* (a).
With T_o = 288 K, the (critical) mass flow = $0.6847 p_o A^*/\sqrt{(RT_o)}$ = **0.0241 kg/s**. *Ans.* (b)

P9.64 Air, from a reservoir at 350 K and 500 kPa, flows through a converging-diverging nozzle. The throat area is 3 cm^2. A normal shock appears, for which the downstream Mach number is 0.6405. (*a*) What is the area where the shock appears? Calculate (*b*) the pressure and (*c*) the temperature downstream of the shock.

Solution: This Ma_2 = 0.6405 occurs right in Table B.2, and we read Ma_1 = 1.70. (*a*) Hence, for isentropic flow up to that point, from Eq. (9.45),

$$\frac{A}{A*} = (\frac{1}{1.70})[\frac{1+0.2(1.70)^2]^3}{1.728} = 1.3376\ , \quad A_{shock} = (1.3376)(3cm^2) = \mathbf{4.0}\ cm^2 \quad Ans.(a)$$

(*b*) Calculate pressure before and after the shock:

Read Table B.1: $\dfrac{p_1}{p_o} = 0.2026\,,\ p_1 = (0.2026)(500\,kPa) = 101.3\ kPa$

Read Table B.2 at $\text{Ma}_1 = 1.7$: $\dfrac{p_2}{p_1} = 3.205;\ p_2 = (3.205)(101.3) = \mathbf{325}\ kPa$ *Ans.*(*b*)

(*c*) Do the same thing as (*b*) to determine temperature downstream of the shock:

Read Table B.1: $\dfrac{T_1}{T_o} = 0.6337\,,\ p_1 = (0.6337)(350\,K) = 222\,K$

Read Table B.2 at $\text{Ma}_1 = 1.7$: $\dfrac{T_2}{T_1} = 1.4583;\ T_2 = (1.4583)(222) = \mathbf{323}\,K$ *Ans.*(*c*)

9.65 Air flows through a converging-diverging nozzle between two large reservoirs, as in Fig. P9.65. A mercury manometer reads *h* = 15 cm. Estimate the downstream reservoir pressure. Is there a shock wave in the flow? If so, does it stand in the exit plane or farther upstream?

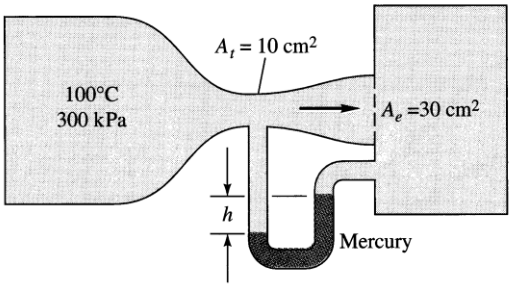


Fig. P9.65

Solution: The manometer reads the pressure drop between throat and exit tank:

$$p_{\text{throat}} - p_{\text{tank\#2}} = (\rho_{\text{mercury}} - \rho_{\text{air}})gh \approx (13550-0)(9.81)(0.15) \approx 19940\ \text{Pa}$$

The lowest possible $p_{\text{throat}} = p^* = 0.5283(300) = 158.5\ \text{kPa}$, for which $p_e \approx 138.5\ \text{kPa}$

But **this** pe is much lower than would occur in the duct for isentropic subsonic flow.

We can check also to see if isentropic *supersonic* flow is a possibility: With Ae/A* = 3.0, the exit Mach number would be 2.64, corresponding to pe = 0.047po ≈ 14 kPa (?). This is much too low, so that case fails also.

Suppose we had supersonic flow with a normal shock wave in the exit plane:

$$A_e/A^* = 3.0,\ \text{Ma}_e \approx 2.64,\ p_e = 14\ \text{kPa},\ \frac{p_{\text{tank\#2}}}{p_e} = \frac{2.8(2.64)^2 - 0.4}{2.4} = 7.95,$$

or: $p_{\text{tank\#2}} = 7.95(14) \approx 113\ \text{kPa}$, compared to p_{tank}(manometer reading) $\approx 138.5\ \text{kPa}$

This doesn't match either, the flow expanded too much before the shock wave. Therefore the correct answer is: a **normal shock wave upstream of the exit plane**. *Ans.*

9.66 In Prob. 9.65 what would be the mercury manometer reading if the nozzle were operating exactly at supersonic "design" conditions?

Solution: We worked out this idealized isentropic-flow condition in Prob. 9.65:

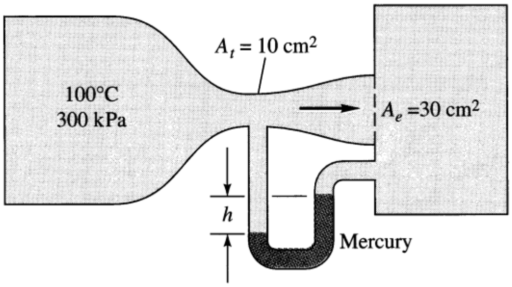


Fig. P9.65

$$\text{Design flow: } \frac{A_e}{A^*} = 3.0, \quad Ma_e = 2.64, \quad p_e = 14\text{ kPa} = p_{tank\,\#2}; \quad p^* = p_{throat} = 158.5\text{ kPa}$$

$$\text{Then } \quad p_t - p_e = 158500 - 14200 = 144300 \approx (13550 - 0)(9.81)h,$$

$$\text{solve} \quad \mathbf{h \approx 1.09\ m} \quad \textit{Ans.}$$

P9.67 A supply tank at 500 kPa and 400 K feeds air to a converging diverging nozzle whose throat area is 9 cm^2. The exit area is 46 cm^2. State the conditions in the nozzle if the pressure outside the exit plane is (*a*) 400 kPa; (*b*) 120 kPa; and (*c*) 9 kPa. (*d*) In each of these cases, find the mass flow.

Solution: For reference purposes, find the design (isentropic supersonic) condition for this nozzle:

$$\frac{A_e}{A^*}\Big|_{design} = \frac{46\,cm^2}{9\,cm^2} = 5.111 \ ; \quad \text{Table B.1: Read} \quad Ma_{design} = 3.20$$

$$p_{exit,design} = \frac{p_o}{(1+0.2Ma_e^2)^{3.5}} = \frac{500{,}000\,Pa}{[1+0.2(3.20)^2]^{3.5}} = 10140\ Pa$$

This immediately establishes what happens in case (*c*):

$p_{\text{outside}} = 9000\text{ Pa} < p_{\text{design}}$, hence **supersonic expansion outside the exit** (Case I, Fig. 9.12)

Now determine the outside pressure if there is a normal shock in the exit plane:

$$Ma_1 = 3.20; \quad \textit{Table B.2}: \quad p_2/p_1 = 11.78 \ , \quad \textit{hence} \quad p_2 = (11.78)(10140) = 119{,}500\,Pa$$

This is close enough to case (*b*), a back pressure of 120 kPa, that we can say there is **a normal shock in the exit**. (case F, Fig. 9.12) *Ans.*(*b*)

Finally, check the situation if the nozzle downstream of the throat is subsonic-isentropic:

$$\frac{A_e}{A^*}\Big|_{subsonic} = \frac{46\,cm^2}{9\,cm^2} = 5.111 \;\; ; \;\; \text{Table B.1: Read} \quad Ma_{subsonic} = 0.114$$

$$p_{exit,subsonic} = \frac{p_o}{(1+0.2Ma_e^2)^{3.5}} = \frac{500{,}000\,Pa}{[1+0.2(0.114)^2]^{3.5}} = 495{,}000\,Pa$$

This is greater than case (*a*), a postulated back pressure of 450,000 Pa.

Thus case (*a*) will result in **a normal shock just downstream of the throat**. (Case D, Fig. 9.12)

In all three cases, the throat is *choked*, hence the mass flow is maximum:

$$\dot{m} = \dot{m}_{\max} = 0.6847\frac{p_o\,A^*}{\sqrt{RT_o}} = 0.6847\frac{(500000)(0.0009)}{\sqrt{(287)(400\,K)}} = \mathbf{0.909\frac{kg}{s}} \quad Ans.(a,b,c)$$

9.68 Air in a tank at 120 kPa and 300 K exhausts to the atmosphere through a 5-cm^2-throat converging nozzle at a rate of 0.12 kg/s. What is the atmospheric pressure? What is the maximum mass flow possible at low atmospheric pressure?

Solution: Let us answer the second question first, to see where 0.12 kg/s stands:

$$\dot{m}_{max} = \frac{0.6847p_oA^*}{\sqrt{RT_o}} = \frac{0.6847(120000)(0.0005)}{\sqrt{287(300)}}$$

$$\approx \mathbf{0.140\ kg/s} \quad \textit{Ans.}\ \text{(b)} \;\; (\text{if } p_{atm} < 63\ \text{kPa})$$

So the given mass flow is about 86% of maximum and patm > 63 kPa. We could just go at it, guess the exit pressure and iterating, or we could express it more elegantly:

$$\dot{m} = \rho AV = \frac{\rho_o}{(1+0.2Ma^2)^{2.5}}A\,Ma\sqrt{kR}\sqrt{\frac{T_o}{1+0.2Ma^2}} = \frac{\text{Const Ma}}{(1+0.2Ma^2)^3},$$

where Const ≈ 0.2419 in SI units. If $\dot{m} = 0.12$ kg/s, we thus solve for Ma:

$$Ma \approx 0.496(1+0.2Ma^2)^3 \quad \text{to obtain } Ma \approx 0.62,\; p_{atm} \approx \mathbf{92.6\ kPa} \quad \textit{Ans.}$$

9.69 With reference to Prob. 3.68, show that the thrust of a rocket engine exhausting into a vacuum is given by

$$F = \frac{p_0 A_e\left(1 + k\,\mathrm{Ma}_e^2\right)}{\left(1 + \frac{k-1}{2}\mathrm{Ma}_e^2\right)^{k/(k-1)}}$$

where A_e = exit area

Ma_e = exit Mach number

p_0 = stagnation pressure in combustion chamber

Note that stagnation temperature does not enter into the thrust.

Solution: In a vacuum, $p_{atm} = 0$, the solution to Prob. 3.68 is

$$\mathrm{F} = \rho_e \mathrm{A_e V_e^2} + \mathrm{A_e}(\mathrm{p_e} - 0) = \mathrm{A_e}\left(\mathrm{p_e} + \rho_e \mathrm{V_e^2}\right),$$

$$\text{but } \rho_e \mathrm{V_e^2} \equiv \mathrm{kp_e Ma_e^2}, \quad \text{hence } \mathrm{F} = \mathrm{p_e A_e}\left(1 + \mathrm{kMa_e^2}\right)$$

$$\text{For isentropic flow, } \mathrm{p_e} = \mathrm{p_o} \Big/ \left(1 + \frac{\mathrm{k}-1}{2}\mathrm{Ma_e^2}\right)^{\mathrm{k/(k-1)}}, \ \therefore\ \mathbf{F} = \frac{\mathbf{p_o A_e\left(1 + kMa_e^2\right)}}{\left(\mathbf{1} + \frac{\mathbf{k-1}}{\mathbf{2}}\mathbf{Ma_e^2}\right)^{\mathbf{k/(k-1)}}} \quad \textit{Ans.}$$

P9.70 Air, with p_o = 500 kPa and T_o = 600 K, flows through a converging-diverging nozzle. The exit area is 51.2 cm^2, and the mass flow is 0.825 kg/s. What is the highest possible back pressure that will still maintain supersonic flow inside the diverging section?

Solution: Naturally assume isentropic flow with k = 1.40. If the flow is to be supersonic, there must be a choked throat. Find its area:

$$\dot{m} = \dot{m}_{\max} = 0.6847\frac{p_o\,A^*}{\sqrt{RT_o}} = 0.6847\frac{(500000)\,A^*}{\sqrt{(287)(600)}} = 0.825\ \frac{kg}{s}$$

$$\textit{Solve for}\quad A^* = A_{throat} = 0.0010\,m^2 = 10\,cm^2$$

With A^* known, we find the desired supersonic Mach number in the exit plane:

$$\frac{A_e}{A^*} = \frac{51.2\,cm^2}{10\,cm^2} = 5.12\ ;\ Table\ B.1\ or\ Eq.(9.48):\ \ Ma_e = 3.2$$

$$p_e = \frac{p_o}{[1+0.2(Ma_e^2)]^{3.5}} = \frac{500,000}{[1+0.2(3.2)^2]^{3.5}} = 10,100\ Pa$$

If 10,100 Pa were the back pressure, that would be the *design condition*. The highest possible back pressure would cause a normal shock in the exit plane and still allow supersonic flow inside:

$$\text{Normal shock}:\ Ma_1 = 3.2\ ;\ Table\ B.2:\ Ma_2 = 0.4643\ ,\ \frac{p_2}{p_1} = 11.78$$

$$\text{Then}\ \ p_2 = p_{b,\max} = (11.78)(10100) = \mathbf{119,000}\ Pa \quad Ans.$$

P9.71 A converging-diverging nozzle has a throat area of 10 cm^2 and an exit area of 20 cm^2. It is supplied by an air tank at 250 kPa and 350 K. (*a*) What is the design pressure at the exit? At one operating condition, the exit properties are p_e = 183 kPa, T_e = 340 K, and V_e = 144 m/s. (*b*) Can this condition be explained by a normal shock inside the nozzle? (*c*) If so, at what Mach number does the normal shock occur? [*Hint*: Use the change in *A** to locate, if necessary, this position.]

Solution: First find the design conditions and also the actual Mach number at the exit:

$$\frac{A_e}{A^*} = \frac{20\,cm^2}{10\,cm^2} = 2.0\ ;\quad \text{Table B.1 or EES}:\quad Ma_{design} = 2.1972 \approx 2.20$$

$$p_{design} = \frac{p_o}{(1+0.2Ma_e^2)^{3.5}} = \frac{250,000}{[1+0.2(2.1972)^2]^{3.5}} = \mathbf{23,500\,Pa} \qquad Ans.(a)$$

$$\text{Actual}\ Ma_e = \frac{V_e}{\sqrt{kRT_e}} = \frac{144\,m/s}{\sqrt{1.4(287)(340K)}} = 0.390 \qquad \text{(subsonic)}$$

So the actual exit flow is nowhere near design conditions. If we were so careful as to check for a normal shock in the exit plane, we would find an exit pressure of 128 kPa, much less than the given value of 183 kPa. Then yes, **there is a normal shock inside the nozzle**. *Ans.*(*b*)
(c) To find where the shock is located, calculate the stagnation pressure and/or the area A^*_2 for the given exit conditions.

$$p_{o2} = p_e(1+0.2Ma_e^2)^{3.5} = (183\,kPa)[1+0.2(0.389)^2]^{3.5} = (183)(1.110) = 203.1\,kPa$$

$$\text{At } Ma_e = 0.389 \quad \frac{A_e}{A_2^*} = 1.627 = \frac{20\,cm^2}{A_2^*}\;;\quad Solve \quad A_2^* = 12.3\,cm^2$$

Either one of these will tell us the Mach number where the normal shock occurred:

$$\frac{p_{o2}}{p_{o1}} = \frac{203.1\,kPa}{250\,kPa} = 0.8124 \;;\quad \text{Table B.2 or EES:}\quad \mathbf{Ma_1 \approx 1.80} \quad Ans.(c)$$

$$\frac{A_2^*}{A_1^*} = \frac{12.3\,cm^2}{10.0\,cm^2} = 1.23 \;;\quad \text{Table B.2 or EES:}\quad \mathbf{Ma_1 \approx 1.80} \quad Ans.(c)$$

9.72 A large tank at 500 K and 165 kPa feeds air to a converging nozzle. The back pressure outside the nozzle exit is sea-level standard. What is the appropriate exit diameter if the desired mass flow is 72 kg/h?

Solution: Given T_o = 500 K and p_o = 165 kPa. The pressure ratio across the nozzle is (101.35 kPa)/(165 kPa) = 0.614 > 0.528. Therefore the flow is not choked but instead exits at a high subsonic Mach number, with $p_{throat} = p_{atm}$ = 101.35 kPa. Equation (9.47) is handy:

$$\frac{\dot{m}}{A}\frac{\sqrt{RT_o}}{p_o} = \sqrt{\frac{2k}{k-1}\left(\frac{p}{p_o}\right)^{2/k}\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} = \frac{(72/3600)}{A}\frac{\sqrt{287(500)}}{165000} = \frac{4.59E\text{–}5\ \mathrm{m}^2}{A}$$

$$= \sqrt{\frac{2(1.4)}{0.4}\left(\frac{101}{165}\right)^{2/1.4}\left[1-\left(\frac{101}{165}\right)^{0.4/1.4}\right]} = 0.673$$

$$\text{Solve for} \quad A_{exit} = 6.82E{-}5\ \mathrm{m}^2 = \frac{\pi}{4}D_{exit}^2, \quad \text{Solve } \mathbf{D_{exit} = 0.0093\ m} \quad Ans.$$

The exit Mach number is approximately 0.86.

9.73 Air flows isentropically in a converging-diverging nozzle with a throat area of 3 cm^2. At section 1, the pressure is 101 kPa, T_1 = 300 K, and V_1 = 868 m/s. (a) Is the nozzle choked? Determine (b) A_1; and (c) the mass flow. Suppose, without changing stagnation conditions of A_1, the flexible throat is reduced to 2 cm^2. Assuming shock-free flow, will there be any changes in the gas properties at section 1? If so, calculate the new p1, V1, and T1 and explain.

Solution: Check the Mach number. If choked, calculate the mass flow:

$$Ma_1 = \frac{V_1}{A_1} = \frac{868}{\sqrt{1.4(287)(300)}} = 2.50 \quad \text{Supersonic: the nozzle is } \textbf{choked.} \quad \textit{Ans.}\text{(a)}$$

$$p_o = 101[1+0.2(2.50)^2]^{3.5} = 1726 \text{ kPa}; \quad T_o = 300[1+0.2(2.50)^2] = 675 \text{ K}$$

$$\text{At } Ma_1 = 2.50,\ \frac{A_1}{A^*} = 2.64 \quad \therefore\ A_1 = 2.64(3) = \mathbf{7.91\ cm^2} \quad \textit{Ans.}\text{ (b)}$$

$$\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{1726000(0.0003)}{\sqrt{287(675)}} = \mathbf{0.805\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (c)}$$

If po and To are unchanged and the throat (A*) is reduced from 3.0 to 2.0 cm^2, the mass flow is cut by one-third and, if A1 remains the same (7.91 cm^2), the area ratio changes and the Mach number will change at section 1:

$$\text{New } \frac{A_1}{A^*_{new}} = \frac{7.91}{2.0} = 3.955; \quad \text{Table B.1:} \quad \text{read } Ma_{1,new} \approx \mathbf{2.928}$$

Since the Mach number changes, all properties at section 1 change:

$$p_{1,new} = \frac{1726000}{[1+0.2(2.928)^2]^{3.5}} = \mathbf{52300\ Pa}$$

$$T_{1,new} = \frac{675}{[1+0.2(2.928)^2]} = \mathbf{249\ K}$$

$$V_{1,new} = 2.928\sqrt{1.4(287)(249)} = \mathbf{926\ \frac{m}{s}}$$

A practical question might be: Does the new, reduced throat shape avoid flow separation and shock waves?

P9.74 Use your strategic ideas, from part (*b*) of Prob. P9.40, to actually carry out the calculations for mass flow of steam, with p_o = 300 kPa and T_o = 600 K, discharging through a converging nozzle of choked exit area 5 cm^2. Take advantage of the new EES thermodynamic function SPEEDSOUND(Steam, p = p1, s = s1).

Solution: Using EES for real steam, we don't have these nice power-law formulas, but we can use the energy equation and the continuity equation and the fact that the entropy is constant:

$$\text{Energy: } h + \frac{1}{2}V^2 = h_o \quad ; \quad \text{Continuity: } \dot{m} = \rho A V = \text{ constant} \; ; \quad \text{s} = s_o = \text{constant}$$

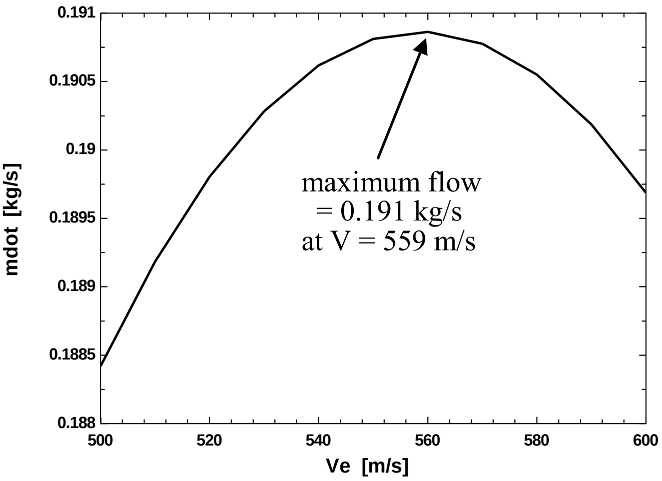
First establish s_o and h_o from the given p_o and T_o:

$$s_o = \text{Entropy}(Steam, p=300, T=600) = 7.7952\, kJ/(kg-K).$$
$$h_o = \text{Enthalpy}(Steam, p=300, T=600) = 3123.8\ kJ/kg$$

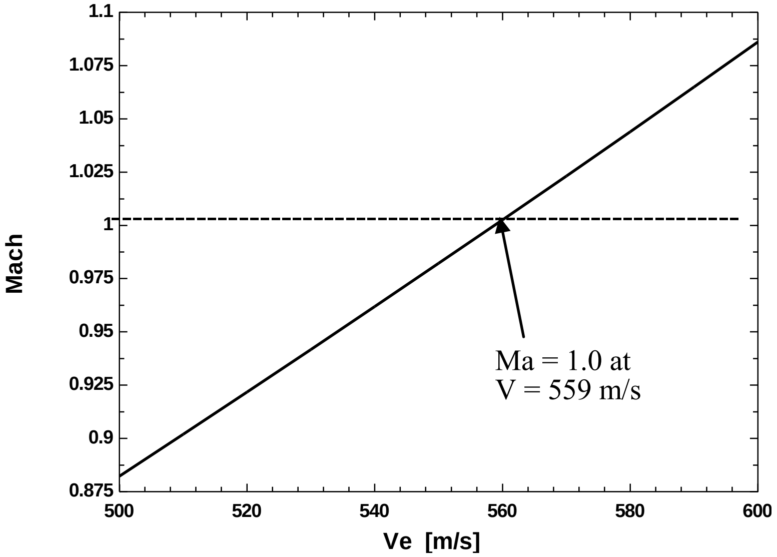
Then guess an exit velocity *V*, which we know, from Prob. 9.40*a,* lies somewhere between 500 and 600 m/s. Compute *h* from energy, then compute $\rho = \rho(h, s_o)$ from the EES thermophysical functions. This enables us to calculate the (guessed) mass flow = $\rho A V$. Is it a maximum? Probably not. Keep changing *V* until you reach a maximum mass flow and a Mach number of 1.0. The final result obtained by the writer is

$$\text{At the exit: } Ma = 1.0,\ V = 559\ m/s,\ \rho = 0.683\, kg/m^3,\ \dot{m} = \dot{m}_{\max} = \mathbf{0.191}\ \frac{kg}{s} \quad Ans.$$

Here is a plot of the exit mass flow versus velocity, from an EES Table:



And here is a plot of the exit Mach number versus velocity, from an EES Table. Each time we changed *V*, we calculated a new SOUNDSPEED and a new Ma = *V*/*a*.



9.75 A double-tank system in Fig. P9.75 has two identical converging nozzles of 1-in^2 throat area. Tank 1 is very large, and tank 2 is small enough to be in steady-flow equilibrium with the jet from tank 1. Nozzle flow is isentropic, but entropy changes between 1 and 3 due to jet dissipation in tank 2. Compute the mass flow. (If you give up, Ref. 14, pp. 288–290, has a good discussion.)

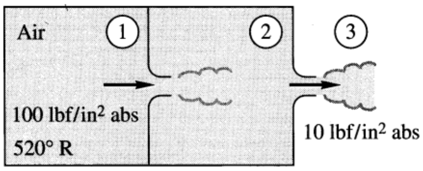


Fig. P9.75

Solution: We know that $\rho_1 V_1 = \rho_2 V_2$ from continuity. Since p_{atm} is so low, we may assume that the second nozzle is choked, but the first nozzle is probably not choked. We may guess values of p_2 and compare the computed values of flow through each nozzle:

$$\text{Assume 2nd nozzle choked:} \quad \rho_3 V_3 = \frac{0.6847 p_2}{\sqrt{(RT_o)}} \quad \text{with } T_o = 520°R = \text{constant}$$

$$\text{Guess} \quad p_2 \approx 80 \text{ psia: } \frac{100}{80} = \left(1 + 0.2\text{Ma}_1^2\right)^{3.5} \quad \text{or Ma}_1 \approx 0.574, \quad \text{then } \rho_1 \approx 0.0138 \frac{\text{slug}}{\text{ft}^3}$$

and $V_1 \approx 621$ ft/s, or $\rho_1 V_1 \approx 8.54\ \text{slug/s·ft}^2$, whereas $\rho_3 V_3 \approx 8.35\ \text{slug/s·ft}^2$

Guess $p_2 \approx 81$ psia: $Ma_1 \approx 0.557$, $\rho_1 V_1 = 8.38\ \dfrac{\text{slug}}{\text{s·ft}^2}$ and $\rho_3 V_3 \approx 8.45\ \dfrac{\text{slug}}{\text{s·ft}^2}$

Interpolate to: $p_2 \approx 80.8$ psia, $Ma_1 \approx 0.561$, $\mathbf{\dot{m} = \rho AV = 0.0585\ \dfrac{slug}{s}}$ *Ans.*

9.76 A large reservoir at 20°C and 800 kPa is used to fill a small insulated tank through a converging-diverging nozzle with 1-cm^2 throat area and 1.66-cm^2 exit area. The small tank has a volume of 1 m^3 and is initially at 20°C and 100 kPa. Estimate the elapsed time when (a) shock waves begin to appear inside the nozzle; and (b) the mass flow begins to drop below its maximum value.

Solution: During this entire time the nozzle is choked, so let's compute the mass flow:

$$\dot{m}_e = \dot{m}_{max} = \frac{0.6847\, p_o A^*}{\sqrt{(RT_o)}} = \frac{0.6847(800000)(0.0001)}{\sqrt{287(293)}} \approx \mathbf{0.189\ kg/s}$$

Meanwhile, a control volume around the small tank reveals a linear pressure rise with time:

$$\dot{m}_e = \frac{d}{dt}(m_{tank}) = \frac{d}{dt}(\rho\upsilon_{tank}) = \frac{d}{dt}\left(\frac{p_{tank}}{RT_o}\upsilon\right), \quad \text{or:} \quad \mathbf{\frac{dp_{tank}}{dt} \approx \frac{RT_o\dot{m}}{\upsilon}}$$

Carrying out the numbers gives $\dfrac{dp}{dt} \approx \dfrac{287(293)(0.189)}{1.0} \approx 15900\ \dfrac{\text{Pa}}{\text{s}} \approx \text{constant}$

We are assuming, for simplicity, that the tank stagnation temperature remains at 293 K. Shock waves move into the nozzle when the tank pressure rises above what would occur if the nozzle exit plane were to have a normal shock:

$$\text{If } \frac{A_e}{A^*} = \frac{1.66}{1.0}, \quad \text{then } Ma_e \approx 1.98, \quad p_1 = \frac{800000}{[1+0.2(1.98)^2]^{3.5}} \approx 105400 \text{ Pa.}$$

$$\text{After the shock, } \frac{p_2}{p_1} = \frac{2.8(1.98)^2 - 0.4}{2.4} \approx 4.41, \quad \therefore\ p_2 = p_{tank} = 4.41(105.4) \approx 465 \text{ kPa}$$

Above this tank pressure, the shock wave moves into the nozzle. The time lapse is

$$\Delta t_{\text{shocks in nozzle}} = \frac{\Delta p_{tank}}{dp/dt} = \frac{465000 - 100000 \text{ Pa}}{15900 \text{ Pa/s}} \approx \mathbf{23\ sec} \quad \textit{Ans.}\ \text{(a)}$$

Assuming the tank pressure rises smoothly and the shocks do not cause any instability or anything, the nozzle ceases to be choked when p_{tank} rises above a subsonic isentropic exit:

$$\frac{A_e}{A^*} = 1.66, \quad \text{than } Ma_e(\text{subsonic}) \approx 0.380, \quad p_e = \frac{800000}{[1+0.2(0.38)^2]^{3.5}} \approx 724300 \text{ Pa}$$

$$\text{Then } \quad \Delta t_{\text{choking stops}} \approx \frac{\Delta p}{dp/dt} = \frac{724300-100000}{15900} \approx \mathbf{39\ sec} \quad \textit{Ans.}\text{ (b)}$$

9.77 A perfect gas (not air) expands isentropically through a supersonic nozzle with an exit area 5 times its throat area. The exit Mach number is 3.8. What is the specific heat ratio of the gas? What might this gas be? If p_o = 300 kPa, what is the exit pressure of the gas?

Solution: We must iterate the area-ratio formula, Eq. (9.44), for *k*:

$$Ma_e = 3.8, \quad \frac{A_e}{A^*} = 5 = \frac{1}{3.8}\left[\frac{1+\frac{k-1}{2}(3.8)^2}{(k+1)/2}\right]^{\frac{k+1}{2(k-1)}}, \quad \text{Solve for} \quad \mathbf{k \approx 1.667} \quad \textit{Ans.}\text{ (a)}$$

Monatomic gas: could be *helium* or *argon*. *Ans*. (b)

$$\text{With } k \text{ known,} \quad p_e = \frac{300 \text{ kPa}}{\left[1+([1.667-1]/2)(3.8)^2\right]^{1.667/0.667}} \approx 3.7\ kPa \quad \textit{Ans.}\text{ (c)}$$

9.78 The orientation of a hole can make a difference. Consider holes A and B in Fig. P9.78, which are identical but reversed. For the given air properties on either side, compute the mass flow through each hole and explain the difference.

Solution: Case B is a converging nozzle

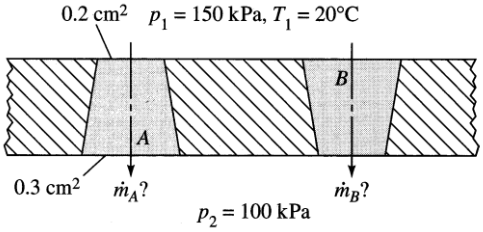


Fig. P9.78

with p2/p1 = 100/150 = 0.667 > 0.528, therefore case B is **not choked**. Case A is **choked at the entrance** and expands to a (subsonic) pressure of 100 kPa, which we may check from a subsonic calculation. The results are:

$$\text{Nozzle } \mathbf{B}\text{:}\quad \frac{p_2}{p_o} = 0.667, \quad \text{read } Ma_2 \approx 0.784, \quad T_e = \frac{293}{[1+0.2(0.784)^2]} \approx 261\,\text{K},$$

$$\rho_e = \frac{100000}{287(261)} = 1.34\ \frac{\text{kg}}{\text{m}^3}, \quad a_e = 324\ \frac{\text{m}}{\text{s}}, \quad V_e = 254\ \frac{\text{m}}{\text{s}},$$

$$\dot{m} = \rho_e A_e V_e \approx \mathbf{0.0068}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\ (\text{B})$$

$$\text{Nozzle } \mathbf{A}\text{:}\quad \dot{m} = \dot{m}_{max} = \frac{0.6847(150000)(0.0002)}{\sqrt{287(293)}} \approx \mathbf{0.071}\ \frac{\mathbf{kg}}{\mathbf{s}}\ \ (5\%\text{ more}) \quad \textit{Ans.}\ (\text{A})$$

9.79 A large reservoir at 600 K supplies air flow through a converging-diverging nozzle with a throat area of 2 cm^2. A normal shock wave forms at a section of area 6 cm^2. Just downstream of this shock, the pressure is 150 kPa. Calculate (a) the pressure in the throat; (b) the mass flow; and (c) the pressure in the reservoir.

Solution: The throat is choked, and just upstream of the shock is a supersonic flow at an area ratio $A/A^* = (6\text{ cm}^2)/(2\text{ cm}^2) = 3.0$. From Table B.1 estimate $Ma1 = 2.64$. That is,

$$\frac{A_1}{A^*} = 3.0 = \frac{\left(1+0.2Ma_1^2\right)^3}{1.728Ma_1}, \quad \text{Solve } Ma_1 = 2.637$$

(a, c) The pressure ratio across the shock is given by Eq. (9.55) or Table B.2:

$$\frac{p_2}{p_1} = \frac{150\,\text{kPa}}{p_1} = \frac{1}{k+1}\left(2kMa_1^2 - k + 1\right)$$

$$= \frac{1}{2.4}[2(1.4)(2.637)^2 - 0.4] = 7.95, \quad \text{or} \quad p_1 = 18.9\text{ kPa}$$

$$p_{tank} = p_o = p_1\left(1+0.2Ma_1^2\right)^{3.5} = (18.9)[1+0.2(2.637)^2]^{3.5} = \mathbf{399\ kPa} \quad \textit{Ans.}\ (\text{c})$$

$$\text{At the throat,} \quad p = p^* = 0.5283p_o = (0.5283)(399\text{ kPa}) = \mathbf{211\ kPa} \quad \textit{Ans.}\ (\text{a})$$

(b) To avoid bothering with density and velocity, Eq. (9.46b) is handy for choked flow.

$$\dot{m}_{\max} = \frac{0.6847 p_o A^*}{\sqrt{RT_o}} = \frac{0.6847(399000\,\text{Pa})(0.0002\ \text{m}^2)}{\sqrt{(287\,\text{m}^2/\text{s}^2\text{K})(600\,\text{K})}} = \mathbf{0.132\ kg/s} \quad \textit{Ans.}\ \text{(b)}$$

9.80 A sea-level automobile tire is initially at 32 lbf/in^2 gage pressure and 75°F. When it is punctured with a hole which resembles a converging nozzle, its pressure drops to 15 lbf/in^2 gage in 12 min. Estimate the size of the hole, in thousandths of an inch.

Solution: The volume of the tire is **2.5 ft^2**. With patm ≈ 14.7 psi, the absolute pressure drops from 46.7 psia to 29.7 psia, both of which are sufficient to cause a **choked** exit. A theory for isothermal blowdown of a choked tank was given in Prob. 9.36:

$$p_{\text{tank}} = p(0)\exp\left[-0.6847\frac{A^*\sqrt{RT_o}}{\upsilon}t\right],$$

$$\text{or:}\quad 29.7 = 46.7\exp\left[-0.6847\frac{A^*\sqrt{1717(535)}}{2.5}12\times 60\right],$$

solve **A* = 2.4E-6 ft^2** (d = 0.00175 *ft* = 21 thousandths of an inch) *Ans.*

P9.81 Air, at p_o = 160 lbf/in^2 and T_o = 300°F, flows isentropically through a converging-diverging nozzle. At section 1, where A_1 = 288 in^2, the velocity is V_1 = 2068 ft/s. Calculate (*a*) Ma$_1$; (*b*) A^*; (*c*) p_1; and (*d*) the mass flow, in slug/s.

Solution: That is a high velocity, 2068 ft/s, even higher than the stagnation speed of sound, $a_o = (kRTo)^{1/2}$ = 1351 ft/s. So the flow at section 1 is *supersonic*. We need to find Ma$_1$ such that

$$V_1 = Ma_1\,a_1 = Ma_1\sqrt{kRT_1} = Ma_1\sqrt{\frac{kRT_o}{1+0.2(Ma_1^2)}} = Ma_1\sqrt{\frac{1.4(1716)(300+460)}{1+0.2(Ma_1^2)}} = 2068\frac{ft}{s}$$

You could iterate, or EES would find the result in a flash: Ma$_1$ = **2.10** Ans.(*a*)

(*b, c, d*) The remaining properties follow from the Mach number:

$$(b)\ Ma_1 = 2.10:\ \text{Table B.1: } \frac{A_1}{A^*} = 1.837\ ,\quad A^* = \frac{288\,in^2}{1.837} = 157\,in^2 = \mathbf{1.089}\ ft^2 \quad Ans.(b)$$

$$(c)\ p_1 = \frac{p_o}{[1+0.2(Ma_1^2)]^{3.5}} = \frac{160\ psia}{[1+0.2(2.1)^2]^{3.5}} = 17.5\ psia = \mathbf{2520}\ \frac{lbf}{ft^2} \quad Ans.(c)$$

$$(d)\ \dot{m} = \dot{m}_{\max} = (0.6847)\frac{(160x144\ psf)(1.089\ ft^2)}{\sqrt{(1716\ ft^2/s^2 -{}^oR)(760^oR)}} = \mathbf{15.04}\ \frac{slug}{s} \quad Ans.(d)$$

9.82 Air at 500 K flows through a converging-diverging nozzle with throat area of 1 cm^2 and exit area of 2.7 cm^2. When the mass flow is 182.2 kg/h, a pitot-static probe placed in the exit plane reads p_0 = 250.6 kPa and p = 240.1 kPa. Estimate the exit velocity. Is there a normal shock wave in the duct? If so, compute the Mach number just downstream of this shock.

Solution: These numbers **just don't add up** to a purely isentropic flow. For example, po/p = 250.6/240.1 yields Ma ≈ **0.248**, whereas A/A* = 2.7 gives Ma ≈ **0.221**. If the mass flow is *maximum*, we can estimate the upstream stagnation pressure:

$$\dot{m} \overset{?}{=} \dot{m}_{\max} = \frac{182.2}{3600} \overset{?}{=} 0.6847\frac{p_{o1}(0.0001)}{\sqrt{287(500)}} \quad \text{if } p_{o1} \approx \mathbf{280\,kPa}$$

This doesn't check with the measured value of 250.6 kPa, nor does an isentropic choked subsonic expansion lead to pexit = 240.1—it gives **271** kPa instead. We conclude that **there is a normal shock wave in the duct** before the exit plane, reducing po:

$$\text{Normal shock: } \frac{p_{o2}}{p_{o1}} = \frac{250.6}{280.0} = 0.895 \quad \text{if } Ma_1 \approx 1.60 \quad \text{and} \quad \mathbf{Ma_2 \approx 0.67} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{This checks with } A_2^* \approx 1.12\ \text{cm}^2,\ \frac{A_e}{A_2^*} = 2.42,\ Ma_e \approx 0.248\ \text{(as above)}$$

$$T_e = \frac{500}{[1+0.2(0.248)^2]} = 494\ \text{K},\quad a_e = \sqrt{kRT_e} = 445\ \frac{m}{s},\quad \mathbf{V_e} = Ma_e a_e \approx \mathbf{110}\ \frac{\mathbf{m}}{\mathbf{s}}$$

9.83 When operating at design conditions (smooth exit to sea-level pressure), a rocket engine has a thrust of 1 million lbf. The chamber pressure and temperature are 600 lbf/in^2 absolute and 4000°R, respectively. The exhaust gases approximate k = 1.38

with a molecular weight of 26. Estimate (a) the exit Mach number and (b) the throat diameter.

Solution: “Design conditions” mean isentropic expansion to p_e = 14.7 psia = 2116 lbf/ft^2:

$$\frac{p_o}{p_e} = \frac{600}{14.7} = 40.8 = \left[1 + \left(\frac{1.38-1}{2}\right)\mathrm{Ma}_e^2\right]^{1.38/0.38}, \quad \text{solve for } \mathbf{Ma_e \approx 3.06} \quad \textit{Ans.}\text{ (a)}$$

From Prob. 3.68, if $p_e = p_a$,

$$F = \rho_e A_e V_e^2 = kp_e A_e \mathrm{Ma}_e^2 = 1.38(2116)A_e(3.06)^2 = 1\text{E}6 \text{ lbf}, \quad \text{solve } A_e \approx 36.6 \text{ ft}^2$$

Assuming an isentropic expansion to $\mathrm{Ma}_e \approx 3.06$, we can compute the throat area:

$$\frac{A_e}{A^*} = \frac{36.6}{A^*} = \frac{1}{3.06}\left[\frac{2+0.38(3.06)^2}{1.38+1}\right]^{\frac{2.38}{2(0.38)}} = 4.65, \quad \text{or} \quad A^* = \frac{36.6}{4.65} = 7.87 \text{ ft}^2 = \frac{\pi}{4}D^{*2}$$

Solve for throat diameter $\mathbf{D^* \approx 3.2 \text{ ft}}$ *Ans.* (b)

9.84 Air flows through a duct as in Fig. P9.84, where A_1 = 24 cm^2, A_2 = 18 cm^2, and A_3 = 32 cm^2. A normal shock stands at section 2. Compute (a) the mass flow, (b) the Mach number, and (c) the stagnation pressure at section 3.

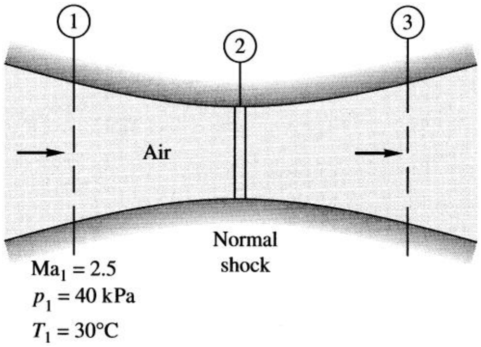


Fig. P9.84

Solution: We have enough information at section 1 to compute the mass flow:

$$a_1 = \sqrt{1.4(287)(30+273)} \approx 349 \text{ m/s}, \quad V_1 = 2.5(349) = 872 \ \frac{\text{m}}{\text{s}}, \quad \rho_1 = \frac{p_1}{RT_1} = 0.46 \ \frac{\text{kg}}{\text{m}^3}$$

$$\text{Then} \quad \dot{m} = \rho_e A_e V_e = 0.46(0.0024)(872) \approx \mathbf{0.96 \text{ kg/s}} \quad \textit{Ans.}\text{ (a)}$$

Now move isentropically from 1 to 2 upstream of the shock and thence across to 3:

$$\mathrm{Ma}_1 = 2.5, \quad \therefore \quad \frac{A_1}{A_1^*} = 2.64, \quad A_1^* = \frac{24}{2.64} = 9.1\ \mathrm{cm}^2, \quad \text{and} \quad \frac{A_2}{A_1^*} = \frac{18}{9.1} = 1.98$$

$$\text{Read} \quad \mathrm{Ma}_{2,\mathrm{upstream}} \approx \mathbf{2.18}, \quad p_{o1} = p_{o2} = 40[1+0.2(2.5)^2]^{3.5} \approx 683\ \mathrm{kPa}, \quad \text{across the}$$

$$\text{shock,} \quad \frac{A_3^*}{A_2^*} = 1.57, \quad A_3^* = 14.3\ \mathrm{cm}^2, \quad \frac{A_3}{A_3^*} = 2.24\,\Big|_{\mathrm{sub}}, \quad \mathbf{Ma_3 \approx 0.27} \quad \textit{Ans.}\ \text{(b)}$$

Finally, go back and get the stagnation pressure ratio across the shock:

$$\text{at } \mathrm{Ma}_2 \approx 2.18, \quad \frac{p_{o3}}{p_{o2}} \approx 0.637, \quad \therefore \quad p_{o3} = 0.637(683) \approx \mathbf{435\ kPa} \quad \textit{Ans.}\ \text{(c)}$$

P9.85 A typical carbon dioxide tank for a paintball gun holds about 12 oz of liquid CO_2. The tank is filled no more than one-third with liquid, which, at room temperature, maintains the gaseous phase at about 850 psia. (*a*) If a valve is opened that simulates a converging nozzle with an exit diameter of 0.050 in, what mass flow and exit velocity result? (*b*) Repeat the calculation for helium.

Solution: For CO_2, from Table A.4, R = 189 J/kg-K and k = 1.30. By "room temperature" we assume T_o = 293 K. Convert p_o = 853 psia = 5.86 MPa. Convert D_{exit} = 0.050 in = 0.00127 m. Assume an outside pressure of 1 atm, which ensures that the flow through the nozzle will be *choked*. The maximum mass flow results, Eq. (9.46a), and $V_{exit} = V^*$, Eq. (9.33):

$$CO_2, k = 1.30: \ \dot{m} = 0.6673\frac{p_o A^*}{\sqrt{RT_o}} = 0.6673\frac{(5.86E6\,Pa)(\pi/4)(0.00127\,m)^2}{\sqrt{(189)(293)}} = 0.021\frac{kg}{s} \quad Ans.(a)$$

$$\text{and} \quad V_{exit} = V^* = \sqrt{\frac{2k}{k+1}RT_o} = \sqrt{\frac{2(1.3)}{(1.3+1)}(189)(293)} = 250\frac{m}{s} \quad Ans.(a)$$

(*b*) For helium, from Table A.4, R = 2077 J/kg-K and k = 1.66. Then

$$Helium, k = 1.66: \ \dot{m} = 0.6673\frac{p_o A^*}{\sqrt{RT_o}} = 0.6673\frac{(5.86E6\,Pa)(\pi/4)(0.00127\,m)^2}{\sqrt{(2077)(293)}} = 0.0069\frac{kg}{s} \quad Ans.(b)$$

$$\text{and} \quad V_{exit} = V^* = \sqrt{\frac{2k}{k+1}RT_o} = \sqrt{\frac{2(1.66)}{(1.66+1)}(2077)(293)} = 872\frac{m}{s} \quad Ans.(b)$$

9.86 Air enters a 3-cm diameter pipe 15 m long at V_1 = 73 m/s, p_1 = 550 kPa, and T_1 = 60°C. The friction factor is 0.018. Compute V_2, p_2, T_2, and p_{02} at the end of the pipe. How much additional pipe length would cause the exit flow to be sonic?

Solution: First compute the inlet Mach number and then get $(fL/D)_1$:

$$a_1 = \sqrt{1.4(287)(60+273)} = 366\ \frac{m}{s},\quad Ma_1 = \frac{73}{366} \approx 0.20,\quad \text{read}\ \left(\frac{fL}{D}\right) = 14.53,$$

for which $p/p^* = 5.4554$, $T/T^* = 1.1905$, $V/V^* = 0.2182$, and $p_o/p_o^* = 2.9635$

Then $(fL/D)_2 = 14.53 - (0.018)(15)/(0.03) \approx \mathit{5.53}$, read $Ma_2 \approx \mathbf{0.295}$

At this new Ma_2, read $p/p^* \approx 3.682$, $T/T^* \approx 1.179$, $V/V^* \approx 0.320$, $p_o/p_o^* \approx 2.067$. Then

$$V_2 = V_1\frac{V_2/V^*}{V_1/V^*} = 73\left(\frac{0.320}{0.218}\right) \approx \mathbf{107\ \frac{m}{s}}\quad \textit{Ans.}\ \text{(a)}$$

$$p_2 = 550\left(\frac{3.682}{5.455}\right) \approx \mathbf{371\ kPa}\quad \textit{Ans.}\ \text{(b)}$$

$$T_2 = 333\left(\frac{1.179}{1.190}\right) \approx \mathbf{330\ K}\quad \textit{Ans.}\ \text{(c)}$$

Now we need p_{o1} to get p_{o2}:

$$p_{o1} = 550[1+0.2(0.2)^2]^{3.5} \approx 566\ \text{kPa},\quad \text{so}\ p_{o2} = 566\left(\frac{2.067}{2.964}\right) \approx \mathbf{394\ kPa}$$

The extra distance we need to choke the exit to sonic speed is $(fL/D)_2 = 5.53$. That is,

$$\Delta L = 5.53\frac{D}{f} = 5.53\left(\frac{0.03}{0.018}\right) \approx \mathbf{9.2\ m}\quad \textit{Ans.}$$

P9.87 Problem C6.9 gives data for a proposed Alaska-to-Canada natural gas (assume CH_4) pipeline. If the design flow rate is 890 kg/s and the entrance conditions are 2500 lbf/in^2 and 140°F, determine the maximum length of adiabatic pipe before choking occurs.

Solution: For CH_4, from Table A.4, R = 518 m^2/s^2-K, k = 1.32, and μ = 1.03E-5 kg/m-s. Convert to SI units: p_1 = 2500 psi = 1.72E7 Pa, D = 52 in = 1.321 m, T_1 = 140°F = 333 K. From the ideal gas law and the known mass flow, find the entrance density and pressure:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1.724E7}{(518)(333)} = 99.9\ kg/m^3$$

$$V_1 = \frac{\dot{m}}{\rho_1(\pi/4)D^2} = \frac{890}{(99.9)(\pi/4)(1.321)^2} = 6.50\ m/s$$

We can now find the Reynolds number and the smooth-wall friction factor:

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(99.9)(6.50)(1.321)}{1.03\mathrm{E}-5} = 8.33\mathrm{E}7$$

$$\text{Smooth wall, Eq.(6.48):}\ \frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right)\ \text{yields}\quad f \approx 0.00608$$

Find the entrance Mach number:

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{6.50}{\sqrt{1.32(518)(333)}} = \frac{6.50}{477.2} = 0.0136$$

Now apply Eq. (9.66) for compressible adiabatic flow with friction, for k = 1.32:

$$\frac{f\,L^*}{D} = \frac{1-Ma^2}{k\,Ma^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma_2}\right] = \mathbf{4074}\ \text{for}\ k=1.32\ \text{and}\ Ma = 0.0136$$

This yields L^*, the pipe length which will cause choking:

$$L^* = \left(\frac{fL^*}{D}\right)\frac{D}{f} = (4074)\left(\frac{1.321}{0.00608}\right) = 885{,}000\,m = \mathbf{550\ miles}\qquad \textit{Ans.}$$

P9.88 Air flows adiabatically through a 2-cm-diameter pipe. Conditions at section 2 are p_2 = 100 kPa, T_2 = 15°C, and V_2 = 170 m/s. The average friction factor is 0.024. At section 1, which is 55 meters upstream, find (*a*) the mass flow; (*b*) p_1; and (*c*) p_{o1}.

Solution: The mass flow is the same at sections 1 and 2, so use section 2 to calculate To find things at section 1, we need compressible pipe-flow theory. It happens that the given data simulates the situation in Examples 9.10 and 9.11 in the text. At section 2,

$$\rho_2 = \frac{p_2}{RT_2} = \frac{100{,}000}{287(288K)} = 1.21\frac{kg}{m^3};\quad \dot{m} = \rho AV = (1.21)\frac{\pi}{4}(0.02)^2(170) = 0.0646\frac{kg}{s}\quad Ans.(a)$$

$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287)(288K)} = 340\,m/s\quad ;\quad Ma_2 = \frac{V_2}{a_2} = \frac{170\,m/s}{340\,m/s} = 0.50$$

$$\text{Table B.3: } \frac{fL}{D}|_2 = 1.0691\ ;\quad Then\ \frac{fL}{D}|_1 = \frac{fL}{D}|_2 + \frac{f\Delta L}{D} = 1.0691 + \frac{0.024(55m)}{0.02m} =$$

$$= 1.0691 + 66.0 = 67.0691 \qquad \text{Table B.3: Read } Ma_1 = 0.100$$

$$\text{Table B.3: } \frac{p_2}{p*} = 2.1381\quad ,\ \frac{p_1}{p*} = 10.9435$$

$$\text{Then } p_1 = p_2\frac{p_1/p*}{p_2/p*} = (100kPa)\frac{10.9435}{2.1381} = 512\,kPa\quad Ans.(b)$$

$$p_{o1} = p_1[1+0.2Ma_1^2]^{3.5} = 512[1+0.2(0.1)^2]^{3.5} = 516\,kPa\quad Ans.(c)$$

We need the pressure ratios to finish the problem, and then calculate p_{o1} from the Mach number

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9.89 Carbon dioxide flows through an insulated pipe 25 m long and 8 cm in diameter. The friction factor is 0.025. At the entrance, $p = 300$ kPa and $T = 400$ K. The mass flow is 1.5 kg/s. Estimate the pressure drop by (a) compressible; and (b) incompressible (Sect. 6.6) flow theory. (c) For what pipe length will the exit flow be choked?

Solution: For CO2, from Table A.4, take $k = 1.30$ and R = 189 J/kg·K. Tough calculation, no appendix tables for CO2, should probably use EES. Find inlet density, velocity, Mach number:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{300000\,\text{Pa}}{(189\,\text{J/kg·K})(400\,\text{K})} = 3.97\ \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = 1.5\ \frac{\text{kg}}{\text{s}} = \rho_1 A_1 V_1 = \left(3.97\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(0.08\ \text{m})^2 V_1, \quad \text{Solve for } V_1 = 75.2\ \text{m/s}$$

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{75.2\,\text{m/s}}{\sqrt{(1.3)(189\,\text{m}^2/\text{s}^2\cdot\text{K})(400\,\text{K})}} = \frac{75.2\,\text{m/s}}{313.5\,\text{m/s}} = \mathbf{0.240}$$

Between section 1 (inlet) and section 2 (exit), the **change** in (fL/D) equals (0.025)(25 m)/(0.08 m) = 7.813. We have to find the correct exit Mach number from this change:

$$fL^*/D = \frac{1 - Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right]$$

For $k = 1.3$ and $Ma_1 = 0.240$ compute $(fL^*/D)_1 = 10.190$

Then $(fL^*/D)_2 = 10.190 - 7.813 = 2.377$ for what Mach number?

Then iterate (or use EES) to the exit value $Ma_2 = \mathbf{0.408}$

Now compute $p_1/p^* = 4.452$ and $p_2/p^* = 2.600$

Then $p_2 = p_1(p_2/p^*)/(p_1/p^*) = (300\,\text{kPa})(2.600)/(4.452) = 175$ kPa

The desired compressible pressure drop $= 300 - 175 = \mathbf{125\,kPa}$ *Ans.* (a)

(b) The incompressible flow theory (Chap. 6) simply predicts that

$$\Delta p_{inc} = \frac{fL}{D}\frac{\rho_1}{2}V_1^2 = (7.813)\left(\frac{3.97\,\text{kg/m}^3}{2}\right)\left(75.2\ \frac{\text{m}}{\text{s}}\right)^2 = 88000\,\text{Pa} = \mathbf{88\ kPa} \quad \textit{Ans. (b)}$$

The incompressible estimate is 30% low. Finally, the inlet value of (fL/D) tells us the maximum possible pipe length for choking at the exit:

$$L_{\max} = \left.\frac{fL^*}{D}\right|_1\left(\frac{D}{f}\right) = (10.19)\left(\frac{0.08\ \text{m}}{0.025}\right) = \mathbf{32.6\ m} \quad \textit{Ans. (c)}$$

P9.90 Air flows through a rough pipe 120 ft long and 3 inches in diameter. Entrance conditions are p = 90 lbf/in^2, T = 68°F, and V = 225 ft/s. The flow chokes at the end of the pipe. (*a*) What is the average friction factor? (*b*) What is the pressure at the end of the pipe?

Solution: Find the Mach number at the entrance:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(1716\,ft^2/s^2-{}^oR)(460+68\,{}^oR)} = 1126\ ft/s$$

$$Ma_1 = \frac{V_1}{a_1} = \frac{225\,ft/s}{1126\,ft/s} = 0.200$$

Excellent, this Mach number is right in Table B.3. Read fL^*/D = 14.5333. Then

$$\bar{f} = (\frac{\bar{f}\,L}{D})\frac{D}{L} = (14.5333)(\frac{0.25\ ft}{120\ ft}) = \mathbf{0.0303} \qquad Ans.(a)$$

Also, in Table B.3, we can read the pressure ratio:

$$Ma = 0.20\ ,\ \frac{p}{p^*} = 5.4554\ ,\ hence\quad p^* = \frac{90\ psia}{5.4554} = \mathbf{16.5}\ \frac{lbf}{in^2}\quad Ans.(b)$$

9.91 Air flows steadily from a tank through the pipe in Fig. P9.91. There is a converging nozzle on the end. If the mass flow is 3 kg/s and the flow is choked, estimate (a) the Mach number at section 1; and (b) the pressure in the tank.

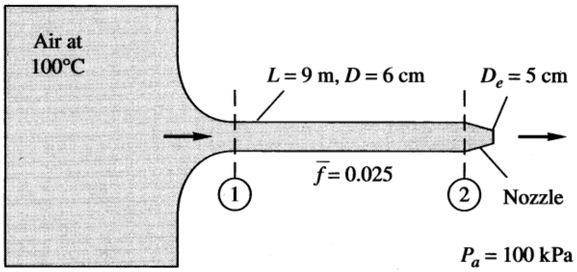


Fig. P9.91

Solution: For adiabatic flow, T* = constant = To/1.2 = 373/1.2 = 311 K. The flow chokes in the small exit nozzle, D = 5 cm. Then we estimate Ma2 from isentropic theory:

$$\frac{A_2}{A^*} = \left(\frac{6\text{ cm}}{5\text{ cm}}\right)^2 = 1.44, \quad \text{read } Ma_2(\text{subsonic}) \approx 0.45, \text{ for which } fL/D|_2 \approx 1.52,$$

$$p_2/p^* \approx 2.388, \quad p_{o2}/p_o^* \approx 1.449, \quad \rho_2/\rho^* \approx 2.070, \quad T_2/T^* = 1.153 \quad \text{or} \quad T_2 \approx 359\text{ K}$$

$$\text{Given} \quad \dot{m} = 3\ \frac{kg}{s} = \rho_2 A_2 V_2 = \frac{p_2}{287(359)}\left(\frac{\pi}{4}\right)(0.06)^2(0.45)\sqrt{1.4(287)(359)},$$

$$\text{Solve for } p_2 \approx 640\text{ kPa.} \quad \text{Then } p^* = 640/2.388 \approx 268\text{ kPa}$$

$$\text{At section 1,} \quad \frac{fL}{D} = \frac{fL}{D}\bigg|_2 + \frac{f\Delta L}{D} = 1.52 + \frac{0.025(9)}{0.06} \approx 5.27, \quad \text{read } Ma_1 \approx \mathbf{0.30} \quad \textit{Ans.}\text{ (a)}$$

$$\text{for which} \quad p_1/p^* \approx 3.6, \quad \text{or} \quad p_1 \approx 3.6(268) \approx 965\text{ kPa.}$$

Assuming isentropic flow in the inlet nozzle,

$$p_{tank} \approx 965[1 + 0.2(0.30)^2]^{3.5} \approx \mathbf{1030\ kPa} \quad \textit{Ans.}\text{ (b)}$$

P9.92 Air enters a 5-cm-diameter pipe at 380 kPa, 3.3 kg/m^3, and 120 m/s. The friction factor is 0.017. Find the pipe length for which the velocity (*a*) doubles; (b) triples; and (*c*) quadruples.

Solution: First find the conditions at the entrance, which we will call section 1:

$$T_1 = \frac{p_1}{R\rho_1} = \frac{380000}{287(3.3)} = 401\,K \ ; \ Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{120}{\sqrt{1.4(287)(401)}} = 0.299$$

$$\text{Table B.3 or EES:} \quad V_1/V^* = 0.3245 \quad , \quad fL/D = 5.353$$

(*a*) Since *V** is constant, we simply double (*V*1/*V**) and find the new Mach number:

$$(a)\ Double\ \frac{V_2}{V^*} = 0.6490 \ ; \ Table\ B.3:\ Ma_2 = 0.6144 \ , \quad fL/D|_2 = 0.4367$$

$$Pipe\ length\ \ L = \frac{D}{f}\left(\frac{fL}{D}|_1 - \frac{fL}{D}|_2\right) = \frac{0.05}{0.017}(5.353 - 0.4367) = 14.46\,m \qquad Ans.(a)$$

(*b*) Again, since *V** is constant, we simply triple (V_1/V^*) and find the new Mach number:

$$(b)\ Triple\ \frac{V_2}{V^*} = 0.9735\ ;\ Table\ B.3:\ Ma_2 = 0.9684\ ,\quad fL/D|_2 = 0.0013$$

$$Pipe\ length\ \ L = \frac{D}{f}(\frac{fL}{D}|_1 - \frac{fL}{D}|_3) = \frac{0.05}{0.017}(5.353 - 0.0013) = 15.74\,m \qquad Ans.(b)$$

(*c*) We are already at choking, **it is impossible to quadruple the velocity for this flow**. *Ans*.(*c*)

9.93 Air flows adiabatically in a 3-cm-diameter duct with f = 0.015. At the entrance, V = 950 m/s and T = 250 K. How far down the tube will (a) the Mach number be 1.8; and (b) the flow be choked?

Solution: (a) Find the entrance Mach number and its value of fL/d:

$$Ma_1 = \frac{950}{\sqrt{1.4(287)(250)}} = 3.00;\quad \text{Table B.3:}\quad \text{read}\ \ f\frac{L_1}{D} = 0.5222;\quad \text{at}\ \ Ma_2 = 1.8,$$

$$\text{read}\quad f\frac{L_2}{D} = 0.2419;\quad \Delta\left(f\frac{L}{D}\right) = 0.5222 - 0.2419 \approx 0.28,$$

$$\Delta L = \frac{0.28(0.03)}{0.015} = \mathbf{0.56\ m} \quad Ans.\ \text{(a)}$$

(b) To go all the way to choking requires the full change

$f\Delta L_1/D = 0.5222$, or: $\Delta L_{choke} = (0.5222)(0.03)/(0.015) =$ **1.04 m** *Ans.* (b)

9.94 Compressible pipe flow with friction, Sec. 9.7, assumes constant stagnation enthalpy and mass flow but variable momentum. Such a flow is often called *Fanno flow*, and a line representing all possible property changes on a temperature-entropy chart is called a *Fanno line*. Assuming an ideal gas with *k* = 1.4 and the data of Prob. 9.86, draw a Fanno line for a range of velocities from very low (Ma <<1) to very high (Ma >>1). Comment on the meaning of the maximum-entropy point on this curve.

Solution: Recall from Prob. 9.86 that, at Section 1 of the pipe, $V_1 = 73$ m/s, $p_1 = 550$ kPa, and $T_1 = 60°C = 333$ K, with $f \approx 0.018$. We can then easily compute $Ma_1 \approx 0.20$, $\rho_1 = 5.76$ kg/m^3, $V_{max} = 822$ m/s, and $T_o = 336$ K. Our basic algebraic equations are:

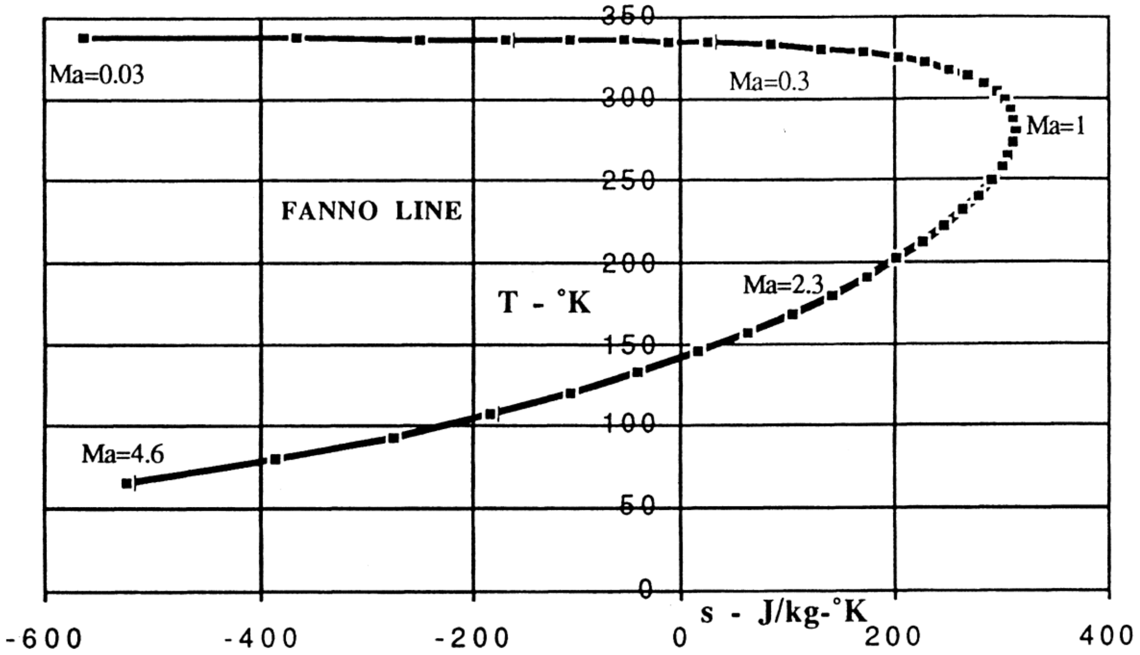
$$\text{Energy:}\quad T = T_o - \frac{V^2}{2c_p},\quad \text{or:}\quad \mathbf{T = 336\,K - \frac{V^2}{2(1005)}} \qquad \textbf{(a)}$$

$$\text{Continuity:}\quad \rho V = \rho_1 V_1 = 5.76(73),\quad \text{or:}\quad \boldsymbol{\rho} \mathbf{= 420/V} \qquad \textbf{(b)}$$

$$\text{Entropy:}\quad \mathbf{s = 718\,ln(T/333) - 287\,ln(}\boldsymbol{\rho}\mathbf{/5.76)} \qquad \textbf{(c)}$$

We simply let *V* vary from, say, 10 m/s to 800 m/s, compute ρ from (b) and *T* from (a) and *s* from (c), then plot *T* versus *s*. [We have arbitrarily set s = 0 at state 1.]

The result of this exercise forms the **Fanno Line** for this flow, shown on the next page. Some Mach numbers are listed, subsonic on the top, supersonic on the bottom, and exactly **sonic** at the right-hand (maximum-entropy) side. *Ans.*



9.95 Helium (Table A.4) enters a 5-cm-diameter pipe at p_1 = 550 kPa, V_1 = 312 m/s, and T_1 = 40°C. The friction factor is 0.025. If the flow is choked, determine (a) the length of the duct and (b) the exit pressure.

Solution: For helium, take k = 1.66 and R = 2077 J/kg·K. We have no tables for k = 1.66, have to do our best anyway. Compute the Mach number at section 1:

$$a_1 = \sqrt{(1.66)(2077)(40+273)} \approx 1039\text{ m/s},\quad \mathrm{Ma}_1 = V_1/a_1 = \frac{312}{1039} \approx \mathbf{0.300}$$

$$\text{Eq. 9.66:}\quad \frac{fL}{D} = \frac{1-(0.3)^2}{1.66(0.3)^2} + \frac{2.66}{2(1.66)}\ln\left[\frac{(2.66)(0.3)^2}{2+(0.66)(0.3)^2}\right] \approx 4.37 \quad \text{at section 1}$$

$$\text{Choked:}\quad fL/D|_2 = 0,\quad \therefore\ L = 4.37D/f = 4.37(0.05)/(0.025) \approx \mathbf{8.7\ m} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Also,}\quad p_1/p^* = \frac{1}{0.3}\left[\frac{2.66}{2+0.66(0.3)^2}\right]^{1/2} \approx 3.79,\quad \therefore\ p_{exit} = p^* = \frac{550}{3.79} \approx \mathbf{145\ kPa} \quad \textit{Ans.}\text{(b)}$$

9.96 Methane (CH_4) flows through an insulated 15-cm-diameter pipe with f = 0.023. Entrance conditions are 600 kPa, 100°C, and a mass flow of 5 kg/s. What lengths of pipe will (a) choke the flow; (b) raise the velocity by 50%; (c) decrease the pressure by 50%?

Solution: For methane (CH4), from Table A.4, take k = 1.32 and R = 518 J/kg·K. Tough calculation, no appendix tables for methane, should probably use EES. Find inlet density, velocity, Mach number:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{600000\,\text{Pa}}{(518\,\text{J/kg·K})(373\,\text{K})} = 3.11\ \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = 5\ \text{kg/s} = \rho_1 A_1 V_1 = (3.11\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.15\ \text{m})^2 V_1, \quad \text{solve for } V_1 = 91.1\ \text{m/s}$$

$$a_1 = \sqrt{kRT_1} = \sqrt{1.32(518)(373)} = 505\ \text{m/s}, \quad Ma_1 = \frac{V_1}{a_1} = \frac{91.1\ \text{m/s}}{505\ \text{m/s}} = 0.180$$

Now we have to work out the pipe-friction relations, Eqs. (9.66) and (9.68), for k = 1.32. We need fL^*/D, V/V^*, and p/p^* at the inlet, Ma = 0.18**:**

$$\frac{fL^*}{D} = \frac{1-Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right] = 19.63 \quad \text{at } Ma_1 = 0.18 \quad \text{and} \quad k = 1.32$$

$$\text{Solve} \quad L^*_{choking} = 19.63\frac{D}{f} = 19.63\left(\frac{0.15\ m}{0.023}\right) = \mathbf{128\,m} \quad \textit{Ans.}\ \text{(a)}$$

(b) First find V/V^* and increase it by 50% to find the new condition:

$$\frac{V_1}{V^*} = Ma_1\left[\frac{k+1}{2+(k-1)Ma_1^2}\right]^{1/2} = 0.1938 \ \text{ if } \ k = 1.32,\ Ma = 0.18$$

$$\text{Then } \frac{V_2}{V^*} = 2(0.1938) = 0.3876\ ; \ \text{ Solve for } Ma_2 = 0.2715,\ fL^*/D = 7.35$$

$$\text{Velocity increases 50\% when } \Delta L = (19.63 - 7.35) * \frac{0.15m}{0.023} = \mathbf{80\,m} \quad \textit{Ans.}(b)$$

(c) Now find p/p^* and decrease it by 50% to find the required pipe length:

$$\frac{p}{p^*} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]^{1/2} = 5.954 \quad \text{at } Ma_1 = 0.18 \quad \text{and} \quad k = 1.32$$

$$\text{Decrease 50\% to:} \quad p/p^* = 2.977 \quad \text{Solve for: } Ma_2 = 0.358,\ fL^*/D = 3.46$$

$$\text{Solve:} \quad \Delta L^* = (19.63 - 3.46)\frac{D}{f} = 16.17\left(\frac{0.15\,\text{m}}{0.023}\right) = \mathbf{105\,m} \quad \textit{Ans.}\ \text{(c)}$$

9.97 By making a few algebraic substitutions, show that Eq. (9.74), or the relation in Prob. 9.96, may be written in the density form

$$\rho_1^2 = \rho_2^2 + \rho^{*2}\left(\frac{2k}{k+1}\frac{\bar{f}L}{D} + 2\ln\frac{\rho_1}{\rho_2}\right)$$

Why is this formula awkward if one is trying to solve for the mass flow when the pressures are given at sections 1 and 2?

Solution: This much less laborious algebraic derivation is left as a student exercise. There are two awkward bits: (1) we don't know ρ_1 and ρ_2; and (2) we don't know ρ^* either, and preliminary computations are necessary.

9.98 Compressible *laminar* flow, $f \approx 64/\text{Re}$, may occur in capillary tubes. Consider air, at stagnation conditions of 100°C and 200 kPa, entering a tube 3 cm long and 0.1 mm in diameter. If the receiver pressure is near vacuum, estimate (a) the average Reynolds number, (b) the Mach number at the entrance, and (c) the mass flow in kg/h.

Solution: The pipe is choked, "receiver pressure near vacuum," so L = L* and we need only to correctly guess the inlet Mach number and iterate until the Table B.3 value of (fL/D) matches the actual value, with f ≈ 64/Re from laminar pipe theory. Since Re = ρVD/μ and ρV is constant due to mass conservation, Re varies only due to the change in μ with temperature (from about 2.1E–5 in the entrance to 1.9E–5 kg/m·s at the exit). We assume μ_{avg} ≈ 2.0E–5 kg/m·s. Try Ma_1 from 0.1 to 0.2 and find **0.12** to be the best estimate:

$$\mathbf{Ma_1 \approx 0.12} \quad \textit{Ans.}\text{(b)} \quad \text{Table B.3:} \quad (\text{fL/D})_1 \approx 45.4\text{, also compute}$$

$$T_1 = 372\text{ K},\ V_1 = 46\text{ m/s},\ \rho_1 = 1.85\ \frac{\text{kg}}{\text{m}^3},\ \mathbf{Re_{avg}} = \frac{1.85(46)(0.0001)}{2.0\text{E}{-5}} = \mathbf{430} \quad \textit{Ans.}\text{(a)}$$

$$\text{Then} \quad f_{laminar} \approx 64/430 = 0.15,\ f(L/D) = 0.15(300) \approx 45.0 \text{ (close enough for me!)}$$

$$\text{The mass flow is } \dot{m} = \rho_1 A_1 V_1 = 1.85(\pi/4)(0.0001)^2(46) \approx \mathbf{6.74E{-}7}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\text{(c)}$$

(0.00243 kg/h)

9.99 A compressor forces air through a smooth pipe 20 m long and 4 cm in diameter, as in Fig. P9.99. The air leaves at 101 kPa and 200°C. The compressor data for pressure rise versus mass flow are shown in the figure. Using the Moody chart to estimate $\bar{f}$, compute the resulting mass flow.

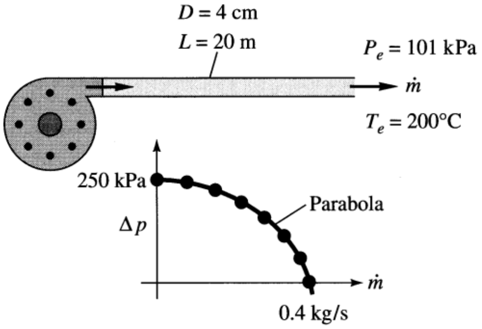


Fig. P9.99

Solution: The compressor performance is approximate by the parabolic relation

$$\Delta p_{compressor} \approx 250 - 1563\,\dot{m}^2, \text{ with } \Delta p \text{ in kPa and } \dot{m} \text{ in kg/s}$$

We must match this to the pressure drop due to friction in the pipe. For preliminaries, compute $\rho_e = p_e/(RT_e) = \mathbf{0.744}$ kg/m^3, and $a_e = \sqrt{(kRT_e)} = \mathbf{436}$ m/s. Guess the mass flow:

$$\dot{m} \overset{?}{=} 0.2 \text{ kg/s}, \quad \text{then } \mathrm{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.2)}{\pi(0.04)(2.5\mathrm{E}{-5})} \approx 255000, \; f_{Moody} \approx 0.015$$

$$V_e = \frac{\mathrm{Re}\mu}{\rho_e D} = \frac{255000(2.5\mathrm{E}{-5})}{0.744(0.04)} \approx 214\ \frac{\mathrm{m}}{\mathrm{s}}, \quad \mathrm{Ma}_e = \frac{214}{436} = 0.491, \quad \text{read } \left.\frac{fL}{D}\right|_e \approx 1.15,$$

$$\text{also read } \quad p_e/p^* \approx 2.18, \; p^* = \frac{101000}{2.18} \approx 46300\ \mathrm{Pa}$$

Then, at the pipe entrance (Sect. 1), we may compute fL/D and find the pressure there:

$$\left.\frac{fL}{D}\right|_1 = 1.15 + \frac{0.015(20)}{0.04} = 8.65, \quad \text{read } \mathrm{Ma}_1 \approx 0.248, \; \frac{p_1}{p^*} \approx 4.39,$$

$$\therefore \; p_2 = 4.39(46300) \quad \text{or} \quad p_2 \approx 203 \text{ kPa},$$

$$\text{or} \quad \Delta p_{pipe} = 203 - 101 \approx 102 \text{ kPa} \quad \text{whereas } \Delta p_{comp} \approx 187\,\text{kPa } (\dot{m} \text{ too small})$$

We increase the mass flow until $\Delta p_{pipe} \approx \Delta p_{compressor}$. The final converged result is:

$$\dot{m} = \mathbf{0.256}\ \frac{\mathrm{kg}}{\mathrm{s}}, \; \mathrm{Re} \approx 326000, \; f \approx 0.0142, \; V_e \approx 274\ \frac{\mathrm{m}}{\mathrm{s}}, \; \mathrm{Ma}_e \approx 0.628, \text{ read}$$

$$fL/D|_e \approx 0.39, \; p_e/p^* \approx 1.68, \; p^* \approx 60.1 \text{ kPa, add } f\Delta L/D = 7.1 \text{ to get} \quad fL/D|_1 \approx \mathbf{7.49},$$

$$\text{read } \mathrm{Ma}_1 \approx 0.263, \; p_1/p^* \approx 4.14, \; p_2 \approx 249 \text{ kPa}, \; \Delta p_{pipe} \approx 148 \text{ kPa} = \Delta p_{compressor} (\text{OK})$$

For these operating conditions, the approximate flow rate is **0.256 kg/s**. *Ans.*

P9.100 Air flows adiabatically through a pipe 10 cm in diameter and 66 m long. Entrance conditions are p_1 = 550 kPa, T_1 = 350 K, and V_1 = 75 m/s. The average friction factor is 0.022. (*a*) Is the pipe flow choked at the exit? (*b*) What is the exit pressure? (*c*) At what distance down the pipe is the velocity 184 m/s?

Solution: Find the inlet Mach number from the given data:

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{75\,m/s}{\sqrt{1.4(287)(350)}} = \frac{75}{375} = 0.20$$

Excellent, this Mach number is right in Table B.3. Read fL^*/D = 14.5333. Then find (fL/D) at the end of the pipe, x = 66 meters:

$$\frac{f\,L}{D}\Big|_{exit} = 14.5333 - \frac{(0.022)(66\,m)}{0.1m} = 14.5333 - 14.52 = 0.0133 \approx 0$$

As close as we can calculate, **Yes, it is choked at the exit.** *Ans.*(*a*)

(*b*) The exit then must be at (sonic) pressure p^*. Calculate this from the inlet pressure:

$$Ma_1 = 0.2\text{:}\ \frac{p}{p^*} = 5.4554\ ,\ \ p_1 = 550\ kPa\ ,\quad \text{thus}\ \ p^* = \frac{550\,kPa}{5.4554} = \mathbf{101}\ kPa \quad Ans.(b)$$

(*c*) Go back and find V/V^* at the inlet. Then we can move downstream to the new velocity:

$$Ma_1 = 0.2\text{:}\ \ V/V^* = 0.2182\ ;\ \ \text{thus}\ \ V^* = 75m/s/0.2182 = 344\,m/s$$

Downstream, $V_2/V^* = (184m/s)/344m/s = 0.5349$; Table B.3: Read $Ma_2 = 0.50$; then

$$fL/D = 1.0691\,,\ f\,\delta L/D = 14.5333 - 1.0691 = 13.46;\ \delta L = (13.46)(0.1)/.022 = \mathbf{61.2}\ m\ \ Ans.(c)$$

9.101 How do the compressible-pipe-flow formulas behave for small pressure drops? Let air at 20°C enter a tube of diameter 1 cm and length 3 m. If $\bar{f} = 0.028$ with p_1 = 102 kPa and p_2 = 100 kPa, estimate the mass flow in kg/h for (a) isothermal flow, (b) adiabatic flow, and (c) incompressible flow (Chap. 6) at the entrance density.

Solution: For a pressure change of only 2%, all three estimates are nearly the same. Begin by noting that fL/D = 0.028(3.0/0.01) = **8.4**, and ρ_1 = 102000/[287(293)] ≈ **1.213** kg/m^3. Take these estimates in order:

$$\text{(a) Isothermal: } \left(\frac{\dot{m}}{A}\right)^2 = \frac{p_1^2 - p_2^2}{RT[fL/D + 2\ln(p_1/p_2)]} = \frac{(102000)^2 - (100000)^2}{287(293)[8.4 + 2\ln(102/100)]} = 569$$

$$\text{Then} \quad \dot{m}/A \approx 23.9,\ \dot{m}_{\text{isothermal}} = 23.9(\pi/4)(0.01)^2 \approx \mathbf{0.00187\ \frac{kg}{s}} \quad \textit{Ans. (a)}$$

(b) Adiabatic: Given $T_o \approx 293$ K, $a_o = \sqrt{kRT_o} = 343$ m/s, use Eqs. 9.74 and 9.75:

$$\text{Converges to } \frac{V_1}{V_2} = 0.9803,\ V_1^2 = \frac{(343)^2[1 - (0.9803)^2]}{1.4(8.4) + 2.4\ln(1.02)} = 388, \text{ or } V_1 \approx 19.7 \text{ m/s}$$

$$\text{Then} \quad \dot{m} = \rho_1 A V_1 = 1.213(\pi/4)(0.01)^2(19.7) = \mathbf{0.00188\ kg/s} \quad \textit{Ans. (b)}$$

$$\text{(c) Incompressible: } \Delta p = (fL/D)(\rho/2)V^2, \text{ or } 2000 = (8.4)(1.213/2)V^2, \text{ or } V \approx 19.8 \text{ m/s.}$$

$$\text{Then} \quad \dot{m}_{\text{incompressible}} = \rho A V = 1.213(\pi/4)(0.01)^2(19.8) \approx \mathbf{0.00189\ kg/s}. \quad \textit{Ans. (c)}$$

9.102 Air at 550 kPa and 100°C enters a smooth 1-m-long pipe and then passes through a second smooth pipe to a 30-kPa reservoir, as in Fig. P9.102. Using the Moody chart to compute *f*, estimate the mass flow through this system. Is the flow choked?

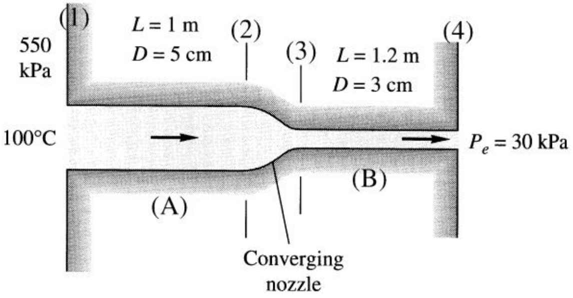


Fig. P9.102

Solution: Label the pipes "A" and "B" as shown. Given (*L/D*)A = 20 and (*L/D*)B = 40. Label the relevant sections 1, 2, 3, 4 as shown. With p_{o1}/p_e = 550/30 = 18.3, these short pipes are *sure* to be choked, with an exit pressure p_4 much larger than 30 kPa. One way is to guess Ma_1 and work your way through to section 4 to require $Ma_4 = 1.0$ (choked). Take a constant average viscosity $\mu = 2.2\text{E}{-5}$ kg/m·s. Assume isentropic expansion to section 1 from the reservoir, frictional flow through pipe A, isentropic expansion from 2 to 3, and a second frictional flow through pipe B to section 4. The correct solution is $Ma_1 \approx 0.18$:

$$Ma_1 = 0.18,\ T_1 = \frac{373\text{ K}}{1+0.2(0.18)^2} = 371\text{ K},\quad p_1 = \frac{450000\text{ Pa}}{[1+0.2(0.18)^2]^{3.5}} = 440000\,\text{Pa},$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{440000}{287(371)} = 4.14\ \frac{\text{kg}}{\text{m}^3},\quad V_1 = Ma_1\sqrt{kRT_1} = 69.5\ \frac{\text{m}}{\text{s}},\quad \text{Re}_1 = \frac{\rho_1 V_1 D_A}{\mu} = 653{,}000$$

$$\text{Moody chart: } f_A \approx 0.0125;\ \text{Table B.3: } \frac{fL}{D}\Big|_1 = 18.54,\ \frac{fL}{D}\Big|_2 = 18.54 - 0.0125(20) = 18.29$$

Pipe A is so short that the Mach number hardly changes. At $(fL/D)_2 = 18.29$, read $Ma_2 \approx 0.181$. Now, at $Ma_2 = 0.181$, determine $A_2/A^* = 3.26$, hence $A_3/A^* = (3.26)(3/5)^2 = 1.17$, read $Ma_3 = 0.613$ and $(fL/D)_3 = 0.442$. Stop to calculate $\rho_3 = 3.49\text{ kg/m}^3$, $V_3 = 229$ m/s, $\text{Re}_B = 1.09\text{E}6$, from the Moody chart, $f_B = 0.0115$. Then $(fL/D)_4 = 0.442 - 0.0115(40) = \mathbf{-0.018}$. (?) This last value should have been exactly $(fL/D)_4 = 0$ if the exit Mach number is 1.0. But we were close. The mass flow follows from the conditions at section 1:

$$\dot{m} = \rho_1 A_1 V_1 = \left(4.14\ \frac{\text{kg}}{\text{m}^3}\right)\left[\frac{\pi}{4}(0.05\text{ m})^2\right]\left(69.5\ \frac{\text{m}}{\text{s}}\right) \approx \mathbf{0.565\ \frac{kg}{s}} \quad \textit{Ans.}$$

EES can barely improve upon this: $Ma_1 = \mathbf{0.1792}$, yielding a mass flow of **0.5616 kg/s**. The exit pressure is $p_4 = 201$ kPa, far larger than the receiving reservoir pressure of 30 kPa.

9.103 Natural gas, with $k \approx 1.3$ and a molecular weight of 16, is to be pumped through 100 km of 81-cm-diameter pipeline. The downstream pressure is 150 kPa. If the gas enters at 60°C, the mass flow is 20 kg/s, and $\bar{f} = 0.024$, estimate the required entrance pressure for (a) isothermal flow and (b) adiabatic flow.

Solution: The gas constant is $R_{gas} = 8314/16 \approx 520$ J/kg·K. First use Eq. 9.73:

(a) Isothermal: $$\left(\frac{\dot{m}}{A}\right)^2 = \left[\frac{20}{(\pi/4)(0.81)^2}\right]^2 = \frac{p_1^2 - p_2^3}{RT[fL/D + 2\ln(p_1/p_2)]}$$

$$= \frac{p_1^2 - (150000)^2}{520(333)[2963 + 2\ln(p_1/150000)]},$$

solve for $\mathbf{p_1 \approx 892\ kPa}$ *Ans.* (a)

Part (a) indicates a low inlet Mach number, ≈ 0.02, so $T_e \approx T_o$, $a_e \approx a_o \approx 475$ m/s. Then use Eqs. (9.74) and (9.75)—the latter simply indicates that the bracket [] ≈ 1.000. Then

$$\frac{V_1}{V_2} \approx \frac{p_2}{p_1} \quad \text{and} \quad V_1^2 = \frac{a_o^2[1-(V_1/V_2)^2]}{k(fL/D) + (k+1)\ln(V_2/V_1)} = \frac{(475)^2[1-(V_1/V_2)^2]}{1.3(2963) + 2.3\ln(V_2/V_1)},$$

plus $\rho_1 V_1 = \rho_2 V_2 = \dot{m}/A = 38.81\ \text{kg/s·m}^2$. Solve for $V_1 = 7.54\ \frac{m}{s}$, $\rho_1 = 5.15\ \frac{kg}{m^3}$,

$$\mathbf{p_1} = \rho_1 R T_1 = 5.15(520)(333) \approx \mathbf{892\ kPa}$$

9.104 A tank of oxygen (Table A.4) at 20°C is to supply an astronaut through an umbilical tube 12 m long and 1.5 cm in diameter. The exit pressure in the tube is 40 kPa. If the desired mass flow is 90 kg/h and f = 0.025, what should be the air pressure in the tank?

Solution: For oxygen, from Table A.4, take $k = 1.40$ and $R = 260$ J/kg·K. Given T_o = 293 K and $f\Delta L/D = (0.025)(12\ \text{m})/(0.015\ \text{m}) = 20$. Use isothermal flow, Eq. (9.73), as a first estimate:

$$\left(\frac{\dot{m}}{A}\right)^2 = \left[\frac{90/3600\ \text{kg/s}}{(\pi/4)(0.015\ \text{m})^2}\right]^2 = \frac{p_1^2 - p_2^2}{RT[fL/D + 2\ln(p_1/p_2)]}$$

$$= \frac{p_1^2 - (40000)^2}{(260)(293)[20 + 2\ln(p_1/40000)]} \quad \text{Solve for } p_1 \approx 192\ \text{kPa}$$

This is a very good estimate of p_1, but we really need *adiabatic* flow, Eqs. (9.66) and (9.68*a*). First estimate the entrance Mach number from p_1 and $T_1 \approx T_o$:

$$\rho_1 \approx \frac{p_1}{RT_o} = \frac{192000}{260(293)} \approx 2.52\ \frac{\text{kg}}{\text{m}^3},\quad \dot{m} = \frac{90\ \text{kg}}{3600\ \text{s}} = \rho_1 A V_1 \approx (2.52)\frac{\pi}{4}(0.015\,\text{m})^2 V_1$$

$$\text{solve}\quad V_1 \approx 56\ \text{m/s},\quad a_1 \approx \sqrt{kRT_o} = \sqrt{1.4(260)(293)} \approx 327\ \text{m/s},\ Ma_1 \approx \frac{56}{327} \approx 0.17$$

We can guess Ma_1 around 0.17, find $(fL^*/D)_1$, subtract $(f\Delta L/D) = 20$, find the new Mach number and p^*, thence back up to obtain p_1. Iterate to convergence. For example:

$Ma_1 = 0.17$, $(fL^*/D)_1 = 21.12$, $p_1/p^* = 6.43$, $(fL^*/D)_2 = 21.12 - 20 = 1.12$, compute

$Ma_2 = 0.49$, $p_2/p^* = 2.16$, $p^* = 40000/2.16 = 18500$ Pa, $p_1 = 6.43(18500) = 119000$ Pa,

$T_1 = T_o/[1 + 0.2(0.17)^2] = 291$ K, $\rho_1 = 1.57$ kg/m^3, mass flow $= \rho_1 A V_1 \approx 55$ kg/h

The mass flow is too low, so try Ma_1 a little higher. The iteration is remarkably sensitive to Mach number because the correct exit flow is close to sonic. The final converged solution is

$Ma_1 = 0.1738$, $Ma_2 = 0.7792$, $\boldsymbol{p_1 = 189.4\ \text{kPa}}$ *Ans.*

This problem is clearly well suited to EES, which converges rapidly to the final pressure.

P9.105 Modify Prob. P9.87 as follows. The pipeline will not be allowed to choke. It will have pumping stations about every two hundred miles. (*a*) Find the length of pipe for which the pressure has dropped to 2000 lbf/in^2. (*b*) What is the temperature at that point?

Solution: From Prob. P9.87, we found the following data. For CH_4, from Table A.4, R = 518 m^2/s^2-K, $k = 1.32$, and $\mu = 1.03\text{E-}5$ kg/m-s. Convert to SI units: $p_1 = 2500$ psi = 1.72E7 Pa, $D = 52$ in = 1.321 m, $T_1 = 140°\text{F} = 333$ K. From the ideal gas law and the known mass flow (890 kg/s), find the entrance density and pressure, Reynolds number, and smooth-wall friction factor:

$$\rho_1 = 99.9\,\frac{kg}{m^3}\ ;\ V_1 = 6.50\frac{m}{s}\ ;\ \text{Re}_D = 8.33E7\ ;\ f_{smooth} \approx 0.00608$$

Then we found the entrance Mach number and thus the factor (fL^*/D):

$$Ma_1 = \frac{V_1}{a_1} = \frac{6.5}{477} = 0.0136 \;;\; k = 1.32 \;;\; Eq.(9.66):\; \frac{f\,L^*}{D} = 4074$$

The present problem involves pressures and temperatures, so we need to calculate p/p^* and T/T^*:

$$Ma_1 = 0.0136;\; k = 1.32,\quad \text{Eqs.(9.68)}:\; \frac{p_1}{p^*} = 79.06\;,\; \frac{T_1}{T^*} = 1.16$$

(*a*) When the pressure p_2 drops to 2000 psia = 1.39E6 Pa, the new pressure ratio will be

$$\frac{p_2}{p^*} = \frac{p_2}{p_1}\frac{p_1}{p^*} = \left(\frac{2000}{2500}\right)(79.06) = 63.25\;;\;\text{ Eq. (9.68}a\text{) yields }\; Ma_2 \approx 0.0170$$

$$\text{At this } Ma_2\;,\; \left(\frac{fL^*}{D}\right) = 2605,\; hence\;\; \delta\left(\frac{fL^*}{D}\right) = 4074 - 2605 = 1469 = \frac{f\;\delta L}{D}$$

$$\text{Finally},\; p_2 = 2000\,psia \text{ at } \delta L = \frac{(1469)(1.321)}{0.00608} = 319{,}000\,m = \mathbf{198\ mi} \quad Ans.(a)$$

(*b*) At this same $Ma_2 = 0.0170$, compute T_2/T^* also equals 1.16, same as T_1/T^*. Therefore the temperature has hardly changed at all:

$$T_2 \approx \mathbf{333K} = 140°\text{F}. \quad \text{Ans.}(b)$$

***P9.106** Air, from a 3 cubic meter tank initially at 300 kPa and 200°C, blows down adiabatically through a smooth pipe 1 cm in diameter and 2.5 m long. Estimate the time required to reduce the tank pressure to 200 kPa. For simplicity, assume constant tank temperature and $f \approx 0.020$.

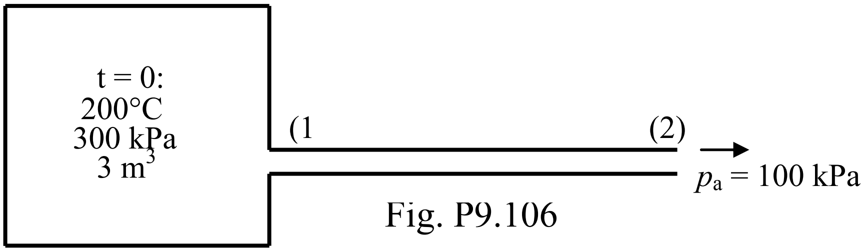


Fig. P9.106

Solution: We know that p_2 at the exit is 100 kPa, and we are given $f = 0.020$, thanks! For the given L and D, we can immediately calculate $f\Delta L/D = (0.02)(2.5\text{m}/0.01\text{m}) = 5.0$. Suggested procedure: Calculate the mass flow at $p_{\text{tank}} = 300$ and 200 kPa and average these two. Guess Ma_2 at the exit, calculate fL/D_2, increase upstream by 5.0 to fL/D_1, find Ma_1 and p_1 when $p_{o1} = p_{\text{tank}}$, estimate ρ_1, V_1, and the mass flow. To start, guess sonic flow at the exit, $\text{Ma}_2 = 1.0$, $fL/D_2 = 0$. Then $fL/D_1 = 5.0$, for which $\text{Ma}_1 = 0.3066$ (Table B.3), $p_1/p^* = 3.54$, hence $p_1 = 354$ kPa and $p_{o1} = 378$ kPa, too high! So the exit Mach numbers are less than 1.0. Iteration (with EES) leads to the following results that converge to a tank pressure of 300 kPa:

$$Ma_2 = 0.814\ ;\ \frac{p_2}{p^*} = 1.2646\ ;\ \frac{fL}{D}|_2 = 0.0605\ ;\frac{fL}{D}|_1 = 5.0605\ ;\ Ma_1 = 0.3052\ ;\ \frac{p_1}{p^*} = 3.5564$$

$$p_1 = \frac{p_1/p^*}{p_2/p^*}(p_2) = \frac{3.5564}{1.2646}(100) = 281.2\,kPa;\ p_{o1} = (281.2)[1+0.2(0.3052)^2]^{3.5} = 300\,kPa$$

Repeat these (laborious) calculations for a tank pressure of 200 kPa:

$$Ma_2 = 0.536\ ;\ \frac{p_2}{p^*} = 1.9874\ ;\ \frac{fL}{D}|_2 = 0.8113\ ;\frac{fL}{D}|_1 = 5.8113\ ;\ Ma_1 = 0.2898\ ;\ \frac{p_1}{p^*} = 3.7491$$

$$p_1 = \frac{p_1/p^*}{p_2/p^*}(p_2) = \frac{3.7491}{1.9874}(100) = 188.6\,kPa;\ p_{o1} = (188.6)[1+0.2(0.2898)^2]^{3.5} = 200\,kPa$$

Now, with inlet conditions known, find V_1 and ρ_1 and thus the two mass flows:

$$p_{o1} = 300\,kPa:\ \rho_1 = 2.110\frac{kg}{m^3};V_1 = 131.8\frac{m}{s}\ ;\ \dot{m} = \rho_1 A V_1 = (2.110)\frac{\pi}{4}(0.01)^2(131.8) = 0.0219\frac{kg}{s}$$

$$p_{o1} = 200\,kPa:\ \rho_1 = 1.413\frac{kg}{m^3};V_1 = 125.3\frac{m}{s}\ ;\ \dot{m} = \rho_1 A V_1 = (1.413)\frac{\pi}{4}(0.01)^2(125.3) = 0.0139\frac{kg}{s}$$

$$Average\ mass\ \ flow = (0.0219 + 0.0139)/2 \approx 0.0179\ kg/s$$

Tank at 300 kPa: $m = [p_o/RT_o](\text{Vol}) = [(300000/287/473)\text{kg/m}^3](3\text{m}^3) = (2.21)(3) = 6.63$ kg

Tank at 200 kPa: $m = [p_o/RT_o](\text{Vol}) = [(200000/287/473)\text{kg/m}^3](3\text{m}^3) = (473)(3) = 4.42$ kg

Time to blow down from 300 to 200 kPa = (6.63-4.42 kg)/(0.0179 kg/s) ≈ **91 s** *Ans.*

9.107 A fuel-air mixture, assumed equivalent to air, enters a duct combustion chamber at *V*1 = 104 m/s and *T*1 = 300 K. What amount of heat addition in kJ/kg will cause the exit flow to be choked? What will be the exit Mach number and temperature if 504 kJ/kg is added during combustion?

Solution: Evaluate stagnation temperature and initial Mach number:

$$T_o = T_1 + \frac{V_1^2}{2c_p} = 300 + \frac{(104)^2}{2(1005)} = 305\text{ K};\quad Ma_1 = \frac{104}{\sqrt{1.4(287)(300)}} \approx 0.30$$

$$\text{Table B.4:}\quad T_o/T_o{}^* = 0.3469,\quad \text{hence}\ \ T_o{}^* \approx \left(\frac{305}{0.3469}\right) \approx 880\text{ K}$$

$$\text{Thus}\quad q_{choke} = c_p\Delta T_{o,max} = 1005(880-305) \approx 5.78\text{E5 J/kg} = \mathbf{578\ kJ/kg}\quad \textit{Ans.}\text{ (a)}$$

A heat addition of 504 kJ/kg is (just barely) less than maximum, should nearly choke:

$$T_{o2} = T_{o1} + \frac{q}{c_p} = 305 + \frac{540000}{1005} \approx 842\text{ K},\quad \frac{T_{o2}}{T_o^*} = \frac{842}{880} = 0.957,\quad \therefore\ Ma_2 \approx \mathbf{0.78}\quad \textit{Ans.}\text{ (b)}$$

Finally, without using Table B.4, $T_2 = 842/[1+0.2(0.78)^2] \approx \mathbf{751\ K}$ *Ans.* (c)

9.108 What happens to the inlet flow of Prob. 9.107 if the combustion yields 1500 kJ/kg heat addition and po1 and To1 remain the same? How much is the mass flow reduced?

Solution: The flow will choke down to a lower mass flow such that $T_{o2} = T_o^*$:

$$T_{o2} = T_o^* = 305 + \frac{1500000}{1005} = 1798\text{ K},\quad \text{thus}\ \ \frac{T_{o1}}{T_o^*} = \frac{305}{1798} = 0.17,\quad Ma_{1,new} \approx \mathbf{0.198}$$

$$(\dot{m}/A)_{new} = \rho_1 V_1 = \rho_1 a_1 Ma_1 = \rho_o a_o Ma_1/[1+0.2Ma_1^2]^3\quad \text{if}\ \ p_{o1},\ T_{o1},\ \rho_{o1}\ \text{are the same.}$$

$$\text{Then}\quad \frac{\dot{m}_{new}}{\dot{m}_{old}} = \frac{0.198}{0.30}\left[\frac{1+0.2(0.30)^2}{1+0.2(0.198)^2}\right] \approx \mathbf{0.68}\ \text{(about 32\% less flow)}\quad \textit{Ans.}$$

9.109 A jet engine at 7000-m altitude takes in 45 kg/s of air and adds 550 kJ/kg in the combustion chamber. The chamber cross section is 0.5 m^2, and the air enters the chamber at 80 kPa and 5°C. After combustion the air expands through an isentropic converging nozzle to exit at atmospheric pressure. Estimate (a) the nozzle throat diameter, (b) the nozzle exit velocity, and (c) the thrust produced by the engine.

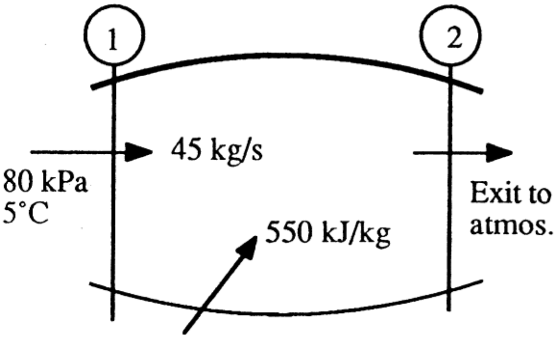


Fig. P9.109

Solution: At 700-m altitude, pa = 41043 Pa, Ta = 242.66 K to use as exit conditions.

$$\rho_1 = \frac{p_1}{RT_1} = \frac{80000}{287(278)} = 1.00\ \frac{kg}{m^3},\quad \dot{m} = 45\ \frac{kg}{s} = \rho AV = 1.00(0.5)V_1,\quad V_1 = 90\,m/s$$

$$Ma_1 = \frac{90}{\sqrt{1.4(287)(278)}} = \mathbf{0.27},\quad \text{Table B.4:}\ \ T_{o1}/T_o^* \approx 0.29,$$

$$T_{o1} = 278 + (90)^2/[2(1005)] \approx 282\ K,\quad \therefore\ T_o^* = 282/0.29 \approx \mathbf{973\ K}$$

$$\text{Add heat:}\quad T_{o2} = 282 + \frac{550000}{1005} \approx 829\ K,\quad \text{thus}\ \frac{T_{o2}}{T_o^*} = \frac{829}{973} \approx 0.85,\quad \text{read}\ \ Ma_2 \approx \mathbf{0.63}$$

$$\text{also read}\quad p_1/p^* \approx 2.18,\ p_2/p^* \approx 1.54,\quad \therefore\quad p_2 = 80(1.54/2.18) \approx 57\ kPa,$$

$$p_{o2} = p_2\left[1 + 0.2\,Ma_2^2\right]^{3.5} = 57[1 + 0.2(0.63)^2]^{3.5} \approx 74\,kPa$$

With data now known at section 2, expand isentropically to the atmosphere:

$$\frac{p_e}{p_{o2}} = \frac{41043}{57000} = 0.72 = \left[1 + 0.2Ma_e^2\right]^{-3.5},\quad \text{solve}\ \ Ma_e = 0.70,\quad \frac{T_e}{T_{o2}} = \frac{T_e}{829} = 0.910\ ,$$

$$\text{Solve}\quad T_e \approx 755\ K,\quad \rho_e = p_e/RT_e \approx 0.189\ kg/m^3,\quad a_e = \sqrt{kRT_e} = 551\ m/s\ ,$$

$$V_e = Ma_e a_e \approx \mathbf{385\ m/s}\quad \textit{Ans.}\ (b)$$

$$\dot{m} = 45 = 0.189(385)\frac{\pi}{4}D_e^2,\quad \text{solve}\ \ D_e \approx \mathbf{0.89\ m}\quad \textit{Ans.}\ (a)$$

Finally, if $p_e = p_{atm}$, from Prob. 3.68, $F_{thrust} = \dot{m}V_e = 45(385) \approx \mathbf{17300\ N}$ *Ans.* (c)

9.110 Compressible pipe flow with heat addition, Sec. 9.8, assumes constant momentum ($p + \rho V^2$) and constant mass flow but variable stagnation enthalpy. Such a flow is often called *Rayleigh flow*, and a line representing all possible property changes on an temperature-entropy chart is called a *Rayleigh line*. Assuming air passing through the flow state $p_1 = 548$ kPa, $T_1 = 588$ K, $V_1 = 266$ m/s, and $A = 1\ \mathrm{m}^2$, draw a Rayleigh curve of the flow for a range of velocities from very low (Ma□ 1) to very high (Ma□ 1). Comment on the meaning of the maximum-entropy point on this curve.

Solution: First evaluate the Mach number and density at the reference state:

$$\rho = \frac{\mathrm{p}}{\mathrm{RT}} = \frac{548000}{287(588)} \approx 3.25\ \frac{\mathrm{kg}}{\mathrm{m}^3}; \quad \mathrm{Ma} = \frac{\mathrm{V}}{\sqrt{\mathrm{kRT}}} = \frac{266}{\sqrt{1.4(287)(588)}} = 0.55$$

Our basic algebraic equations are then:

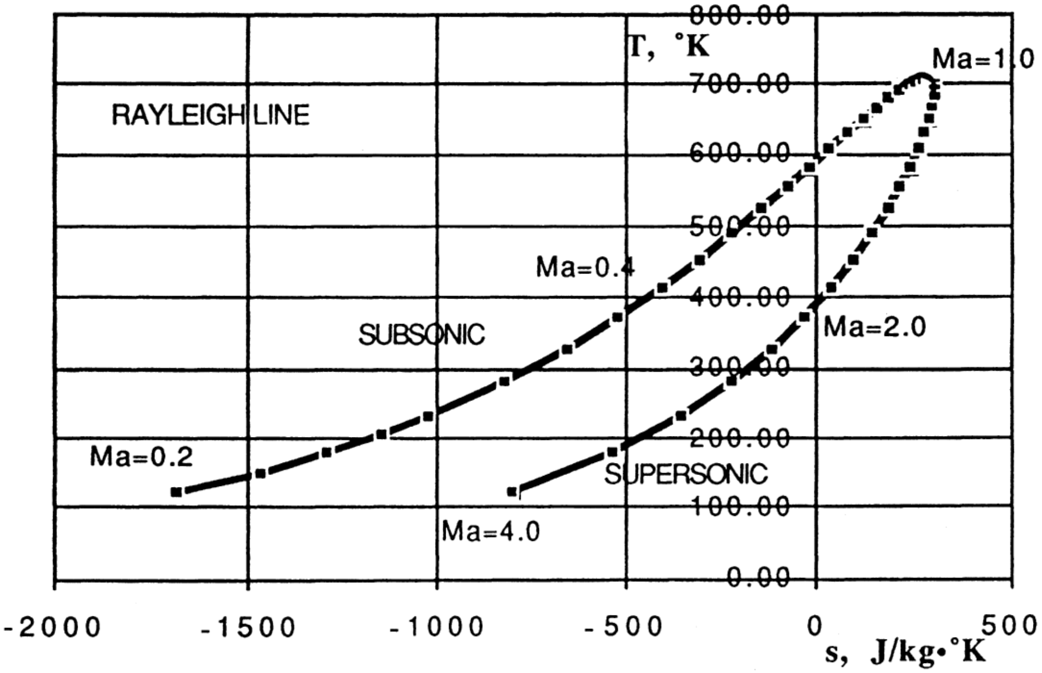
$$\text{Momentum:} \quad \mathbf{p} + \boldsymbol{\rho}\mathbf{V}^2 = 548000 + 3.25(266)^2 = \mathbf{778000} \tag{a}$$

$$\text{Continuity:} \quad \rho\mathrm{V} = 3.25(266), \quad \text{or:} \quad \rho = \mathbf{864/V} \tag{b}$$

$$\text{Entropy:} \quad \mathbf{s = 718\ln(T/588) - 287\ln(\boldsymbol{\rho}/3.25)} \tag{c}$$

We simply let V vary from, say, 10 m/s to 800 m/s, compute ρ from (b), p from (a), $T = \mathrm{p}/\rho\mathrm{T}$, and s from (c), then plot T versus s. [We have arbitrarily set s = 0 at state 1.]

The result of this exercise forms the **Rayleigh Line** for this flow, shown below. Some Mach numbers are listed, subsonic on the top, supersonic on the bottom, and exactly **sonic** at the right-hand (maximum-entropy) side. *Ans.*



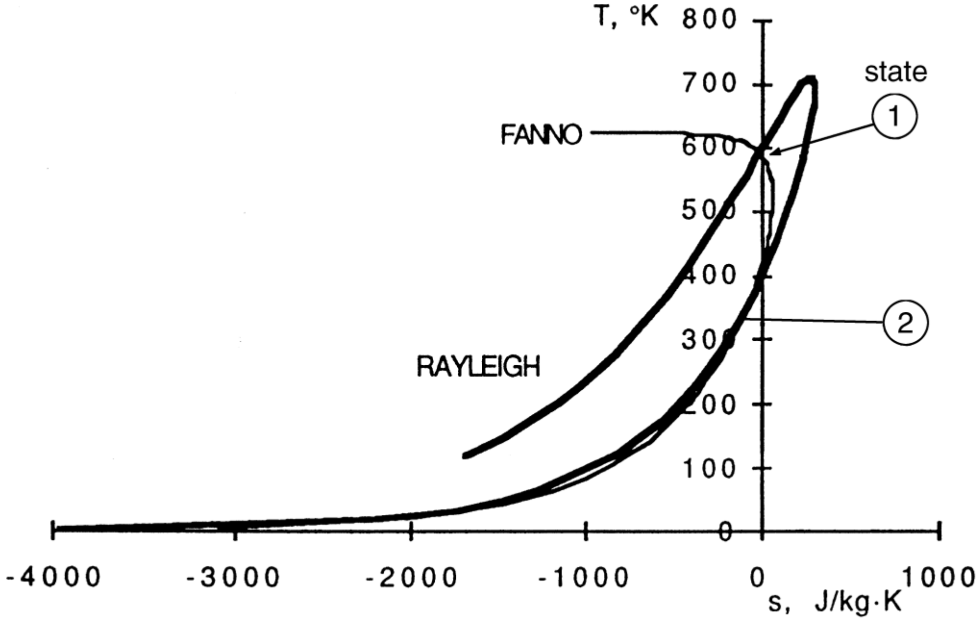
***9.111** Add to your Rayleigh line of Prob. 9.110 a Fanno line (see Prob. 9.94) for stagnation enthalpy equal to the value associated with state 1 in Prob. 9.110. The two curves will intersect at state 1, which is subsonic, and also at a certain state 2, which is supersonic. Interpret these two cases vis-a-vis Table B.2.

Solution: For $T_1 = 588$ K and $V_1 = 266$ m/s, the stagnation temperature is

$$T_o = T_1 + \frac{V^2}{2c_p} = 588 + \frac{(266)^2}{2(1005)} \approx \mathbf{623\ K};\quad \text{Elsewhere, } T = 623 - \frac{V^2}{2(1005)} \qquad \textbf{(d)}$$

Also, from Prob. 9.110, $\rho = 864/V$ **(b)** and $s = 728\ln\left(\frac{T}{588}\right) - 287\ln\left(\frac{\rho}{3.25}\right)$ **(c)**

By varying V and computing (T, ρ, s) from (b, c, d), we plot the Fanno line and add it to the previous Rayleigh line. The composite graph is as follows:



The subsonic intersection is state 1, Ma1 ≈ **0.55**, T2 ≈ 588 K, and the supersonic intersection is at Ma2 ≈ **2.20**, where, for example, T2 ≈ 316 K. Also, s1 > s2. These two points thus correspond to the two sides of a **normal shock wave**, where "2" is the supersonic upstream and "1" the subsonic downstream condition. We may check these results in Table B-2, where, at Ma ≈ **2.20**, the temperature ratio across the shock is 1.857—for *our* calculations, this ratio is 588 K/316 K ≈ 1.86 (agreement would be perfect if we kept more significant figures). Shock flow satisfies **all** the four equations of Rayleigh and Fanno flow combined—continuity, momentum, energy, and the equation of state.

9.112 Air enters a duct subsonically at section 1 at 1.2 kg/s. When 650 kW of heat is added, the flow chokes at the exit at p2 = 95 kPa and T2 = 700 K. Assuming frictionless heat addition, estimate (a) the velocity; and (b) the stagnation pressure at section 1.

Solution: Since the exit is choked, p2 = p* and T2 = T* and, of course, Ma2 = 1.0. Then

$$V_2 = V^* = \sqrt{kRT^*} = \sqrt{1.4(287)(700)} \approx 530 \text{ m/s}, \quad \text{and} \quad q = \dot{Q}/\dot{m} = \frac{650}{1.2} = 542 \ \frac{\text{kJ}}{\text{kg}}$$

$$\text{Also,} \quad T_o^* = 1.2T_2 = 1.2(700) = 840 \text{ K}; \quad \text{hence } T_{o1} = 840 - \frac{542000}{1005} \approx 301 \text{ K}$$

$$\text{Then}\quad T_{o1}/T_o^* = \frac{301}{840} = 0.358; \quad \text{Table B.4: read } Ma_1 \approx \mathbf{0.306}, \text{ read } V_1/V^* \approx 0.199$$

$$\text{So}\quad V_1 = 530(0.199) \approx \mathbf{105\ m/s} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Also read}\quad p_{o1}/p_o^* \approx 1.196, \quad \text{where } p_o^* = p_2/0.5283 \approx 180\text{ kPa},$$

$$\text{Hence}\quad p_{o1} \approx 1.196(180) \approx \mathbf{215\ kPa} \quad \textit{Ans.}\text{ (b)}$$

9.113 Air enters a constant-area duct at p1 = 90 kPa, V1 = 520 m/s, and T1 = 558°C. It is then *cooled* with negligible friction until it exists at p2 = 160 kPa. Estimate (a) V2; (b) T2; and (c) the total amount of cooling in kJ/kg.

Solution: We have enough information to estimate the inlet Ma1 and go from there:

$$a_1 = \sqrt{1.4(287)(558+273)} = 578\ \frac{m}{s}, \quad \therefore\ Ma_1 = \frac{520}{578} \approx \mathbf{0.90}, \quad \text{read } \frac{p_1}{p^*} = 1.1246,$$

$$\text{or}\quad p^* = \frac{90}{1.1246} = 80.0\text{ kPa}, \quad \text{whence } \frac{p_2}{p^*} = \frac{160}{80} \approx 2.00, \quad \text{read } Ma_2 \approx \mathbf{0.38},$$

$$\text{read}\quad T_2/T^* = 0.575,\ V_2/V^* = 0.287,\ T_{o2}/T_o^* \approx 0.493$$

We have to back off to section 1 to determine the critical (*) values of T, V, To:

$$Ma_1 = 0.9,\ T_1/T^* = 1.0245,\ T^* = \frac{558+273}{1.0245} = 811\text{ K},\ T_2 = 0.575(811) \approx \mathbf{466\ K} \quad \textit{Ans.}\text{ (b)}$$

$$\text{also,}\quad V_1/V^* = 0.911,\ V^* = \frac{520}{0.911} = 571\text{ m/s}, \quad \text{so } V_2 = 0.287(571) \approx \mathbf{164\ m/s} \quad \textit{Ans.}\text{ (a)}$$

$$T_{o1}/T_o^* = 0.9921, \text{ where } T_{o1} = T_1 + V_1^2/2c_p = 966\text{ K},\ T_o^* = \frac{966}{0.9921} = 973\text{ K}$$

$$\text{Finally,}\quad T_{o2} = 0.493(973) = 480\text{ K},$$

$$\mathbf{q_{cooling}} = c_p\Delta T_o = 1.005(966-480) \approx \mathbf{489\ \frac{kJ}{kg}} \quad \textit{Ans.}\text{ (c)}$$

P9.114 The scramjet of Fig. 9.30 operates with supersonic flow throughout. An idealized sketch is in Fig. C9.8, at the end of this chapter. Assume that the heat addition of 500 kJ/kg, between sections 2 and 3, is frictionless and at constant area of 0.2 m^2. Given $Ma_2 = 4.0$, $p_2 = 260$ kPa, and $T_2 = 420$ K. Assume airflow at $k = 1.40$. At the combustion section exit, find (*a*) Ma_3, (*b*) p_3, and (*c*) T_3.

Solution: Use frictionless heat-addition theory to get from section 2 to section 3:

$$Ma_2 = 4.0: \frac{T_{o2}}{T_o^*} = 0.5891, \; T_{o2} = (420K)[1+0.2(4.0)^2] = 1764\,K$$

$$T_o^* = \frac{1764}{0.5891} = 2994\,K\;;\; \frac{T_2}{T^*} = 0.1683\;,\; T^* = \frac{420}{0.1683} = 2496\,K$$

$$\frac{p_2}{p^*} = 0.1026\;,\; p^* = \frac{260\,kPa}{0.1026} = 2534\,kPa\;;\; T_{o3} = T_{o2} + \frac{Q}{c_p} = 1764 + \frac{500000}{1005} = 2262\,K$$

Now find the new stagnation temperature ratio and hence the properties at section 3:

$$\frac{T_{o3}}{T_o^*} = \frac{2262}{2994} = 0.7552\;,\; \text{thus}\;\; Ma_3 = \mathbf{2.205} \quad Ans.(a)\;;$$

$$\frac{p_3}{p^*} = 0.3074\;,\; p_3 = (0.3074)(2534\,kPa) = \mathbf{779}\;kPa \quad Ans.(b)$$

$$\frac{T_3}{T^*} = 0.4594\;,\; T_3 = (0.4594)(2496\,K) = \mathbf{1146}\;K \quad Ans.(c)$$

P9.115 Air enters a 5-cm-diameter pipe at 380 kPa, 3.3 kg/m^3, and 120 m/s. Assume frictionless flow with heat addition. Find the amount of heat addition for which the velocity (*a*) doubles; (b) triples; and (*d*) quadruples.

Solution: First find the conditions at the entrance, which we will call section 1:

$$T_1 = \frac{p_1}{R\rho_1} = \frac{380000}{287(3.3)} = 401\,K \;;\; Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{120}{\sqrt{1.4(287)(401)}} = 0.299$$

$$\text{Table B.4 or EES:} \quad V_1/V^* = 0.1905 \quad , \quad T_{o1}/T_o^* = 0.3448$$

$$Then\ T_{o1} = T_1(1+0.2Ma_1^2) = (401K)[1+0.2(0.299)\text{^}2] = 408\,K$$

$$And\quad T_o^* = T_{o1}/(T_{o1}/T_o^*) = 408K/(0.3448) = 1184\,K$$

Since V_1 is less than one-fifth of V*, it looks like we can make it up to quadruple velocity:

$$(a)\ V_2/V^* = 2(0.1905) = 0.3810 \;;\; Table\,B.4:\ Ma_2 = 0.452\ ,\ T_{o2}/T_o^* = 0.6168$$

$$T_{o2} = 1184(0.6168) = 731K \;;\; q = c_p(T_{o2}-T_{o1}) = 1005(731-408) = 324{,}000\,J/kg \quad Ans.(a)$$

$$(b)\ V_3/V^* = 3(0.1905) = 0.5715 \;;\; Table\,B.4:\ Ma_3 = 0.598\ ,\ T_{o3}/T_o^* = 0.8164$$

$$T_{o3} = 1184(0.8164) = 967K \;;\; q = c_p(T_{o3}-T_{o1}) = 1005(967-408) = 561{,}000\,J/kg \quad Ans.(b)$$

$$(c)\ V_4/V^* = 4(0.1905) = 0.7620 \;;\; Table\,B.4:\ Ma_4 = 0.756\ ,\ T_{o4}/T_o^* = 0.9434$$

$$T_{o4} = 1184(0.9434) = 1117K \;;\; q = c_p(T_{o4}-T_{o1}) = 1005(1117-408) = 713{,}000\,J/kg \quad Ans.(c)$$

Note that we could even *quintuple*, going up to $5.25V_1$ before choking due to heat addition.

9.116 An observer at sea level does not hear an aircraft flying at 12000 ft standard altitude until it is 5 statute miles past her. Estimate the aircraft speed in ft/sec.

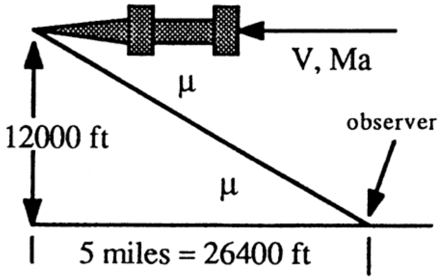


Fig. P9.116

Solution: The average temperature over this range is 498°R, hence

$$\bar{a} = \sqrt{kRT} = \sqrt{1.4(1717)(498)} = 1094 \text{ ft/s}, \quad \text{and} \quad \tan\mu = \frac{12000}{26400} = 0.455,$$

$$\text{or:} \quad \mu \approx 24.4°, \ \text{Ma}_{\text{plane}} = \csc\mu \approx 2.42, \ \text{V}_{\text{plane}} = 2.42(1094) \approx 2640 \ \frac{\text{ft}}{\text{s}} \quad \textit{Ans.}$$

P9.117 A tiny scratch in the side of a supersonic wind tunnel creates a very weak wave of angle 17°, as shown in Fig. P9.117, after which a normal shock occurs. The air temperature in region (1) is 250 K. Estimate the temperature in region (2).

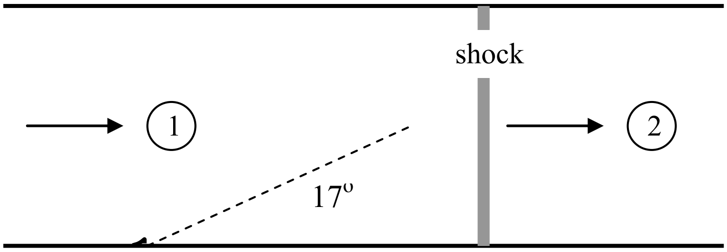


Fig. P9.117

Solution: The weak wave is a *Mach wave*, hence the Mach number in region 1 is $\text{Ma}_1 = 1/\sin(17°) = 3.42$. The normal shock relations, Table B.2 or Eq. (9.58), thus give the temperature ratio:

$$Ma_1 = 3.42: \quad T_2/T_1 = 3.2069 \quad \textit{Thus} \quad T_2 = 3.2069(250K) \approx 800\,K \quad \textit{Ans.}$$

9.118 A particle moving at uniform velocity in sea-level standard air creates the two disturbance spheres shown in Fig. P9.118. Compute the particle velocity and Mach number.

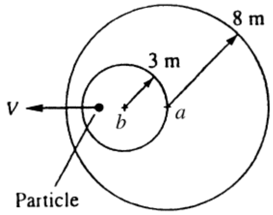


Fig. P9.118

Solution: If point "a" represents t = 0 units, the particle reaches point "b" in 8 – 3 = 5 units. But the distance from *a* to *b* is only 3 units. Therefore the (subsonic) Mach number is

$$\mathrm{Ma} = \frac{V\,\Delta t}{a\,\Delta t} = \frac{3\text{ units}}{5\text{ units}} \approx \mathbf{0.6} \quad \textit{Ans.}\text{ (a)}$$

$$V_{\text{particle}} = \mathrm{Ma}(a) = 0.6\sqrt{1.4(287)(288\text{ K})} \approx \mathbf{204\ m/s} \quad \textit{Ans.}\text{ (b)}$$

9.119 The particle in Fig. P9.119 is moving supersonically in sea-level standard air. From the two disturbance spheres shown, compute the particle (a) Mach number; (b) velocity; and (c) Mach angle.

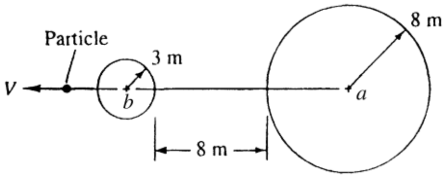


Fig. P9.119

Solution: If point "a" represents t = 0 units, the particle reaches point "b" in 8 – 3 = 5 units. But the distance from *a* to *b* is 8 + 8 + 3 = **19** units. Therefore the Mach number is

$$\mathrm{Ma} = \frac{V\,\Delta t}{a\,\Delta t} = \frac{19\text{ units}}{5\text{ units}} \approx \mathbf{3.8}\quad \textit{Ans.}\text{ (a)}\qquad \mu_{\text{wave}} = \sin^{-1}\left(\frac{1}{3.8}\right) \approx \mathbf{15.3^\circ}\quad \textit{Ans.}\text{ (c)}$$

$$V_{\text{particle}} = \mathrm{Ma}(a) = 3.8\sqrt{1.4(287)(288\text{ K})} \approx \mathbf{1290\ m/s}\quad \textit{Ans.}\text{ (b)}$$

9.120 The particle in Fig. P9.120 is moving in sea-level standard air. From the two disturbance spheres shown, estimate (a) the position of the particle at this instant; and (b) the temperature in °C at the front stagnation point of the particle.

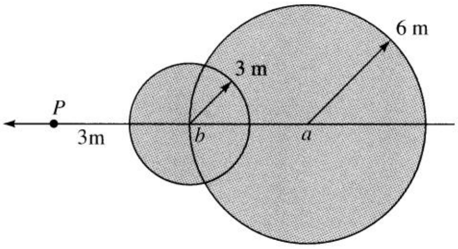


Fig. P9.120

Solution: Given sea-level temperature = 288 K. If point "a" represents t = 0 units, the particle reaches point "b" in 6 – 3 = 3 units. But the distance from *a* to *b* is **6** units. Therefore the particle Mach number is

$$\mathrm{Ma} = \frac{6}{3} = \mathbf{2.0},\quad \therefore\ T_{\text{stagnation}} = T_o = 288[1+0.2(2.0)^2] \approx \mathbf{518\ K}\quad \textit{Ans.}\text{ (b)}$$

$\mu = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$, and the particle is at point "P" at **6 meters ahead of "b."** *Ans.* (a)

9.121 A thermistor probe, in the shape of a needle parallel to the flow, reads a static temperature of –25°C when inserted in the stream. A conical disturbance of half-angle 17° is formed. Estimate (a) the Mach number; (b) the velocity; and (c) the stagnation temperature of the stream.

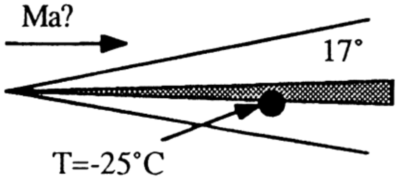


Fig. P9.121

Solution: If the needle is “very thin,” it reads the *stream* static temperature, $T_\infty \approx -25°C = 248$ K. We are given the Mach angle, $\mu = 17°$, so everything else follows readily:

$$\mathrm{Ma}_\infty = \csc\mu = \csc 17° = \mathbf{3.42} \quad \textit{Ans.}\ \text{(a)}$$

$$T_o = 248[1 + 0.2(3.42)^2] = 828\ \mathrm{K} = \mathbf{555°C} \quad \textit{Ans.}\ \text{(c)}$$

$$U_\infty = \mathrm{Ma}_\infty a_\infty = 3.42\sqrt{1.4(287)(248)} = 3.42(316) \approx \mathbf{1080\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

9.122 Supersonic air takes a 5° compression turn, as in Fig. P9.122. Compute the downstream pressure and Mach number and wave angle, and compare with small-disturbance theory.

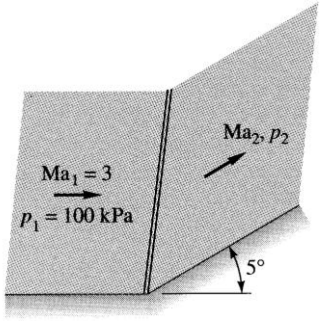


Fig. P9.122

Solution: From Fig. 9.23, $\beta \approx 25°$, and we can iterate Eq. (9.86) to a closer estimate:

$$\mathrm{Ma}_1 = 3.0,\ \theta = 5°, \text{compute } \beta = \mathbf{23.133°},$$

$$p_2/p_1 = 1.454, \quad \mathbf{p_2 = 145.4\ Pa}$$

From Eq. 9.83f, compute $\mathbf{Ma_2 = 2.750}$ *Ans.* (a, b, c) Exact oblique shock theory.

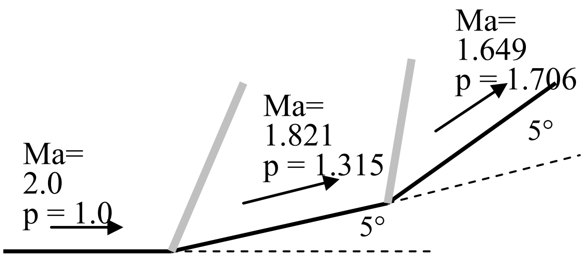
This is a small deflection. The linear theory of Eqs. 9.88 and 9.89 is reasonably accurate:

$$\mu = \sin^{-1}\left(\frac{1}{3}\right) = 19.47°, \quad \sin\beta = \sin\mu + \frac{(k+1)\tan\theta}{4\cos\mu} + \cdots = \frac{1}{3} + 0.0557 + \cdots \approx 0.389$$

$$\text{or:} \quad \beta_{\text{linear}} \approx \mathbf{22.9°}\ \text{(1\% low)} \quad \frac{\Delta p}{p_1} \approx \frac{1.4(3)^2}{\sqrt{3^2 - 1}}\tan 5° = 0.39,$$

$$p_2 \approx 100(1.39) \approx \mathbf{139\ kPa}\ \text{(4\% low)}$$

P9.123 The 10° deflection in Ex. 9.17 caused a final Mach number of 1.641 and a pressure ratio of 1.707. Compare this with the case of the flow passing through two 5° deflections. Comment on the results and why they might be higher or lower in the second case.



Solution: The sketch shows both the intermediate and the final states. The final Mach number is slightly higher, Ma = 1.649. The final pressure is slightly lower, 1.706. They are close to the 10° case because ten degrees is small, nearly isentropic. *Comment*: The closer the flow is to isentropic, the higher the final Mach number and the lower the pressure.

9.124 When a sea-level air flow approaches a ramp of angle 20°, an oblique shock wave forms as in Figure P9.124. Calculate (a) Ma_1; (b) p_2; (c) T_2 ; and (d) V_2.

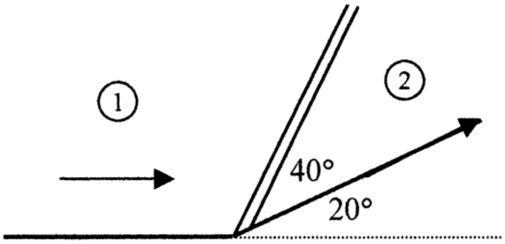


Fig. P9.12

Solution: For sea-level air, take p_1 = 101.35 kPa, T_1 = 288.16 K, and ρ_1 = 1.2255 kg/m^3. (a) The approach Mach number is determined by the specified angles, $\beta = 60°$ and $\theta = 20°$. From Fig. 9.23 we read that Ma_1 is slightly less than 2.0. More accurately, use Eq. (9.86):

$$\tan\theta = \frac{2\cot\beta\left(Ma_1^2\sin^2\beta - 1\right)}{Ma_1^2(k+\cos 2\beta)+2} \quad \text{for} \quad \theta = 20° \quad \text{and} \quad \beta = 60°$$

Iterate, or use EES, to find that $\mathrm{Ma}_1 = \mathbf{1.87}$. *Ans.* (a)

(b, c) With Ma_1 known, use Eqs. (9.83*a, c*) to find p_2 and T_2:

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left(2kMa_1^2\sin^2\beta - k + 1\right) = 2.893, \; p_2 = 2.893(101.35\text{ kPa}) = \mathbf{293\text{ kPa}} \quad \textit{Ans.}\text{ (b)}$$

$$\frac{T_2}{T_1} = \left[2+(k-1)Ma_1^2\sin^2\beta\right]\frac{2kMa_1^2\sin^2\beta - k + 1}{(k+1)^2 Ma_1^2\sin^2\beta} = 1.401,$$

$$T_2 = 1.401(288.16\text{ K}) = \mathbf{404\text{ K}} \quad \textit{Ans.}\text{ (c)}$$

(d) Finally, to find V_2, first find V_1 from the approach Mach number, then use Eq. (9.83b):

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340\ \frac{\text{m}}{\text{s}},\ V_1 = a_1 Ma_1 = (340)(1.87) = 636\ \frac{\text{m}}{\text{s}}$$

$$\frac{V_2}{V_1} = \frac{\cos\beta}{\cos(\beta-\theta)} = \frac{\cos 60°}{\cos 40°} = 0.653,\ V_2 = 0.653(636\text{ m/s}) = \mathbf{415}\ \frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}\text{ (d)}$$

P9.125 We saw in the text that, for k = 1.40, the maximum possible deflection caused by an oblique shock wave occurs at infinite approach Mach number and is $\theta_{\max}$ = 45.58°. Assuming an ideal gas, what is $\theta_{\max}$ for (*a*) argon; and (*b*) carbon dioxide?

Solution: In the limit as $\text{Ma}_1 \to \infty$, the normal-velocity ratio across a shock wave, Eq. (9.58), is

$$\frac{V_1}{V_2} \to \frac{k+1}{k-1}$$

It is this limiting ratio that we use to calculate $\theta_{\max}$ for the two gases above, with Eq. (9.85).

$$\theta_{\max}\,|_{Ma\to\infty} = \tan^{-1}(\sqrt{r}) - \tan^{-1}\left(\frac{1}{\sqrt{r}}\right)\quad ,\quad \text{where}\ \ r = \frac{k+1}{k-1}$$

The results, using Table B.4 to determine k, are as follows:

(*a*) Argon, k = 1.67, r = 3.985: $\theta_{\max}$ = 63.392° - 26.608° = **36.78°** *Ans.*(*a*)

(*b*) CO_2, k = 1.30, r = 7.667: $\theta_{\max}$ = 70.142° - 19.858° = **50.28°** *Ans.*(*b*)

9.126 Consider airflow at $\text{Ma}1$ = 2.2. Calculate, to two decimal places, (a) the deflection angle for which the downstream flow is sonic; and (b) the maximum deflection angle.

Solution: We are near the peak of the (invisible) curve for Ma1 = 2.2 in Fig. 9.23. The wave angles are ≈ 65°, which we guess for finding the sonic downstream condition:

$$\mathrm{Ma}_1 = 2.2,\ \text{guess}\ \beta \approx 65°,\ \text{and using Eq. 9.86,}$$

$$\text{compute}\ \theta \approx 26.1°\ \text{and}\ \mathrm{Ma}_2 = 0.92\ \text{(not quite sonic)}$$

$$\text{Converges to}\quad \mathrm{Ma}_2 = 1.000\ \text{when}\ \beta \approx 61.9°\ \text{and}\ \theta \approx \mathbf{25.9°} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Maximum deflection occurs at}\quad \beta \approx 64.6°\ \text{and}\ \theta_{\max} \approx \mathbf{26.1°} \quad \textit{Ans.}\ \text{(b)}$$

9.127 Do the Mach waves upstream of an oblique-shock wave intersect with the shock? Assuming supersonic downstream flow, do the downstream Mach waves intersect the shock? Show that for small deflections the shock-wave angle β lies halfway between $\mu 1$ and $\mu 2 + \theta$ for any Mach number.

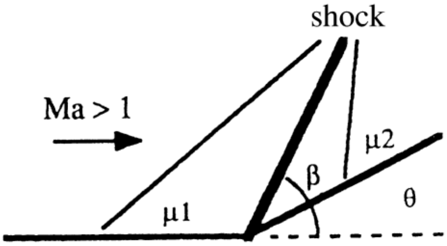


Fig. P9.127

Solution: Yes, Mach waves both upstream and downstream will intersect the shock:

$$\text{Linear theory:}\quad \beta \approx \mu_1 + \frac{(k+1)\mathrm{Ma}_1^2}{4\sqrt{(\mathrm{Ma}_1^2 - 1)}}\theta \quad \text{and} \quad \beta - \theta \approx \mu_2 - \frac{(k+1)\mathrm{Ma}_1^2}{4\sqrt{(\mathrm{Ma}_1^2 - 1)}}\theta$$

Thus, to first order (small deflection), the shock wave angle β will lie **halfway between** $\mu 1$ and $(\mu 2 + \theta)$, as sketched in the figure above.

9.128 Air flows past a two-dimensional wedge-nosed body as in Fig. P9.128. Determine the wedge half-angle δ for which the horizontal component of the total pressure force on the nose is 35 kN/m of depth into the paper.

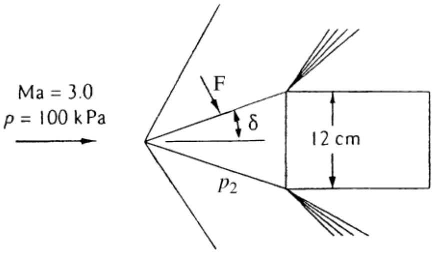


Fig. P9.128

Solution: *Regardless* of the wedge angle δ, the horizontal force equals the pressure inside the shock times the projected vertical area of the nose:

$$F_{horiz} = p_2 A_{vert\ proj} = p_2(0.12\,m)(1.0\,m) = 35000\,N, \quad \text{or} \quad p_2 = 291700\ Pa$$

$$\text{Eq. (9.83a):}\ \frac{p_2}{p_1} = \frac{291700}{100000} = 2.917 = \frac{1}{2.4}\left[2.8Ma_{1n}^2 - 0.4\right], \quad \text{solve}\ Ma_1 \sin\beta = 1.63$$

$$\text{or} \quad \beta = \sin^{-1}(1.63/3.0) = 32.81° \quad \text{Use Eq. (9.86) to compute}\ \delta_{wedge} \approx 15.5° \quad \textit{Ans.}$$

9.129 Air flows at supersonic speed toward a compression ramp, as in Fig. P9.129. A scratch on the wall at *a* creates a wave of 30° angle, while the oblique shock has a 50° angle. What is (a) the ramp angle θ; and (b) the wave angle ϕ caused by a scratch at *b*?

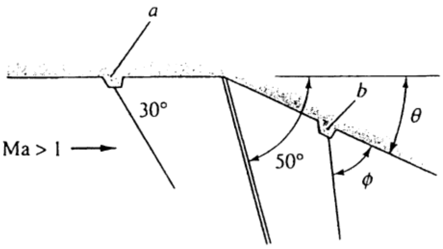


Fig. P9.129

Solution: The two "scratches" cause *Mach waves* which are directly related to Mach No.:

$$\mu_1 = 30°, \quad Ma_1 = \csc 30° = \mathbf{2.0}, \quad \beta = 50°, \quad \text{Eq. 9.86 yields}\ \theta \approx \mathbf{18.13°} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Then} \quad Ma_{2n} = 0.690 = Ma_2 \sin(50 - 18.13°),$$

$$Ma_2 = 1.307, \ \phi = \sin^{-1}\left(\frac{1}{1.307}\right) \approx \mathbf{49.9°} \quad \textit{Ans.}\ \text{(b)}$$

P9.130 A supersonic airflow, at a temperature of 300 K, strikes a wedge and is deflected 12°. If the resulting shock wave is attached, and the temperature after the shock is 450 K, (*a*) estimate the approach Mach number and wave angle. (*b*) Why are there two solutions?

Solution: We search through the oblique-shock solutions until we find a θ= 12° deflection for which T_2/T_1 = 450K/300K = **1.50**.

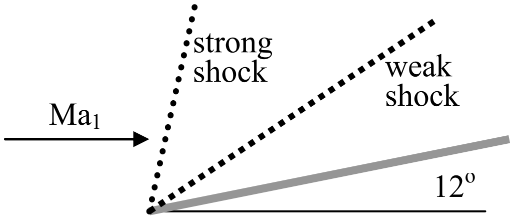


Fig. P9.130

(*a*, *b*) There are two solutions, weak and strong:

$Ma_1 = 4.69$, $\beta = 22°$ (weak shock)

$Ma_1 = 1.78$, $\beta = 80°$ (strong shock) *Answers* (*a*)

9.131 The following formula has been suggested as an alternate to Eq. (9.86) to relate upstream Mach number to the oblique shock wave angle β and turning angle θ:

$$\sin^2\beta = \frac{1}{Ma_1^2} + \frac{(k+1)\sin\beta\sin\theta}{2\cos(\beta-\theta)}$$

Can you prove or disprove this relation? If not, try a few numerical values and compare with the results from Eq. (9.86).

Solution: The formula is quite correct and serves as an interesting alternative to Eq. (9.86). Notice that one can immediately solve for Ma1 if β and θ are known, which would have been a great help in Prob. 9.124. For details of the proof, see page 371 of R. M. Olson, *Essentials of Engineering Fluid Mechanics*, 4th ed., Harper and Row, New York, 1980.

9.132 Air flows at Ma = 3 and p = 10 psia toward a wedge of 16° angle at zero incidence, as in Fig. P9.132. (a) If the pointed edge is forward, what is the pressure at point A? If the blunt edge is forward, what is the pressure at point B?

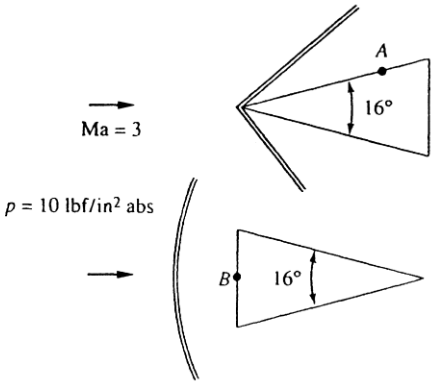


Fig. P9.132

Solution: For Ma = 3, $\theta = 8°$, Eq. 9.86:

$$\beta = 25.61°,$$

$$p_A/p_1 = \frac{2.8(3\sin 25.61°)^2 - 0.4}{2.4} = 1.80,$$

$$\therefore \quad p_A \approx \mathbf{18.0\ psia} \quad \textit{Ans. (a)}$$

(b) A normal shock forms, and $p_B = p_{o2}$ inside the shock. Given $p_{o1} = p_1/0.0272 = 367$ psia, Table B.2, Ma = 3: $p_{o2}/p_{o1} = 0.3283$, hence $p_{o2} = 0.3283(367) =$ **121 psia**. *Ans.* (b)

9.133 Air flows supersonically toward the double-wedge system in the figure. The (x,y) coordinates of the tips are given. Both wedges have 15° deflection angles. The shock wave of the forward wedge strikes the tip of the aft wedge. What is the freestream Mach number?

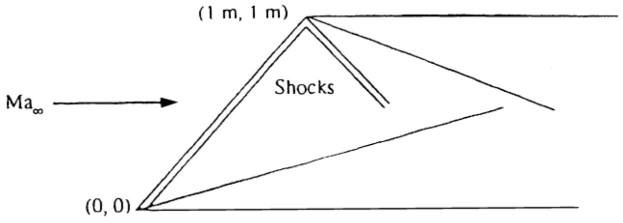


Fig. P9.133

Solution: However tricky the shock reflection might be at the upper (aft) wedge, the problem is solved by knowing the shock angle at the lower (forward) wedge:

$$\beta = \mathbf{tan}^{-1}(1.0) = 45°, \quad \theta = 15°, \quad Eq.(9.86)\, yields \quad \mathbf{Ma_\infty = 2.01} \quad Ans.$$

9.134 When an oblique shock strikes a solid wall, it reflects as a shock of sufficient strength to cause the exit flow Ma_3 to be parallel to the wall, as in Fig. P9.134. For airflow with $Ma_1 = 2.5$ and $p_1 = 100$ kPa, compute Ma_3, p_3, and the angle ϕ.

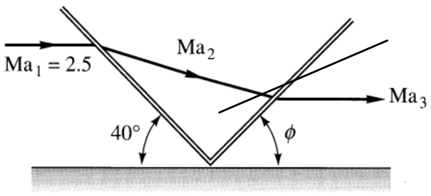


Fig. P9.134

Solution: With $\beta_1 = 40°$, we can compute the first shock deflection, which then must turn back the same amount through the second shock:

$$Ma_{1n} = 2.5 \sin 40° = 1.607; \quad Eq.(9.86): \theta_1 = 17.68°, Ma_{2n} = 0.666, Ma_2 = 1.754,$$

$$\text{Also } \theta_2 = 17.68°, \quad \text{solve } \beta_2 = 60.45°, Ma_{2n} = 1.526, Ma_{3n} = 0.692 = Ma_2 \sin(\beta_2 - \theta_2),$$

$$\text{Finally} \quad \mathbf{Ma_3 \approx 1.02} \quad Ans.\text{ (a)} \quad p_2/p_1 = 2.85, p_2 = 285 \text{ kPa}, p_3/p_2 = 2.55.$$

$$\text{Keep going:} \quad p_3 = 2.55(285) \approx \mathbf{727\ kPa} \quad Ans.\text{ (b)}$$

$$\text{Finally,} \quad \phi = \beta_2 - \theta_2 = 60.56 - 17.68 \approx \mathbf{42.8°} \quad Ans.\text{ (c)}$$

9.135 A bend in the bottom of a supersonic duct flow induces a shock wave which reflects from the upper wall, as in Fig. P9.135. Compute the Mach number and pressure in region 3.

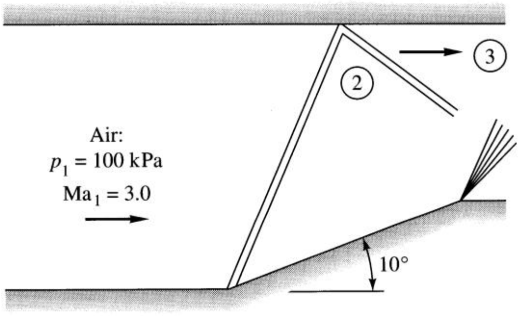


Fig. P9.135

Solution: Given $\theta = 10°$, find state 2:

$$\mathrm{Ma}_1 = 3.0,\quad \theta_1 = 10°,$$
$$\text{Eq. 9.86 predicts } \beta_1 \approx 27.38°,$$
$$\mathrm{Ma}_{1n} = 1.380,\quad \mathrm{Ma}_{2n} = 0.748,$$

$$\therefore\ \mathrm{Ma}_2 = 2.505,\quad \theta_2 = \theta_1 = 10°,\quad \beta_2 = 31.80°,\quad \mathrm{Ma}_{2n} = 1.32,\quad \mathrm{Ma}_{3n} = 0.776,$$

$$\therefore\ \mathrm{Ma}_3 = \mathbf{2.09}\quad \textit{Ans.}$$

$$\text{Meanwhile,}\quad p_2/p_1 = 2.054,\quad \text{or}\quad p_2 = 205.4\text{ kPa},$$

$$\text{and}\quad p_3/p_2 = 1.866,\quad p_3 = 1.866(205.4) \approx \mathbf{383\ kPa}\quad \textit{Ans.}$$

9.136 Figure P9.136 is a special application of Prob. 9.135. With careful design, one can orient the bend on the lower wall so that the reflected wave is exactly canceled by the return bend, as shown. This is a method of reducing the Mach number in a channel (a supersonic diffuser). If the bend angle is $\phi = 10°$, find (a) the downstream width *h* and (b) the downstream Mach number. Assume a weak shock wave.

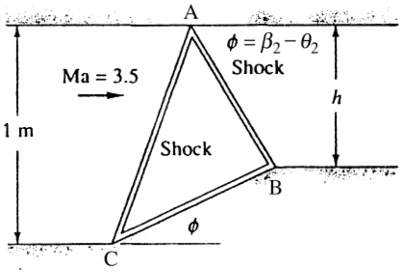


Fig. P9.136

Solution: The important thing is to find the angle ϕ between the *second* shock and the upper wall, as shown in the figure. With initial deflection = 10°, proceed forward to "3":

$$\mathrm{Ma}_1 = 3.5, \theta_1 = 10°,\quad \text{compute } \beta_1 = 24.384°,\ \mathrm{Ma}_2 = 2.904,\ \theta_2 = \theta_1 = 10°,\ \beta_2 = 28.096°,$$

$$\phi_{\text{upper wall}} = \beta_2 - \theta_2 = 28.096 - 10 = 18.096°,\ \mathrm{Ma}_3 = 2.427\quad \textit{Ans.}\text{(b)}$$

Length CB in the figure = (1 m)/sin(24.384°) = 2.422 m, angle ACB = 14.384°, angle CBA = $\beta 2$ = 28.096°, by the law of sines, AB/sin(14.384°) = 2.422/sin(28.096°) or the length AB = 1.278 m. Finally, duct width *h* = 1.278sin(18.096°) ≈ **0.40 m**. *Ans.* (a)

The horizontal distance from one lower corner to the next is 3.42 m. The length of CB is 3.47 m. Thus the shocks are not drawn to scale in the figure.

9.137 A 6° half-angle wedge creates the reflected shock system in Fig. P9.137. If $Ma_3 = 2.5$, find (a) Ma_1; and (b) the angle α.

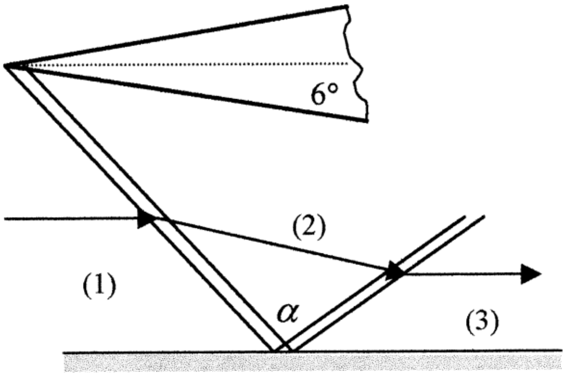


Fig. P9.137

Solution: (a) We have to go backward from region 3 to regions 2 and 1, using Eq. (9.86). In both cases the turning angle is $\theta = 6°$. EES is of course excellent for this task, otherwise the iteration will be laborious. The results are:

$$\mathrm{Ma}_2 = 2.775, \quad \beta_2 = 25.66°, \quad \mathbf{Ma_1 = 3.084}, \quad \beta_1 = 23.36° \quad \textit{Ans. } \text{(a)}$$

Since the wall is horizontal, it is clear from the geometry of Fig. P9.137 that

$$\alpha = 180° - 25.66° - 23.36° = \mathbf{130.98°} \quad \textit{Ans. } \text{(b)}$$

9.138 The supersonic nozzle of Fig. P9.138 is *overexpanded* (case G of Fig. 9.12) with $A_e/A_t = 3.0$ and a stagnation pressure of 350 kPa. If the jet edge makes a 4° angle with the nozzle centerline, what is the back pressure p_r in kPa?

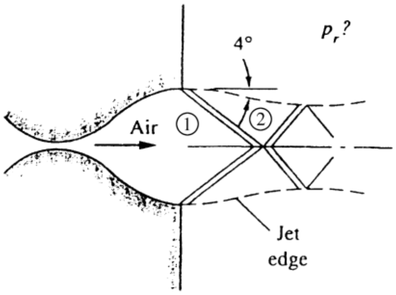


Fig. P9.138

Solution: The nozzle is clearly choked because there are shock waves downstream. Thus

$$\frac{A_e}{A^*} = 3.0, \quad \text{read } \mathrm{Ma}_e = \mathrm{Ma}_1 = 2.64, \ p_e = 350/[1 + 0.2(2.64)^2]^{3.5} = 16.5 \text{ kPa}$$

$$\theta = 4°, \quad \text{Eq. 9.86 gives } \beta_2 = 25.3°, \mathrm{Ma}_{1n} = 2.64 \sin 25.3° = 1.125, \ p_2/p_1 = 1.311$$

$$\text{Thus} \quad p_2 = p_{\text{receiver}} = 1.311(16.5) \approx \mathbf{21.7\ kPa} \quad \textit{Ans.}$$

9.139 Airflow at Ma = 2.2 takes a compression turn of 12° and then another turn of angle θ in Fig. P9.139. What is the maximum value of θ for the second shock to be *attached*? Will the two shocks intersect for any θ less than θ_{max}?

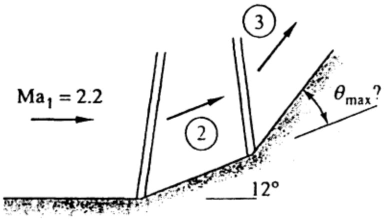


Fig. P9.139

Solution: First get the conditions in section (2) and then iterate for θ_{max}:

$$Ma_1 = 2.2,\ \theta = 12°,\ \text{Eq. 9.86: } \beta_1 = 37.87°,\ Ma_{1n} = 1.351,\ Ma_{2n} = 0.762 = Ma_2 \sin(\beta - \theta)$$

$$\text{or: } Ma_2 \approx \mathbf{1.745}.\ \text{For this Mach number, estimate } \theta_{max} \approx 18° \text{ from Fig. 9.23}$$

$$\text{Iterate, by trial and error, find } \quad \boldsymbol{\theta_{max} \approx 18.02°} \quad \textit{Ans.}$$

NOTE: The two shocks β_1 and β_2 **always** intersect for any $\theta_2 < \theta_{max}$. *Ans.*

9.140 The solution to Prob. 9.122 is Ma_2 = 2.750 and p_2 = 145.5 kPa. Compare these results with an isentropic compression turn of 5°, using Prandtl-Meyer theory.

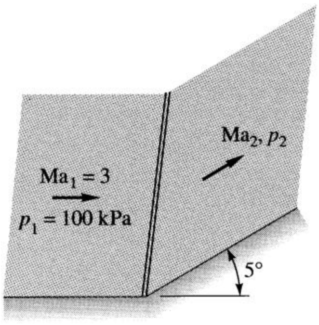


Fig. P9.122

Solution: Find ω for Ma = 3 and subtract 5°:

$$Ma_1 = 3.0, \quad \text{Table B.5: } \omega_1 = 49.76°,$$

$$\omega_2 = 49.76 - 5 = 44.76°,\ \text{read } Ma_2 \approx \mathbf{2.753}$$

$$\text{Then} \quad p_2 = p_1\left(\frac{p_2/p_o}{p_1/p_o}\right) = 100\left[\frac{1+0.2(2.753)^2}{1+0.2(3.0)^2}\right]^{-3.5} \approx \mathbf{145.4\ kPa} \quad \textit{Ans.}$$

This is almost identical to the shock wave result, because a 5° turn is nearly isentropic.

9.141 Supersonic airflow takes a 5° expansion turn, as in Fig. P9.141. Compute the downstream Mach number and pressure and compare with small-disturbance theory.

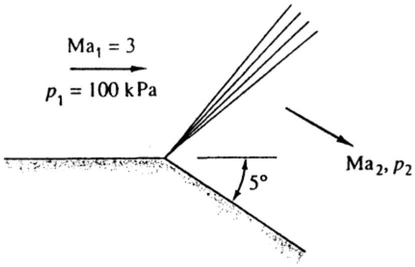


Fig. P9.141

Solution: Find ω for Ma = 3 and *add* 5°:

$$Ma_1 = 3.0, \quad \text{Table B.5: } \omega_1 = 49.76°,$$

$$\omega_2 = 49.76 + 5 = 54.76°,$$

$$\text{Read} \quad Ma_2 \approx \mathbf{3.274} \quad \textit{Ans.}$$

$$\text{Then}\quad p_2 = p_1\left(\frac{p_2/p_o}{p_1/p_o}\right) = 100\left[\frac{1+0.2(3.274)^2}{1+0.2(3.0)^2}\right]^{-3.5} \approx \mathbf{66.7\ kPa}\quad \textit{Ans.}$$

The linear theory is not especially accurate because even a 5° turn is slightly nonlinear:

$$\text{Eq. 9.89:}\quad \frac{\Delta p}{p} \approx \frac{k\,\mathrm{Ma}^2}{\sqrt{\mathrm{Ma}^2-1}}\tan\theta_2 = \frac{1.4(3)^2}{\sqrt{3^2-1}}\tan(-5°) = -0.390,$$

$$p = 100(1-0.390) \approx \mathbf{61\ kPa}\quad \textit{Ans.}\ (9\%\ \text{low})$$

9.142 A supersonic airflow at Ma1 = 3.2 and *p*1 = 50 kPa undergoes a compression shock followed by an isentropic expansion turn. The flow deflection is 30° for each turn. Compute Ma2 and *p*2 if (a) the shock is followed by the expansion and (b) the expansion is followed by the shock.

Solution: The solution is given in the form of the two sketches below. A shock wave with a 30° turn is a hugely non-isentropic flow, so the final conditions are nowhere near the original and they do not agree with each other either.

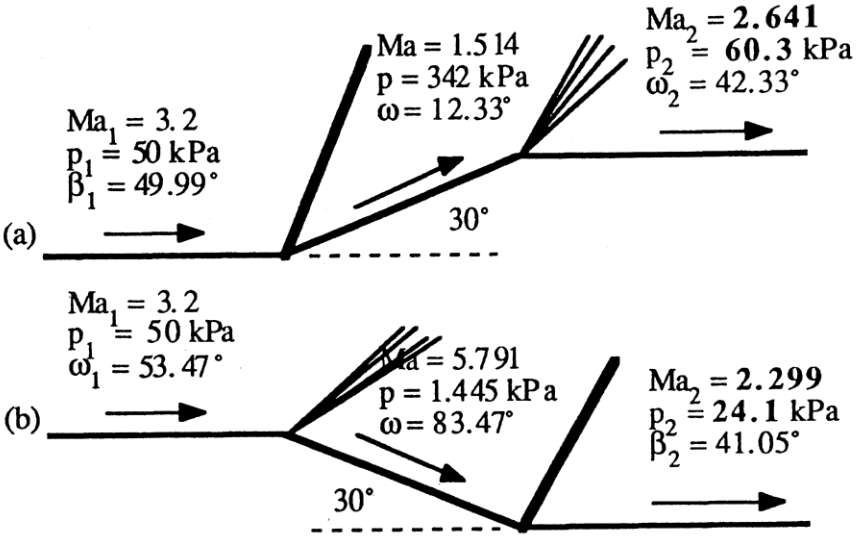


Fig. P9.142

9.143 Airflow at Ma1 = 3.2 passes through a 25° oblique-shock deflection. What isentropic expansion turn is required to bring the flow back to (a) Ma1 and (b) *p*1?

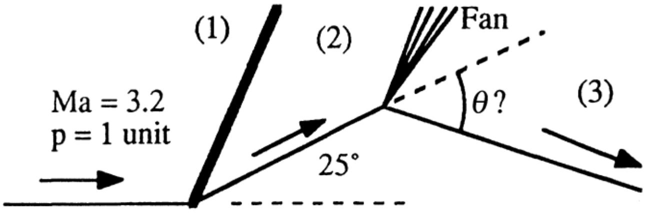


Fig. P9.143

Solution: First work out state (2):

$$\mathrm{Ma}_1 = 3.2,\ \beta_1 = 42.56°,\ \mathrm{Ma}_2 = 1.83,\ \mathrm{p}_2 = 5.30\ \text{units},\quad \omega_2 = 21.59°$$

(a) $\mathrm{Ma}_3 = 3.2$ means $\omega_2 = 53.47°$, or $\theta = 53.47 - 21.59 \approx \mathbf{31.9°}$ *Ans.* (a)

(b) $\mathrm{p}_3 = 1$ unit means $\mathrm{Ma}_3 = 2.906,\ \omega_3 = 47.90°,\ \theta = 47.90 - 21.59 = \mathbf{26.3°}$ *Ans.* (b)

P9.144 The 10° deflection in Ex. 9.17 caused the Mach number to drop to 1.64. (*a*) What turn angle will create a Prandtl-Meyer fan and bring the Mach number back up to 2.0?
(*b*) What will be the final pressure?

②

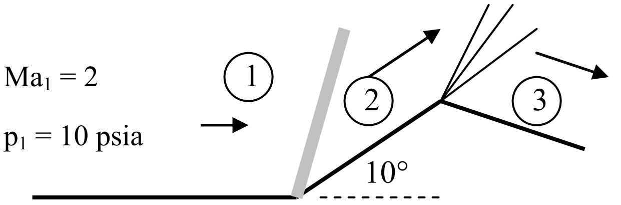


Fig. P9.144

Solution: Recall from Ex. 9.17 that the shock wave caused $\mathrm{Ma}_2 = 1.64$ and $\mathrm{p}_2 = 17.07$ psia. First calculate the (reduced) stagnation pressure in section 2, plus the Prandtl-Meyer angle ω_2:

$$\text{Table B.5 or Eq.(9.99)}:\ Ma_2 = 1.64\ ;\ p_2 = 17.07\ psia\ ;\ \omega_2 = 16.04^o$$

$$\text{Then}\quad p_{o2} = p_2(1+0.2Ma_2^2)^{3.5} = (17.07)[1+0.2(1.64)^2]^{3.5} = 77.0\ psia$$

Now find the Prandtl-Meyer angle in section 3, knowing that $\mathrm{Ma}_3 = 2.0$:

$$Ma_3 = 2.0:\quad \text{Table B.5}:\quad \omega_3 = 26.38^o$$

$$Thus\quad \delta\theta = \omega_3 - \omega_2 = 26.38^o - 16.04^o = \mathbf{10.34^o}\quad Ans.(a)$$

$$p_{o3} = p_{o2} = 77\,psia:\ p_3 = (77)/[1+0.2(2.0)^2]^{3.5} = \mathbf{9.84\ psia}\quad Ans.(b)$$

The final wall is just slightly down from horizontal, and the final pressure is just slightly reduced from the original value of 10 psia. The 10 deflection angle is almost isentropic.

9.145 Air at $Ma_1 = 2.0$ and $p_1 = 100$ kPa undergoes an isentropic expansion to a downstream pressure of 50 kPa. What is the desired turn angle in degrees?

Solution: This is a real 'quickie' compared to what we have been doing for the past few problems. Isentropic expansion to a new pressure specifies the downstream Mach number:

$$p_o = p_1\left[1+0.2\,Ma_1^2\right]^{3.5} = 100[1+0.2(2)^2]^{3.5} = 782\text{ kPa}$$

$$p_2/p_o = \frac{50}{782} = 0.0639,\quad \text{read } Ma_2 \approx 2.44,\quad \text{read } \omega_2 \approx 37.79^\circ,$$

$$\text{while}\quad \omega_1 \approx 26.38^\circ,\quad \therefore\ \Delta\theta = 37.79 - 26.38 \approx \mathbf{11.4^\circ}\quad \textit{Ans.}$$

9.146 Air flows supersonically over a surface which changes direction twice, as in Fig. P9.146. Calculate (a) Ma_2; and (b) p_3.

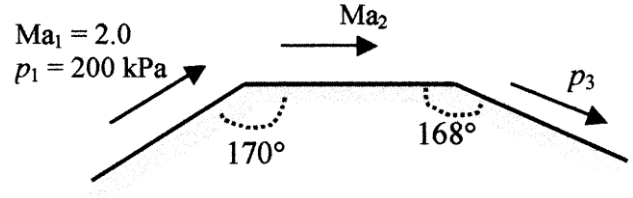


Fig. P9.146

Solution: (a) At the initial condition $Ma_1 = 2.0$, from Table B.5 read $\omega_1 = 26.38^\circ$. The first turn is 10°, so $\omega_2 = 26.38 + 10 = 36.38^\circ$. From Table B.5 read $Ma_2 = 2.38$. For more accuracy, use Eq. (9.99) to obtain $\mathbf{Ma_2 = 2.385}$. *Ans.* (a)

(b) The second turn is 12°, so $\omega_3 = 36.38 + 12 = 48.38^\circ$. From Table B.5 read $Ma_3 = 2.93$. For more accuracy, use Eq. (9.99) to obtain $Ma_2 = 2.9296$. (Not worth the extra effort.) To find pressures, we need the stagnation pressure, which is constant:

$$p_o = p_1\left(1+0.2\,Ma_1^2\right)^{3.5} = (200\text{ kPa})[1+0.2(2.0)^2]^{3.5} = 1565\text{ kPa}$$

$$\text{Then}\quad p_3 = p_o\Big/\left(1+0.2\,Ma_3^2\right)^{3.5} = (1565)/[1+0.2(2.9296)^2]^{3.5} = \mathbf{47.4\text{ kPa}}\quad \textit{Ans.}\text{ (b)}$$

9.147 A converging-diverging nozzle with a 4:1 exit-area ratio and $p0$ = 500 kPa operates in an underexpanded condition (case 1 of Fig. 9.12) as in Fig. P9.147. The receiver pressure is pa = 10 kPa, which is less than the exit pressure, so that expansion waves form outside the exit. For the given conditions, what will the Mach number Ma2 and the angle ϕ of the edge of the jet be? Assume $k = 1.4$ as usual.

Solution: Get the Mach number in the exit and then execute a Prandtl-Meyer expansion:

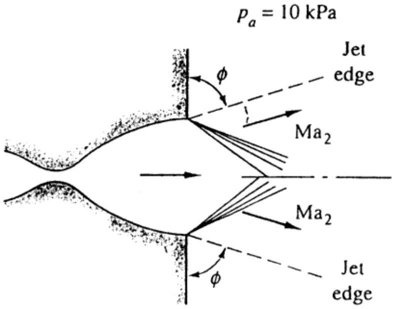


Fig. P9.147

$$\frac{A_e}{A^*} = 4.0, \quad \text{read } Ma_e \approx 2.94, \quad \text{Table B.5: } \omega_1 = 48.59°, \quad p_{o1} = p_{o2} = 500 \text{ kPa}$$

$$p_o/p_2 = \frac{500}{10} = 50, \quad \text{read } Ma_2 \approx \mathbf{3.21} \quad \textit{Ans.}\text{ (a)} \quad \text{Read } \omega_2 = 53.61°,$$

$$\therefore \quad \Delta\theta = 53.61 - 48.59 = 5.02°, \quad \text{or} \quad \phi_{\text{see figure above}} = 90 - \Delta\theta \approx \mathbf{85.0°} \quad \textit{Ans.}\text{ (b)}$$

9.148 Air flows supersonically over a circular-arc surface as in Fig. P9.148. Estimate (a) the Mach number Ma2 and (b) the pressure $p2$ as the flow leaves the circular surface.

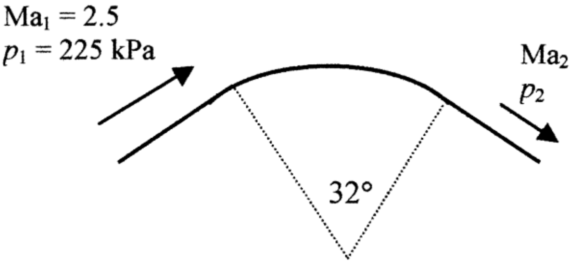


Fig. P9.148

Solution: (a) At the initial condition Ma1 = 2.5, from Table B.5 read $\omega 1$ = 39.12°. (a) Circular arc or not, the turn angle is 32°, so $\omega 2$ = 39.12 + 32 = 71.12°. From Table B.5 read Ma2 = 4.44. For more accuracy, use Eq. (9.99) to obtain **Ma2 = 4.437**. *Ans.* (a)

(b) To find $p2$, first find the stagnation pressure:

$$p_o = p_1\left(1+0.2\,Ma_1^2\right)^{3.5} = (150\,\text{kPa})[1+0.2(2.5)^2]^{3.5} = 2563\,\text{kPa}$$

Then $$p_2 = p_o\Big/\left(1+0.2\,Ma_2^2\right)^{3.5} = (2563)/[1+0.2(4.437)^2]^{3.5} = \mathbf{9.6\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

P9.149 Air flows at Ma$_\infty$ = 3.0 past a doubly symmetric diamond airfoil whose front and rear included angles are both 24°. For zero angle of attack, compute the drag coefficient with shock-expansion theory and compare with Ackeret theory.

Solution: The airfoil and its front and rear pressures are shown below.

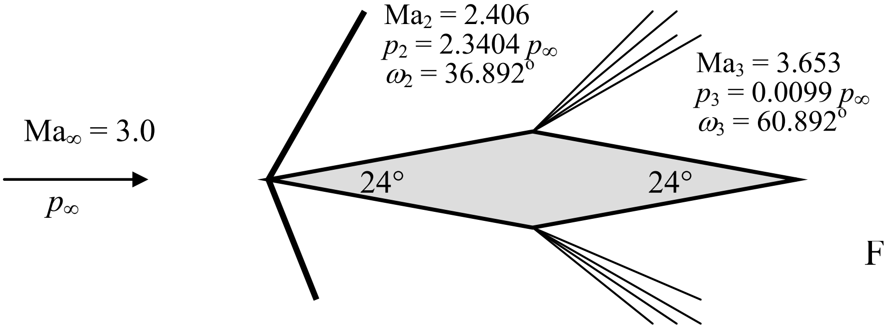


Fig. P9.149

The foil thickness is $C[\tan(12°)] = 0.2126C$. The drag coefficient is:

$$C_D = \frac{Drag}{(k/2)Ma_\infty^2\,p_\infty bC} = \frac{(2.3404\,p_\infty - 0.0099\,p_\infty)b(0.2126C)}{(1.4/2))(3.0)^2\,p_\infty bC} = \mathbf{0.0787} \quad \textit{Ans.}$$

From Ackeret theory, Eq. (9.107), the drag coefficient prediction is somewhat lower:

$$C_{D,Ackeret} = \frac{4}{\sqrt{Ma_\infty^2 - 1}}[\alpha^2 + (\frac{dy}{dx})^2_{avg}] = \frac{4}{\sqrt{3^2-1}}[0^2 + \tan^2(12^o)] = 0.0639 \quad \textit{Ans.}$$

Ackeret (small-disturbance) theory is 19% low because the airfoil is too thick (21%).

9.150 A flat plate airfoil with chord C = 1.2 m is to have a lift of 30 kN/m when flying at 5000-m standard altitude with U_∞ = 641 m/s. Using Ackeret theory, estimate (a) the angle of attack; and (b) the drag force in N/m.

Solution: At 5000 m, $\rho = 0.7361$ kg/m^3, T = 256 K, and p = 54008 Pa. Compute Ma:

$$\mathrm{Ma} = \frac{641}{\sqrt{1.4(287)(256)}} = 2.00, \quad C_L = \frac{30000}{(1/2)(0.7361)(641)^2(1.2)(1)} = 0.1653 \approx \frac{4\alpha}{\sqrt{(2)^2 - 1}}$$

$$\text{Solve for} \quad \alpha = 0.0716 \text{ rad} \approx \mathbf{4.10^\circ} \quad \textit{Ans.}$$

With angle of attack known, Ackeret theory simply predicts that

$$D = L\tan\alpha = 30000\tan(4.10^\circ) \approx \mathbf{2150\ N/m} \quad \textit{Ans.}\ \text{(b)}$$

9.151 Air flows at Ma = 2.5 past a half-wedge airfoil whose angles are 4°, as in Fig. P9.151. Compute the lift and drag coefficients at α equal to (a) 0°; and (b) 6°.

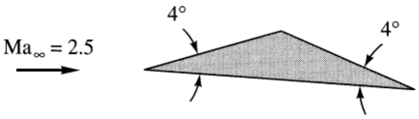


Fig. P9.151

Solution: Let's use Ackeret theory here:

$$\text{(a) } \alpha = 0^\circ\text{:} \quad C_L \approx \mathbf{0}; \quad C_D \approx \frac{4}{\sqrt{(2.5)^2 - 1}}\left[0^2 + \frac{1}{2}(\tan^2 4^\circ + 0)\right] \approx \mathbf{0.00427} \quad \textit{Ans.}\text{(a)}$$

[A complete shock-expansion calculation gives $C_L \approx -0.0065$, $C_D \approx 0.00428$.]

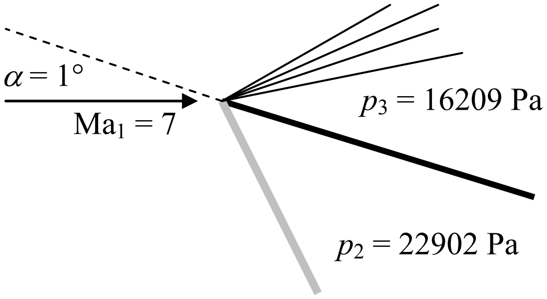
$$\text{(b) } \alpha = 6^\circ = 0.105 \text{ rad:} \quad C_L = \frac{4(0.105)}{\sqrt{5.25}} \approx \mathbf{0.183},$$

$$C_D = \frac{4}{\sqrt{5.25}}[(0.105)^2 + (1/2)(\tan^2 4^\circ + 0)] \approx \mathbf{0.0234} \quad \textit{Ans.}\text{(b)}$$

[A complete shock-expansion calculation gives $C_L \approx 0.179$, $C_D \approx 0.0219$.]

P9.152 The X-43 model A scramjet aircraft in Fig. 9.30 is small, W = 3000 lbf, and unmanned, only 12.33 ft long and 5.5 ft wide. The aerodynamics of a slender arrowhead-shaped hypersonic vehicle is beyond our scope. Instead, let us assume it is a flat plate airfoil of area 2.0 m^2. Let Ma = 7 at 12,000 m standard altitude. Estimate the drag, by shock-expansion theory. [HINT: Use Ackeret theory to estimate the angle of attack.]

Solution: From Table B.6 at 12,000 m, p_1 = 19312 Pa and T_1 = 216.66 K. Knowing W = 3000 lbf = 13345 N, we can estimate $C_L = W/[0.5kp\text{Ma}1^2]$ = $(13345\text{N})/[0.5(1.4)(19312)(7)^2(2.0)] = 0.0101$. Use Ackeret theory, Eq. (9.104), to estimate α:



$$C_L = 0.0101 = 4\alpha/(\text{Ma}^2 - 1)^{1/2}\ ,\ \text{solve for}\ \alpha \approx 1.0\ \text{degree.}$$

Assume α = 1 degree, as in the figure. From shock theory, Eqs. (9.86) and (9.83*a*), calculate

$$\text{Ma}_2 = 5.582,\ \beta = 8.848°,\ p_2/p = 1.186,\ p_2 = 1.186(19312) = 22902\ \text{Pa}$$

The Prandtl-Meyer expansion is on the top, with a turning angle of 1 degree. From Eq. (9.99),

$$Ma_1 = 7\ ,\ \omega_1 = 90.97^o\ ,\ \omega_2 = 90.97 + 1^o = 91.97^o\ ,\ \text{read}\ \ Ma_3 = 7.195$$

$$p_{o1} = (19312\,Pa)[1+0.2(7)^2]^{3.5} = 7.995\text{E}7\,Pa = p_{o3}$$

$$p_3 = (7.995\text{E}7\,Pa)/[1+0.2(7.195)^2]^{3.5} = 16212\ Pa$$

The (hypothesized) wing area is 2.0 m^2. The total force, lift, and drag of the flat plate is thus

$$F = (p_2 - p_3)A_p = (22902 - 16209\,Pa)(2.0\,m^2) = 13386\,N = 3009\,lbf \approx Lift$$

$$Drag = F\sin(1^o) = (13386\,N)(0.01745) = 234\,N = \mathbf{53}\ lbf \qquad \textit{Ans.}\ \text{(close enough!)}$$

We could iterate, but we are already within 0.3% of the correct lift. Ackeret theory was surprisingly accurate for such a high Mach number, because of the very small deflection.

9.153 A supersonic transport has a mass of 65 Mg and cruises at 11-km standard altitude at a Mach number of 2.25. If the angle of attack is 2° and its wings can be approximated by flat plates, estimate (a) the required wing area; and (b) the thrust.

Solution: At 11 km (Table B.6), take $p_\infty = 22612$ Pa. (a) Use linearized theory:

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(2\pi/180)}{\sqrt{2.25^2 - 1}} = 0.0693 = \frac{W}{\frac{k}{2}pMa^2 A} = \frac{65000(9.81)}{0.7(22612)(2.25)^2 A},$$

$$\mathbf{A = 115\ m^2} \quad \textit{Ans. (a)}$$

(b) According to linearized (Ackeret) theory, if there is no thickness drag, then

$$\text{Drag} = \text{Lift} \times \alpha, \quad \text{or} \quad \text{Drag} = \text{Thrust} = 65000(9.81)(2\pi/180) = \mathbf{22{,}300\ N} \quad \textit{Ans. (b)}$$

9.154 A symmetric supersonic airfoil has its upper and lower surfaces defined by a sine waveshape:

$$y = \frac{t}{2}\sin\frac{\pi x}{C}$$

where *t* is the maximum thickness, which occurs at $x = C/2$. Use Ackeret theory to derive an expression for the drag coefficient at zero angle of attack. Compare your result with Ackeret theory for a symmetric double-wedge airfoil of the same thickness.

Solution: Evaluate the mean-square surface slope and then use Eq. (9.107):

$$y' = \frac{\pi t}{2C}\cos\left(\frac{\pi x}{C}\right), \quad \overline{y'^2} = \frac{1}{C}\int_0^C \left(\frac{\pi t}{2C}\right)^2 \cos^2\left(\frac{\pi x}{C}\right)dx = \frac{\pi^2}{8}\left(\frac{t}{C}\right)^2$$

$$\text{At } \alpha = 0, \quad C_D = \frac{4}{\sqrt{(Ma_\infty^2 - 1)}}\left[0^2 + \frac{1}{2}\left\{2\left(\frac{\pi^2}{8}\frac{t^2}{C^2}\right)\right\}\right],$$

$$\text{or:} \quad \mathbf{C_D = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left(\frac{\pi^2}{8}\right)\left(\frac{t}{C}\right)^2} \quad \textit{Ans.}$$

Meanwhile, for a double-wedge of the same thickness *t*, from Prob. 9.152,

$$C_{D,\text{double-wedge}} = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left(\frac{t}{C}\right)^2 \quad \text{(19\% less)} \quad \textit{Ans.}$$

P9.155 The F-35 airplane in Fig. 9.29 has a wingspan of 10 m and a wing area of 41.8 m^2. It cruises at about 10 km altitude with a gross weight of about 200 kN. At that altitude, the engine develops a thrust of about 50 kN. Assume the wing has a symmetric diamond airfoil with a thickness of 8%, and accounts for all lift and drag. Estimate the cruise Mach number of the airplane. For extra credit, explain why there are *two* solutions.

Solution: At 10 km standard altitude, from Table B.6, ρ = 0.4125 kg/m^3, and atmospheric pressure is p_∞ = 26416 Pa. We need to match lift and drag to the given weight/thrust data:

$$Lift = W = 200{,}000\,N = C_L \frac{k}{2} Ma^2\, p_\infty A\ , \quad \text{where } C_L = \frac{4\alpha}{\sqrt{Ma^2-1}}\ , \ \alpha \text{ in radians}$$

$$Drag = Thrust = 50{,}000\,N = C_D \frac{k}{2} Ma^2\, p_\infty A\ , \quad \text{where } C_D = \frac{4}{\sqrt{Ma^2-1}}[\alpha^2 + (\frac{dy}{dx})^2_{avg}]$$

For a symmetric 8% diamond airfoil, as in Fig. E9.21, the slope everywhere is $|dy/dx|$ = 0.08, so the numbers for lift and drag are as follows:

$$W = 200{,}000\,N = \frac{4\alpha}{\sqrt{Ma^2-1}}(\frac{1.4}{2})Ma^2\,(26416\,Pa)(41.8m^2)$$

$$T = 50{,}000\,N = \frac{4}{\sqrt{Ma^2-1}}[\alpha^2 + (0.08)^2](\frac{1.4}{2})Ma^2\,(26416\,Pa)(41.8m^2)$$

to be solved for α and Ma. It was a bit of a surprise to the writer that there are ***two solutions*** to this system, both quite realistic:

(1) High velocity, low lift coefficient: α = 0.02895 rad (1.66°), Ma = **1.90** *Ans.*(1)

(2) Low velocity, high lift coefficient: α = 0.02895 rad (1.66°), Ma = **1.176** *Ans.*(2)

NOTE: The true Mach number of the F-35A would include drag and lift of the fuselage and tail.

EXTRA CREDIT: *Why are there two solutions*? Because the lift and drag vary as $Ma^2/(Ma^2\text{-}1)^{1/2}$, which, for a given α, comes down and then back up again, like this:

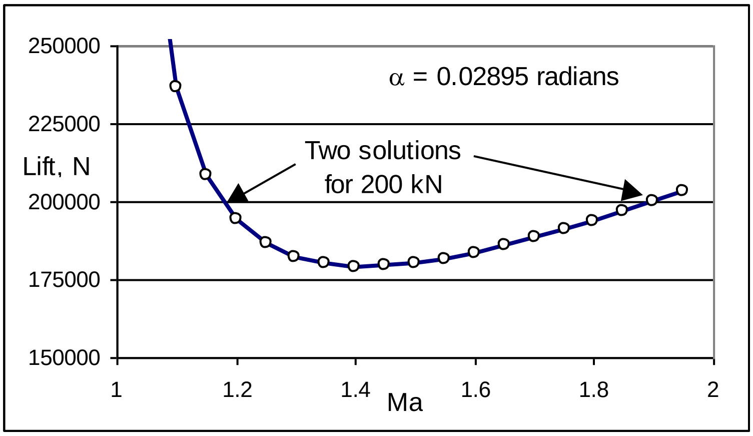


Fig. P9.155

To match both lift *and* drag requires α = 0.02895 radians, then solving the lift as a quadratic equation yields the two solutions, Ma = 1.176 and 1.900. COMMENT: Ma = 1.176 is close to the *transonic* regime, for which Ackeret theory falters.

9.156 A thin circular-arc airfoil is shown in Fig. P9.156. The leading edge is parallel to the free stream. Using linearized (small-turning-angle) supersonic-flow theory, derive a formula for the lift and drag coefficient for this orientation, and compare with Ackeret-theory results for an angle of attack $\alpha = \tan^{-1}(h/L)$.

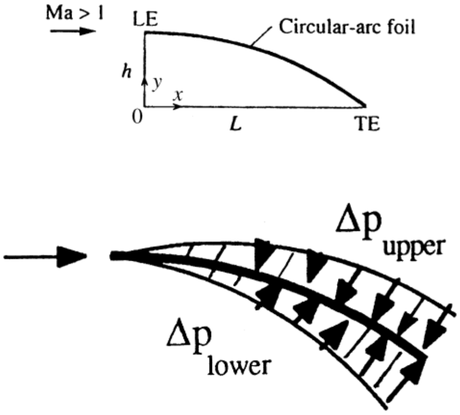


Fig. P9.156

Solution: For the (x,y) coordinate system shown, the formula for the plate surface is

$$y_{foil} = \sqrt{R^2 - x^2} - R + h,$$

$$\text{where } R = \frac{L^2 + h^2}{2h}, \quad \text{and} \quad \frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}}$$

If $h \ll L$ (small disturbances), $R \ll L, h$. Since the flow approaches parallel to the leading edge, the pressure distribution on the airfoil looks like the bottom figure on the previous page. We compute this pressure distribution from the linearized expression, Eq. (9.89):

$$\frac{\Delta p}{p_\infty} \approx B\frac{dy}{dx}, \quad \text{where } B = \frac{kMa_\infty^2}{\sqrt{Ma_\infty^2 - 1}}; \quad \text{Thus } \Delta p_{lower} - \Delta p_{upper} = \frac{2Bx}{\sqrt{R^2 - x^2}}p_\infty$$

$$\text{Lift} = \int_{foil} \Delta p_{total}\, dA_{foil} = 2Bp_\infty \int_0^L x(R^2 - x^2)^{-1/2} \cos\theta\, b\, dx,$$

$$\text{where} \quad \cos\theta = \left[1 + (dy/dx)^2\right]^{-1/2}$$

Carry out this integration, assuming that $h \ll L$, $R \ll L$, chord length $C \approx L$, to obtain

$$\text{Lift} \approx \frac{2kMa_\infty^2}{\sqrt{Ma_\infty^2 - 1}}p_\infty bh, \quad \text{or} \quad \mathbf{C_L} = \frac{\text{Lift}}{(k/2)Ma_\infty^2 p_\infty bL} \approx \frac{\mathbf{4}}{\sqrt{\mathbf{Ma_\infty^2 - 1}}}\frac{\mathbf{h}}{\mathbf{L}} \quad \textit{Ans.}\text{(a)}$$

$$\text{But} \quad h/L \approx \alpha \text{radians}, \quad \text{therefore } C_L \approx \left[4\Big/\sqrt{Ma_\infty^2 - 1}\right]\alpha, \text{ which is exactly Ackeret theory.}$$

$$\text{Similarly,} \quad \text{Drag} = 2Bp_\infty \int_0^L x(R^2 - x^2)^{-1/2} \sin\theta\, b\, dx \approx \left[2kMa_\infty^2\left(Ma_\infty^2 - 1\right)^{-1/2}\right] p_\infty bh\left(\frac{4h}{3L}\right)$$

$$\text{or:} \quad \mathbf{C_D} \approx \left[4/\sqrt{Ma_\infty^2 - 1}\right](h/L)^2\left(1 + \frac{1}{3}\right) \approx \mathbf{C_L}\boldsymbol{\alpha}^{\mathbf{2}}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{3}}\right) \quad \textit{Ans.}\text{(b)}$$

This is exactly the same as Ackeret theory. The extra term "1/3" is the "thickness-drag" contribution (actually the **camber** slope contribution) from Eqs. (9.106, 107) of the text.

P9.157 The Ackeret airfoil theory of Eqs. (9.104-5) is meant for *moderate* supersonic speeds, 1.2 < Ma < 4. How does it fare for *hypersonic* speeds? To illustrate, calculate (*a*) C_L and (*b*) C_D for a flat-plate airfoil at $\alpha = 5°$, with $Ma_\infty = 8.0$, using shock-expansion theory, and compare with Ackeret theory. Comment.

Solution: Like Example 9.19, we need only calculate a (steep) shock wave on the bottom and a (steep) expansion wave fan on the top. The results are shown in the figure below.

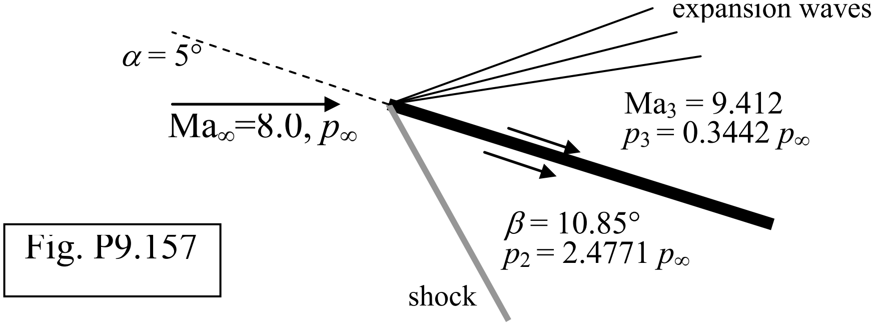


Fig. P9.157

If the airfoil has length C and span b, the force on the plate is $F = (p_2 - p_3)bC$. Then we compute Lift = $F\cos\alpha$ and Drag = F $\sin\alpha$. The two force coefficients are:

$$C_L = \frac{(2.4771\,p_\infty - 0.3442\,p_\infty)bC\cos 5^o}{(k/2)p_\infty Ma_\infty^2 bC} = \frac{2.125}{(1.4/2)(8.0)^2} = 0.0474 \quad Ans.(a)$$

$$C_D = \frac{(2.4771\,p_\infty - 0.3442\,p_\infty)bC\sin 5^o}{(k/2)p_\infty Ma_\infty^2 bC} = \frac{0.186}{(1.4/2)(8.0)^2} = 0.00415 \quad Ans.(b)$$

If we apply Ackeret theory to the same data, we obtain, much more simply,

$$C_{L,Ackeret} \approx \frac{4\alpha}{\sqrt{Ma_\infty^2 - 1}} = \frac{4(5\pi/180)}{\sqrt{8^2-1}} = 0.0440 \quad Ans.(a) - 7\%\,low$$

$$C_{D,Ackeret} \approx \frac{4\alpha^2}{\sqrt{Ma_\infty^2 - 1}} = \frac{4(5\pi/180)^2}{\sqrt{8^2-1}} = 0.00384 \quad Ans.(b) - 8\%\,low$$

Even at hypersonic Mach numbers, Ackeret theory is not bad – and temptingly simple! The theory is helped here by the fact that the angle of attack and foil thickness are small.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

In the following problems, assume one-dimensional flow of ideal air, R = 287 J/(kg·K) and k = 1.4.

FE9.1 For steady isentropic flow, if the absolute temperature increases 50%, by what ratio does the static pressure increase?

(a) 1.12 (b) 1.22 (c) 2.25 (d) 2.76 **(e) 4.13**

FE9.2 For steady isentropic flow, if the density doubles, by what ratio does the static pressure increase?

(a) 1.22 (b) 1.32 (c) 1.44 **(d) 2.64** (e) 5.66

FE9.3 A large tank, at 500 K and 200 kPa, supplies isentropic air flow to a nozzle. At section 1, the pressure is only 120 kPa. What is the Mach number at this section?

(a) 0.63 (b) 0.78 **(c) 0.89** (d) 1.00 (e) 1.83

FE9.4 In Prob. FE9.3 what is the temperature at section 1?

(a) 300 K (b) 408 K (c) 417 K **(d) 432 K** (e) 500 K

FE9.5 In Prob. FE9.3, if the area at section 1 is 0.15 m^2, what is the mass flow?

(a) 38.1 kg/s **(b) 53.6 kg/s** (c) 57.8 kg/s (d) 67.8 kg/s (e) 77.2 kg/s

FE9.6 For steady isentropic flow, what is the maximum possible mass flow through the duct in Fig. FE9.6? (T_o = 400 K, p_o = 300 kPa)

(a) 9.5 kg/s (b) 15.1 kg/s (c) 26.2 kg/s **(d) 30.3 kg/s** (e) 52.4 kg/s

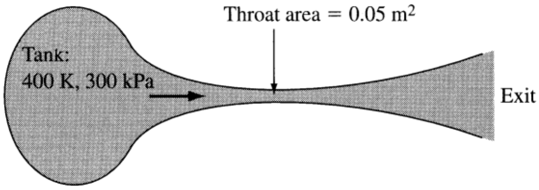


Fig. FE9.6

FE9.7 If the exit Mach number in Fig. FE9.6 is 2.2, what is the exit area?

(a) 0.10 m^2 (b) 0.12 m^2 (c) 0.15 m^2 (d) 0.18 m^2 (e) 0.22 m^2

FE9.8 If there are no shock waves and the pressure at one duct section in Fig. FE9.6 is 55.5 kPa, what is the velocity at that section?

(a) 166 m/s (b) 232 m/s **(c) 554 m/s** (d) 706 m/s (e) 774 m/s

FE9.9 If, in Fig. FE9.6, there is a normal shock wave at a section where the area is 0.07 m^2, what is the air density just upstream of that shock?

(a) 0.48 kg/m^3 **(b) 0.78 kg/m^3** (c) 1.35 kg/m^3 (d) 1.61 kg/m^3 (e) 2.61 kg/m^3

FE9.10 In Prob. FE9.9, what is the Mach number just downstream of the shock wave?

(a) 0.42 (b) 0.55 **(c) 0.63** (d) 1.00 (e) 1.76

COMPREHENSIVE PROBLEMS

C9.1 The converging-diverging nozzle in the figure has a design Mach number of 2.0 at the exit plane for isentropic flow from tank *a* to *b*. (a) Find the exit area Ae and back pressure pb which will allow design conditions. (b) The back pressure grows as tank *b* fills with air, until a normal shock wave appear in the exit plane. At what back pressure does this occur? (c) If tank *b* remains at constant T = 20°C, how long will it take for the flow to go from condition (a) to condition (b)?

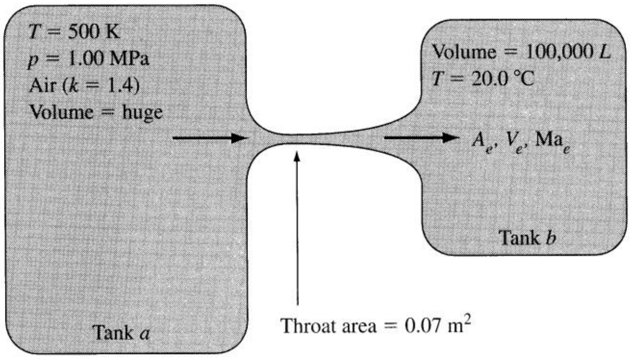


Fig. C9.1

Solution: (a) Compute the isentropic pressure ratio and area ratio for Ma = 2.0:

$$Ma_e = 2.0\text{: Table B.1: } \frac{p_e}{p_o} = 0.1278, \quad p_e = 0.1278(1E6) = \mathbf{128{,}000\ Pa} \quad \textit{Ans.}\text{(a)}$$

$$\frac{A_e}{A^*} = 1.6875, \quad A_e = 1.6875(0.07) = \mathbf{0.118\ m^2} \quad \textit{Ans.}\text{(a)}$$

(b) Compute the pressure ratio across a shock for Ma = 2.0:

$$Ma_e = 2.0\text{: Table B.2: } \frac{p_b}{p_e} = 4.5, \quad p_b = 4.5(128000) = \mathbf{575{,}000\ Pa} \quad \textit{Ans.}\text{(b)}$$

(c) Compute the (constant) mass flow and the mass needed to fill the tank:

$$\dot{m} = \dot{m}_{\max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{(1E6)(0.07)}{\sqrt{287(500)}} = 126.5\ \frac{\text{kg}}{\text{s}} = \textit{constant}$$

$$\Delta m_{tank} = (\rho_{final} - \rho_{initial})\upsilon_{tank} = \left[\frac{575000}{287(293)} - \frac{128000}{287(293)}\right](100\,\text{m}^2) = 531.9\,\text{kg}$$

Since mass flow is constant, the time required is simply the ratio of these two:

$$\Delta t_{required} = \frac{\Delta m}{\dot{m}} = \frac{531.9\,\text{kg}}{126.5\,\text{kg/s}} = \mathbf{4.2\ sec} \quad \textit{Ans.}\text{(c)}$$

C9.2 Two large air tanks, one at 400 K and 300 kPa and the other at 300 K and 100 kPa, are connected by a straight tube 6 m long and 5 cm in diameter. The average friction factor is 0.0225. Assuming adiabatic flow, estimate the mass flow through the tube.

Solution: The higher-pressure tank denotes the inlet *stagnation* conditions, $p_o = 300$ kPa and $T_o = 400$ K. The flow will be subsonic, but we have no idea whether it is choked. Assume that the tube exit pressure equals the receiver pressure, 100 kPa. We must iterate—an ideal job for EES! We *do* know ($f\,\Delta L/D$):

$$f\frac{\Delta L}{D} = 0.0225\left(\frac{6.0}{0.05}\right) = 2.70$$

If $p_2 = 100$ kPa, we must ensure that the inlet Mach number is just sufficient that the inlet stagnation pressure $p_{o1} = 300$ kPa:

Guess Ma_2, back off $(f\Delta L/D) = 2.70$, find Ma_1, check p^*, p_1, and p_{o1}. Example:

$$Ma_2 = 1.0,\quad p^* = p_2 = 100\,\text{kPa},\quad f\frac{L_2^*}{D} = 0,\quad f\frac{L_1^*}{D} = 2.70,$$

$$\text{Read}\quad Ma_1 = \mathbf{0.380},\quad \frac{p_1}{p^*} = 2.84\quad \text{Then}\quad p_1 = 2.84(100) = 284\,\text{kPa},$$

$$\frac{p_1}{p_{o1}} = [1+0.2(0.380)^2]^{-3.5},\quad \text{solve}\ \ p_{o1} = 314\,\text{kPa} \neq 300$$

So we back off and try values of $Ma_2 < 1.0$ and proceed until the inlet matches. The solution (performed by the author using EES) is

$$Ma_2 = 0.962,\quad Ma_1 = 0.380,\quad p_2 = p^* = 100\,\text{kPa},\quad p_1 = 271.5\,\text{kPa},\quad T_1 = 389\,\text{K},$$

$$\rho_1 = 2.434\ \frac{\text{kg}}{\text{m}^3},\quad V_1 = 150\ \frac{\text{m}}{\text{s}},\quad \dot{m} = \rho_1\frac{\pi}{4}D^2V_1 = \mathbf{0.718\ \frac{kg}{s}}\qquad \textit{Ans.}$$

So the tube is *nearly*, but not quite, choked.

C9.3 Fig. C9.3 shows the exit of a converging-diverging nozzle, where an oblique shock pattern is formed. In the exit plane, which has an area of 15 cm^2, the air pressure is 16 kPa and the temperature is 250 K. Just outside the exit shock, which makes an angle of 50° with the exit plane, the temperature is 430 K. Estimate (a) the mass flow; (b) the throat area; (c) the turning angle of the exit flow; and, in the tank supplying the air, (d) the pressure and (e) the temperature.

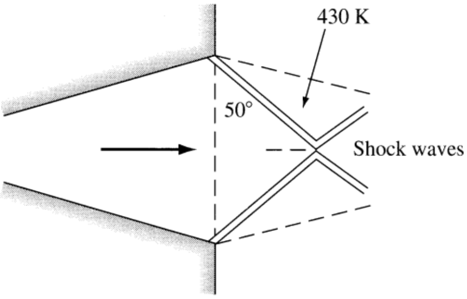


Fig. C9.3

Solution: We know the temperature ratio and the shock wave angle, so we can muddle through oblique-shock-wave theory to find the shock conditions:

$$\frac{T_2}{T_1} = \frac{430}{250} = 1.72; \quad \beta = 40° \quad \textit{Iterate Eqns.}(9.83) \textit{ or use EES!}$$

$$\textit{Solution}: \quad Ma_{exit} = 3.17; \quad \rho_1 = 0.223 \ \frac{\text{kg}}{\text{m}^3}; \quad \frac{A_1}{A^*} = 4.99 = \frac{15}{A_{throat}},$$

$$\mathbf{A_{throat} = 3.00\ cm^2} \quad \textit{Ans.}\ \text{(b)}$$

$$V_1 = Ma_1 a_1 = 1006\ \frac{\text{m}}{\text{s}}, \quad \dot{m} = \rho_1 A_1 V_1 = \mathbf{0.336}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(a)} \quad \theta_{shock} = \mathbf{22.88°} \quad \textit{Ans.}\ \text{(c)}$$

With the exit Mach number known, it is easy to compute stagnation conditions:

$$p_{o,tank} = 16[1 + 0.2(3.17)^2]^{3.5} = \mathbf{760\ kPa} \quad \textit{Ans.}\ \text{(d)}$$

$$T_{o,tank} = 250[1 + 0.2(3.17)^2] = \mathbf{753\ K} \quad \textit{Ans.}\ \text{(e)}$$

C9.4 The properties of a dense gas (high pressure and low temperature) are often approximated by van der Waals' equation state [Refs. 17 and 18]:

$$p = \frac{\rho R T}{1 - b_1 \rho} - a_1 \rho^2$$

$$\text{where } a_1 = \frac{27 R^2 T_c^2}{64 p_c} = 9E5 \frac{ft - lb^4}{slug^2} \text{ for air}$$

$$\text{and } b_1 = \frac{R T_c}{8 p_c} = 0.65 \frac{ft^3}{slug} \text{ for air}$$

Find an analytic expression for the speed of sound of a van der Waals gas. Assuming *k* = 1.4, compute the speed of sound of air, in ft/s, at –100°F and 20 atm, for (*a*) a perfect gas, and (*b*) a van der Waals gas. What percentage higher density does the van der Waals relation predict? NOTE: 1 atm = 2116 lbf/ft^3.

Solution: For air, take *R* = 1716 ft-lbf/slug°R. First evaluate the densities, T = 360°R:

$$\text{Ideal gas: } \rho = \frac{p}{RT} = \frac{20 \times 2116\, lbf/ft^2}{(1716\, ft-lbf/slug^o R)(360^o R)} = 0.0684 \frac{slug}{ft^3}$$

van der Waals:

$$20 \times 2116 = \frac{\rho(1716)(360)}{1-0.65\rho} - 9E5\rho^2\text{, Solve for } \rho = 0.0724 \frac{slug}{ft^3}$$

The van der Waals relation predicts air density **5.7% higher**.

(*a*) For an ideal gas,

$$a_{ideal} = \sqrt{kRT} = \sqrt{(1.4)(1716)(360)} = \mathbf{930\,ft/s} \qquad Ans.(a)$$

(*b*) For the van der Waals gas,

$$a^2 = k\left(\frac{\partial p}{\partial \rho}\right)_T = k\left[\frac{RT}{1-b_1\rho} + \frac{\rho RTb_1}{(1-b_1\rho)^2} - 2a_1\rho\right]$$

$$\text{Plug in the numbers: } a_{van\ der\ Waals} = \mathbf{877\ ft/s} \qquad Ans.(b)$$

C9.5 Consider one-dimensional steady flow of a non-ideal gas, steam, in a converging nozzle. Stagnation conditions are p_o = 100 kPa and T_o = 200°C. The nozzle exit diameter is 2 cm. If the nozzle exit pressure is 70 kPa, (a) calculate the mass flow and the exit temperature for real steam, from the Steam Tables or using EES. (As a first estimate, assume steam to be an ideal gas from Table A.4.) Is the flow choked? (b) Find the nozzle exit pressure and mass flow for which the steam flow *is* choked, using EES or the Steam Tables.

Solution: (a) For steam as an ideal gas, from Table A.4, *k* = 1.33 and *R* = 461 J/kg·K. First use this approximation to find the exit Mach number:

$$\frac{p}{p_o} = \frac{70\text{ kPa}}{100\text{ kPa}} \approx \left(1 + \frac{0.33}{2} Ma_e^2\right)^{-1.33/0.33}, \quad \text{solve for } Ma_{exit} \approx 0.75 \quad \text{Flow is not choked}$$

For real steam, we use EES. The nice Power-law ideal-gas formulas, Eqs. (9.26–9.28), are invalid, but the energy equation (9.22) is valid, and the nozzle flow is isentropic. First evaluate

$$h_o = \text{ENTHALPY(steam, p = 100, T = 473)} = 2875 \text{ kJ/kg·K}$$

$$s_o = \text{ENTROPY(steam, p = 100, T = 473)} = 7.833 \text{ kJ/kg·K}$$

Then use the energy equation with the known pressure at the exit:

$$h_o = 2875 = h_e + \frac{V_e^2}{2(1000)} \quad \text{with } h_e = \text{ENTHALPY}(steam, p = 70, s = 7.833)$$

EES returns the result $h_e = 2800\,\text{kJ/kg}$ and $V_e = 385$ m/s

The specific request was for the exit temperature and the mass flow:

$$T_e = \text{TEMPERATURE(steam, p = 70, s = 7.833)} = \mathbf{434\ K} \quad \textit{Ans.}\ \text{(a)}$$

$$\rho_e = \text{DENSITY(steam, p = 70, s = 7.833)} = 0.351 \text{ kg/m}^3$$

$$\text{mass flow} = \rho_e A_e V_e = (0.351 \text{ kg/m}^3)(\pi/4)(0.02 \text{ m})^2(385 \text{ m/s}) = \mathbf{0.0425\ kg/s} \quad \textit{Ans.}\ \text{(a)}$$

EES now has a speed of sound function, so we can compute

$$a_e = \text{SOUNDSPEED(steam, p = 70, s = 7.833)} = 512 \text{ m/s}$$

Hence the exit Mach number is $Ma_e = (385 \text{ m/s})/(512 \text{ m/s}) = \mathbf{0.75}$
The agreement with perfect-gas theory is excellent.

(b) We are asked to determine p_e for which the flow ***is*** choked, using EES. Ideal gas theory for $k \approx 1.33$ predicts from Eq. (9.32) that $p^*/p_o \approx 0.54$. For EES, real steam, we use the same procedure as in part (a) above and reduce p_{exit} gradually until the mass flow is a maximum. The final result is $p_e = 54.2$ kPa:

$$p_e/p_o = 0.542, \quad Ma_e \approx 1.00, \quad \mathbf{\dot{m}_{max} = 0.04517\ kg/s} \quad \textit{Ans.}\ \text{(b)}$$

C9.6 Extend Prob. C9.5 as follows. Let the nozzle be converging-diverging, with an exit diameter of 3 cm. Assume isentropic flow. (a) Find the exit Mach number, pressure, and temperature for an ideal gas, from Table A.4. Does the mass flow agree with the value of 0.0452 kg/s in Prob. C9.5? (b) Investigate, briefly, the use of EES for this problem and explain why part (a) is unrealistic and poor convergence of EES is obtained. [HINT: Study the pressure and temperature state predicted by part (a).]

Solution: (a) For steam as an ideal gas, from Table A.4, $k = 1.33$ and $R = 461$ J/kg·K.

$$\frac{A_{exit}}{A*} = \frac{(\pi/4)(0.03)^2}{(\pi/4)(0.02)^2} = 2.25 = \frac{1}{Ma_e}\left\{\frac{\left[1+0.5(k-1)Ma_e^2\right]}{0.5(k+1)}\right\}^{0.5(k+1)/(k-1)} \quad \text{for } k = 1.33$$

Solve for $\mathbf{Ma_e = 2.27}$ *Ans.* (a)

For the exit pressure, temperature, and mass flow, use the ideal Power-law relations:

$$T_e = T_o\Big/\left[1+0.5(k-1)Ma_e^2\right] = 473/[1+0.165(2.27)^2] = \mathbf{256\ K} \quad \textit{Ans. (a)}$$

$$p_e = p_o\Big/\left[1+0.5(k-1)Ma_e^2\right]^{k/(k-1)} = 100/[1+0.165(2.27)^2]^{4.03} = \mathbf{8.4\ kPa} \quad \textit{Ans. (a)}$$

$$\rho_e = \frac{p_e}{RT_e} = 0.0713\ \frac{\text{kg}}{\text{m}^3}; \quad V_e = Ma_e a_e = 898\ \frac{\text{m}}{\text{s}}; \quad \dot{\mathbf{m}} = \rho_e A_e V_e = \mathbf{0.0452\ kg/s} \quad \textit{Ans. (a)}$$

The mass flow does equal the choked-flow value from Prob. C9.5, as expected.

(b) Recall from Prob. C9.5 that $p_o = 100$ kPa and $T_o = 200$°C, which corresponds for real steam (EES) to $h_o = 2875$ kJ/kg·K and $s_o = 7.833$ kJ/kg. We proceed through the nozzle, using the energy equation, $h_o = h + V^2/2$, plus the condition of constant entropy. We also know that the flow is choked at 0.0452 kg/s with a throat diameter of 2 cm. The results are, for real steam,

$Ma_e = \mathbf{2.22}$; $p_e = \mathbf{10.2\ kPa}$; $T_e = \mathbf{319\ K}$; $\rho_e = \mathbf{0.0725\ kg/m^3}$; Quality = **96%**

The ideal-gas theory is still reasonably accurate at this point, but it is unrealistic, since the real steam has entered the *two-phase (wet) region*.

C9.7 Professor Gordon Holloway and his student, Jason Bettle, of the University of New Brunswick, obtained the following tabulated data for blow-down air flow through a converging-diverging nozzle similar in shape to Fig. P3.22. The supply tank pressure and temperature were 29 psig and 74°F, respectively. Atmospheric pressure was 14.7 psia. Wall pressures and centerline stagnation pressures were measured in the expansion section, which was a frustrum of a cone. The nozzle throat is at $x = 0$.

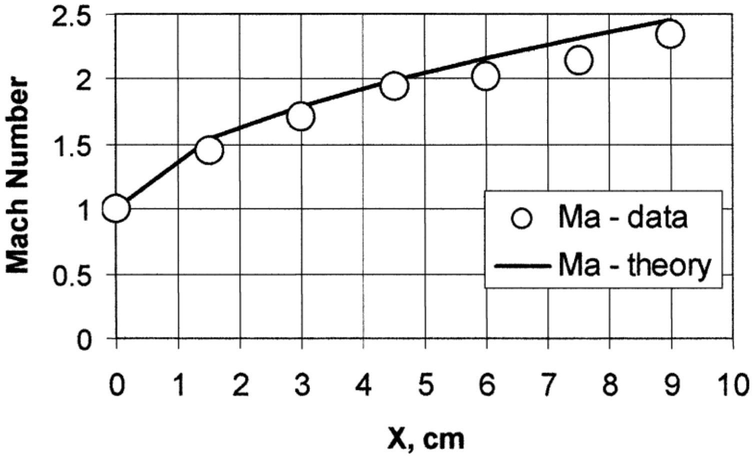
x (cm):	0	1.5	3	4.5	6	7.5	9
Diameter (cm):	1.00	1.098	1.195	1.293	1.390	1.488	1.585
p_{wall} (psig):	7.7	–2.6	–4.9	–7.3	–6.5	–10.4	–7.4
$p_{\text{stagnation}}$ (psig):	29	26.5	22.5	18	16.5	14	10

Use the stagnation pressure data to estimate the local Mach number. Compare the measured Mach numbers and wall pressures with the predictions of one-dimensional theory. For $x > 9$ cm, the stagnation pressure data was not thought by Holloway and Bettle to be a valid measure of Mach number. What is the probable reason?

Solution: From the cone's diameters we can determine A/A* and compute theoretical Mach numbers and pressures from Table B.1. From the measured stagnation pressures we can compute measured (supersonic) Mach numbers, because a normal shock forms in front of the probe. The ratio p_{o2}/p_{o1} from Eq. (9.58) or Table B.2 is used to estimate the Mach number.

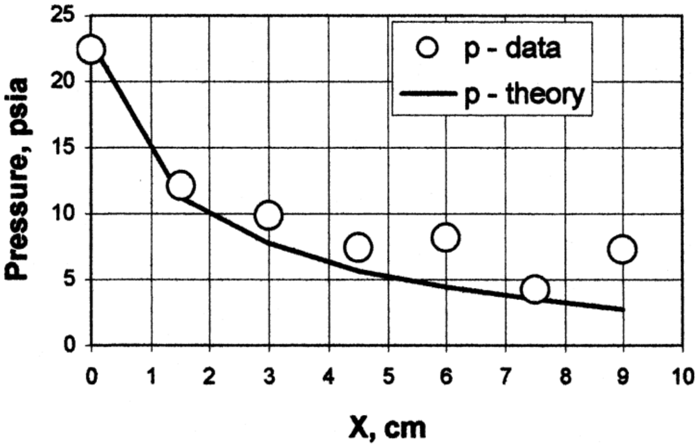
x (cm):	0	1.5	3	4.5	6	7.5	9
Ma-theory:	1.00	1.54	1.79	1.99	2.16	2.31	2.45
p_w-theory (psia):	23.1	11.2	7.72	5.68	4.36	3.44	2.77

The comparison of measured and theoretical Mach number is shown in the graph below.



Problem C9.7

The comparison of measured and theoretical static pressure is shown in the graph below.



Holloway and Bettle discounted the data for x > 9 cm, which gave Mach numbers and pressures widely divergent from theory. It is probably that a normal shock formed in the duct.

C9.8 Engineers call the supersonic combustion, in the scramjet of Fig. 9.30, almost miraculous, "like lighting a match in a hurricane". Figure C9.8 is a crude idealization of the engine. Air enters, burns fuel in the narrow section, then exits, all at supersonic speeds. There are no shock waves. Assume areas of 1 m^2 at sections 1 and 4 and 0.2 m^2 at sections 2 and 3. Let the entrance conditions be $Ma_1 = 6$, at 10,000 m standard altitude. Assume isentropic flow from 1 to 2, frictionless heat transfer from 2 to 3 with Q = 500 kJ/kg, and isentropic flow from 3 to 4. Calculate the exit conditions and the thrust produced.

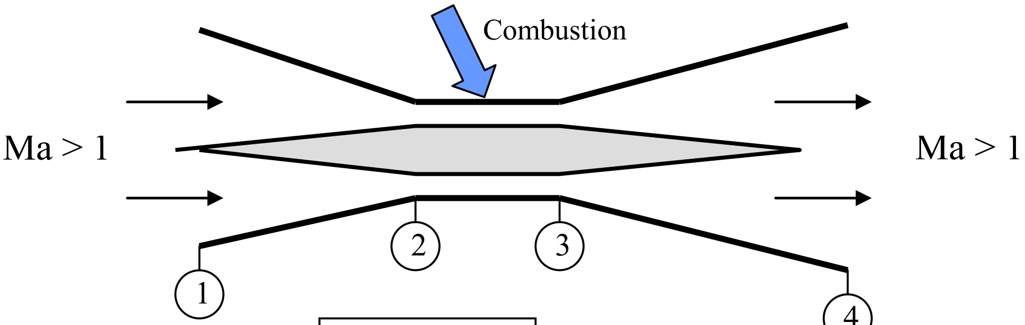


Fig. C9.8

Solution: From Table B.6 at 10,000 m, p_1 = 26416 Pa, T_1 = 223.16 K, ρ_1 = 0.4125 kg/m^3, and a_1 = 299.5 m/s. Calculate the inlet velocity and mass flow:

$$V_1 = Ma_1\, a_1 = (6.0)(299.5) = 1797\, m/s\ , \quad \dot{m} = \rho_1 A_1 V_1 = (0.4125)(1.0)(1797) = 741\, kg/s$$

Now find conditions at section 2, assuming one-dimensional isentropic steady flow:

$$T_{o1} = T_{o2} = (223.16)[1+0.2(6)^2] - 1830\,K;\quad p_{o1} = p_{o2} = (26416)[1+0.2(6)^2]^{3.5} = 4.17\text{E}7\, Pa$$

$$\text{At } Ma_1 = 6.0\ ,\ \frac{A_1}{A*} = 53.18\ ;\ \text{then } \frac{A_2}{A*} = (53.18)\left(\frac{0.2\,m^2}{1.0\,m^2}\right) = 10.64 \text{ for which } Ma_2 = 3.991$$

$$T_2 = \frac{1830\,K}{[1+0.2(3.991)^2]} = 437\,K\ \ ;\ \ p_2 = \frac{4.171\text{E}7\,Pa}{[1+0.2(3.991)^2]^{3.5}} = 278,000\,Pa$$

Use frictionless heat-addition theory to get from section 2 to section 3:

$$Ma_2 = 3.991:\ \frac{T_{o2}}{T_o^*} = 0.5895\ ,\ \ T_o^* = 3104\,K\ ;\ \ \frac{T_2}{T*} = 0.169\ ,\ T* = 2587\,K$$

$$\frac{p_2}{p*} = 0.103\ ,\ \ p* = 2.698\text{E}6\ Pa\ ;\ T_{o3} = T_{o2} + \frac{Q}{c_p} = 1830 + \frac{500000}{1005} = 2327\,K$$

$$\frac{T_{o3}}{T_o^*} = \frac{2327}{3104} = 0.7497\ ,\ \text{thus } Ma_3 = 2.238\ ;\ \frac{p_3}{p*} = 0.2996\ ,\ \ p_3 = 808,500\,Pa$$

Now move isentropically from section 3 to section 4 (staying supersonic, of course):

$$p_{o3} = p_{o4} = p_3[1+0.2(Ma_3^2)]^{3.5} = (808500)[1+0.2(2.238)^2]^{3.5} = 9.17\text{E}6\ Pa$$

$$Ma_3 = 2.2376\ ,\ \frac{A_3}{A_3^*} = 2.073\ ,\ \text{ then } \frac{A_4}{A_3^*} = (2.073)(\frac{1.0\,m^2}{0.2\,m^2}) = 10.37 \text{ and } Ma_4 = 3.963$$

$$p_4 = \frac{9.17\text{E}6\ Pa}{[1+0.2(3.963)^2]^{3.5}} = 63{,}480\,Pa\ ;\ T_4 = \frac{2327\,K}{[1+0.2(3.963)^2]} = 562\,K$$

$$\rho_4 = \frac{p_4}{RT_4} = 0.394\frac{kg}{m^3}\ ;\ V_4 = Ma_4\sqrt{kRT_4} = (3.963)(475) = 1883\ \frac{m}{s}$$

Check the mass flow = (1883)(1.0)(0.394) =741 kg/s, Yes. For a control volume around the scramjet, we find that the thrust is due to both velocity changes and pressure changes:

$$Thrust = F = \dot{m}(V_4 - V_1) + A_1(p_4 - p_1) =$$

$$= (741\frac{kg}{s})(1883 - 1797\,\frac{m}{s}) + (1.0\,m^2)(63480 - 26416\,Pa) =$$

$$= 64150 + 37060 = \mathbf{101{,}200}\,N = 22{,}750\ lbf \qquad Ans.$$

Excellent force, but the combustion chamber has to withstand high pressures (8 atm) and high temperatures (2600 K). For this simple one-dimensional model, the thrust goes up linearly with Q.

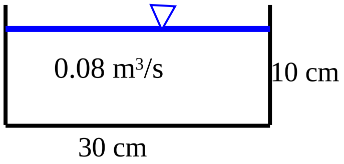
Chapter 10 · Open Channel Flow

10.1 The formula for shallow-water wave propagation speed, Eq. (10.9) or (10.10), is independent of the physical properties of the liquid, i.e., density, viscosity, or surface tension. Does this mean that waves propagate at the same speed in water, mercury, gasoline, and glycerin? Explain.

Solution: The shallow-water wave formula, $c_o = \sqrt{(gy)}$, is valid for any fluid except for *viscosity and surface tension effects*. If the wave is very small, or "capillary" in size, its propagation may be influenced by surface tension and Weber number [Ref. 3−7]. If the fluid is very viscous, its speed may be influenced by Reynolds number. The formula is accurate for water, mercury, and gasoline, in larger channels, but would be inaccurate for *glycerin*.

P10.2 Water at 20°C flows in a 30-cm-wide rectangular channel at a depth of 10 cm and a flow rate of 80,000 cm³/s. Estimate (*a*) the Froude number; and (*b*) the Reynolds number.

Solution: For water, take $\rho = 998\ \text{kg/m}^3$ and μ = 0.001 kg/m-s. The surface wave speed is



$$c_o = \sqrt{gy} = \sqrt{(9.81\,m/s^2)(0.1\,m)} = 0.99\,m/s$$

$$V = \frac{Q}{A} = \frac{(80{,}000\,cm^3/s)}{(30\,cm)(10\,cm)} = 267\,\frac{cm}{s} = 2.67\,\frac{m}{s}$$

$$\text{Froude number:}\quad Fr = \frac{V}{c_o} = \frac{2.67\,m/s}{0.99\,m/s} = \mathbf{2.69}\quad \text{(supercritical)}\quad Ans.(a)$$

The average velocity is determined from the given flow rate and area:

The Reynolds number should be, for this writer, based upon hydraulic radius:

10.3 Narragansett Bay is approximately 21 (statute) mi long and has an average depth of 42 ft. Tidal charts for the area indicate a time delay of 30 min between high tide at the mouth of the bay (Newport, Rhode Island) and its head (Providence, Rhode Island). Is

$$R_h = \frac{A}{P} = \frac{(0.3m)(0.1m)}{(0.3+0.1+0.1m)} = 0.06\,m$$

$$\text{Reynolds number:}\quad \mathrm{Re}_{R_h} = \frac{\rho V R_h}{\mu} = \frac{(998)(2.67)(0.06)}{(0.001)} = \mathbf{160{,}000}\ (turbulent)\quad Ans.(b)$$

this delay correlated with the propagation of a shallow-water tidal-crest wave through the bay? Explain.

Solution: If it *is* a simple shallow-water wave phenomenon, the time delay would be

$$\Delta t = \frac{\Delta L}{c} = \frac{(21\ \text{mi})(5280\ \text{ft/mi})}{\sqrt{32.2(42)}} \approx 3015\ \text{s} \approx \mathbf{50\ min} \quad \textit{Ans.}???$$

This doesn't agree with the measured Δt ≈ 30 min. In reality, tidal propagation in estuaries is a *dynamic* process, dependent on estuary shape, bottom friction, and tidal period.

10.4 The water-channel flow in Fig. P10.4 has a free surface in three places. Does it qualify as an open-channel flow? Explain. What does the dashed line represent?

Fig. P10.4

Solution: No, this is ***not*** an open-channel flow. The open tubes are merely piezometer or pressure-measuring devices, there is no flow in them. The dashed line represents the pressure distribution in the tube, or the "hydraulic grade line" (HGL).

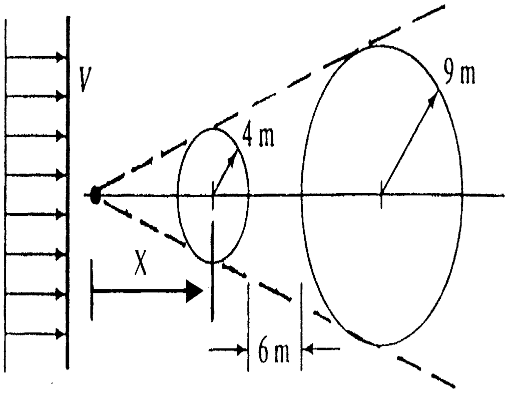
P10.5 Water flows down a rectangular channel that is 4 ft wide and 3 ft deep. The flow rate is 15,000 gal/min. Estimate the Froude number of the flow.

Solution: Convert the flow rate from 15,000 gal/min ÷ 448.83 = 33.4 ft³/s. The water velocity and wave speed may be calculated:

$$V = Q/A = (33.4\,ft^3/s)/(3\times4\,ft^2) = 2.78\ ft/s\ ; \quad c = \sqrt{g\,y} = \sqrt{(32.2\,ft/s^2)(3\,ft)} = 9.83\,ft/s$$

$$\text{Thus} \quad Fr = V/c = 2.78/9.83 = \mathbf{0.28}\ \text{(subcritical)} \quad \textit{Ans.}$$

10.6 Pebbles dropped successively at the same point, into a water-channel flow of depth 42 cm, create two circular ripples, as



in Fig. P10.6. From this information, estimate (a) the Froude number; and (b) the stream velocity.

Fig. P10.6

Solution: The center of each circle moves at stream velocity V. For the small circle,

$$\text{small circle:}\quad X = \frac{4V}{c_o};\quad \text{large circle:}\ X + 4 + 6 + 9 = \frac{9V}{c_o};$$

$$\text{Solve}\quad \mathbf{Fr} = \frac{V}{c_o} = \mathbf{3.8}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{Compute } c_o = \sqrt{gh} = \sqrt{9.81(0.42)} = 2.03\ \frac{m}{s},\quad \mathbf{V_{current}} = 3.8c_o \approx \mathbf{7.7}\ \frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(b)}$$

Fig. P10.7

10.7 Pebbles dropped successively at the same point, into a water-channel flow of depth 65 cm, create two circular ripples, as in Fig. P10.7. From this information, estimate (a) the Froude number; and (b) the stream velocity.

Solution: If the pebble-drop-site is at distance X ahead of the small-circle center,

$$\text{small circle:}\quad X = \frac{3V}{c};\quad \text{large circle:}\ X + 4 = \frac{9V}{c};\quad \text{solve}\ \mathbf{Fr} = \frac{V}{c} = \frac{\mathbf{2}}{\mathbf{3}}\quad \textit{Ans.}\ \text{(a)}$$

$$c_o = \sqrt{gh} = \sqrt{9.81(0.65)} = 2.53\ \frac{m}{\,};\quad \mathbf{V_{current}} = \frac{2}{3}c_o \approx \mathbf{1.68}\ \frac{\mathbf{m}}{\,}\quad \textit{Ans.}\ \text{(b)}$$

10.8 An earthquake near the Kenai Peninsula, Alaska, creates a single “tidal” wave (called a ‘tsunami’) which propagates south across the Pacific Ocean. If the average ocean depth is 4 km and seawater density is 1025 kg/m^3, estimate the time of arrival of this tsunami in Hilo, Hawaii.

Solution: Everyone get out your Atlases, how far is it from Kenai to Hilo? Well, it’s about 2800 statute miles (4480 km), and seawater *density* has nothing to do with it:

$$\Delta t_{travel} = \frac{\Delta x}{c} = \frac{4480E3\ m}{\sqrt{9.81(4000\ m)}} \approx 22600\ s \approx \mathbf{6.3\ hours}\quad \textit{Ans.}$$

So, given warning of an earthquake in Alaska (by a seismograph), there is plenty of time to warn the people of Hilo (which is very susceptible to tsunami damage) to take cover.

10.9 Equation (10.10) is for a single disturbance wave. For *periodic* small-amplitude surface waves of wavelength λ and period T, inviscid theory [5 to 9] predicts a wave propagation speed

$$c_0^2 = \frac{g\lambda}{2\pi}\tanh\frac{2\pi y}{\lambda}$$

where y is the water depth and surface tension is neglected. (a) Determine if this expression is affected by the Reynolds number, Froude number, or Weber number. Derive the limiting values of this expression for (b) $y \ll \lambda$ and (c) $y \gg \lambda$. (d) Also for what ratio y/λ is the wave speed within 1 percent of limit (c)?

Solution: (a) Obviously there is **no effect** in this theory for Reynolds number or Weber number, because viscosity and surface tension are not present in the formula. There *is* a Froude number effect, and we can rewrite it as Froude number versus dimensionless depth:

$$\mathbf{Fr_{wave}} = \frac{\mathbf{c_o}}{\sqrt{\mathbf{g\lambda}}} = \sqrt{\frac{\mathbf{1}}{\mathbf{2\pi}}\,\mathbf{tanh}\left(\frac{\mathbf{2\pi y}}{\boldsymbol{\lambda}}\right)} = \mathbf{fcn}\left(\frac{\mathbf{y}}{\boldsymbol{\lambda}}\right) \quad \textit{Ans. } \text{(a)}$$

(b)

$$\mathrm{y} \ll \lambda:\ \tanh\zeta \approx \zeta \ \text{ if } \ \zeta \ll 1:\ \mathrm{c}^2_{\text{o,long waves}} \approx \frac{\mathrm{g}\lambda}{2\pi}\frac{2\pi\mathrm{y}}{\lambda} \approx \mathbf{gy} \quad \text{(same as Eq. 10.10)} \quad \textit{Ans. } \text{(b)}$$

(c)

$$\mathrm{y} \gg \lambda:\ \tanh\zeta \approx 1 \ \text{ if } \ \zeta \gg 1:\ \mathrm{c}^2_{\text{o,short waves}} \approx \frac{\mathbf{g\lambda}}{\mathbf{2\pi}} \quad \text{(periodic deep-water waves)} \quad \textit{Ans. } \text{(c)}$$

(d) $\mathrm{c_o} = 0.99\mathrm{c_{o,deep}}$ if $\tanh(2\pi\mathrm{y}/\lambda) \approx 0.995 \approx \tanh(3)$, or $\mathbf{y/\boldsymbol{\lambda} \approx 0.48}$. *Ans.* (d).

10.10 If surface tension Y is included in the analysis of Prob. 10.9, the resulting wave speed is [Refs. 3 to 9]:

$$c_0^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi Y}{\rho\lambda}\right)\tanh\frac{2\pi y}{\lambda}$$

(a) Determine if this expression is affected by the Reynolds number, Froude number, or Weber number. Derive the limiting values of this expression for (b) $y \ll \lambda$ and (c) $y \gg \lambda$. (d) Finally determine the wavelength λ_{crit} for a minimum value of c_0, assuming that $y \gg \lambda$.

Solution: (a) Obviously there is **no effect** in this theory for Reynolds number, because viscosity is not present in the formula. There *are* Froude number and Weber number effects, and we can rewrite it as Froude no. versus Weber no. and dimensionless depth:

$$\mathbf{Fr_{wave}} = \frac{\mathbf{c_o}}{\sqrt{\mathbf{g\lambda}}} = \sqrt{\left(\frac{\mathbf{1}}{\mathbf{2\pi}} + \frac{\mathbf{2\pi Y}}{\mathbf{\rho g\lambda^2}}\right)\mathbf{tanh}\left(\frac{\mathbf{2\pi y}}{\mathbf{\lambda}}\right)} = \mathbf{fcn}\left(\mathbf{We}, \frac{\mathbf{y}}{\mathbf{\lambda}}\right), \quad \mathrm{We} = \frac{\rho g\lambda^2}{Y} \quad \textit{Ans. (a)}$$

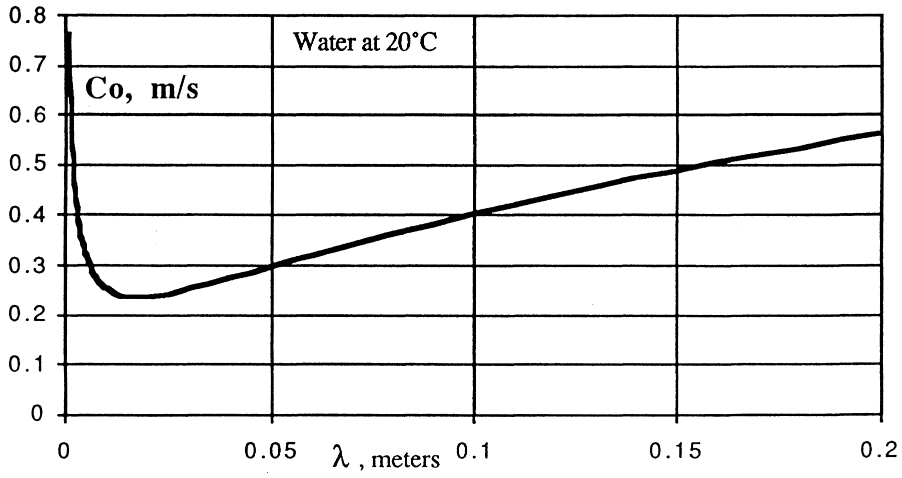
$$\text{(b)}\quad y \ll \lambda: \quad \tanh\zeta \approx \zeta \quad \text{if} \quad \zeta \ll 1: \quad c^2_{o,\text{long waves}} \approx \left(\mathbf{gy} + \frac{\mathbf{4\pi^2 Yy}}{\mathbf{\rho\lambda^2}}\right) \quad \textit{Ans. (b)}$$

$$\text{(c)}\quad y \gg \lambda: \quad \tanh\zeta \approx 1 \quad \text{if} \quad \zeta \gg 1: \quad c^2_{o,\text{short waves}} \approx \left(\frac{\mathbf{g\lambda}}{\mathbf{2\pi}} + \frac{\mathbf{2\pi Y}}{\mathbf{\rho\lambda}}\right) \quad \textit{Ans. (c)}$$

For a deep-water wave, part (c) applies, and we can differentiate with respect to λ:

$$\frac{dc_o^2}{d\lambda} = \frac{g}{2\pi} - \frac{2\pi Y}{\rho\lambda^2} = 0 \quad \text{if} \quad \boldsymbol{\lambda_{crit} = 2\pi\sqrt{\frac{Y}{\rho g}}} \quad (\text{where } c_o = c_{o,min}) \quad \textit{Ans. (d)}$$

For water at 20°C, we may compute that $\lambda_{crit} \approx 0.018$ m = 1.8 cm, as shown below.



10.11 A rectangular channel is 2 m wide and contains water 3 m deep. If the slope is 0.85° and the lining is corrugated metal, estimate the discharge for uniform flow.

Fig. P10.11

Solution: For corrugated metal, take Manning's *n* ≈ 0.022. Get the hydraulic radius:

$$R_h = \frac{A}{P} = \frac{2(3)}{3+2+3} = 0.75\text{ m};\quad Q \approx \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1}{0.022}(6)(0.75)^{2/3}[\tan(0.85°)]^{1/2}$$

$$\text{or:}\quad \mathbf{Q \approx 27\ m^3/s}\quad \textit{Ans.}$$

10.12 (a) For laminar draining of a wide thin sheet of water on pavement sloped at angle θ, as in Fig. P4.36, show that the flow rate is given by

$$Q = \frac{\rho g b h^3 \sin\theta}{3\mu}$$

where b is the sheet width and h its depth. (b) By (somewhat laborious) comparison with Eq. (10.13), show that this expression is compatible with a friction factor $f = 24/\text{Re}$, where $\text{Re} = V_{av}h/\nu$.

Solution: The velocity and flow rate were worked out in detail in Prob. 4.36:

$$\text{x-Mom. yields}\quad u=\frac{\rho g\sin\theta}{2\mu}y(2h-y),\quad Q=\int_0^h ub\,dy=\frac{\boldsymbol{\rho g b h^3\sin\theta}}{\boldsymbol{3\mu}}\quad \textit{Ans.}\text{ (a)}$$

$$\text{for } 0<y<h.\ \text{Then}\quad V_{avg}=\frac{2}{3}u_{max}=\frac{h^2 g\sin\theta}{3\nu}\quad\text{and}\quad R_h\big|_{wide\ channel}\approx h$$

Interpreting "sinθ" as "S_o," the channel slope, we compare Q above with Eq. 10.13:

$$Q=\frac{gbh^3S_o}{3\nu}=\sqrt{\frac{8g}{f}}(bh)h^{1/2}S_o^{1/2},\quad\text{solve for}\ \mathbf{f}=\frac{72\nu^2}{h^3gS_o}=\frac{24\nu}{V_{avg}h}=\frac{24}{Re_h}\quad \textit{Ans.}$$

P10.13 A large pond drains down an asphalt rectangular channel that is 2 ft wide. The channel slope is 0.6 degrees. If the flow is uniform, at a depth of 21 inches, estimate the time to drain 1 acre-foot of water.

Solution: From Table 10.1, for asphalt, Manning's *n* = 0.016. The Chézy formula is

$$Q\approx\frac{1.486}{n}AR_h^{2/3}S_o^{1/2}\ ,\ A=by=(2)(1.75)=3.5\,ft^2\ ,\ R_h=\frac{A}{P}=\frac{3.5}{2+2(1.75)}=0.636\,ft$$

$$\text{Thus}\ Q=\frac{1.486}{0.016}(3.5\,ft^2)(0.636\,ft)^{2/3}[\tan(0.6^o)]^{1/2}\ =\ 24.6\ ft^3/s$$

Convert 1 acre-ft = (43560 ft^2)(1 ft) = 43560 ft^3. Then the time to drain this volume is

$$t_{drain}\ =\ \frac{43560\,ft^3}{24.6\,ft^3/s}\ =\ 1770\ s\ \approx\ \mathbf{29.5}\ minutes\qquad \textit{Ans.}$$

This is a fast flow! The Froude number is approximately 0.94.

10.14 The Chézy formula (10.18) is independent of fluid density and viscosity. Does this mean that water, mercury, alcohol, and SAE 30 oil will all flow down a given open channel at the same rate? Explain.

Solution: The Chézy formula, $V = (1.0/n)(R_h)^{2/3}\sqrt{S_o}$, appears to be independent of fluid properties, with *n* only representing surface roughness, but in fact it requires that the channel flow be "fully rough" and turbulent, i.e., at high Reynolds number ≥ 1E6 at least. Even for low-viscosity fluids such as water, mercury, and alcohol, this requires reasonable **size** for the channel, R_h of the order of 1 meter or more if the slope is small $(S_o \ll 1)$. SAE 30 oil is so viscous that it would need R_h > 10 m to approach the Chézy formula.

10.15 The painted-steel channel of Fig. P10.15 is designed, without the barrier, for a flow rate of 6 m^3/s at a normal depth of 1 m. Determine (a) the design slope of the channel and (b) the percentage of reduction in flow if the painted-steel barrier is installed.

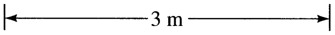


Fig. P10.15

Solution: For painted steel, n ≈ 0.014. Evaluate the hydraulic radius and then S_o:

$$R_h = \frac{A}{P} = \frac{3m^2}{3m + 1m + 1m} = 0.6m$$

$$Q = 6\frac{m^3}{s} = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.014}(3m^2)(0.6m)^{2/3} S_o^{1/2} \;;\; \text{Solve } S_o = \mathbf{0.00155} \quad Ans.(a)$$

(b) Add the barrier, work with one-half channel and double the result:

$$R_{h2} = \frac{1.5m^2}{3.5m} = 0.429m \;;\; Q_2 = 2\left[\frac{1}{0.014}(1.5m^2)(0.429m)^{2/3}\sqrt{0.00155}\,\right]$$

$$Q_2 = 2(2.40\frac{m^3}{s}) = 4.80\frac{m^3}{s} \;;\; Ratio = \frac{4.80}{6.0} = 0.80 \quad \text{or}: \; \mathbf{20\%} \text{ reduction in flow} \quad Ans.(b)$$

Interesting, this reduction is independent of slope and of *n*, as long as both have the same values:

P10.16 A brickwork rectangular channel is 120 cm wide and laid on a slope of 2 m per km. (*a*) Find the normal flow rate when the water depth is 40 cm. (*b*) For the same slope, find the water depth which will double the normal flow rate in part (a). Comment.

Solution: For brickwork, from Table 10.1, *n* = 0.015. (*a*) Use the Chézy formula:

$$\frac{Q_{barrier}}{Q_{no\,barrier}} = \frac{2[(1/n)(1.5)(0.429)^{2/3}S_o^{1/2}]}{(1/n)(3.0)(0.6)^{2/3}S_o^{1/2}} = 0.80 \quad (20\%\ reduction)$$

$$Q = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1}{0.015}[1.2(0.4)][\frac{1.2(0.4)}{1.2+2(0.4)}](\frac{2}{1000})^{1/2} = 0.553\frac{m^3}{s} \quad Ans.(a)$$

(*b*) Double the flow rate to 1.106 m^3/s. This time the depth *y* is the unknown.

$$Q_b = 2Q_a = 1.106\frac{m^3}{s} = \frac{1}{0.015}(1.2y)[\frac{1.2y}{1.2+2y}](\frac{2}{1000})^{1/2}\ ;\ Solve \quad y = 0.666\,m \quad Ans.(b)$$

Comment: The variation of *Q* with *y* is nonlinear. The depth is only increased by 67%.

10.17 The trapezoidal channel of Fig. P10.17 is made of brickwork and slopes at 1:500. Determine the flow rate if the normal depth is 80 cm.

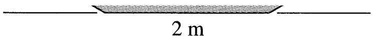


Fig. P10.17

Solution: For brickwork, *n* ≈ 0.015. Evaluate the hydraulic radius with y = 0.8 m:

$$A = 2y + y^2\cot\theta = 2(0.8) + (0.8)^2\cot 30° = 2.71\ m^2$$

$$P = 2 + 2(0.8)\csc 30° = 5.2\ m, \quad R_h = A/P = 2.71/5.2 \approx 0.521\ m$$

$$Q = \frac{1.0}{n}AR_h^{2/3}S_o^{1/2} = \frac{1.0}{0.015}(2.71)(0.521)^{2/3}\left(\frac{1}{500}\right)^{1/2} \approx \mathbf{5.23\ m^3/s} \quad Ans.$$

P10.18 A vee-shaped painted steel channel, similar to Fig. E10.6, has an included angle of 90 degrees. If the slope, in uniform flow, is 3 m per km, and the depth is 2 m, estimate (*a*) the flow rate, in m^3/s; and (*b*) the average wall shear stress.

Solution: For painted steel, from Table 10.1, $n = 0.014$. The properties of the 90° vee are easy:

$$P = 2y\sqrt{2} = 2(2m)\sqrt{2} = 5.66m \;;\; A = y^2 = (2m)^2 = 4m^2 \;;\; R_h = \frac{A}{P} = \frac{4}{5.66} = 0.707\,m$$

$$\text{Then}\quad Q = \frac{1.0}{n} A R_h^{2/3}\sqrt{S_o} = \frac{1.0}{n}(4m^2)(0.707m)^{2/3}\sqrt{3/1000} = \mathbf{12.4}\,\frac{m^3}{s} \quad Ans.(a)$$

(*b*) We could find the average shear stress in two ways. One would be to find the friction factor *f*, from Eq. (10.15), assuming fully rough flow. From Table 10.1 for painted steel, $\varepsilon = 2.4$ mm.

$$f = [2.0\log(\frac{14.8R_h}{\varepsilon})]^{-2} = [2.0\log(\frac{14.8(0.707m)}{0.0024m})]^{-2} = 0.0189$$

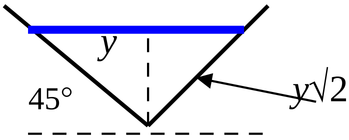
$$\text{Then}\quad \tau_{average} = \frac{f}{8}\rho(\frac{Q}{A})^2 = (\frac{0.0189}{8})(998\frac{kg}{m^3})(\frac{12.4m^3/s}{4m^2})^2 \approx \mathbf{22.6}\,Pa \quad Ans.(b)$$

This is only *approximate*, because we are using an *estimate* of wall roughness to find the friction factor. A more accurate result is the third of Eqs. (10.14), **which you are asked to prove, later, from a force balance, in Problem P10.28**:

$$\tau_{average} = \rho g R_h S_o = (998\frac{kg}{m^3})(9.81\frac{m}{s^2})(0.707m)(0.003) = \mathbf{21}\,Pa \quad \text{(better)}\,Ans.(b)$$

P10.19 Modify Prob. P10.18, the 90° vee-channel, to let the surface be clean earth, which erodes if the average velocity exceeds 6 ft/s. Find the maximum depth that avoids erosion. The slope is still 3 m per km.

Solution: From Table 10.1, for clean earth, $n = 0.022$. The vee-channel has easy algebra:



$$R_h = \frac{A}{P} = \frac{y^2}{2y\sqrt{2}} = \frac{y}{2\sqrt{2}}$$

$$V = 6\frac{ft}{s} = 1.829\frac{m}{s} = \frac{1}{0.022}\left(\frac{y}{2\sqrt{2}}\right)^{2/3}\sqrt{0.003}$$

Solve for $y = 1.78$ m. Keep the depth less than **1.78 m = 4.44 ft** to avoid erosion. *Ans*.

10.20 A circular corrugated-metal storm drain is flowing half-full over a slope of 4 ft/mile. Estimate the normal discharge if the drain diameter is 8 ft.

Fig. P10.20

Solution: For corrugated metal, $n \approx 0.022$. Evaluate the hydraulic radius, etc.:

$$A = (\pi/2)R^2 = 25.13\ ft^2;\quad P = \pi R = 12.56\ ft,\quad R_h = A/P = R/2 = 2\ ft$$

$$Q = \frac{1.486}{n} A R_h^{2/3} S_o^{1/2} = \frac{1.486}{0.022}(25.13)(2.0)^{2/3}\left(\frac{4}{5280}\right)^{1/2} \approx \mathbf{74\ \frac{ft^3}{s}}\quad \textit{Ans.}$$

10.21 An engineer makes careful measurements with a weir (see Sect. 10.7 later) which monitors a rectangular unfinished concrete channel laid on a slope of 1°. She finds, perhaps with surprise, that when the water depth doubles from 2 ft 2 inches to 4 ft 4 inches, the normal flow rate more than doubles, from 200 to 500 ft^3/s. (a) Is this plausible? (b) If so, estimate the channel width.

Solution: (a) **Yes**, Q always more than doubles for this situation where the depth doubles. *Ans*. (a)

(b) For unfinished concrete, take n = 0.014. Apply the normal-flow formula (10.19) to this data:

$$Q=\frac{1.486}{n}AR_h^{2/3}S_o^{1/2}=\frac{1.486}{0.014}(bh)\left(\frac{bh}{b+2h}\right)^{2/3}\sqrt{\sin 1^\circ}=200(or\ 500)\ \frac{\text{ft}^3}{\text{s}}$$

The two pieces of flow rate data give us two equations to solve for width b. It is unusual, but true, that both round-number flow rates converge to the same width ***b* = 5.72 ft**. *Ans*. (b)

10.22 A trapezoidal aqueduct has b = 5 m and θ = 40° and carries a normal flow of 60 m^3/s when y = 3.2 m. For clay tile surfaces, estimate the required elevation drop in m/km.

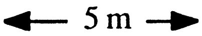


Fig. P10.22

Solution: For clay tile, take $n \approx 0.014$. The geometry leads to these values:

$$A=by+y^2\cot\theta=28.2\ m^2;\quad P=b+2y\csc\theta=14.96\ m,\quad R_h=A/P=1.886\ m$$

$$Q=60\ m^3/s=\frac{1.0}{0.014}(28.2)(1.886)^{2/3}S_o^{1/2},\quad \text{solve for } S_o=0.00038=\mathbf{0.38\ m/km}\quad \textit{Ans.}$$

10.23 It is desired to excavate a clean-earth channel as a trapezoidal cross-section with θ = 60° (see Fig. 10.7). The expected flow rate is 500 ft^3/s, and the slope is 8 ft per mile. The uniform flow depth is planned, for efficient performance, such that the flow cross-section is half a hexagon. What is the appropriate bottom width of the channel?

Solution: For clean earth, take n = 0.022. For a half-hexagon, from Fig. 10.7 of the text, depth $y = \sin(60°)b = 0.866b$, and

$$A = by + y^2\cot 60° = b(0.866b) + (0.866b)^2(0.577) = 1.299b^2; \quad P = 3b$$

$$Q = 500\ \frac{ft^3}{s} = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1.486}{0.022}(1.299b^2)\left(\frac{1.299b^2}{3b}\right)^{2/3}(8/5280)^{1/2} = 1.955b^{8/3}$$

Solve for **b = 8.0 ft** *Ans.*

P10.24 A rectangular channel, laid out on a 0.5° slope, delivers a flow rate of 5,000 gal/min in uniform flow when the depth is 1 ft and the width is 3 ft. (*a*) Estimate the value of Manning's factor *n*. (*b*) What water depth will triple the flow rate?

Solution: We have enough data to figure everything except *n* in Manning's formula:

$$A = (3)(1) = 3\,ft^2\ ;\ P = 3+1+1 = 5\,ft\ ;\ R_h = \frac{A}{P} = \frac{3\,ft^2}{5\,ft} = 0.6\,ft$$

$$Q = 5000\frac{gal}{min} = 11.14\frac{ft^3}{s} = \frac{1.486}{n}A\,R_h^{2/3}\sqrt{S_o} = \frac{1.486}{n}(3\,ft^2)(0.6\,ft)^{2/3}\sqrt{\tan 0.5^o}$$

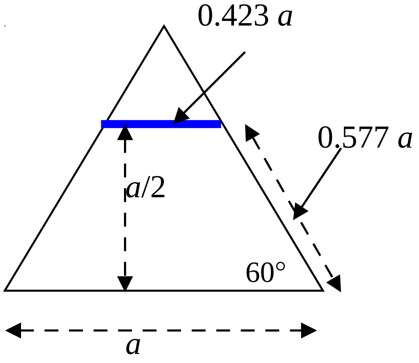
$$Solve\ \ for\quad n \approx \mathbf{0.0266}\qquad Ans.(a)$$

(*b*) If we triple the flow rate, the unknown is the (higher) depth *y*:

$$Q\ =\ 15{,}000\ \frac{gal}{min} = 33.42\frac{ft^3}{s} = \frac{1.486}{0.266}(3\,y)\left(\frac{3\,y}{3+2y}\right)\sqrt{\tan 0.5^o}$$

$$\text{Solve by iteration or EES:}\qquad y \approx \mathbf{2.28}\,ft\qquad Ans.(b)$$

P10.25 The equilateral-triangle channel in Fig. P10.25 has constant slope S_o and constant Manning factor *n*. If $y = a/2$, find an analytic expression for the flow rate Q.



Solution: The geometry is a bit awkward, but you only have to do it once. The width of the water line is

$a - 2(a/2)\cot 60° = 0.423\,a$

The length of one slanted wet side is

$a/2/\sin 60° = 0.577\,a$

Thus the perimeter, area, and hydraulic radius are

$$P = a + 2(0.577a) = 2.155\,a$$

$$A = (a/2)(a + 0.423a)/2 = 0.356\,a^2$$

$$R_h = A/P = (0.356a^2)/(2.155a) = 0.165\,a$$

$$Q = \frac{1}{n} A R_h^{2/3}\sqrt{S_o} = \frac{1}{n}(0.356a^2)(0.165a)^{2/3}\sqrt{S_o} = \mathbf{\frac{0.107}{n} a^{8/3}\sqrt{S_o}} \quad \textit{Ans.}$$

The formula is dimensionally incorrect, but that's what you get with Manning's formula.

10.26 In the spirit of Fig. 10.6*b*, analyze a rectangular channel in uniform flow with constant area $A = by$, constant slope, but varying width b and depth y. Plot the resulting flow rate Q, normalized by its maximum value $Q_{\max}$, in the range $0.2 < b/y < 4.0$, and comment on whether it is crucial for discharge efficiency to have the channel flow at a depth exactly equal to half the channel width.

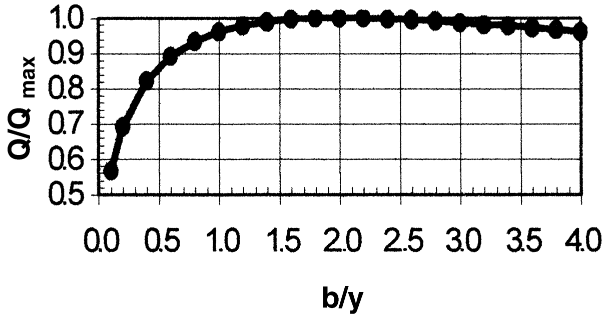
Solution: The Manning formula for a rectangular channel is:

$$Q = \frac{\alpha}{n} A R_h^{2/3} S_o^{1/2} \quad \text{where} \quad R_h = \frac{A}{b+2y} \quad \text{and} \quad A = by$$

$$Q = Q_{\max} \quad \text{when} \quad b = 2y, \quad \textit{or}: \; R_h = A/2b$$

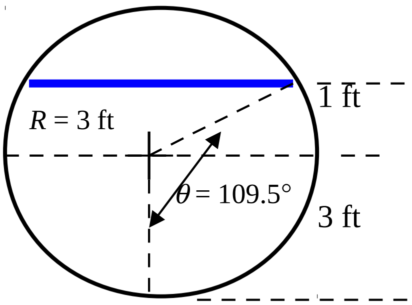
Then A cancels in the ratio $Q/Q_{\max} = [2b/(b+2y)]^{2/3}$

Plot $Q/Q_{\max}$ versus (b/y) making sure that area is constant, that is, $b = A/y$. The results are shown in the graph below. The curve is very flat near $b = 2y$, so **depth is not crucial**.



P10.27 A circular corrugated metal water channel has a slope of 1:800 and a diameter of 6 ft. (*a*) Estimate the normal discharge, in gal/min, when the water depth is 4 ft. (*b*) For this condition, calculate the average wall shear stress.

Solution: The geometry of this circular partly-full channel is given in Fig. 10.6*a* of the text. The angle θ (see Fig. 10.6*a*) is 109.5° = 1.911 radians. The area and hydraulic radius are given in the text:



$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = (3\,ft)^2\left[1.911 - \frac{\sin(2x1.911)}{2}\right] = 20.0\,ft^2$$

$$R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right) = \frac{3\,ft}{2}\left[1 - \frac{\sin(2x1.911)}{2(1.911)}\right] = 1.747\,ft$$

(*a*) Then the flow rate, by Manning's formula, with $n = 0.022$ for corrugated metal, is

$$Q = \frac{1.486}{0.022}(20.0\,ft^2)(1.747\,ft)^{2/3}\left(\frac{1}{800}\right)^{1/2} = 69\,\frac{ft^3}{s} \approx \mathbf{31{,}000}\ \frac{gal}{min} \quad Ans.(a)$$

(*b*) The average wall shear stress is best calculated by the formula from Prob. P10.28:

$$\tau_{average} = \rho\, g\, R_h\, S_o = \left(1.94\frac{slug}{ft^3}\right)\left(32.2\frac{ft}{s^2}\right)(1.747\,ft)\left(\frac{1}{800}\right) \approx \mathbf{0.136}\ \frac{lbf}{ft^2} \quad Ans.(b)$$

An alternate solution would use friction factor f from Eq. (10.15), for $\varepsilon = 0.12$ ft, Table 10.1:

$$f = [2\log(\frac{14.8(1.747\,ft)}{0.12\,ft}]^{-2} = 0.0459 \;;\; V = \frac{Q}{A} = \frac{69\,ft^3/s}{20\,ft^2} = 3.45\frac{ft}{s}$$

$$\text{Then}\quad \tau_{average} = \frac{f}{8}\rho V^2 = \frac{0.0459}{8}(1.94\frac{slug}{ft^3})(3.45\frac{ft}{s})^2 \approx \mathbf{0.133}\,\frac{lbf}{ft^2}\quad Ans.(b)$$

10.28 Show that, for any straight, prismatic channel in uniform flow, the average wall shear stress is given by

$$\tau_{avg} \approx \rho g R_h S_o$$

Use this result in Prob. 10.27 also.

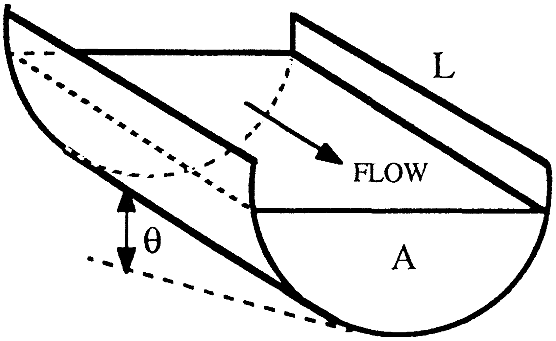


Fig. P10.28

Solution: For a control volume enclosing the fluid prism of length L as shown in the figure,

$$\sum F_{\text{along flow}} = W_{\text{fluid}}\sin\theta - \tau_{avg}A_{\text{wall}},\quad \text{or}\quad \tau_{avg}PL = \rho g A L\sin\theta$$

But $\sin\theta = S_o$ by definition, L cancels, leaving $\boldsymbol{\tau_{avg} = \rho\, g\, R_h \sin\theta}$ *Ans.*

10.29 Suppose that the trapezoidal channel of Fig. P10.17 contains sand and silt that we wish not to erode. According to an empirical correlation by A. Shields in 1936, the average wall shear stress τ_{crit} required to erode sand particles of diameter d_p is approximated by

$$\frac{\tau_{\text{crit}}}{(\rho - \rho)g\,d} \approx 0.5$$

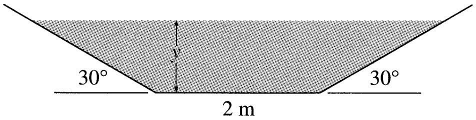


Fig. P10.17

where $\rho_s \approx 2400\ \text{kg/m}^3$ is the density of sand. If the slope of the channel in Fig. P10.17 is 1:900 and $n \approx 0.014$, determine the maximum water depth to keep from eroding particles of 1-mm diameter.

Solution: We relate the Shields critical shear stress to our result in Prob. 10.28 above:

$$\tau_{\text{crit}} = 0.5(2400 - 998)(9.81)(0.001\ \text{m}) \approx 6.88\ \text{Pa} = \rho g R_h S_o = (9790)R_h\left(\frac{1}{900}\right)$$

Solve for $R_{h.\text{crit}} \approx 0.632\ \text{m} = \frac{A}{P}$, where $A = by + y^2\cot 30°$ and $P = b + 2y\csc 30°$

By iteration (b = 2 m), we solve for water depth **y < 1.02 m** to avoid erosion. *Ans.*

10.30 A clay tile V-shaped channel, with an included angle of 90°, is 1 km long and is laid out on a 1:400 slope. When running at a depth of 2 m, the upstream end is suddenly closed while the lower end continues to drain. Assuming quasi-steady normal discharge, find the time for the channel depth to drop to 20 cm.

Fig. P10.30

Solution: We assume quasi-steady uniform flow at any instant. For a control volume enclosing the entire channel of length L = 1 km, we obtain

$$\frac{d}{dt}(m_{CV}) = -\dot{m}_{out}, \quad \text{or,} \quad \text{cancelling } \rho, \quad \frac{d}{dt}(LA) = -Q_{out} = -\frac{1}{\;}AR_h^{2/3}S_o^{1/2}$$

$$\text{For a Vee-channel,} \quad A = y^2\cot 45° \quad \text{and} \quad R_h = \frac{y}{2}\cos 45°, \quad L = 1000\text{ m}, \quad S_o = \frac{1}{400}$$

Clean this up, separate the variables, and integrate:

$$\int^{y}\frac{dy}{y^{5/3}} = -\text{const}\int^{t}dt, \quad \text{or} \quad y_o^{-2/3} - y^{-2/3} = -Ct, \quad \text{where } C = \frac{(1/n)(\cos 45°/2)^{2/3}S_o^{1/2}}{(2/3)L}$$

$$\text{or:} \quad C = 0.00119 \quad \text{for our case. Set } (2.0)^{-2/3} - (0.2)^{-2/3} = -0.00119t_{drain}$$

Solve for $\mathbf{t_{drain}}$ = 1927 sec ≈ **32 min** *Ans.*

P10.31 An unfinished-concrete 6-ft-diameter sewer pipe flows half full. What is the appropriate slope to deliver 50,000 gal/min of water in uniform flow?

Solution: For unfinished concrete, from Table 10.1, $n \approx 0.022$. For a half-full circle,

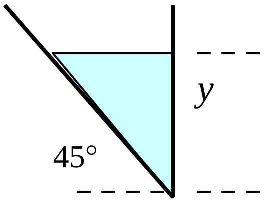
$$A = \frac{\pi}{2}R^2 = \frac{\pi}{2}(3\,ft)^2 = 14.14\,ft^2\,;\; P = \pi R = \pi(3\,ft) = 9.43\,ft\,;\; R_h = \frac{A}{P} = 1.50\,ft$$

$$Q = 50{,}000\frac{gal}{min} = 111.4\frac{ft^3}{s} = \frac{1.486}{n}AR_h^{2/3}\sqrt{S_o} = (\frac{1.486}{0.022})(14.14\,ft^2)(1.50\,ft)^{2/3}\sqrt{S_o}$$

Solve for slope $S_o \approx$ **0.0032** *Ans.*

P10.32 Does half a V-shaped channel perform as well as a full V-shaped channel? The answer to Prob. 10.18 is Q = 12.4 m³/s for y = 2m. (Do not reveal this to your friends still working on P10.18.) For the painted-steel half-V in Fig. P10.32, at the same slope of 3:1000, find the flow area that gives the same Q and compare with P10.18.

Solution: For painted steel, from Table 10.1, $n = 0.014$.
The properties of the half-V are easy:



$$P = y\sqrt{2} + y = 2.414y$$
$$A = 0.5y^2$$
$$R_h = A/P = 0.207y$$

The unknown is the depth y:

$$Q = 12.4\frac{m^3}{s} = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{0.014}(0.5y^2)(0.207y)^{2/3}\sqrt{0.003}$$

Solve for $y^{8/3} = 18.12$, $y = 2.96m$, $A = (0.5)(2.96m)^2 = \mathbf{4.39}m^2$ *Ans.*

The area in P10.18 was 4 m^2. This half-V area is **10% larger**, not as efficient.

10.33 Five 2-m-diameter clay tile sewer pipes, each running normal and half-full on a slope of 0.25°, empty into a single asphalt pipe, also laid out at 0.25°. If the large pipe is also to run half-full and normal, what should be its diameter?

Solution: First compute the small-pipe flow rate. For clay tile, $n = 0.014$.

$$Q_{small} = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{0.014}[\frac{\pi}{2}(1m)^2](\frac{1m}{2})^{2/3}\sqrt{\sin(0.25^o)} = 4.67\frac{m^3}{s}$$

For asphalt, take $n \approx 0.016$. The large-pipe radius R is unknown:

$$Q = 5Q_{small} = 5(4.67)\ \frac{m^3}{s} = \frac{1}{0.016}\left(\frac{\pi}{2}R^2\right)\left(\frac{R}{2}\right)^{2/3}\sqrt{\sin 0.25^\circ},\quad \text{solve } R \approx 1.92\text{ m}$$

10.34 A brick rectangular channel, with a slope of 0.002, is designed to carry 230 ft^3/s of water in uniform flow. There is an argument over whether the channel width should be 4 ft or 8 ft. Which design needs fewer bricks? By what percentage?

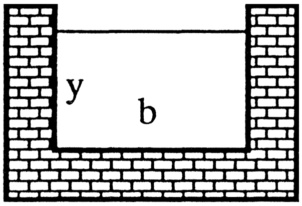


Fig. P10.34

Solution: For brick, take $n \approx 0.015$. For both designs, $A = by$ and $P = b + 2y$. Thus

$$Q = 230\frac{ft^3}{s} = \frac{1.486}{0.015}(by)\left(\frac{by}{b+2y}\right)^{2/3}(0.002)^{1/2}$$

(a) If b = **4** ft, solve for $y \approx 9.31$ ft or perimeter $P \approx$ **22.62 ft** *Ans.* (a)

(b) If b = **8** ft, solve $y \approx 4.07$ ft or $P \approx$ **16.14 ft** *Ans.* (b)

For a given channel-wall thickness, the number of bricks is proportional to the *perimeter*. Thus the 8-ft-wide channel has 16.14/22.62 = 71% as many, or **29% fewer bricks**.

10.35 In flood stage a natural channel often consists of a deep main channel plus two floodplains, as in Fig. P10.35. The floodplains are often shallow and rough. If the channel has the same slope everywhere, how would you analyze this situation for the discharge? Suppose that $y_1 = 20$ ft, $y_2 = 5$ ft, $b_1 = 40$ ft, $b_2 = 100$ ft, $n_1 = 0.020$, $n_2 = 0.040$, with a slope of 0.0002. Estimate the discharge in ft^3/s.

Fig. P10.35

Solution: We compute the flow rate in three pieces, with the dashed lines in the figure above serving as "water walls" which are not counted as part of the perimeter:

$$\text{(a) Deep channel: } Q_1 = \frac{1.486}{0.02}(25\times 40)\left(\frac{25\times 40}{20+40+20}\right)^{2/3}(0.0002)^{1/2} \approx 5659\ ft^3/s$$

$$\text{(b) Flood plains: } 2Q_2 = 2\left(\frac{1.486}{0.04}\right)(5\times 100)\left(\frac{5\times 100}{5+100+0}\right)^{2/3}(0.0002)^{1/2} \approx 1487\ ft^3/s$$

$$\text{Total discharge } \quad \mathbf{Q} = Q_1 + 2Q_2 = \mathbf{7150\ ft^3/s} \quad \textit{Ans.}$$

10.36 The Blackstone River in northern Rhode Island normally flows at about 25 m^3/s and resembles Fig. P10.35 with a clean-earth center channel, $b_1 \approx 20$ m and $y_1 \approx 3$ m. The bed slope is about 2 ft/mi. The sides are heavy brush with $b_2 \approx 150$ m. During hurricane Carol in 1955, a record flow rate of 1000 m^3/s was estimated. Use this information to estimate the maximum flood depth y_2 during this event.

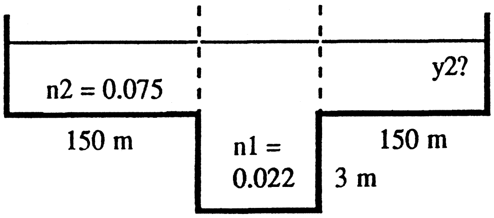


Fig. P10.36

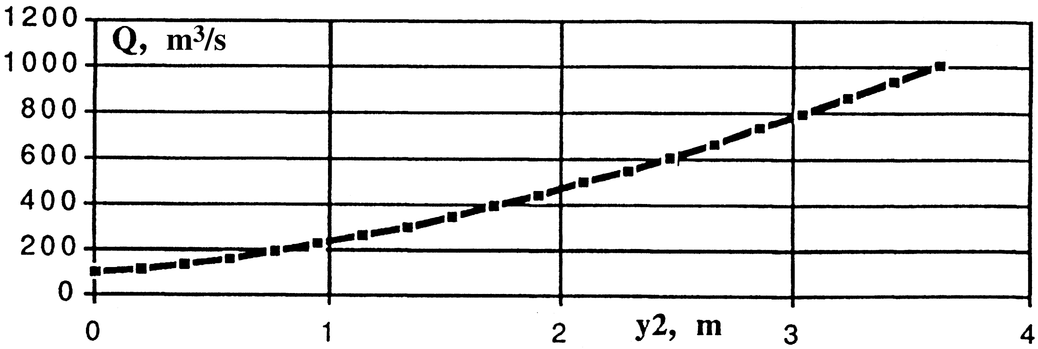
Solution: For heavy brush, $n_2 = 0.075$ and for clean earth, $n_1 = 0.022$, as shown in the figure. Use the same “zero-perimeter water-wall” scheme as in Prob. 10.35:

$$Q = Q_1 + 2Q_2 = \frac{1}{0.022} A_1 R_{h1}^{2/3} S_o^{1/2} + 2\frac{1}{0.075} A_2 R_{h2}^{2/3} S_o^{1/2} = \left(1000\ \frac{m^3}{s}\right)$$

$$\text{where} \quad S_o = \frac{2}{5280}, \quad A_1 = (3+y_2)20, \quad R_{h1} = \frac{A_1}{6+20}, \quad A_2 = 150y_2, \quad \text{and} \quad R_{h2} = \frac{A_2}{y_2+150}$$

Solve by iteration for $\mathbf{y_2 \approx 3.6\ m}$. *Ans.*

This heavy rainfall overflowed the flood plains and was the worst in Rhode Island history. A graph of flow rate versus flood-plain depth y_2 is shown below.



10.37 A triangular channel (see Fig. E10.6) is to be constructed of corrugated metal and will carry 8 m^3/s on a slope of 0.005. The supply of sheet metal is limited, so the engineers want to minimize the channel surface. What is (a) the best included angle θ for the channel; (b) the normal depth for part (a); and (c) the wetted perimeter for part (b).

Solution: For corrugated metal, take n = 0.022. From Ex. 10.5, for a vee-channel, recall that

Fig. P10.37

$$A = y^2\tan(\theta/2); \quad P = 2y\sec(\theta/2);$$

$$R_h = 0.5y\sin(\theta/2)$$

Manning's formula (10.19) predicts that:

$$Q = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = 8\ \frac{\mathrm{m}^3}{\ } = \frac{1}{0.022}\left(y^2\tan\frac{\theta}{2}\right)\left(\frac{y}{2}\sin\frac{\theta}{2}\right)^{2/3}\sqrt{0.005}$$

(a) Eliminate y in terms of P and set dP/dθ = 0. The algebra is not too bad, and the result is:

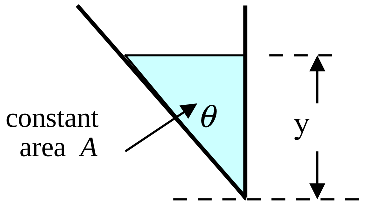
Minimum perimeter P occurs for a given flow rate at $\theta = \mathbf{90°}$ *Ans.* (a)

(b) Insert $\theta = 90°$ in the formula for $Q = 8$ m^3/s above and solve for:

$\boldsymbol{y} = \mathbf{1.83\ m}$ *Ans.* (b) and $\boldsymbol{P}_{\mathbf{min}} = \mathbf{5.16\ m}$. *Ans.* (c)

P10.38 For the half-Vee channel in Fig. P10.32, let the interior angle of the Vee be θ. For a given value of area, slope, and n, find the value of θ for which the flow rate is a maximum. To avoid cumbersome algebra, simply plot Q versus θ for constant A.

Solution: Write out Manning's formula and solve for y in terms of A and θ:

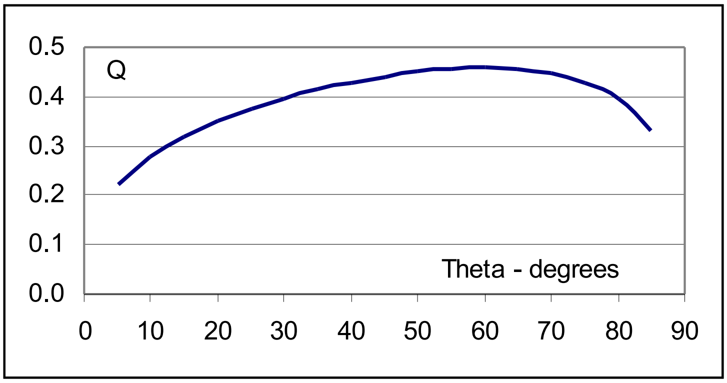


$$A = (1/2)\,y^2\tan\theta \ ; \quad P = y(1+\sec\theta)$$

$$Q = \frac{1}{n}A\left(\frac{A}{P}\right)^{2/3}\sqrt{S_o}\ , \quad y = \sqrt{\frac{2A}{\tan\theta}}$$

Holding A, n, and S_o constant, plot Q versus θ.

The writer simply took $[(1/n)A(S_o)^{1/2}]$ equal to unity. The resulting plot is as follows:



Maximum flow rate occurs at θ equal to 60 degrees. The curve is very flat in that region.

10.39 A trapezoidal channel has $n = 0.022$ and $S_0 = 0.0003$ and is made in the shape of a half-hexagon for maximum efficiency. What should the length of the side of the hexagon be if the channel is to carry 225 ft^3/s of water? What is the discharge of a semicircular channel of the same cross-sectional area and the same S_o and n?

Solution: The half-hexagon corresponds to Fig. 10.7 with $\theta = 60°$. Its properties are

$$A = \frac{3b^2}{2}\sin 60°, \quad R_h = \frac{b}{2}\sin 60°,$$

$$\text{or:} \quad b^{8/3} \approx 259, \quad \mathbf{b \approx 8.03\ ft} \quad \textit{Ans.} \quad (\text{for which } A_{hexagon} \approx 83.79\ \text{ft}^2)$$

A semicircular channel of the same area has $D = [8(83.79)/\pi]^{1/2} \approx 14.6$ ft. Its hydraulic radius and flow rate are

$$R_{h,semicircle} = D/4 \approx 3.65\ \text{ft}, \quad Q = \frac{1.486}{0.022}(83.8)(3.65)^{2/3}(0.0003)^{1/2}$$

10.40 Using the geometry of Fig. 10.6a, prove that the most efficient circular open channel (maximum hydraulic radius for a given flow area) is a semicircle.

Solution: Maximum hydraulic radius means minimum perimeter. Using Eq. 10.20,

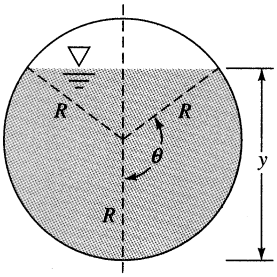


Fig. 10.6(a)

$$A = R^2\left[\theta - \frac{1}{2}\sin 2\theta\right],\quad P = 2R\theta,\ \text{eliminate R:}\quad P = \frac{2\theta\sqrt{A}}{\sqrt{\theta - \sin(2\theta)/2}}$$

$$\text{Differentiate:}\quad \left.\frac{dP}{d\theta}\right|_{\text{constant A}} = 0 \quad \text{if} \quad \boldsymbol{\theta = 90°} \quad \textit{Ans.}$$

10.41 Determine the most efficient value of θ for the vee-shaped channel of Fig. P10.41.

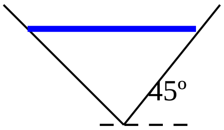
Fig. P10.41

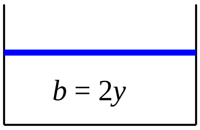
Solution: Given the (simple) geometric properties

$$A = y^2\cot\theta;\quad P = 2y\csc\theta;\quad \text{Eliminate y:}\ P = 2\csc\theta\sqrt{A\tan\theta}$$

$$\text{Set}\quad \left.\frac{dP}{d\theta}\right|_{\text{constant A}} = 0 \quad \text{if} \quad \boldsymbol{\theta = 45°} \quad \textit{Ans.}$$

P10.42 It is desired to deliver 30,000 gal/min of water in a brickwork channel laid on a slope of 1:100. Which would require fewer bricks, in uniform flow: (*a*) a V-channel with $\theta = 45°$, as in Fig. P10.41, or (*b*) an efficient rectangular channel with $b = 2y$?





Solution: For brickwork, from Table 10.1, $n = 0.015$. The answer may be surprising, but it is hard to think it through analytically, at least for this writer. So let's just work through the numbers:
For the vee-shaped channel,

$$\text{Vee-channel:} \quad A = y^2, \ P = 2\sqrt{2}\,y, \ R_h = \frac{y}{2\sqrt{2}}$$

$$Q = 30{,}000\frac{gal}{min} = 66.84\frac{ft^3}{s} = \frac{1.486}{0.015}(y^2)\left(\frac{y}{2\sqrt{2}}\right)^{2/3}\sqrt{0.01}, \quad \text{solve for} \quad y = 2.653\,ft$$

$$\text{Vee-channel perimeter} \quad P = 2\sqrt{2}\,(2.653\,ft) = \mathbf{7.50}\,ft \qquad Ans.(a)$$

(*b*) For the efficient rectangular channel,

$$\text{Rectangular channel:} \quad A = by = 2y^2, \ P = b + 2y = 4y, \ R_h = y/2$$

$$Q = 66.84 = \frac{1.486}{0.015}(2y^2)\left(\frac{y}{2}\right)^{2/3}\sqrt{0.01}, \ \text{solve} \ y = 1.876\,ft, \ P = 4y = \mathbf{7.50}\,ft \quad Ans.(b)$$

Same perimeter, same number of bricks! These two designs are equally successful.

10.43 What are the most efficient dimensions for a riveted-steel *rectangular* channel to carry 4.8 m^3/s of water at a slope of 1:900?

Solution: For riveted steel, take n ≈ 0.015. We know from Eq. (10.26) that

Best rectangle: $b = 2y$; $A = 2y^2$; $R_h = y/2$. So the flow rate is

$$Q = 4.8 = \frac{1}{0.015}(2y^2)(y/2)^{2/3}\left(\frac{1}{900}\right)^{1/2}, \quad \text{solve} \ \mathbf{y \approx 1.22\,m, b \approx 2.45\,m} \quad Ans.$$

10.44 What are the most efficient dimensions for a *half-hexagon* cast-iron channel to carry 15000 gal/min of water at a slope of 0.16°?

Solution: For cast iron, take $n \approx 0.013$. We know from Fig. 10.7 for a half-hexagon that

$$A = \frac{3b^2}{}\sin 60°, \quad R_h = \frac{b}{}\sin 60°, \quad \text{hence} \quad Q = \frac{15000}{}\ \frac{ft^3}{} = \frac{1.486}{} A R_h^{2/3}\sqrt{\sin 0.16°}$$

Solve for side length **b ≈ 2.12 ft** *Ans.*

10.45 The calculus tells us that the most efficient wall angle for a V-shaped channel (Fig. P10.41) is $\theta = 45°$. It yields the highest normal flow rate for a given area. But is this a sharp or a flat maximum? For a flow area of 1 m² and an unfinished-concrete channel of slope 0.004, plot the normal flow rate Q, in m³/s, versus angle for the range $30° \le \theta \le 60°$ and comment.

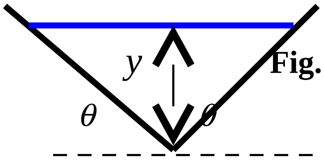


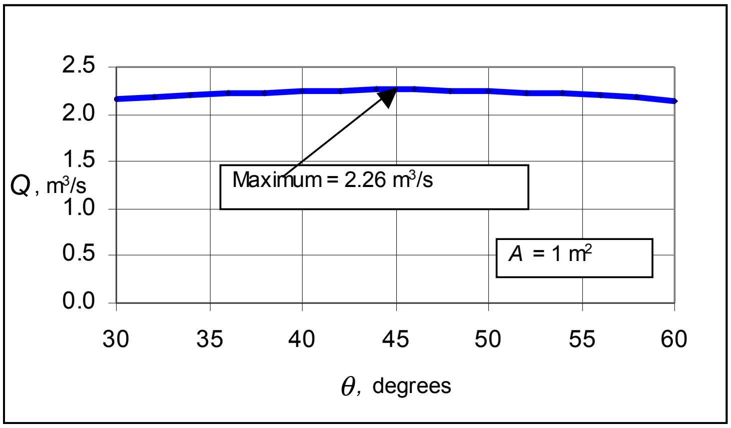
Fig. P10.45

Solution: The area is $A = y^2\cot(\theta)$, so, if $A = 1\ m^2$, the depth is $y = [1m^2(\tan\theta)]^{1/2}$. The wetted perimeter is $P = 2y\csc(\theta)$. The hydraulic radius is

$$R_h = \frac{A}{P} = \frac{1m^2\sin(\theta)}{2\sqrt{1m^2\tan\theta}}, \quad \text{and} \quad Q = \frac{1.0}{n} A R_h^{2/3} S_o^{1/2},$$

$$\text{or}: \quad Q(m^3/s) = \frac{1.0}{0.014}(1m^2)\left[\frac{\sin\theta}{2\sqrt{\tan\theta}}\right]^{2/3}(0.004)^{1/2}$$

Plot flow rate versus θ in the requested range and the result is a flat, flat curve:



Yes, 45° is the best, but the flow rate only varies 5% over the entire range 30° ≤ $\theta \le 60°$.

10.46 It is suggested that a channel which reduces erosion has a **parabolic shape**, as in Fig. P10.46. Formulas for area and perimeter of the parabolic cross-section are as follows [Ref. 7 of Chap. 10]:

Fig. P10.46

$$A = \frac{2}{3}bh_0; \quad P = \frac{b}{2}\left[\sqrt{1+\alpha^2} + \frac{1}{\alpha}\ln\left(\alpha + \sqrt{1+\alpha^2}\right)\right], \quad \text{where } \alpha = \frac{4h_0}{b}$$

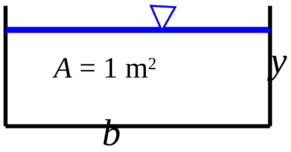
For uniform flow conditions, determine the most efficient ratio h_0/b for this channel (minimum perimeter for a given constant area).

Solution: We are to minimize *P* for constant *A*, and this time, unlike Prob. 10.37, the algebra is too heavy, what with logarithms and square roots, to solve for *P* in terms of *A* and h_0/b. The writer backed off and simply used a spreadsheet (or EES) to find the minimum *P* numerically. The answer is

Minimum *P* (maximum *Q* for a given *A*) occurs at $\mathbf{h_o/b \approx 0.486}$ *Ans.*

[NOTE: For a sine-wave shape, instead of a parabola, the answer is $h_0/b \equiv 1/2$.]

10.47 The calculus tells us that the most efficient water depth for a rectangular channel (such as Fig. E10.1) is $y/b = 1/2$. It yields the highest normal flow rate for a given area. But is this a sharp or a flat maximum? For a flow area of 1 m² and a clay tile channel with a slope of 0.006, plot the normal flow rate *Q*, in m³/s, versus y/b for the range $0.3 \le y/b \le 0.7$ and comment.



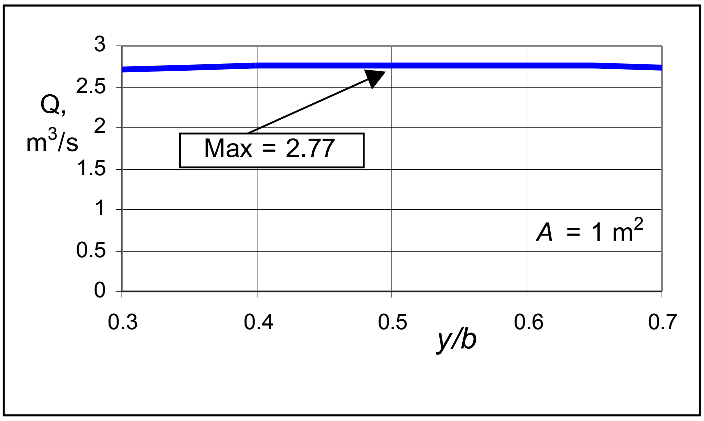
Solution: The area = 1 m² = (*y*)(*b*), so $y/b = A/b^2$.

For clay tile, from Table 10.1, $n = 0.014$. The flow rate is

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.014}(1m^2)[\frac{1}{b(1+2y/b)}]^{2/3}(0.006)^{1/2} \quad where \quad b = \sqrt{\frac{1m^2}{y/b}}$$

$$Example: \frac{y}{b} = 0.3: \; b = 1.825\,m, \; Q = \frac{1}{0.014}(1m^2)[\frac{1}{1.825+2(0.548)}]^{2/3}\sqrt{0.006} = 2.71\frac{m^3}{s}$$

Plot flow rate versus *y*/*b* in the requested range and the result is a flat, flat curve:



Yes, *y*/*b* = 0.5 is the best, but the flow rate only varies 2% over the entire range $0.3 \le y/b \le 0.7$.

10.48 A wide, clean-earth river has a flow rate $q = 150\ \text{ft}^3/(\text{s}\cdot\text{ft})$. What is the critical depth? If the actual depth is 12 ft, what is the Froude number of the river? Compute the critical slope by (a) Manning's formula and (b) the Moody chart.

Solution: For clean earth, take $n \approx 0.030$ and roughness $\varepsilon \approx 0.8$ ft. The critical depth is

$$y_c = (q^2/g)^{1/3} = [(150)^2/32.2]^{1/3} \approx \mathbf{8.87\ ft} \quad \textit{Ans.}$$

$$\text{If} \quad y_{actual} = 12\ \text{ft}, \quad Fr = \frac{V}{V} = \frac{q/y}{\sqrt{gy}} = \frac{150/12}{\sqrt{32.2(8.87)}} = \frac{12.5}{16.9} \approx \mathbf{0.739} \quad \textit{Ans.}$$

The critical slope is easy to compute by Manning and somewhat harder by the Moody chart:

$$\text{(a) Manning:}\quad S_c = \frac{gn}{\;^{1/3}} = \frac{32.2(0.030)}{2.208(8.87)^{1/3}} \approx \mathbf{0.00634}\ \text{Manning}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b) Moody:}\quad \frac{1}{\sqrt{f}} \approx -2\log_{10}\left(\frac{0.8}{3.7(4)(8.87)}+\right),$$

$$\text{or}\quad f \approx 0.0509,\quad S_c = \frac{f}{8} \approx \mathbf{0.00637}\ \text{Moody}\quad \textit{Ans.}\ \text{(b)}$$

10.49 Find the critical depth of the brick channel in Prob. 10.34 for both the 4- and 8-ft widths. Are the normal flows sub- or supercritical?

Fig. P10.49

Solution: For brick, take *n* ≈ 0.015. Recall and extend our results from Prob. 10.34:

$$\text{(a)}\ b = 4\ \text{ft:}\quad y_c = \left[\frac{Q}{\;}\right] = \left[\frac{(230)}{\;}\right] = \mathbf{4.68\ ft}\quad [y_n = 9.31\ \text{ft is } \mathbf{subcritical}]$$

$$\text{(b)}\ b = 8\ \text{ft:}\quad y_c = \left[\frac{(230)^2}{\;}\right]^{1/3} = \mathbf{2.95\ ft}\quad [y_n = 4.07\ \text{ft is } \mathbf{subcritical}]\quad \textit{Ans.}\ \text{(a, b)}$$

10.50 A pencil point piercing the surface of a rectangular channel flow creates a 25° half-angle wedgelike wave, as in Fig. P10.50. If the channel surface is painted steel and the depth is 35 cm, determine (a) the Froude no.; (b) the critical depth; and (c) the critical slope.

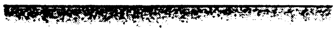


Fig. P10.50

Solution: For painted steel, take *n* ≈ 0.014. The wave angle and depth give

$$\text{Fr} = \csc(25^\circ) = \mathbf{2.37}\quad \textit{Ans.}\ \text{(a)}\quad \therefore\ V = \text{Fr}\,V_c = 2.37\sqrt{9.81(0.35)} = 4.38\ \frac{m}{\;}$$

$$\text{Flow rate}\quad q = Vy = 4.38(0.35) = 1.53\ \frac{ft^3}{\;},$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(1.53)^2}{9.81}\right]^{1/3} \approx \mathbf{0.62\ m} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Finally,}\quad S_c = \frac{gn^2}{\xi y_c^{1/3}} = \frac{9.81(0.014)^2}{(1.0)(0.62)^{1/3}} = \mathbf{0.0023} \qquad \textit{Ans.}(c)$$

10.51 An asphalt circular channel, of diameter 75 cm, is flowing half-full at an average velocity of 3.4 m/s. Estimate (a) the volume flow rate; (b) the Froude number; and (c) the critical slope.

Solution: For an asphalt channel, take n = 0.016. For a half-full channel, $A = \pi R^2/2$, $P = \pi R$, $R_h = R/2$, and $b_o = 2R$. The volume flow is easy, and Froude number and critical slope are not hard either:

$$Q = VA = \left(3.4\ \frac{m}{s}\right)[\pi(0.375\ m)^2/2] = \mathbf{0.75\ m^3/s} \quad \textit{Ans.}\ \text{(a)}$$

$$V_c = \sqrt{\frac{gA_c}{b_o}} = \sqrt{\frac{(9.81\ m/s^2)[\pi(0.375\ m)^2/2]}{0.75}} = 1.70\ \frac{m}{s}, \quad \mathbf{Fr} = \frac{3.4\ m/s}{1.7\ m/s} = \mathbf{2.00} \quad \textit{Ans.}\ \text{(b)}$$

$$S_c = \frac{n^2 V_c^2}{\xi^2 R_h^{4/3}} = \frac{(0.016)^2(1.7\ m/s)^2}{(1)^2(0.1875\ m)^{4/3}} = \mathbf{0.0069} \quad \textit{Ans.}\ \text{(c)}$$

10.52 Water flows full in an asphalt half-hexagon channel of bottom width W. The flow rate is 12 m^3/s. Estimate W if the Froude number is exactly 0.6.

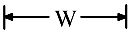


Fig. P10.52

Solution: For asphalt, n = 0.016, but we don't need n because *critical* flow is independent of roughness. Work out the properties of a half-hexagon:

$$y = W\sin 60°, \quad A = Wy + y^2\cot 60° = 1.299W^2, \quad b_o = W + 2W\cos 60° = 2W$$

$$V_c = \sqrt{\frac{gA}{b_o}} = \sqrt{\frac{9.81(1.299W^2)}{2W}} = 2.524W^{1/2}, \quad V = FrV_c = 0.6V_c = 1.515W^{1/2}$$

$$Q = 12\ \frac{m^3}{s} = AV = (1.299W^2)(1.515W^{1/2}) = 1.967W^{2.5}, \quad \text{solve } \mathbf{W = 2.06\ m} \quad \textit{Ans.}$$

10.53 For the river flow of Prob. 10.48, find the depth y_2 which has the same specific energy as the given depth $y_1 = 12$ ft. These are called *conjugate depths*. What is Fr_2?

Solution: Recall from Prob. 10.48 that the flow rate is $q = 150\ ft^3/(s\cdot ft)$. Hence

$$E = y_1 + \frac{V_1^2}{2g} = 12 + \frac{(150/12)^2}{2(32.2)} = 14.43\ ft \overset{?}{=} y_2 + \frac{V_2^2}{2g} = y_2 + \frac{(150/y_2)^2}{2(32.2)}$$

This is a cubic equation which has only one realistic solution: $\mathbf{y_2 \approx 6.74\ ft}$ *Ans.*

$$V_2 = \frac{150}{6.74} = 22.2\ \frac{ft}{s}, \quad Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{22.2}{\sqrt{32.2(6.74)}} \approx \mathbf{1.51} \quad \textit{Ans.} \quad (\text{compared to } Fr_1 = 0.64)$$

10.54 A clay tile V-shaped channel has an included angle of 70° and carries 8.5 m^3/s. Compute (a) the critical depth, (b) the critical velocity, and (c) the critical slope for uniform flow.

Fig. P10.54

Solution: For clay tile, take $n \approx 0.014$. The cross-section properties are

$$P = 2y\csc 55°; \quad A = y^2\cot 55°; \quad R_h = \frac{y}{2}\cos 55°; \quad b_o = 2y\cot 55°$$

$$A_c = \left(\frac{b_oQ^2}{g}\right)^{1/3} = \left[\frac{2y_c(\cot 55°)(8.5)^2}{9.81}\right]^{1/3} = y_c^2\cot 55°, \quad \text{solve for } \mathbf{y_c \approx 1.975\ m} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Compute} \quad A_c = 2.731\ m^2, \quad b_{oc} = 2.766\ m, \quad V_c = \left[\frac{9.81(2.731)}{2.766}\right]^{1/2} \approx \mathbf{3.11\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Compute} \quad R_h = 0.566\ m, \quad S_c = \frac{n^2gA_c}{\alpha^2 b_o R_h^{4/3}} = \frac{(0.014)^2(9.81)(2.731)}{(1)(2.766)(0.566)^{4/3}} \approx \mathbf{0.00405} \quad \textit{Ans.}\text{ (c)}$$

---

b = 1 m

Fig. P10.55

10.55 A trapezoidal channel resembles Fig. 10.7 with b = 1 m and $\theta = 50°$. The water depth is 2 m and Q = 32 m^3/s. If you stick your fingernail in the surface, as in Fig. P10.50, what half-angle wave might appear?

Solution: The cross-section properties are

$$A = by + y^2 \cot 50° = 5.36\ m^2, \quad \text{hence} \ \ V = Q/A = \frac{32}{5.36} \approx 5.97\ m/s$$

$$b_o = b + 2y\cot 50° = 4.36\ m, \quad \text{hence} \ \ V_c = (gA/b_o)^{1/2} = \left[\frac{9.81(5.36)}{4.36}\right]^{1/2} = 3.47\ m/s$$

$$\text{Thus} \quad Fr = V/V_c = \frac{5.97}{3.47} = 1.72 = \csc\theta, \quad \text{or:} \quad \boldsymbol{\theta_{wave} \approx 35.5°} \quad \textit{Ans.}$$

The flow is definitely supercritical, and a 'fingernail wave' will indeed appear.

P10.56 A 4-ft-diameter finished-concrete sewer pipe is half full of water. (*a*) In the spirit of Fig. 10.4*a*, estimate the speed of propagation of a small-amplitude wave propagating along the channel. (*b*) If the water is flowing at 14,000 gal/min, calculate the Froude number.

Solution: For finished concrete, *n* = 0.012, but we don't need this number! (*a*) From the critical-flow analysis, Eq. (10.37*b*), the critical velocity is the desired speed of wave propagation:

$$V_c = c_{wave} = \sqrt{\frac{g\,A}{b_o}} = \sqrt{\frac{(32.2\,ft/s^2)[(\pi/2)(2\,ft)^2]}{4\,ft}} = \mathbf{7.11}\ \frac{ft}{s} \qquad \textit{Ans.}(a)$$

(*b*) Convert 14,000 gal/min ÷ 448.83 = 31.2 ft^3/s. Then the velocity and Froude number are

$$V = \frac{Q}{A} = \frac{31.2\,ft^3/s}{(\pi/2)(2\,ft)^2} = 4.96\frac{ft}{s}\ ; \quad Fr = \frac{V}{c} = \frac{4.96}{7.11} = \mathbf{0.70}\ \ \text{Subcritical} \quad \textit{Ans.}(b)$$

10.57 Consider the vee-shaped channel of arbitrary angle in Fig. P10.41. If the depth is *y*, (*a*) find an analytic expression for the propagation speed c_o of a small-disturbance wave along this channel. [HINT: Eliminate flow rate from the analyses in Sec. 10.4.] If $\theta = 45°$ and the depth is 1 m, determine (*b*) the propagation speed; and (*c*) the flow rate if the channel is running at a Froude number of 1/3.

Solution: The algebra is given in Ex.10.6:

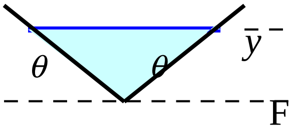


Fig. P10.57

$$A = y^2\cot\theta \quad ; \quad b_o = 2y\cot\theta$$

$$\text{Eq.(10.37a)}: \; gA^3 = b_oQ^2 \quad ; \quad \text{Solve} \quad Q = \left(\frac{gA^3}{b_o}\right)^{1/2} = \left(\frac{g\,y^5}{2\tan^2\theta}\right)^{1/2}$$

$$\text{Then} \quad V_c = c_o = \frac{Q}{A} = \frac{g^{1/2}y^{5/2}/(2^{1/2}\tan\theta)}{y^2\cot\theta} = \sqrt{\frac{gy}{2}} \qquad Ans.(a)$$

This answer may or may not surprise you. The propagation speed c_o is independent of θ.

(*b*) If y = 1 m, never mind θ or velocity, $c_o = [9.81\text{m/s}^2)(1\text{m})/2]^{1/2} = \mathbf{2.21\ m/s}$ *Ans*.(*b*)

If Fr = 1/3, then V = (1/3) c_o = 0.738 m/s. The flow rate for this condition is

$$Q = VA = (0.738\,m/s)[(1m)^2\cot(45^\circ)] = \mathbf{0.738\ m^3/s} \qquad Ans.(c)$$

P10.58 To follow up the half-full circular channel of Prob. P10.56, calculate (*a*) the critical slope; and (*b*) the actual slope, if the uniform flow rate is 14,000 gal/min.

Solution: For finished concrete, n = 0.012. (*a*) The critical slope follows from Eq. (10.38):

$$R = 2\,ft\ ;\ A = \frac{\pi}{2}R^2 = 6.28\,ft^2\ ;\ P = \pi R = 6.28\,ft\ ;\ R_h = \frac{A}{P} = 1.0\,ft\ ;\ b_o = 4\,ft$$

$$S_c = \frac{n^2\,g\,A}{\alpha^2\,b_o\,R_h^{4/3}} = \frac{(0.012)^2(32.2\,ft/s^2)(6.28ft^2)}{(1.486)^2(4\,ft)(1.0\,ft)^{4/3}} \approx \mathbf{0.0033} \qquad Ans.(a)$$

(*b*) The actual slope, at 14,000 gal/min, is found from the Manning equation:

$$Q = 14{,}000\,gpm = 31.2\frac{ft^3}{s} = \frac{1}{n}A R_h^{2/3}\sqrt{S_o} = (\frac{1.486}{0.012})(6.28ft^2)(1.0ft)^{2/3}\sqrt{S_o}$$

$$Solve\ for \qquad S_o \approx \mathbf{0.0016} \qquad Ans.(b)$$

10.59 Uniform water flow in a wide brick channel of slope 0.02° moves over a 10-cm bump as in Fig. P10.59. A slight depression in the water surface results. If the minimum depth over the bump is 50 cm, compute (a) the velocity over the bump; and (b) the flow rate per meter of width.

Fig. P10.59

Solution: For brickwork, take *n* ≈ 0.015. Since the water level decreases over the bump, the upstream flow is *subcritical*. For a wide channel, $R_h = y/2$, and Eq. 10.39 holds:

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{} = 0, \quad q = V_1 y_1, \quad E_2 = \frac{V_1^2}{} + y_1 - \Delta h, \quad \Delta h = 0.1\text{ m}, \quad y_2 = 0.5\text{ m}$$

$$\text{Meanwhile, for uniform flow,} \quad q = \frac{1}{0.015}y_1(y_1/2)^{2/3}\sqrt{\sin 0.02°} = 0.785 y_1^{5/3}$$

Solve these two simultaneously for $y_1 = 0.608$ m, $\mathbf{V_1 = 0.563}$ **m/s** *Ans.* (a), and $\mathbf{q = 0.342}$ $\mathbf{m^3/s\cdot m}$. *Ans.* (b) [The upstream flow is subcritical, $Fr_1 \approx 0.23$.]

P10.60 Water, flowing in a rectangular channel 2 m wide, encounters a bottom bump 10 cm high. The approach depth is 60 cm, and the flow rate 4.8 m³/s. Determine (*a*) the water depth, (*b*) velocity, and (*c*) Froude number above the bump. [HINT: The change in water depth is rather slight, only about 8 cm.]

Solution: First compute the approach velocity and Froude number:

$$V_1 = \frac{Q}{b_o y_1} = \frac{4.8}{(2.0)(0.6)} = 4.0\frac{m}{s}\ ; \quad Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{4.0}{\sqrt{9.81(0.6)}} = 1.65$$

The approach is *supercritical*. Hence we expect the water depth to *rise* as it passes over the bump. We are to solve the cubic equation, Eq. (10.39):

$$y_2^3 - E_2\, y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0\ ,\ E_2 = y_1 + \frac{V_1^2}{2g} - \Delta h = 0.6 + \frac{(4.0)^2}{2(9.81)} - 0.10 = 1.315\ ;$$

$$\frac{V_1^2 y_1^2}{2g} = \frac{(4)^2(0.6)^2}{2(9.81)} = 0.294 \quad ; \quad \text{Solve} \quad y_2^3 - 1.315\, y_2^2 + 0.294 = 0$$

The supercritical solution is

$$y_2 = \mathbf{0.679}\, m\ ;\ V_2 = Q/(b_o y_2) = \mathbf{3.53}\, m/s\ ;\ Fr_2 = V_2/\sqrt{g y_2} = \mathbf{1.37} \qquad Ans.(a,b,c)$$

The bump caused supercritical approach flow to ***rise*** about 8 cm, like the bottom leg of Fig. E10.7.

10.61 Modify Prob. 10.59 as follows: Again assuming uniform subcritical approach flow V_1, find (a) the flow rate *q*; and (b) the height y_2 for which the Froude number Fr_2 at the crest of the bump is exactly 1.0 (critical). Recall that $S_o = 0.02°$ and Δh = 10 cm.

Solution: The basic analysis above, for uniform upstream flow plus a bump, still holds:

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{2g} = 0, \quad q = V_1 y_1, \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h, \quad \Delta h = 0.1\ \text{m}$$

Meanwhile, for uniform flow, $q = \frac{1}{0.015} y_1 (y_1/2)^{2/3}\sqrt{\sin 0.02°} = 0.785 y_1^{5/3}$

This time, however, y_2 is unknown, and we need $Fr_2 = V_2/\sqrt{(g y_2)} = \mathbf{1.0}$. [At the crest in Prob. 10.60, $Fr_2 \approx 0.8$.] The iteration proceeds laboriously to the result:

$$Fr_2 = 1.0 \quad \text{if} \quad y_1 = 0.1916\ \text{m}; \quad V_1 = 0.261\ \frac{\text{m}}{};$$

$$\mathbf{y_2 \approx 0.0635\ m} \quad \textit{Ans.}\ (a); \quad \mathbf{q = 0.0500\ \frac{m}{}} \quad \textit{Ans.}\ (b)$$

[Finding the *critical* point is more difficult than finding a purely subcritical solution.]

10.62 Consider the flow in a wide channel over a bump, as in Fig. P10.62. One can estimate the water-depth change or *transition* with frictionless flow. Use continuity and the Bernoulli equation to show that

$$\frac{dy}{dx} = -\frac{dh/dx}{1 - V^2/(gy)}$$

Is the drawdown of the water surface realistic in Fig. P10.62? Explain under what conditions the surface might rise above its upstream position y_o.

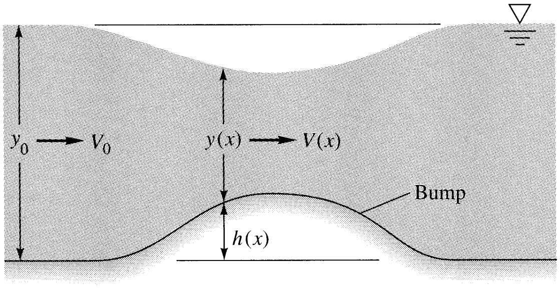


Fig. P10.62

Solution: This is a form of frictionless "gradually-varied" flow theory (Sect. 10.6). Use the frictionless energy equation from upstream to any point along the bump section:

$$\frac{p_{atm}}{} + \frac{V_o^2}{2g} + y_o = \frac{p_{atm}}{} + \frac{V^2}{2g} + h + y, \quad \text{differentiate: } 0 = \frac{VdV}{} + dy + dh;$$

$$\text{Continuity: } Vy = \text{ const} \quad , \quad \frac{dV}{V} = -\frac{dy}{y}$$

$$\text{Solve for: } \mathbf{\frac{dy}{dx} = -\frac{dh/dx}{1 - V^2/(gy)}} \quad \textit{Ans.}$$

Assuming dh/dx > 0 in front (a 'bump'), dy/dx will be positive (a *rise* in water level) if the flow is *supercritical* (i.e., Fr > 1 or $V^2 > gy$).

10.63 In Fig. P10.62, let $V_o = 1$ m/s and $y_o = 1$ m. If the maximum bump height is 15 cm, estimate (a) the Froude number over the top of the bump; and (b) the maximum depression in the water surface.

Solution: Here we don't need to differentiate, just apply Eq. 10.39 directly:

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2} = 0, \quad \text{where } E_2 = \frac{V_1^2}{2} + y_1 - h_{max} = \frac{(1)^2}{2(9.81)} + 1 - 0.15 = 0.901 \text{ m}$$

$$\text{Insert} \quad V_1 y_1 = 1.0 \quad \text{to get } y_2^3 - 0.901 y_2^2 + 0.051 = 0, \quad \text{solve for } y_2 \approx 0.826 \text{ m}$$

Thus the center depression is $\Delta\mathbf{z}$ = 1 – 0.15 – 0.826 ≈ **0.024 m**. *Ans.* (b)

Also, V_2 = 1.21 m/s. The bump Froude number is $Fr_2 = 1.21/[9.81(0.826)]^{1/2} \approx$ **0.425** *Ans.* (a).

P10.64 For the rectangular channel in Prob. P10.60, the Froude number over the bump is about 1.37, which is 17% less than the approach value. For the same entrance conditions, find the bump height *Δh* that causes the bump Froude number to be 1.00.

Solution: Recall that b_o = 2 m, Q = 4.8 m³/s, and y_1 = 60 cm. The bump height is the essential unknown in Eq. (10.39):

$$y_2^3 - E_2\, y_2^2 + \frac{V_1^2\, y_1^2}{2g} = 0\ ,\ E_2 = y_1 + \frac{V_1^2}{2g} - \Delta h = 0.6 + \frac{(4.0)^2}{2(9.81)} - \Delta h = 1.415 - \Delta h\ ;$$

$$\frac{V_1^2\, y_1^2}{2g} = \frac{(4)^2 (0.6)^2}{2(9.81)} = 0.294 \quad ; \quad \text{Solve} \quad y_2^3 - (1.415 - \Delta h)\, y_2^2 + 0.294 = 0$$

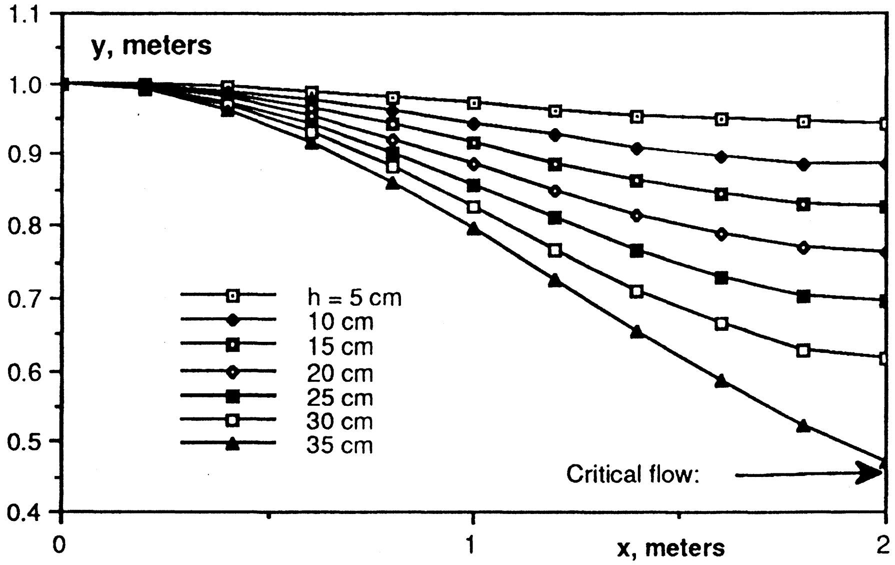
The object is to solve this cubic equation, varying Δ*h*, until the velocity $V_2 = Q/(b_o y_2)$ is such that $Fr_2 = 1$, that is, $V_2/(g y_2)^{1/2} = 1.0$. The problem statement is helpful, showing that a bump height Δ*h* = 10 cm results in $Fr_2 = 1.37$, so we need maybe 5 to 10 more centimeters of bump.

EES is excellent for this, of course, but the writer also was able to use an Excel spreadsheet that varied Δ*h* until $Fr_2 = 1$. The final result is:

$Fr_2 = 1$, y_2 = 0.837 m , V_2 = 2.87 m/s , Δ*h* = **15.94** cm *Ans.*

10.65 Program and solve the differential equation of "frictionless flow over a bump," from Prob. 10.62, for entrance conditions $V_o = 1$ m/s and $y_o = 1$ m. Let the bump have the convenient shape $h = 0.5h_{max}[1 - \cos(2\pi x/L)]$, which simulates Fig. P10.62. Let $L = 3$ m, and generate a numerical solution for $y(x)$ in the bump region $0 < x < L$. If you have time for only one case, use $h_{max} = 15$ cm (Prob. 10.63), for which the maximum Froude number is 0.425. If more time is available, it is instructive to examine a complete family of surface profiles for $h_{max} \approx 1$ cm up to 35 cm (which is the solution of Prob. 10.64).

Solution: We solve the differential equation $dy/dx = -(dh/dx)/[1 - V^2/(gy)]$, with $h = 0.5h_{max}[1 - \cos(2\pi x/L)]$, plus continuity, $Vy = 1\ m^2/s$, subject to initial conditions $V = 1.0$ and $y = 1.0$ at $x = 0$. The plotted water profiles for various bump heights are as follows:



The Froude numbers at the point of maximum bump height are as follows:

h_{max}, cm:	0	5	10	15	20	25	30	35
Fr_{bump}:	0.319	0.348	0.383	0.425	0.479	0.550	0.659	1.000

10.66 In Fig. P10.62 let $V_o = 6$ m/s and $y_o = 1$ m. If the maximum bump height is 35 cm, estimate (a) the Froude number over the top of the bump; and (b) the maximum increase in the water-surface level.

Solution: This is a straightforward application of Eq. 10.39 for *supercritical* approach:

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{\;}} = \frac{6}{\sqrt{9.81(1)}} = 1.92 > 1. \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h = \frac{(6)^2}{2(9.81)} + 1 - 0.35 = 2.48\ \mathrm{m}$$

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2} = y_2^3 - 2.48 y_2^2 + 1.835 = 0; \quad \text{solve } y_2 \approx \mathbf{1.19\ m} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Then} \quad V_2 = 6(1)/1.19 = 5.03\ \mathrm{m/s} \quad \text{and} \quad \mathrm{Fr}_2 = V_2/\sqrt{(g y_2)} \approx \mathbf{1.47} \quad \textit{Ans.}\ \text{(a)}$$

P10.67 Modify Prob. P10.63 so that the 15-cm change in bottom level is a *depression*, not a bump. Estimate (*a*) the Froude number above the depression; and (*b*) the maximum change in water depth.

Solution: The upstream Froude number is $V_1/(gy_1)^{1/2} = 0.319$, subcritical. Use Eq. (10.39):

$$y_2^3 - E_2\, y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0\ , \ E_2 = y_1 + \frac{V_1^2}{2g} - \Delta h = 1.0 + \frac{(1.0)^2}{2(9.81)} - (-0.15m) = 1.201m$$

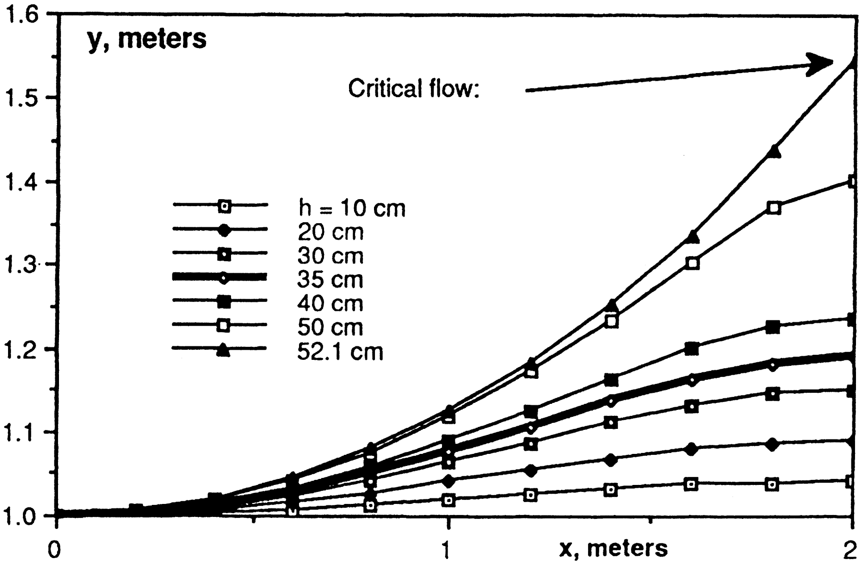
$$\frac{V_1^2 y_1^2}{2g} = \frac{(1.0)^2 (1.0)^2}{2(9.81)} = 0.051\ , \ \text{Solve} \ \ y_2^3 - (1.201) y_2^2 + 0.0501 = 0$$

$$\text{Solution}: \quad y_2 = 1.163\ m \ \ ; \ V_2 = 0.86 \frac{m}{s} \ \ ; \ Fr_2 = \mathbf{0.255} \ \ \textit{Ans.}(a)\ ; \ y_2 - y_1 = \mathbf{0.163}\, m \ \ \textit{Ans.}(b)$$

The bottom depression causes the water level to rise *higher* than the upstream flow.

10.68 Modify Prob. 10.65 to have a supercritical approach condition $V_o = 6$ m/s and $y_o = 1$ m. If you have time for only one case, use $h_{\max} = 35$ cm (Prob. 10.66), for which the maximum Froude number is 1.47. If more time is available, it is instructive to examine a complete family of surface profiles for 1 cm < $h_{\max}$ < 52 cm (which is the solution to Prob. 10.67).

Solution: This is quite similar to the subcritical display in Prob. 10.65. The new family of supercritical-flow profiles is shown below:



The Froude numbers at the point of maximum bump height are as follows:

h_{max}, cm:	0	10	20	30	35	40	50	52.1
Fr_{bump}:	1.92	1.80	1.68	1.55	1.47	1.39	1.15	1.000

10.69 Given is the flow of a channel of large width *b* under a sluice gate, as in Fig. P10.69. Assuming frictionless steady flow with negligible upstream kinetic energy, derive a formula for the dimensionless flow ratio $Q^2/(y_1^3 b^2 g)$ as a function of the ratio y_2/y_1. Show by differentiation that the maximum flow rate occurs at $y_2 = 2y_1/3$.

Fig. P10.69

Solution: With upstream kinetic energy neglected, the energy equation becomes

$$y_1 \approx y_2 + \frac{V_2^2}{2} = y_2 + \frac{(Q/by_2)^2}{2}; \quad \text{rearrange and multiply by } \left(y_2^2/y_1^3\right):$$

$$\frac{\mathbf{Q^2}}{\ {}^{2}\ {}^{3}} = \mathbf{2(y_2/y_1)^2 - 2(y_2/y_1)^3} \quad \textit{Ans.}$$

Differentiate this with respect to (y_2/y_1) to find maximum Q at **$y_2/y_1 = 2/3$** *Ans.*

10.70 The spectacular water release in the chapter-opener photo flows through a giant sluice gate. Assume the gate is 23 m wide with an opening 8 m high, with the water depth far upstream assumed equal to 32 m. Assuming free discharge, estimate the volume flow rate through the gate.

Solution: Let us proceed, although the writer has no firm idea if the sluice gate formula,

Eq. (10.41), is accurate for such large dimensions. Apply the formula anyway:

$$Q = C_d H b\sqrt{2gy_1}\ ,\quad C_d = \left(\frac{0.61}{\sqrt{1+0.61H/y_1}}\right) = \left(\frac{0.61}{\sqrt{1+0.61(8m)/32m}}\right) = 0.57$$

$$\text{or:}\qquad Q = (0.57)(8m)(23m)\sqrt{2(9.81m/s^2)(32m)} \approx \mathbf{2600\ \frac{m^3}{s}} \qquad \textit{Ans.}$$

This is the flow rate reported by the Internet site. The "data" were made up by the writer.

10.71 In Fig. P10.69 let y_1 = 95 cm and y_2 = 50 cm. Estimate the flow rate per unit width if the upstream kinetic energy is (a) neglected; and (b) included.

Solution: The result of Prob. 10.69 gives an excellent answer to part (a):

$$\text{Neglect } \frac{V_1^2}{}:\quad \frac{q^2}{} = \left(\frac{y_2}{}\right)^2 - \left(\frac{y_2}{}\right)^3 = \left(\frac{50}{}\right)^2 - \left(\frac{50}{}\right)^3 = 0.1312 = \frac{q^2}{}$$

$$\text{Solve for}\quad \mathbf{q \approx 1.49\ \frac{m^3}{s{\times}m}}\qquad \textit{Ans.}\ \text{(a)}$$

$$\text{(b) Exact:}\quad y_2^3 - \left(\frac{V_1^2}{2g} + y_1\right)y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0,$$

$$\text{or:}\quad V_1^2 = \frac{2gy_2^2(y_1 - y_2)}{} = \frac{2(9.81)(0.5)^2(0.95-0.5)}{} = 3.383,$$

10.72 Water approaches the wide sluice gate in the figure, at V_1 = 0.2 m/s and y_1 = 1 m. Accounting for upstream kinetic energy, estimate, at outlet section 2, (a) depth; (b) velocity; and (c) Froude number.

Fig. P10.72

Solution: (a) If we assume frictionless flow, the gap size is immaterial, and Eq. (10.40) applies:

EES yields 3 solutions: y = 1.0 m (trivial); 0.0442 m (impossible);

(b) From continuity, $V_2 = \frac{V_1 y_1}{y_2} = \frac{(1.0)(0.2)}{0.0462} = \mathbf{4.33\ \frac{m}{s}}$ *Ans.* (b)

(c) The Froude number is $Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{4.33}{\sqrt{9.81(0.0462)}} = \mathbf{6.43}$ *Ans.* (c)

P10.73 In Fig. P10.69, let y_1 = 6 ft and the gate width b = 8 ft. Find (*a*) the gate opening H that would allow a free-discharge flow of 30,000 gal/min; and (*b*) the depth y_2.

Solution: (*a*) The gate-opening problem is handled by Eq. (10.41):

$$Q = C_d H b\sqrt{2 g y_1}\ ,\quad where\quad C_d \approx \frac{0.61}{\sqrt{1+0.61H/y_1}}$$

Everything is known except H, and we know $C_d \approx 0.6$, so very little iteration is necessary:

$$Q = 30{,}000\frac{gal}{min} = 66.84\ \frac{ft^3}{3} = [\frac{0.61}{\sqrt{1+0.61H/6ft}}]H(8ft)\sqrt{2(32.2)(6ft)}$$

Solve for $H \approx \mathbf{0.72}\,ft$ *Ans.*(*a*) (*b*) The cubic polynomial, Eq. (10.40), enables us to solve for downstream water depth. First solve for $V_1 = Q/b/y_1 =$ (66.84)/[8(6)] = 1.393 ft/s. Then

$$y_2^3 - (y_1 + V_1^2/2g)\,y_2^2 + V_1^2 y_1^2/2g = 0\,, \quad or\ \ y_2^3 - (6.03)\,y_2^2 + 1.084 = 0$$

$$\text{Solve for the supercritical result:}\ \ y_2 \approx \mathbf{0.44}\,ft \qquad (Fr_2 \approx 5.03) \qquad Ans.(b)$$

10.74 With respect to Fig. P10.69, show that, for frictionless flow, the upstream velocity may be related to the water levels by

$$V_1 = \sqrt{\frac{2g(y_1 - y_2)}{K^2 - 1}} \qquad \text{where } K = y_1/y_2.$$

Solution: We have already shown this beautifully in Prob. 10.71*b*:

$$\text{Eq. 10.40:}\quad y_2^3 - \left(y_1 + V_1^2/2g\right) y_2^2 + (V_1 y_1)^2/2g = 0;$$

10.75 A tank of water 1 m deep, 3 m long, and 4 m wide into the paper has a closed sluice gate on the right side, as in Fig. P10.75. At $t = 0$ the gate is opened to a gap of 10 cm. Assuming quasi-steady sluice-gate theory, estimate the time required for the water level to drop to 50 cm. Assume free outflow.

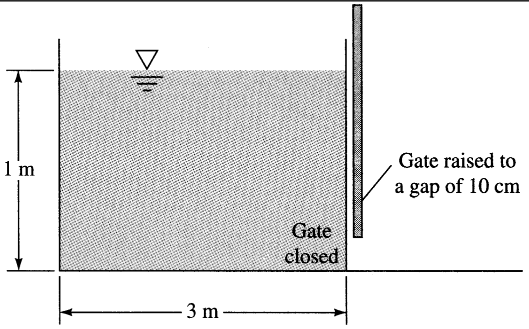


Fig. P10.75

Solution: Use a control volume surrounding the tank with Eq. 10.41 for the gate flow:

$$\frac{d}{dt}(\text{tank water}) = \frac{d}{dt}(bLy_1) = -Q_{out} = -C_d Hb\sqrt{2gy_1},\ \ C_d \approx 0.61/\sqrt{1 + 0.61H/y_1}$$

Because y_1 drops from 1.0 to 0.5 m, C_d also drops slowly from 0.592 to 0.576. Assume approximately constant $C_d \approx$ **0.584**, separate the variables and integrate:

$$\int^{y_1} \frac{dy_1}{\sqrt{y_1}} = -\frac{H}{L} C_d \sqrt{2g} \int^{t} dt, \quad \text{or:} \quad y_1 \approx \left(y_o^{1/2} - Kt\right)^2, \ y_o = 1\text{ m} \quad \text{and} \quad K = \frac{C_d H \sqrt{2g}}{2L}$$

$$K = \frac{0.584(0.1)\sqrt{2(9.81)}}{2(3.0)} = 0.0431. \quad \text{Set } y_1 = 0.5\text{ m}, \quad \text{solve for } \mathbf{t \approx 6.8\ s} \quad \textit{Ans.}$$

10.76 Figure P10.76 shows a horizontal flow of water through a sluice gate, a hydraulic jump, and over a 6-ft sharp-crested weir. Channel, gate, jump, and weir are all 8 ft wide unfinished concrete. Determine (*a*) the flow rate in cubic feet per second, and (*b*) the normal depth.

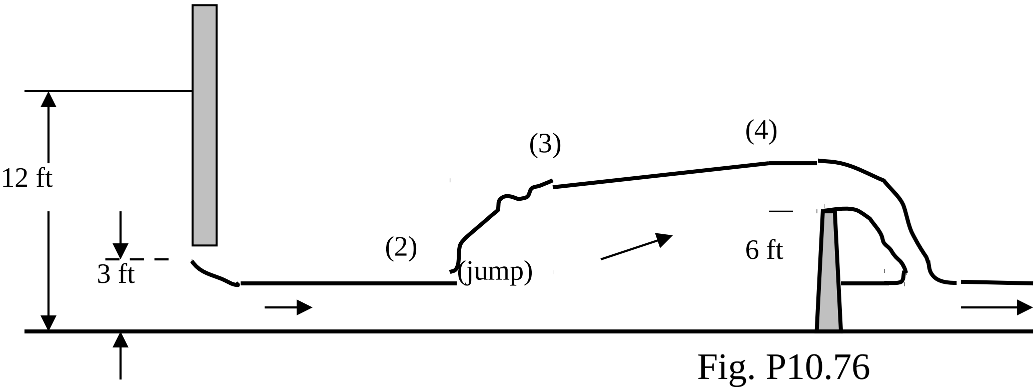


Fig. P10.76

Solution: This problem is concerned only with the sluice gate. (Other questions will be asked later.) (*a*) The flow rate follows from the sluice gate correlation, Eq. (10.41):

$$Q = C_d H b \sqrt{2 g y_1}\ , \quad C_d = \left(\frac{0.61}{\sqrt{1 + 0.61 H_1 / y_1}}\right) = \left(\frac{0.61}{\sqrt{1 + 0.61(3ft)/12ft}}\right) = 0.568$$

$$\text{or :} \qquad Q = (0.568)(3ft)(8ft)\sqrt{2(32.2ft/s^2)(12ft)} \approx \mathbf{379\ \frac{ft^3}{s}} \qquad \textit{Ans.}(a)$$

(*b*) Find the normal depth. This is a trick question! Ordinarily, if you know the flow rate, you can find the normal depth. But this channel is *horizontal*! **It has no normal depth**. *Ans.*(*b*)

P10.77 Equation (10.41) for sluice gate discharge is for free outflow. If the outflow is *drowned*, as in Fig. 10.10*c*, there is dissipation, and C_d drops sharply, as shown in Fig. P10.77, taken from Ref. 2. Use this data to restudy Prob. 10.73, with H = 9 inches. Plot the estimated flow rate, in gal/min, versus y_2 in the range 0.5 ft < y_2 < 5 ft.

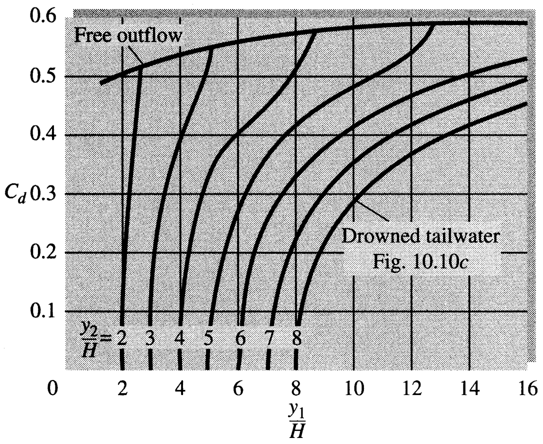
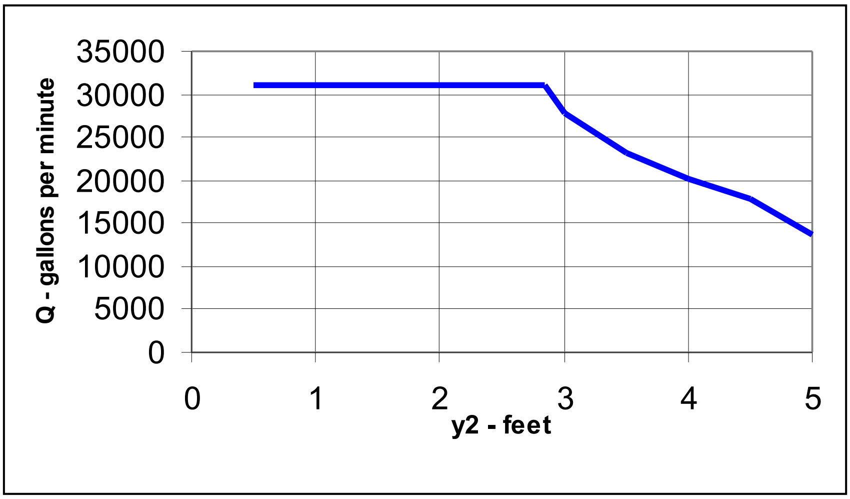


Fig. P10.77[*from Ref. 2 of Chap. 10*]

Solution: We pick all our discharge coefficients from the single vertical line y_1/H = 6 ft/0.75 ft = 8 in Fig. P10.77. We see that the flow is *not* drowned until $y_2/H > 3.8$, or $y_2 >$ 2.85 ft. Below that tailwater depth, $C_d \approx 0.58$. We can then move down the vertical line in Fig. P10.77, getting lower and lower values of C_d as y_2 rises. The formula is

$$Q = C_d H b\sqrt{2gy_1} = C_d\,(0.75ft)(8.0ft)\sqrt{2(32.2ft/s^2)(6ft)} = 117.9\,C_d \quad (ft^3/s)$$

The resulting plot is shown here:



P10.78 In Fig. P10.73, free discharge, a gate opening of 0.72 ft will allow a flow rate of 30,000 gal/min. Recall y_1 = 6 ft and the gate width b = 8 ft. Suppose that the gate is drowned (Fig. P10.77), with y_2 = 4 ft. What gate opening would then be required?

Solution: We need to find C_d, which varies considerably in Fig.P10.77, such that

$$Q = 30,000\frac{gal}{min} = 66.84\frac{ft^3}{s} = C_d\,H\,b\sqrt{2\,g\,y_1} = C_d\,H\,(8\,ft)\sqrt{2(32.2\,ft/s^2)(6\,ft)} = 157.3\,C_d\,H$$

We need to find the point in Fig. P10.77 where C_dH = 66.84/157.3 = 0.425. We already know that H = 0.72 ft is *not* drowned. So guess a little higher: H = 1 ft. Then y_1/H = 6, and y_2/H = 4. From the figure, read $C_d \approx 0.41$. Calculate this condition:

$$Q = 157.3\,C_d\,H \approx (157.3\,ft/s)(0.41)(1.0\,ft) \approx 64.5\frac{ft^3}{s} \quad \text{(only 3.5\% low)}$$

A slight upward adjustment matches $66.84\frac{ft^3}{s} = 30,000\frac{gal}{min}$: $H \approx \mathbf{1.01}\,ft$ *Ans.*

10.79 Show that the Froude number downstream of a hydraulic jump will be given by $Fr_2 = 8^{1/2}Fr_1/[(1+8Fr_1^2)^{1/2}-1]^{3/2}$. Does the formula remain correct if we reverse subscripts 1 and 2? Why?

Solution: Take the ratio of Froude numbers, use continuity, and eliminate y_2/y_1:

$$\frac{Fr_2}{\;} = \frac{V_1}{\;}\frac{\sqrt{gy_2}}{\;} = \frac{V_1}{\;}\sqrt{\frac{y_2}{\;}}, \quad \text{but } \frac{V_1}{\;} = \frac{y_2}{\;} \text{ from continuity, so } \frac{Fr_2}{\;} = \left(\frac{y_2}{\;}\right)^{3/2}$$

$$\text{From Eq. 10.43,} \quad \frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2}-1\right],$$

$$\mathbf{Fr_1\sqrt{8}}$$

The formula is indeed *symmetric*, you can reverse “1” and “2.” *Ans.*

10.80 Water, flowing in a channel at 30-cm depth, undergoes a hydraulic jump of dissipation 71%. Estimate (a) the downstream depth; and (b) the volume flow.

Solution: We use the jump and dissipation relations, Eqs. (10.43) and (10.45):

$$\frac{2y_2}{\;} = -1+\sqrt{1+8Fr_1^2}, \quad Fr_1^2 = \frac{V_1^2}{\;}, \quad h_f = \frac{(y_2-y_1)^3}{\;} = 0.71\left(y_1+\frac{V_1^2}{\;}\right), \quad y_1 = 0.3\text{ m}$$

Solve by EES: $V_1 = 16.1$ m/s; $Fr_1 = 9.38$; $y_2 =$ **3.83 m** *Ans.* (a);

q = **4.83 m³/s·m** *Ans.* (b)

10.81 Water flows in a wide channel at $q = 25$ ft³/s·ft and $y_1 = 1$ ft and undergoes a hydraulic jump. Compute y_2, V_2, Fr_2, h_f, the percentage dissipation, and the horsepower dissipated per unit width. What is the critical depth?

Solution: This is a series of straightforward calculations:

$$V_1 = \frac{q}{y_1} = \frac{25}{1} = 25\ \frac{ft}{s};\quad Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{25}{\sqrt{32.2(1)}} = 4.41;\quad E_1 = y_1 + \frac{V_1^2}{2g} \approx 10.7\ ft$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(4.41)^2} - 1\right] = 5.75,\quad \text{or}\quad \mathbf{y_2 \approx 5.75\ ft}\quad \textit{Ans.}\ (a)$$

$$V_2 = q/y_2 = 25/5.75 \approx \mathbf{4.35\ \frac{ft}{s}};\quad Fr_2 = \frac{4.35}{\sqrt{32.2(5.75)}} \approx \mathbf{0.32}\quad \textit{Ans.}\ (b, c)$$

$$h_f = (5.75-1)^3/[4(5.75)(1)] \approx 4.66\ ft,\quad \mathbf{\%\ dissipated} = 4.66/10.7 \approx \mathbf{44\%}\quad \textit{Ans.}\ (d)$$

$$\text{Power dissipated} = \rho g q h_f = (62.4)(25)(4.66) \div 550 \approx \mathbf{13.2\ hp/ft}\quad \textit{Ans.}\ (e)$$

$$\text{Critical depth}\quad y_c = (q^2/g)^{1/3} = [(25)^2/32.2]^{1/3} \approx \mathbf{2.69\ ft}\quad \textit{Ans.}\ (f)$$

10.82 Downstream of a wide hydraulic jump the flow is 4 ft deep and has a Froude number of 0.5. Estimate (a) y_1; (b) V_1; (c) Fr_1; (d) the percent dissipation; and (e) y_c.

Solution: As shown in Prob. 10.79, the hydraulic jump formulas are reversible. Thus, with Fr_1 known,

$$Fr_1 = \frac{Fr_2\sqrt{8}}{\left[\sqrt{1+8Fr_2^2} - 1\right]^{3/2}} = \frac{(0.5)\sqrt{8}}{\left[\sqrt{1+8(0.5)^2} - 1\right]^{3/2}} = \mathbf{2.26}\quad \textit{Ans.}\ (c)$$

$$2\frac{y_1}{y_2} = 2\left(\frac{y_1}{4\ ft}\right) = -1 + \sqrt{1+8(0.5)^2} = 0.732,\quad \mathbf{y_1 = 1.46\ ft}\quad \textit{Ans.}\ (a)$$

$$V_1 = Fr_1\sqrt{gy_1} = 2.26\sqrt{(32.2\ ft/s^2)(1.46\ ft)} = \mathbf{15.5\ ft/s}\quad \textit{Ans.}\ (b)$$

The percent dissipation follows from Eq. (10.45):

$$h_f = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(4.0\ ft - 1.46\ ft)^3}{4(4.0\ ft)(1.46\ ft)} = 0.696\ ft;\quad E_1 = y_1 + \frac{V_1^2}{2g} = 1.46\ ft + \frac{(15.5)^2}{2(32.2)} = 5.20\ ft$$

$$\text{Percent dissipation} = \frac{h_f}{E_1} = \frac{0.696\ ft}{5.20\ ft} = 0.13\quad \text{or}\quad \mathbf{13\%}\quad \textit{Ans.}\ (d)$$

Finally, the critical depth for a wide channel is given by Eq. (10.30):

$$q = V_1 y_1 = (15.5\text{ ft/s})(1.46\text{ ft}) = 22.7\text{ m}^2/\text{s};\quad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(22.7)^2}{32.2}\right]^{1/3} = \mathbf{2.52\ ft}\quad \textit{Ans.}\text{ (e)}$$

10.83 A wide channel flow undergoes a hydraulic jump from 40 cm to 140 cm. Estimate (a) V1; (b) V2; (c) the critical depth; and (d) the percent dissipation.

Solution: With the jump-height-ratio known, use Eq. 10.43:

$$\frac{2y_2}{y_1} = \frac{2(140)}{40} = 7 = \sqrt{1+8\mathrm{Fr}_1^2} - 1,\quad \text{solve for } \mathrm{Fr}_1 \approx 2.81$$

$$V_1 = \mathrm{Fr}_1\sqrt{gy_1} = 2.81\sqrt{9.81(0.4)} \approx \mathbf{5.56\ \frac{m}{s}}\quad \textit{Ans.}\text{ (a)};\quad V_2 = \frac{V_1 y_1}{y_2} \approx \mathbf{1.59\ \frac{m}{s}}\quad \textit{Ans.}\text{ (b)}$$

$$q = V_1 y_1 = 5.56(0.4) = 2.22\ \frac{\text{m}^3}{\text{s}},\quad y_{crit} = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(2.22)^2}{9.81}\right]^{1/3} \approx \mathbf{0.80\ m}\quad \textit{Ans.}\text{ (c)}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.4 + \frac{(5.56)^2}{2(9.81)} = 1.98\text{ m};\quad h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.4-0.4)^3}{4(0.4)(1.4)} = 0.45\text{ m}$$

$$\%\ \text{dissipation} = h_f/E_1 = 0.45/1.98 \approx \mathbf{23\%}\quad \textit{Ans.}\text{ (d)}$$

10.84 Consider the flow under the sluice gate of Fig. P10.84. If y_1 = 10 ft and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percentage of dissipation, and sketch the flow to scale with the EGL included. The channel is horizontal and wide.

Fig. P10.84

Solution: First get the conditions at "2" by assuming a frictionless acceleration:

$$E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{(2)^2}{2(32.2)} = 10.062\text{ ft} = E_2 = y_2 + \frac{V_2^2}{2g},\quad \text{Also, } V_1y_1 = V_2y_2 = 20$$

$$\text{Solve for}\quad V_2 \approx 24.4\ \text{ft/s};\ y_2 \approx \mathbf{0.820\ ft}\quad \textit{Ans.}\ (a)\qquad Fr_2 = \frac{24.4}{\sqrt{32.2(0.820)}} \approx 4.75$$

$$\text{Jump:}\quad \frac{y_3}{y_2} = \frac{1}{2}\left[\sqrt{1+8Fr_2^2} - 1\right] \approx 6.23,\quad y_3 \approx \mathbf{5.11\ ft}\quad \textit{Ans.}\ (b)$$

$$E_2 = 10.062\ \text{ft};\quad h_f = \frac{(y_3 - y_2)^3}{4y_2y_3} = \frac{(5.11-0.82)^3}{4(0.82)(5.11)} \approx 4.71\ \text{ft},$$

10.85 In Prob. 10.72 the exit velocity from the sluice gate is 4.33 m/s. If there is a hydraulic jump at "3" just downstream of section 2, determine the downstream (a) velocity; (b) depth; (c) Froude number; and (d) percent dissipation.

Solution: If $V_2 = 4.33$ m/s, then $y_2 = V_1y_1/V_2 = (1.0)(0.2)/4.33 = 0.0462$ m, and the Froude number is $Fr_2 = V_2/[gy_2]^{1/2} = 6.43$. Now use hydraulic jump theory:

$$\frac{2y_3}{y_2} = -1 + \sqrt{1+8(6.43)^2} = 17.2,\quad \text{or:}\quad \mathbf{y_3 = 0.398\ m}\quad \textit{Ans.}\ (b)$$

$$V_3 = \frac{q}{y_3} = \frac{(1.0)(0.2)}{0.398} = \mathbf{0.503}\ \frac{\mathbf{m}}{\mathbf{s}}\quad \textit{Ans.}\ (a)$$

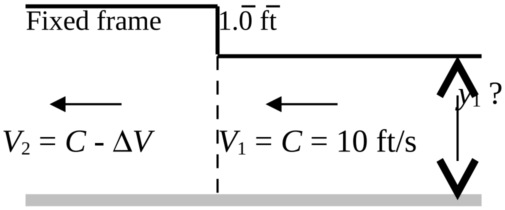
$$Fr_3 = \frac{V_3}{\sqrt{gy_3}} = \frac{0.503}{\sqrt{9.81(0.398)}} = \mathbf{0.255}\quad \textit{Ans.}\ (c)$$

$$h_f = \frac{(0.398-0.046)^3}{4(0.398)(0.046)} = 0.592\ \text{m};\quad \frac{h_f}{E_2} = \frac{0.592}{0.046 + (4.33)^2/2} = 0.59\quad \textit{or}\quad \mathbf{59\%}\quad \textit{Ans.}\ (d)$$

P10.86 A *bore* is a hydraulic jump that propagates upstream into a still or slower-moving fluid, as in Fig. P10.86, on the Sée-Sélune channel, near Mont Saint Michel in northwest France. The bore is moving at about 10 ft/s and is about one foot high. Estimate (*a*) the depth of the water in this area, and (*b*), the velocity induced by the wave.

Fig. P10.86

Fig. P10.86. Tidal bore on the Sée-Sélune river channel in northwest France.
[*Courtesy of Prof. Hubert Chanson, University of Queensland*]



Solution: The bore moves at speed C =10 ft/s and induces a velocity ΔV behind it. If viewed in a *frame fixed to the wave*, (*a*) The depth y_1 is unknown, but the bore height is known, 1.0 ft, thus $y_2 = y_1 + 1.0$, and Eq. (10.43) applies:

$$\frac{y_2}{y_1} = \frac{y_1 + 1.0\,ft}{y_1} = \frac{1}{2}[\sqrt{1+8Fr_1^2} - 1] = \frac{1}{2}[\sqrt{1+8(10\,ft/s)^2/(32.2y_1)} - 1]$$

$$\text{Iteration or EES will yield} \quad y_1 \approx \mathbf{1.18}\ ft \qquad Ans.(a)$$

Equation (10.9), with $\delta y = 1.0$ ft and $C = 10$ ft/s, will also iterate to give $y_1 = 1.18$ ft. The Froude number $Fr_1 = 1.62$, yes, supercritical.

(*b*) The induced velocity is substantial and is easy to find from continuity and the figure above:

$$V_2 = V_1\,y_1/y_2 = (10\,ft/s)(1.18\,ft/2.18\,ft) = 5.42\ ft/s\ ;\ \Delta V = C - V_2 = 10 - 5.42 = \mathbf{4.58}\,ft/s \quad Ans.(b)$$

10.87 A *tidal bore* may occur when the ocean tide enters an estuary against an oncoming river discharge, such as on the Severn River in England. Suppose that the tidal bore is 10 ft deep and propagates at 13 mi/h upstream into a river which is 7 ft deep. Estimate the river current V$_r$ in kn.

Fig. P10.87

Solution: Modify the analysis in 10.86 by superimposing a river velocity V$_r$ onto the flow. Then, as shown, the approach velocity is V$_1$ = C + V$_r$, where C = 13 mi/h = 19.06 ft/s. We may again use Eq. 10.43 to find the Froude number:

$$\frac{y_2}{y_1}=\frac{10}{7}=\frac{1}{2}\left[\sqrt{1+8Fr_1^2}-1\right],\quad \text{solve for } Fr_1=1.32$$

$$\text{Then}\quad V_1=C+V_r=Fr_1\sqrt{gy_1}=1.32\sqrt{32.2(7)}=19.77\text{ ft/s}=19.06+V_r$$

$$\text{Thus}\quad \mathbf{V_r}=0.71\text{ ft/s}\approx\mathbf{0.42\ knots}\quad \textit{Ans.}$$

10.88 For the situation in Fig. P10.84, suppose that at section 3 the depth is 2 m and the Froude number is 0.25. Estimate (a) the flow rate per meter of width; (b) y_c; (c) y_1; (d) the percent dissipation in the jump; and (e) the gap height *H* of the gate.

Solution: We have enough information to immediately calculate the flow rate:

$$Fr_3=0.25=\frac{V_3}{\sqrt{gy_3}}=\frac{V_3}{\sqrt{(9.81\text{ m/s}^2)(2\text{ m})}},$$

The critical velocity is $y_c=(q^2/g)^{1/3}=[(2.22\text{ m}^2/\text{s})^2/(9.81\text{ m/s}^2)]^{1/3}=\mathbf{0.794\ m}$. *Ans.* (b) We have to work our way back through the jump and sluice gate to find y_1:

$$\frac{2y_2}{y_3}=-1+\sqrt{1+8Fr_3^2}=\frac{2y_2}{(2\text{ m})}=-1+\sqrt{1+8(0.25)^2},\quad \text{solve } y_2=0.225\text{ m}$$

$$V_2=q/y_2=(2.22)/(0.225)=9.86\text{ m/s},\quad E_2=y_2+\frac{V_2^2}{2g}=0.225+\frac{(9.86)^2}{2(9.81)}=5.18\text{ m}$$

$$E_2=E_1=5.18\text{ m}=y_1+\frac{V_1^2}{2g},\quad q=V_1y_1=2.22\frac{\text{m}^3}{\text{s}},\quad \text{solve } \mathbf{y_1=5.17\ m}\quad \textit{Ans.}\text{ (c)}$$

The head loss in the jump leads to percent dissipation:

$$h_f = \frac{(y_3 - y_2)^3}{4y_3y_2} = \frac{(2\text{ m} - 0.225\text{ m})^3}{4(2\text{ m})(0.225\text{ m})} = 3.11\text{ m},$$

Finally, the gap height *H* follows from Eq. (10.41), assuming free discharge:

$$q = C_d H\sqrt{2gy_1} = 2.22\ \frac{\text{m}^3}{\text{s}\cdot\text{m}} = \left[\frac{0.61}{\sqrt{1+0.61H/5.17\text{ m}}}\right]H\sqrt{2(9.81)(5.17\text{ m})},$$

10.89 Water 30 cm deep is in uniform flow down a 1° unfinished-concrete slope when a hydraulic jump occurs, as in Fig. P10.89. If the channel is very wide, estimate the water depth y_2 downstream of the jump.

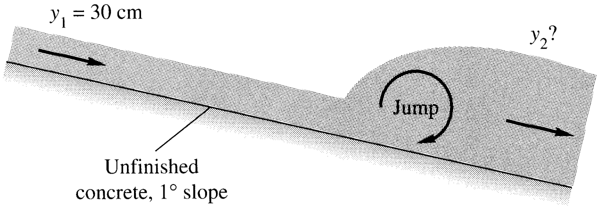


Fig. P10.89

Solution: For unfinished concrete, take $n \approx 0.014$. Compute the upstream velocity:

$$V_1 = \frac{1}{n}R_h^{2/3}S_o^{1/2} = \frac{1}{0.014}(0.3)^{2/3}(\sin 1°)^{1/2} \approx 4.23\ \frac{\text{m}}{\text{s}};\quad Fr_1 = \frac{4.23}{\sqrt{9.81(0.3)}} \approx 2.465$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(2.465)^2} - 1\right] \approx 3.02,\quad \mathbf{y_2} = 3.02(0.3) \approx \mathbf{0.91\ m}\quad \textit{Ans.}$$

10.90 For the gate/jump/weir system sketched earlier in Fig. P10.76, the flow rate was determined to be 379 ft³/s. Determine (*a*) the water depths y_2 and y_3, and (*b*) the Froude numbers Fr_2 and Fr_3 before and after the hydraulic jump.

Solution: With the flow rate known, we can compute V_1 and E_1 and equate them to section 2 values:

$$\frac{Q}{b} = \frac{379\, ft^3/s}{8\, ft} = 47.39\frac{ft^2}{s} = V_1 y_1 = V_1(12\, ft)\ , \quad V_1 = 3.95\frac{ft}{s}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 12\, ft + \frac{(3.95\, ft/s)^2}{2(32.2\, ft/s^2)} = 12.24\, ft$$

$$Thus \quad \frac{Q}{b} = V_2 y_2 = 47.39\frac{ft^2}{s} \quad and \quad E_2 = y_2 + \frac{V_2^2}{2g} = 12.24\, ft$$

As we did in Chapter 3 Bernoulli problems, we can solve the last two equations for

$$y_2 = \mathbf{1.83\, ft} \quad Ans.(a) \quad ; \quad V_2 = 25.9\frac{ft}{s} \quad ; \; Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{25.9}{\sqrt{32.2(1.83)}} = \mathbf{3.37} \quad Ans.(b)$$

As expected, the flow in section 2 is highly *supercritical*. Now use hydraulic jump theory:

$$\frac{2y_3}{y_2} = \frac{2y_3}{(1.83\, ft)} = -1 + \sqrt{1 + 8Fr_2^2} = -1 + \sqrt{1 + 8(3.37)^2} = 8.593; \; Solve \quad y_3 = \mathbf{7.86 ft} \quad Ans.(a)$$

$$V_3 = \frac{Q/b}{y_3} = \frac{47.39\, ft^2/s}{7.86\, ft} = 6.03\frac{ft}{s} \quad ; \quad Fr_3 = \frac{V_3}{\sqrt{g y_3}} = \frac{6.03}{\sqrt{32.2(7.86)}} = \mathbf{0.38} \quad Ans.(b)$$

Again as expected, the flow in section 3 is *subcritical*.

10.91 No doubt you used the horizontal-jump formula (10.43) to solve Probs. 10.89 and 10.90, which is reasonable since the slope is so small. However, Chow [ref. 3, p. 425] points out that hydraulic jumps are *higher* on sloped channels, due to “the weight of the fluid in the jump.” Make a control-volume sketch of a sloping jump to show why this is so. The sloped-jump chart given in Chow’s figure 15-20 may be approximated by the following curve fit:

$$\frac{2y_2}{y} \approx \left[\left(1+8\,\mathrm{Fr}_1^2\right)^{1/2}-1\right]e^{3.5S_0}$$

where $0 < S_o < 0.3$ are the channel slopes for which data are available. Use this correlation to modify your solution to Prob. 10.89. If time permits, make a graph of y_2/y_1 (≤ 20) versus Fr_1 (≤ 15) for various S_o (≤ 0.3).

Solution: Include the water weight in a control volume around the jump:

$$\sum F_x = \frac{\rho g}{2}y_1^2 b - \frac{\rho g}{2}y_2^2 b + W\sin\theta$$
$$-\dot{m}(V - V)$$

Clean this up and rearrange:

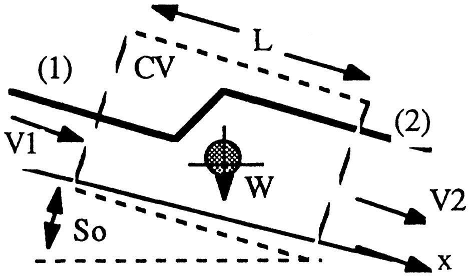


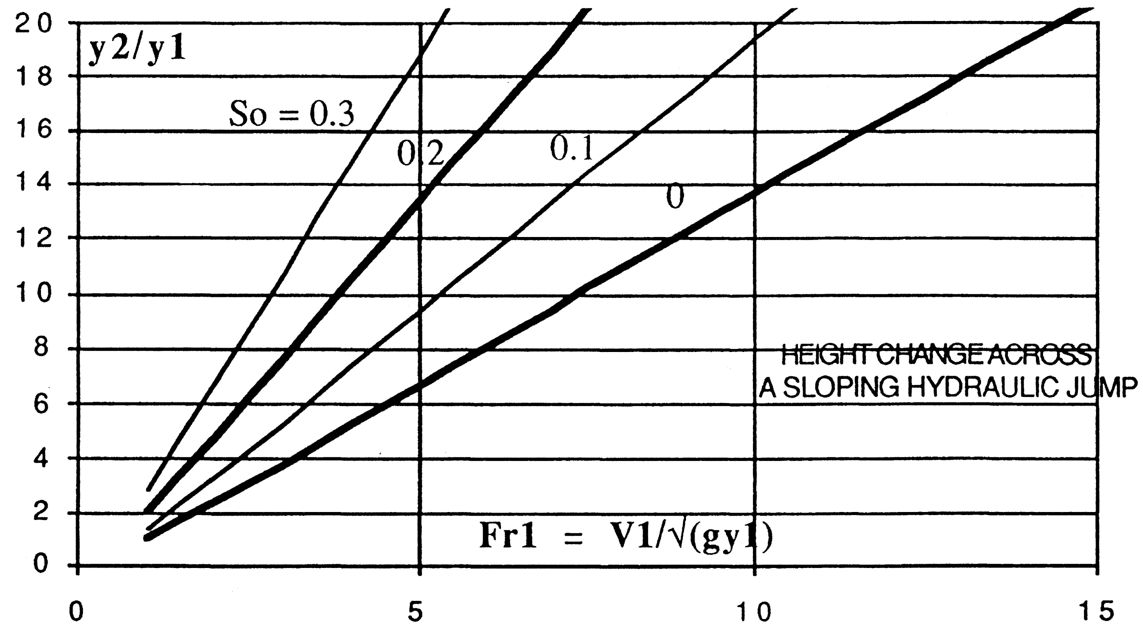
Fig. P10.91

$$r^3(1-KS_o) - r(\alpha+1) + \alpha = 0, \quad \text{where } r = \frac{y_2}{y}, \quad \alpha = 2\mathrm{Fr}_1^2, \quad K = \frac{L}{y} \approx 4 \quad \text{or so}$$

The solution to this cubic equation gives the jump height ratio *r*. If K = 0 (horizontal jump), the solution is Eq. 10.43. If K > 0 (sloping jump), the jump height increases roughly as Chow's formula predicts. There is only a slight change to the result of Prob. 10.89:

$$y_{2,\text{new}} \approx y_{2,\text{Prob.10.89}}e^{3.5S_o} \approx 0.91e^{3.5\sin 1^\circ} \approx \mathbf{0.97\ m} \quad \textit{Ans.}$$

A plot of Chow's equation is shown below.



P10.92 A familiar sight is the circular hydraulic jump formed by a faucet jet falling onto a flat sink surface, as in Fig. P10.92. Because of the shallow depths, this jump is strongly dependent on bottom friction, viscosity, and surface tension [Watson, 1964]. It is also unstable and can form remarkable non-circular shapes, as shown in the website http://web.mit.edu/jeffa/www/jump.htm. For this problem, assume that two-dimensional inviscid jump theory is valid. If the water depth outside the jump is 5 mm, the radius at which the jump appears is R = 3 cm, and the faucet flow rate is 100 cm^3/s, find the conditions just upstream of the jump.

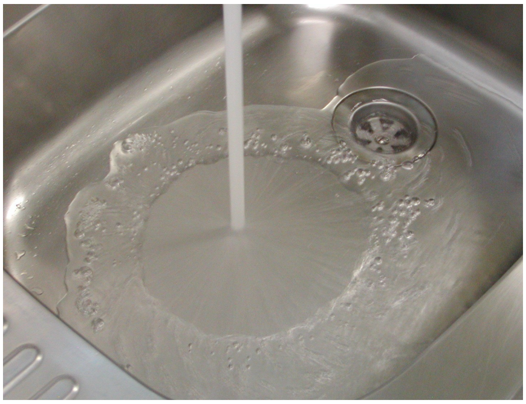


Figure P10.92 A circular hydraulic jump in a kitchen sink.
[*Courtesy of Prof. Hubert Chanson, University of Queensland*]

Solution: Compute the upstream Froude number, jump theory, and one-dimensional continuity:

$$Fr_1 = \frac{V_1}{\sqrt{g\,y_1}} > 1 \quad ; \quad \frac{2y_2}{y_1} = -1 + \sqrt{1+8Fr_1^2} \quad ; \quad V_1 = \frac{Q}{2\pi R\,y_1}$$

We are given $y_2 = 0.005$ m, $R = 0.03$ m, $Q = 0.0001$ m^3/s, and, of course, $g = 9.81$ m/s^2. The only unknowns are y_1 and V_1, plus we must ensure that $Fr_1 > 1$. Solve by EES or iteration:

y_1 = **1.71** mm ; V_1 = **0.310** m/s ; Fr_1 = **2.395** *Ans.*

10.93 Water in a horizontal channel accelerates smoothly over a bump and then undergoes a hydraulic jump, as in Fig. P10.93. If $y_1 = 1$ m and $y_3 = 40$ cm, estimate (a) V_1; (b) V_3; (c) y_4; and (d) the bump height h.

Solution: Assume frictionless flow except in the jump. From point 1 to point 3:

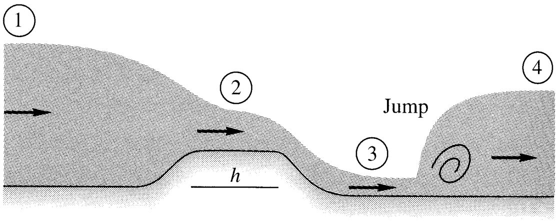


Fig. P10.93

$$\text{Energy: } E_1 = 1 + \frac{V_1}{2(9.81)} = E_3 = 0.4 + \frac{V_3}{2(9.81)}; \quad \text{Continuity: } V_1(1.0) = V_3(0.4)$$

Solve simultaneously by iteration for $V_1 \approx \mathbf{1.50}\ \frac{\mathbf{m}}{}$ *Ans.* (a) $V_3 = \mathbf{3.74}\ \frac{\mathbf{m}}{}$ *Ans.* (b)

The flow after the bump is supercritical: $Fr_3 = 3.74/\sqrt{[9.81(0.4)]} \approx \mathbf{1.89}$. For the jump,

$$\frac{y_4}{y} = \frac{1}{2}\left[\sqrt{1+8(1.89)^2} - 1\right] = 2.22, \quad y_4 = 2.22(0.4) \approx \mathbf{0.89\ m} \quad \textit{Ans. } (c)$$

Finally, use energy from “1” to “2” and note that the bump flow must be *critical*:

$$E_1 = 1.115 = E_2 = y_2 + \frac{V_2^2}{2} + h; \quad V_1 y_1 = 1.50 = V_2 y_2; \quad \text{and:} \quad V_2 = \sqrt{9.81 y_2}$$

Solve simultaneously for $y_2 \approx 0.61$ m; $V_2 \approx 2.45$ m/s; $\mathbf{h \approx 0.20\ m}$ *Ans.* (d)

P10.94 In Fig. 10.11, the upstream flow is only 2.65 cm deep. The channel is 50 cm wide, and the flow rate is 0.0359 m^3/s. Determine (*a*) the upstream Froude number; (*b*) the downstream velocity; (*c*) the downstream depth; and (*d*) the percent dissipation.

Solution: (*a*) We have enough information to find the upstream Froude number:

$$V_1 = \frac{Q}{b\,y_1} = \frac{0.0359\,m^3/s}{(0.5m)(0.0265m)} = 2.71\frac{m}{s}\ ; \ Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2.71}{\sqrt{9.81(.0265)}} = \mathbf{5.32}\ \ \textit{Ans.}(a)$$

(*b, c*) Jump theory, Eqs. (10.43,44) gives us the downstream depth and velocity:

$$\frac{y_2}{y_1} = \frac{1}{2}[(1+8\,Fr_1^2)^{0.5}-1] = \frac{1}{2}[(1+8\{5.32\}^2)^{0.5}-1] = 7.04\ ;\ y_2 = 7.04(0.0265m) = \mathbf{0.187}\,m \quad Ans.(c)$$

$$V_2 = \frac{V_1 y_1}{y_2} = \frac{(2.71)(0.0265)}{0.187} = \mathbf{0.385}\ \frac{m}{s} \quad Ans.(b)$$

(*d*) The head loss and percent dissipation follow from Eq. (10.45):

$$h_f = \frac{(y_2-y_1)^3}{4\,y_1\,y_2} = \frac{(0.187-0.0265)^3}{4(0.0265)(0.187)} = 0.208\,m$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.0265 + \frac{(2.71)^2}{2(9.81)} = 0.401\ m\ ;\ \text{Dissipation} = \frac{0.208}{0.401} = 0.52\ \text{or}\ \mathbf{52\%} \quad Ans.(d)$$

This is a strong, steady jump, similar to Fig. 10.12*d* of the text.

10.95 A 10-cm-high bump in a wide horizontal channel creates a hydraulic jump just upstream and the flow pattern in Fig. P10.95. Neglect losses except in the jump. If y_3 = 30 cm, estimate (a) V_4; (b) y_4; (c) V_1; and (d) y_1.

Bump: *h* = 10 cm

Fig. P10.95

Solution: Since section "2" is subcritical and "4" is supercritical, assume "3" is *critical*:

$$V_3 = \sqrt{gy_3} = \sqrt{9.81(0.3)} = 1.72\ \frac{m}{s},\quad \text{thus}\ \ q = V_i y_i\big|_{2.3.4} = 1.72(0.3) = 0.515\ m^3/s{\times}m$$

$$y_3 + \frac{V_3^2}{2g} + h = 0.3 + \frac{(1.72)^2}{2(9.81)} + 0.1 = 0.55\ m = E_4 = y_4 + \frac{V_4^2}{2(9.81)};\quad \text{and}\quad V_4 y_4 = 0.515$$

Solve for $y_4 \approx$ **0.195 m** *Ans.* (b); $V_4 \approx \mathbf{2.64}\ \frac{m}{s}$ *Ans.* (a)

Also, $E_2 = 0.55 = y_2 + \frac{V_2^2}{2g}$ and $V_2 y_2 = 0.515$, solve $y_2 \approx 0.495$ m, $V_2 \approx 1.04$ m/s

$$\mathrm{Fr}_2 = \frac{1.04}{\sqrt{9.81(0.495)}} \approx 0.472, \quad \text{Jump: } \frac{y_1}{y_2} = \frac{1}{2}\left|\sqrt{1+8(0.472)^2} - 1\right| = 0.334 = \frac{y_1}{0.495}$$

$$\text{Thus} \quad y_1 \approx \mathbf{0.165\ m} \quad \textit{Ans.}\ \text{(d)}; \quad V_1 = \frac{0.515}{0.165} \approx \mathbf{3.11\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(c)}$$

P10.96 For the circular hydraulic jump in Figure P10.92, let the water depths before and after the jump be 2 mm and 4 mm, respectively. Assume that two-dimensional jump theory is valid. If the faucet flow rate is 150 cm^3/s, estimate the radius *R* at which the jump will appear.

Solution: The proper radius *R* must yield the upstream and downstream velocities that satisfy two-dimensional jump theory:

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} > 1 \ ; \quad \frac{2y_2}{y_1} = -1 + \sqrt{1 + 8Fr_1^2} \ ; \quad V_1 = \frac{Q}{2\pi R\, y_1} \ ; \ V_2 = \frac{Q}{2\pi R\, y_2}$$

Given *Q* = 0.00015 m^3/s, y_1 = 0.002 m, and y_2 = 0.004 m. The only unknown is *R*, which then establishes the velocities. EES will solve the problem, but a direct solution is possible:

$$\frac{2y_2}{y_1} = \frac{2(0.004m)}{(0.002m)} = 4 = -1 + \sqrt{1+8Fr_1^2} \ , \quad \text{solve for} \quad Fr_1 = 1.732 = \frac{V_1}{\sqrt{9.81(0.002)}}$$

$$\text{Solve for } V = 0.243\frac{m}{s} = \frac{0.00015\,m^3/s}{2\pi R(0.002m)}, \quad \text{solve for} \quad R = \mathbf{0.0492\,m} \quad \textit{Ans.}$$

10.97 A brickwork rectangular channel 4 m wide is flowing at 8.0 m^3/s on a slope of 0.1°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 1.5 m; (c) 2 m?

Solution: For brickwork, take $n \approx 0.015$. Then, with $q = Q/b = 8/4 = 2.0$ m^3/s·m,

$$Q = 8\ \frac{m^3}{s} = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.015}(4y_n)\left(\frac{4y_n}{4+2y_n}\right)^{2/3} (\sin 0.1°)^{1/2},$$

whereas $y_c = (q^2/g)^{1/3} = [(2)^2/9.81]^{1/3} \approx 0.742$ m. Since $y_n > y_c$, **slope is mild** *Ans.*

All three of the given depths—1.0, 1.5, and 2.0 meters—are above y_n on Fig. 10.14*c*, hence **all three are on M-1 curves**. *Ans.*

10.98 A gravelly-earth wide channel is flowing at 10.0 m^3/s per meter on a slope of 0.75°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 2 m; (c) 3 m?

Solution: For gravelly earth, take *n* ≈ 0.025. Then, with $R_h = y$ itself,

$$q = 10\,\frac{m^3}{s} = \frac{1}{n}\,A\,R_h^{2/3}S_o^{1/2} = \frac{1}{0.025}(y_n)y_n^{2/3}(\sin 0.75^\circ)^{1/2}, \quad \text{solve for } y_n \approx 1.60\text{ m}$$

whereas $y_c = (q^2/g)^{1/3} = [(10)^2/9.81]^{1/3} \approx 2.17$ m. Since $y_c > y_n$, **slope is steep** *Ans.*

The three given depths fits nicely into the spaces between y_n and y_c in Fig. 10.14*a*:

$y = 1\text{ m} < y_n < y_c$: **S-3 curve** *Ans.* (a) $y_n < y = 2\text{ m} < y_c$: **S-2 curve** *Ans.* (b)

$y_n < y_c < y = 3\text{ m}$: **S-1 curve** *Ans.* (c)

10.99 A clay tile V-shaped channel, of included angle 60°, is flowing at 1.98 m^3/s on a slope of 0.33°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 2 m; (c) 3 m?

Solution: For clay tile, take *n* ≈ 0.014. For a 60° Vee-channel, from Example 10.5 of the text, $A = y^2\cot 60^\circ$ and $R_h = (y/2)\cos 60^\circ$. For uniform flow,

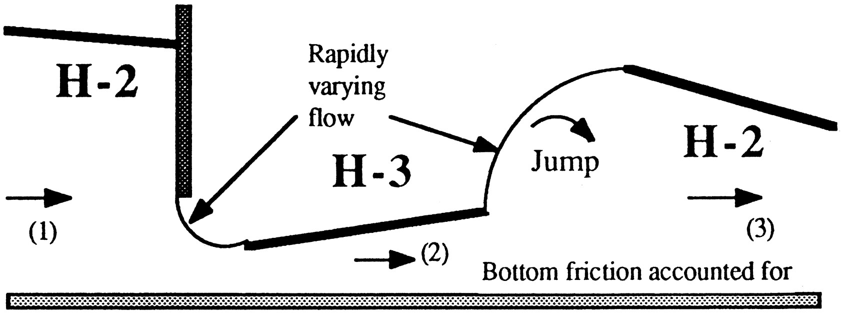
$$Q = 1.98 = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1}{0.014}(y^2\cot 60^\circ)\left(\frac{y}{2}\cos 60^\circ\right)^{2/3}(\sin 0.33^\circ)^{1/2},$$

solve for $y_n \approx 1.19$ m; whereas $y_c = \left(\frac{2Q^2}{g\tan^2 60^\circ}\right)^{1/5} \approx 1.19$ also. Slope is **critical**. *Ans.*

Fig. 10.14b: $y = 1\text{ m} < y_c$: **C-3 curve** *Ans.* (a); $y = 2 \text{ or } 3\text{ m} > y_c$:**C-1 curves** *Ans.* (b, c)

10.100 If bottom friction is included in the sluice-gate flow of Prob. 10.84, the depths (y_1, y_2, y_3) will vary with *x*. Sketch the type and shape of gradually-varied solution curve in each region (1,2,3) and show the regions of rapidly-varying flow.

Solution: The expected curves are all of the “H” (horizontal) type and are shown below:



Ans.

Fig. P10.100

10.101 Consider the gradual change from the profile beginning at point *a* in Fig. P10.101 on a mild slope S_{o1} to a *mild* but steeper slope S_{o2} downstream. Sketch and label the gradually-varied solution curve(s) *y*(x) expected.

Solution: There are two possible profiles, depending upon whether or not the initial M-2 profile slips below the new normal depth y_{n2}. These are shown on the next page:

steeper

Fig. P10.101

10.102 The wide channel flow in Fig. P10.102 changes from a steep slope to one even steeper. Beginning at points a and b, sketch and label the water surface profiles which are expected for gradually-varied flow.

Fig. P10.102

Solution: The point-*a* curve will approach each normal depth in turn. Point-*b* curves, depending upon initial position, may approach y_{n2} either from above or below, as shown in the sketch below.

10.103 A circular painted-steel channel, of radius 50 cm, is running half-full at 1.2 m^3/s on a slope of 5 m/km. Determine (a) whether the slope is mild or steep; and (b) what type of gradually-varied solution applies at this point. (c) Use the approximate method of Eq. (10.52), and a single depth increment $\Delta y = 5$ cm, to calculate the estimated Δx for this new y.

Solution: (a) To classify the slope, we need to compute y_n and y_c. Take $n = 0.014$. The geometric properties of the partly-full circular duct are taken from the discussion of Eq. (10.20):

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right); \quad P = 2R\theta; \quad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right); \quad y = R[1 + \sin(90° - \theta)]$$

where θ is measured from the bottom of the circle (see Fig. 10.6*a*). For normal flow,

$$Q = 1.2\ \frac{m^3}{s} = \frac{1}{n} A R_h^{2/3}\sqrt{S_o} = \frac{1}{0.014}\left[(0.5)^2\left(\theta - \frac{\sin 2\theta}{2}\right)\right]\left[\frac{0.5}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3}\sqrt{0.005}$$

EES seems indicated, and the solution is $\theta_n = 107.9°$ and $\boldsymbol{y_n = 0.654}$ **m**. Next, for this non-rectangular channel, critical flow occurs when

$$A_c = \left(\frac{b_o Q^2}{g}\right)^{1/3} \quad \text{where } b_o = 2R\cos(90° - \theta) \quad \text{and} \quad Q = 1.2\ \frac{m^3}{s}$$

Again, EES is handy, and the solution is $\theta_c = 104.8°$ and $\boldsymbol{y_c = 0.628}$ **m**.

(a) Thus $y_c < y_n$, and the channel slope is therefore **mild**. *Ans.* (a)

(b) From Fig. 10.14c, since we are starting at $y = 0.5$ m, which is less than y_c, we will proceed for Fr > 1 along an **M-3 curve**. *Ans.* (b)

(c) We are to find Δx required to move from $y = 0.5$ m to $y = 0.55$ m in one step ($\Delta y = 0.05$ m), using the numerical method of Eq. (10.52). At the initial depth,

$$y_1 = 0.5\text{ m};\ V_1 = 3.06\text{ m/s};\ E_1 = 0.976\text{ m};\ R_{h1} = 0.25\text{ m};\ S_1 = n^2V^2/R_h^{4/3} = 0.0116$$

Similarly, at the final depth,

$$y_2 = 0.55\text{ m};\ V_2 = 2.71\text{ m/s};\ E_2 = 0.925\text{ m};\ R_{h2} = 0.265\text{ m};\ S_2 = n^2V^2/R_h^{4/3} = 0.00847$$

The numerical approximation, Eq. (10.52), then predicts:

$$\Delta x = \frac{E_2 - E_1}{S_o - S} = \frac{0.925 - 0.976\text{ m}}{0.005 - (0.0116 + 0.00847)/2} = \frac{-.051}{0.00504} \approx \mathbf{10.1\ m} \quad \textit{Ans.}$$

10.104 The rectangular channel flow in Fig. P10.104 expands to a cross-section 50% wider. Beginning at points *a* and *b*, sketch and label the water-surface profiles which are expected for gradually-varied flow.

Solution: Three types of dual curves are possible: S2/S2, S3/S2, and S3/S3, as shown:

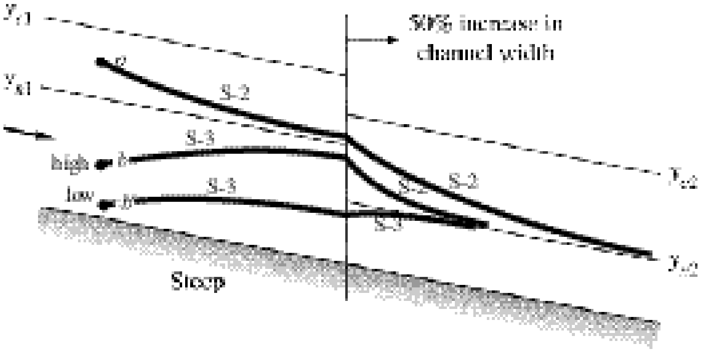


Fig. P10.104

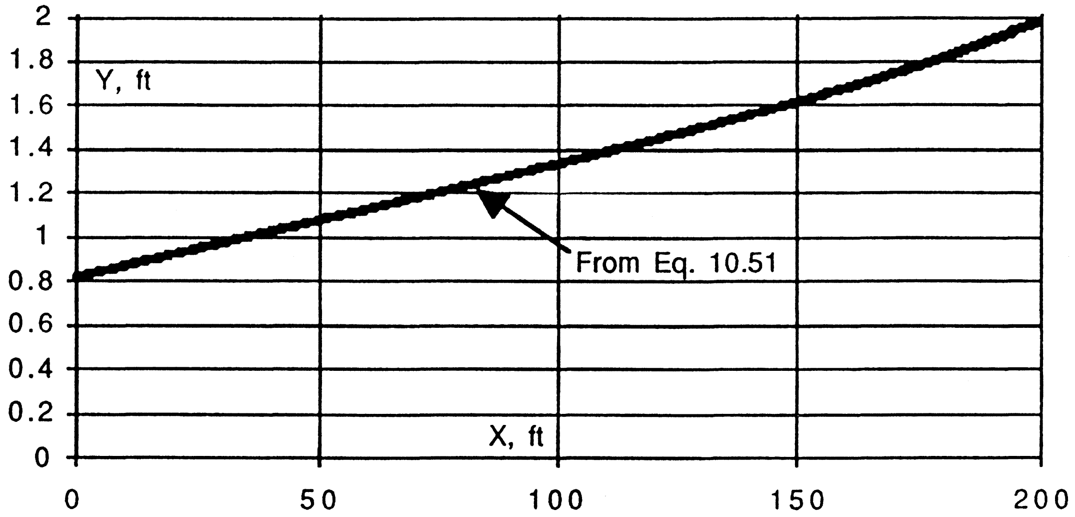
10.105 In Prob. 10.84 the frictionless solution is $y_2 = 0.82$ ft, which we denote as $x = 0$ just downstream of the gate. If the channel is horizontal with $n = 0.018$ and there is no hydraulic jump, compute from gradually-varied theory the downstream distance where $y = 2.0$ ft.

Solution: Given $q = Vy = 20\ \text{ft}^3/\text{s}\cdot\text{ft}$, the critical depth is $y_c = (q^2/32.2)^{1/3} = 2.32$ ft, hence we are on an **H-2** curve (see Fig. 10.14d) which will approach y_c from below. We solve the basic differential equation 10.51 for horizontal wide-channel flow ($S_o = 0$):

$$\frac{dy}{\;} = \frac{-n^2q^2/(\alpha^2 y^{10/3})}{\;} = -\frac{(0.018)^2(20)^2/(2.208y^{10/3})}{\;} \quad \text{for } y_o = 0.82 \text{ ft at } x = 0 \text{ ft.}$$

The water level increases until **y = 2.0 ft at x ≈ 200 ft**. *Ans.*

The complete solution y(x) is shown below.



10.106 A rectangular channel with $n = 0.018$ and a constant slope of 0.0025 increases its width linearly from b to $2b$ over a distance L, as in Fig. P10.106. (a) Determine the variation y(x) along the channel if b = 4 m, L = 250 m, y(0) = 1.05 m, and Q = 7 m^3/s. (b) Then, if your computer program is working well, determine y(0) for which the exit flow will be exactly critical.

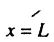


Fig. P10.106

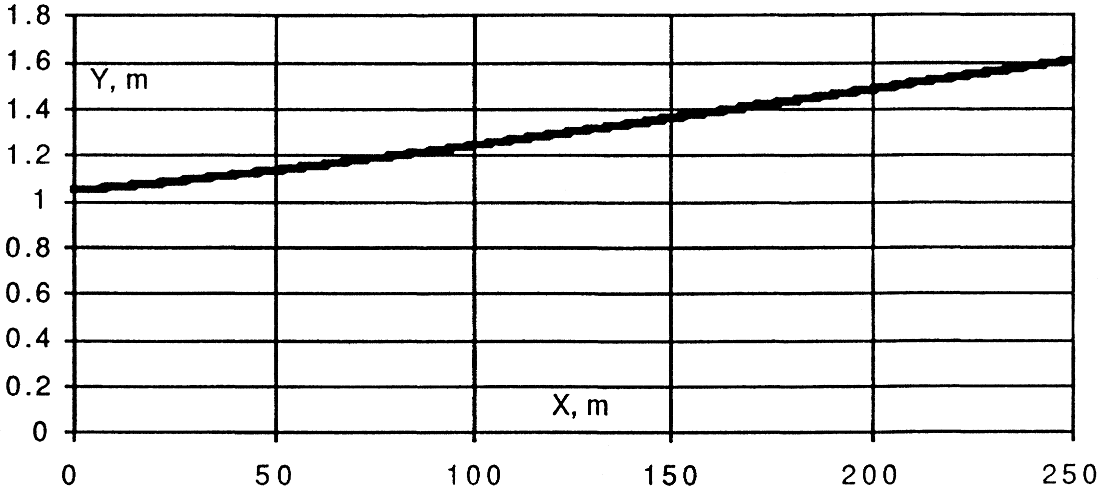
Solution: We are to solve the gradually-varied-flow relation, Eq. 10.51:

$$\frac{dy}{dx} = \frac{S_o - S}{1 - V^2/(9.81y)}, \quad \text{where } S = \frac{n^2V^2}{R^{4/3}}, \quad V = \frac{Q}{by}, \quad R_h = \frac{by}{b+2y}, \quad b = 4\left(1 + \frac{x}{250}\right)$$

For reference purposes, compute $y_c = 0.68$ m and $y_n = 0.88$ m at x = 0, compared to $y_c = 0.43$ m and $y_n = 0.53$ m at x = 250 m. For initial depth y(0) = 1.05, we are on an M-2 curve (see Fig. 10.14*c*) and we compute **y(L) ≈ 1.61 m**. *Ans.* (a)

The curve y(x) is shown below.

The writer **cannot find any y(0) for which the exit flow is critical**. *Ans.* (b)



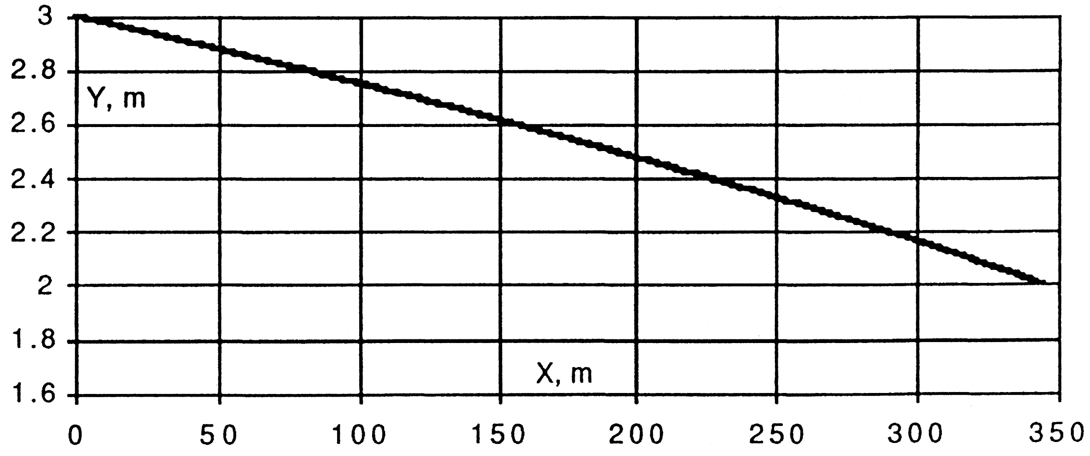
This figure shows y(x) for Prob. 10.106 when y(0) = 1.05 m.

10.107 A clean-earth wide-channel flow is flowing up an *adverse* slope with $S_o = -0.002$. If the flow rate is $q = 4.5$ m^3/s·m, use gradually-varied theory to compute the distance for the depth to drop from 3.0 to 2.0 meters.

Solution: For clean earth, take $n \approx 0.022$. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/(\alpha^2 y^{10/3})}{1 - q^2/(gy^3)},\quad S_o = -0.002,\ \alpha = 1.0,\ q = 4.5,\ n = 0.022,\ y(0) = 3.0\text{ m}$$

The complete graph y(x) is shown below. The depth = 2.0 m when **x = 345 m**. *Ans.*



10.108 Water flows at 1.5 m³/s along a straight, riveted-steel 90° V-channel (see Fig. 10.41, θ = 45°). At section 1, the water depth is 1.0 m. (*a*) As we proceed downstream, will the water depth rise or fall? Explain. (*b*) Depending upon your answer to part (*a*), calculate, in one numerical swoop, from gradually varied theory, the distance downstream for which the depth rises (or falls) 0.1 m.

Solution: For riveted steel, from Table 10.1, $n = 0.015$. Calculate y_n and y_c at section 1:

$$90^\circ\,Vee: A = y^2,\ R_h = \frac{y}{\sqrt{8}};\ gA_c^3 = b_oQ^2 \text{ becomes } y_c = \left(\frac{2Q^2}{g}\right)^{1/5} = \left[\frac{2(1.5)^2}{9.81}\right]^{1/5} = 0.856\,m$$

$$\text{Chezy}: Q = 1.5\frac{m^3}{s} = \frac{1}{0.015}(y_n^2)\left(\frac{y_n}{\sqrt{8}}\right)^{2/3}\sqrt{0.011}\ ,\quad solve\ for\quad y_n = 0.728\,m$$

Since $y_c > y_n$, we are on a *steep slope*. Since the given initial $y_1 = 1.0$ m is greater than both y_c and y_n, we will proceed along an **S-1 curve**, Fig. 10.14*a*. The water depth will rise. *Ans.*(*a*)

(b) For $y_1 = 1$ m, compute the quantities we need for gradually varied theory, Eq. (10.49):

$$R_h = \frac{1m}{\sqrt{8}} = 0.354\,m\ ;\ V_1 = \frac{Q}{A_1} = \frac{1.5m^3/s}{(1m)^2} = 1.5\frac{m}{s}\ ;\quad S_1 = \frac{n^2V_1^2}{\alpha^2 R_h^{4/3}} = \frac{(0.015)^2(1.5)^2}{(1.0)(0.354)^{4/3}} = 0.00203$$

$$Fr_1^2 = \frac{2V_1^2}{gy_1} = \frac{2(1.5)^2}{(9.81)(1.0)} = 0.459\ ;\quad \frac{dy}{dx}|_1 = \frac{S_o - S_1}{1 - Fr_1^2} = \frac{0.011 - 0.00203}{1 - 0.459} = 0.00163$$

Make the same calculations at section 2, where $y_2 = y_1 + 0.1$ m = 1.1 m:

$$R_h = \frac{1.1m}{\sqrt{8}} = 0.389\,m\ ;\ V_2 = \frac{Q}{A_2} = \frac{1.5m^3/s}{(1.1m)^2} = 1.24\frac{m}{s}\ ;\quad S_2 = \frac{n^2V_2^2}{\alpha^2 R_h^{4/3}} = \frac{(0.015)^2(1.24)^2}{(1.0)(0.389)^{4/3}} = 0.00122$$

$$Fr_2^2 = \frac{2V_2^2}{gy_2} = \frac{2(1.24)^2}{(9.81)(1.1)} = 0.285\ ;\quad \frac{dy}{dx}|_2 = \frac{S_o - S_2}{1 - Fr_2^2} = \frac{0.011 - 0.00122}{1 - 0.285} = 0.00137$$

For one swoop Δx, take the average slope and use it to estimate the horizontal distance:

$$\frac{dy}{dx}|_{avg.1-2} = \frac{1}{2}(0.00166 + 0.00137) = 0.00151\ ,\ \Delta x_{1-2} \approx \frac{1.1 - 1.0m}{0.00151} \approx \mathbf{6.6\,m} \quad Ans.(b)$$

10.109 Figure P10.109 illustrates a free overfall or *dropdown* flow pattern, where a channel flow accelerates down a slope and falls freely over an abrupt edge. As shown, the flow reaches critical just before the overfall. Between y_c and the edge the flow is rapidly varied and does not satisfy gradually varied theory. Suppose that the flow rate is $q = 1.3\ m^3/(s\cdot m)$ and the sur-face is unfinished cement. Use Eq. (10.51) to estimate the water depth 300 m upstream as shown.

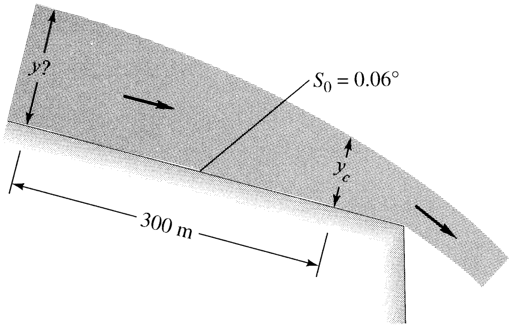


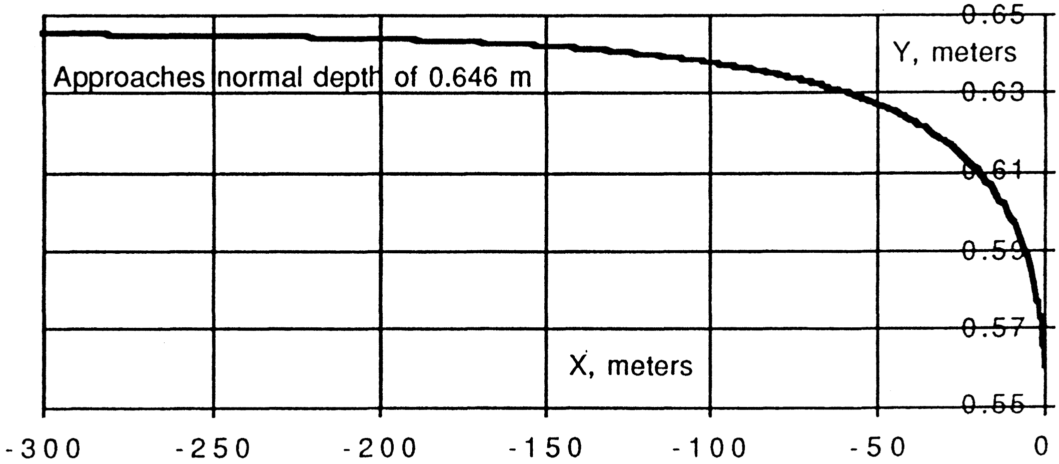
Fig. P10.109

Solution: For unfinished cement, take *n* ≈ 0.012. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/y^{10/3}}{1 - q^2/(gy^3)}, \quad S_o = \sin(0.06°) = 0.00105, \quad q = 1.3\ \frac{m^3}{s\cdot m}, \quad n = 0.012,$$

$$y(0) = y_{critical} = (q^2/g)^{1/3} = [(1.3)^2/9.81]^{1/3} = 0.556\ \text{m}, \quad \text{integrate for } \Delta x < 0.$$

The solution grows rapidly at first and then approaches, at about 150 m upstream, the normal depth of **0.646 m** for this flow rate and roughness. The profile is shown on the next figure.



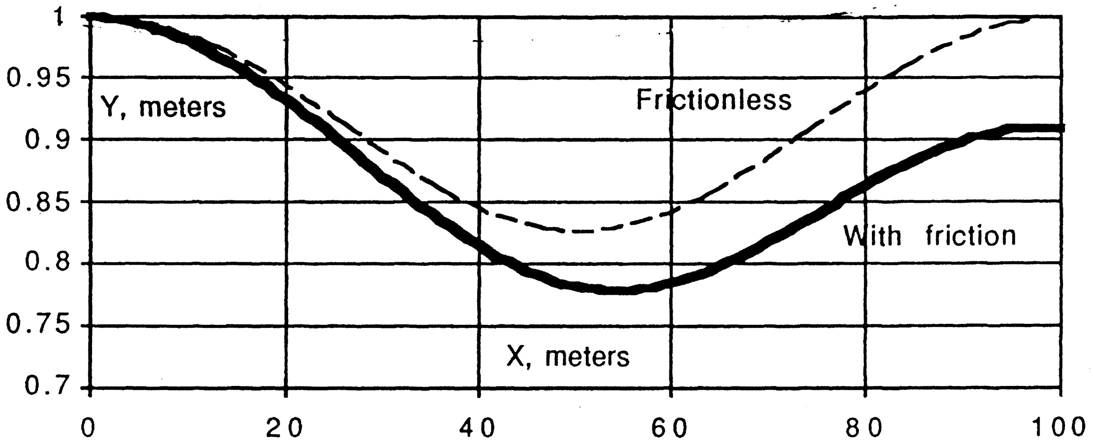
10.110 We assumed frictionless flow in solving the bump case, Prob. 10.65, for which $V_2 = 1.21$ m/s and $y_2 = 0.826$ m over the crest when $h_{max} = 15$ cm, $V_1 = 1$ m/s, and $y_1 = 1$ m. However, if the bump is long and rough, friction may be important. Repeat Prob. 10.65 for the same bump shape, h = $0.5h_{max}[1 - \cos(2\pi x/L)]$, to compute conditions (a) at the crest and (b) at the end of the bump, $x = L$. Let $h_{max} = 15$ cm and $L = 100$ m, and assume a clean-earth surface.

Solution: For clean earth, take $n = 0.022$. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2 q^2 y^{-10/3}}{1 - q^2/(gy^3)}, \quad q = 1.0\ \frac{\text{m}^3}{\text{s}\cdot\text{m}},$$

$$S_o = \frac{dh}{dx}\bigg|_{bump} = -\frac{\pi h_{\text{max}}}{2L}\sin\left(\frac{2\pi x}{L}\right), \quad y(0) = 1\ \text{m}$$

We integrate this for clean earth ($n = 0.022$) and also for *frictionless* flow (Prob. 10.65), $n = 0$. The results are shown on the next page. The frictionless profile drops to $y = 0.826$ m at the crest and returns to $y = 1.0$ m at the end, x = L = 100 m. The *frictional* flow drops lower, to **y = 0.782 m at the crest** [*Ans.* (a)] and even lower, to y = 0.778 m at x = 54 m, and then does not recover fully, ending up at **y = 0.909 m at**
x = L. [*Ans.* (b)]



10.111 Solve Prob. 10.105 (a horizontal variation along an H-3 curve) by the approximate method of Eq. (10.52), beginning at $(x, y) = (0, 0.82\text{ ft})$ and using a depth increment $\Delta y = 0.2$ ft. (The final increment should be $\Delta y = 0.18$ ft to bring us exactly to $y = 2.0$ ft.)

Solution: The procedure is explained in Example 10.10 of the text. Recall that $n = 0.018$ and the flow rate is $q = 20\text{ ft}^3/\text{s/ft}$. The numerical method uses Eq. (10.52) to compute Δx for a given Δy. The "friction slope" $S = n^2V^2/(2.208y^{4/3})$. The bed slope $S_o = 0$ (horizontal). The tabulated results below indicate that a depth of 2.0 ft is reached at a distance $x \approx$ **195 ft** downstream.

[NOTE: Prob. 10.105 had a more accurate numerical solution $x \approx 200$ ft.]

***y*, ft**	***V* = 20/*y***	***E* = *y* + V^2/2g**	***S***	***S*-avg**	**Δ*x*, ft**	***x* = ∑Δ*x*, ft**
0.82	24.390	10.057	0.1137	n/a	n/a	0.000
1.02	19.608	6.990	0.0549	0.084	36.37	36.37
1.22	16.393	5.393	0.0303	0.043	37.49	73.86
1.42	14.085	4.500	0.0182	0.024	36.82	110.68
1.62	12.346	3.987	0.0118	0.015	34.25	144.93
1.82	10.989	3.695	0.0080	0.010	29.56	174.49
2.00	10.000	3.553	0.0058	0.007	20.63	**195.12**

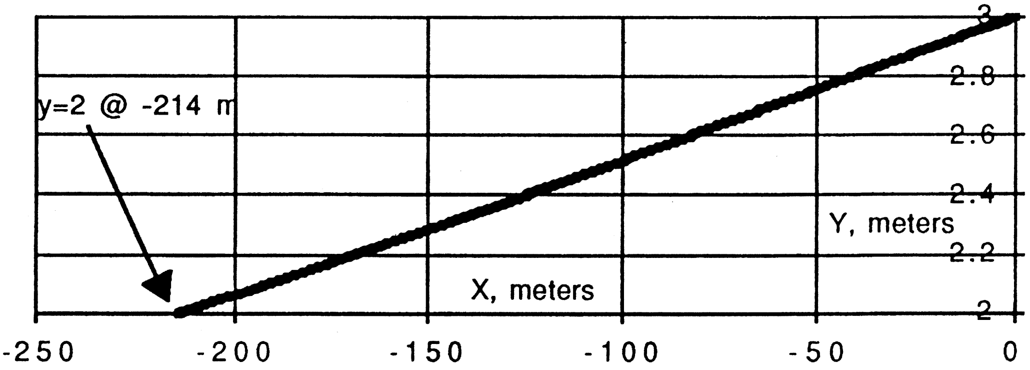
10.112 The clean-earth channel in Fig. P10.112 is 6 m wide and slopes at 0.3°. Water flows at 30 m^3/s in the channel and enters a reservoir so that the channel depth is 3 m just before the entry. Assuming gradually-varied flow, how far is L to a point upstream where y = 2m? What type of curve is the water surface?

Fig. P10.112

Solution: For clean earth, take *n* ≈ 0.022. The differential equation is Eq. 10.51:

$$\frac{dy}{\;} = \frac{S_o - n^2Q^2/(A^2R_h^{4/3})}{\;}, \quad \text{where } S_o = 0.3°,\ Q = 30\ \frac{m^3}{\;},\ b = 6\text{ m},\ A = by,\ y(0) = 3.0$$

To begin, compute $y_n \approx 1.51$ m and $y_c \approx 1.37$ m, hence $y_c < y_n < y$: we are on a *mild* slope above the normal depth, hence we are on an **M-1** curve. *Ans.*
Begin at y(0) = 3 m and integrate backwards ($\Delta x < 0$) until y = 2 m at **L = 214 m**. *Ans.*



10.113 Figure P10.113 shows a channel contraction section often called a *venturi flume* [from Ref. 23 of Chap. 10], because measurements of y_1 and y_2 can be used to meter the flow rate. Show that if losses are neglected and the flow is one-dimensional and subcritical, the flow rate is given by

Fig. P10.113

$$Q = \left[\frac{2g(y_1 - y_2)}{\;}\right]^{1/2}$$

Apply this to the special case $b_1 = 3$ m, $b_2 = 2$ m, and $y_1 = 1.9$ m. Find the flow rate (a) if $y_2 = 1.5$ m; and (b) find the depth y_2 for which the flow becomes critical in the throat.

Solution: Given the water depths, continuity and energy allow us to eliminate one velocity:

$$\text{Continuity: } Q = V_1 y_1 b_1 = V_2 y_2 b_2; \quad \text{Energy: } y_1 + \frac{V_1^2}{} = y_2 + \frac{V_2^2}{}$$

$$\text{Eliminate } V_1 \text{ to obtain} \quad V_2 = [2g(y_1 - y_2)/(1-\alpha^2)]^{1/2} \quad \text{where } \alpha = (y_2 b_2)/(y_1 b_1)$$

$$\text{or:} \quad \mathbf{Q} = V_2 y_2 b_2 = \left[\mathbf{2g(y_1 - y_2)} \big/ \left\{\mathbf{b_2^{-2} y_2^{-2} - b_1^{-2} y_1^{-2}}\right\}\right]^{\mathbf{1/2}} \quad \textit{Ans.}$$

Evaluate the solution we just found:

$$Q = \left[\frac{2(9.81)(1.9-1.5)}{\phantom{(3)^{-2}}}\right]^{1/2} \approx \mathbf{9.88}\ \frac{\mathbf{m^3}}{} \quad \textit{Ans.}\text{ (a)}$$

For this part (a), $Fr_2 = V_2/\sqrt{(gy_2)} \approx$ **0.86**.
(b) To find critical flow, keep reducing y_2 until $Fr_2 = 1.0$. This converges to $\mathbf{y_2 \approx 1.372}$ **m**. [for which Q = **10.1** m^3/s] *Ans.* (b)

10.114 For the gate/jump/weir system sketched earlier in Fig. P10.76, the flow rate was determined to be 379 ft^3/s. Determine the water depth y_4 just upstream of the weir.

Solution: With the flow rate and weir width known, we can go directly to Eqs. (10.55, 56)

$$C_{d,sharp-crested} = 0.564 + 0.0846\frac{H_4}{Y_{weir}} = 0.564 + 0.0846(\frac{H_4}{6ft}) \quad (= 0.642 \text{ after iteration})$$

$$Q = 379\frac{ft^3}{s} \approx C_d b\sqrt{g}\,H_4^{3/2} = C_d(8ft)\sqrt{32.2ft/s^2}\,H_4^{3/2}$$

$$\text{Iteration or EES: } H_4 = 5.53ft \;;\; C_d = 0.642 \;;\; y_4 = 5.53 + 6ft = \mathbf{11.53\,ft} \qquad \textit{Ans.}$$

10.115 Gradually varied theory, Eq. (10.49), neglects the effect of *width* changes, *db*/*dx*, assuming that they are small. But they are not small for a short, sharp contraction such as the venturi flume in Fig. P10.113. Show that, for a rectangular section with *b* = *b*(*x*), Eq. (10.49) should be modified as follows:

$$\frac{dy}{dx} \approx \frac{S_o - S + [V^2/(gb)](db/dx)}{1 - Fr^2}$$

Investigate a criterion for reducing this relation to Eq. (10.49).

Solution: We use the same energy equation, 10.47, but modify continuity, 10.47:

$$\text{Energy: } \frac{dy}{dx} + \frac{V}{g}\frac{dV}{dx} = S_o - S; \quad \text{continuity: } V = \frac{Q}{by}, \quad \therefore \frac{dV}{dx} = -\frac{Q}{by^2}\frac{dy}{dx} - \frac{Q}{yb^2}\frac{db}{dx}$$

$$\text{or: } \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx} - \frac{V}{b}\frac{db}{dx}; \quad \text{combine: } \mathbf{\frac{dy}{dx}\left(1 - \frac{V^2}{gy}\right) \approx S_o - S + \frac{V^2}{gb}\frac{db}{dx}} \quad \textit{Ans.}$$

Obviously, we can neglect the last term (width expansion) and obtain Eq. 10.49 if

$$\frac{V^2}{gb}\frac{db}{dx} >> S_o - S = O\left(\frac{f}{4R_h}\frac{V^2}{2g}\right), \quad \text{or: } \mathbf{\frac{db}{dx}\frac{\Delta b}{L} \gg \frac{f}{8}\frac{b}{R_h}} \quad \textit{Ans.} \text{ (approximate)}$$

Since (f/8) = O(0.01) and (b/R_h) = O(1), we are OK unless Δb ≈ L (large expansion).

P10.116 A Cipolletti weir, popular in irrigation systems, is trapezoidal, with sides sloped at 1:4 horizontal to vertical, as in Fig. P10.116. The following are flow-rate values, from the U. S. Dept. of Agriculture, for a few different system parameters:

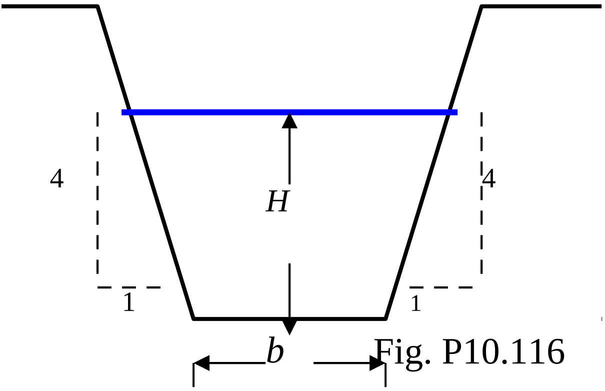


Fig. P10.116

H, ft	0.8	1.0	1.35	1.5
b, ft	1.5	2.0	2.5	3.5
Q, gal/min	1610	3030	5920	9750

C_d (calculated)	**0.589**	**0.595**	**0.593**	**0.595**

Use this data to correlate a Cipolletti weir formula with a reasonably constant weir coefficient.

Solution: We have done this, and added a bottom row to the table. The correlation is

$$Q = C_d\, b\sqrt{g}\, H^{3/2};\quad Example:\ C_d = \frac{1610\,gpm(0.002228\,ft^3/s-gpm)}{(1.5\,ft)\sqrt{32.2\,ft/s^2}\,(0.8\,ft)^{3/2}} = 0.589$$

The correlation is so good, <1%, that the writer suspects the "data" is calculated from a formula

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10.117 A popular flow-measurement device in agriculture is the *Parshall flume* [33], Fig. P10.117, named after its inventor, Ralph L. Parshall, who developed it in 1922 for the U. S. Bureau of Reclamation. The subcritical approach flow is driven, by a steep constriction, to go critical ($y = y_c$) and then supercritical. It gives a constant reading H for a wide range of tailwaters. Derive a formula for estimating Q from measurement of H and knowledge of constriction width b. Neglect the entrance velocity head.

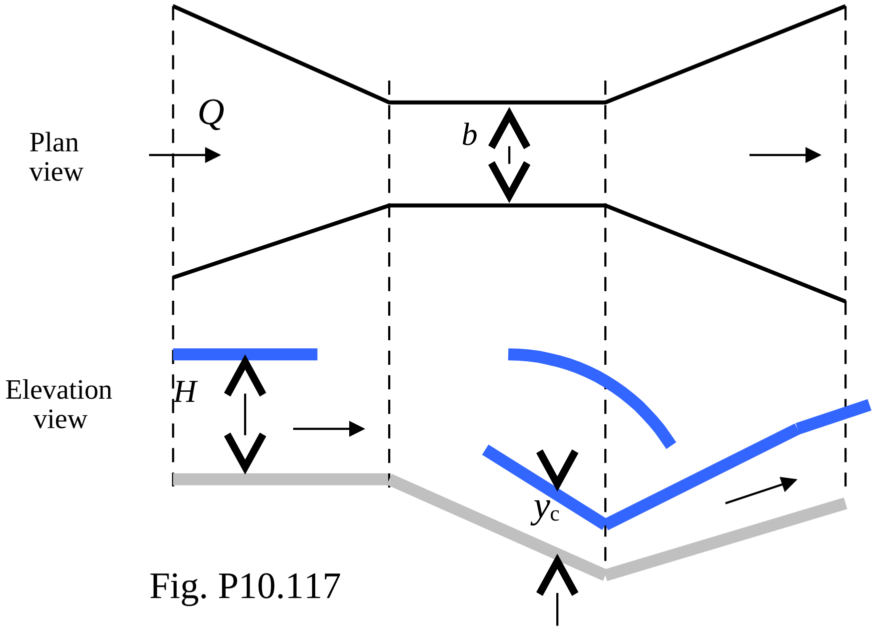


Fig. P10.117

Solution: The rectangular constriction fits the specific energy conditions of Eq. (10.31):

$$E_{critical} = \frac{3}{2}y_c = H + \frac{V_e^2}{2g} \approx H$$

$$\text{Eq.(10.30)}: \; Q = b\sqrt{gy_c^3} \approx b\sqrt{g(2H/3)^3} = (\frac{2}{3})^{3/2} b\sqrt{g}\, H^{3/2} \quad \textit{Ans.}$$

where $(2/3)^{3/2} = 0.544$. In practice, experimental correlations are recommended. In Ref. 33, depending upon flume size, measured values of the exponent on *H* vary from 1.5 to 1.6.

10.118 Using a Bernoulli-type analysis similar to Fig. 10.16*a*, show that the theoretical discharge of the V-shaped weir in Fig. P10.118 is given by

Fig. P10.118

$$Q = 0.7542\, g^{1/2} \tan\alpha\; H^{5/2}$$

Solution: As in Eq. 10.52, assume that velocity V in any strip of height *dz* and width *b*, where *z* is measured down from the top, is $V \approx \sqrt{(2gz)}$ and integrate for the flow rate:

$$Q = \int V\,dA = \int^{H} \sqrt{2gz}\; b\,dz, \quad \text{where } b = b_o(1 - z/H) \quad \text{and} \quad b_o = \text{top width.}$$

$$\text{Thus} \quad Q = \int_0^H \sqrt{2gz}\; b_o\left(1 - \frac{z}{H}\right)dz = \frac{4}{15}\sqrt{2g}\,\frac{b_o}{H}H^{5/2},$$

$$\text{Finally, then,} \quad Q_{\text{V-notch}} = \frac{8}{15}\sqrt{2g}\tan\alpha H^{5/2} \approx \mathbf{0.7542}\sqrt{\mathbf{g}}\,\mathbf{tan}\boldsymbol{\alpha}\,\mathbf{H^{5/2}} \quad \textit{Ans.}$$

10.119 Data by A. T. Lenz for water at 20°C (reported in Ref. 19) show a significant increase of discharge coefficient of V-notch weirs (Fig. P10.118) at low heads. For $\alpha = 20°$, some measured values are as follows:

H, ft:	0.2	0.4	0.6	0.8	1.0
C_d:	0.499	0.470	0.461	0.456	0.452

Determine if these data can be correlated with Reynolds and Weber numbers vis-à-vis Eq. (10.61). If not, suggest another correlation.

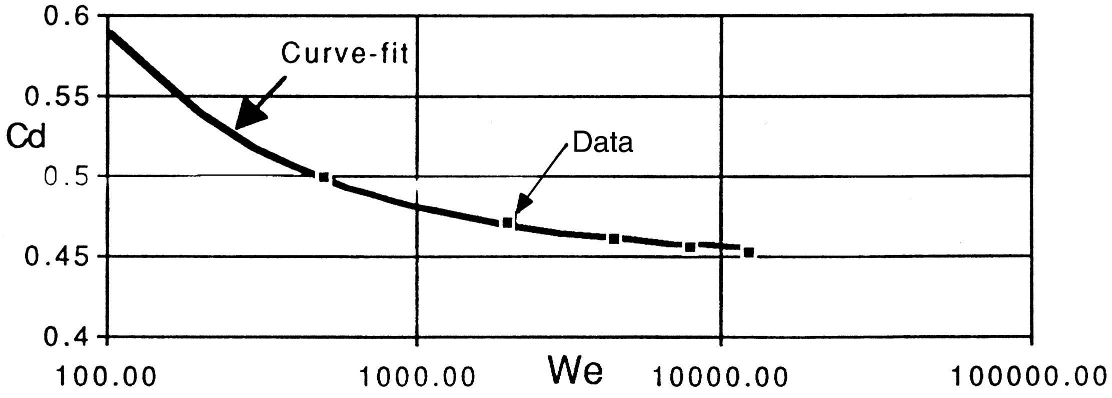
Solution: There is little or no Reynolds number effect. We can ascribe the entire effect to surface tension Y, or Weber number We = $\rho gH^2/Y$.

A Power-law curve-fits the raw data and hence also fits the Weber-number form:

$$C_d = 0.449 + \frac{0.0060}{[H\ \text{in ft}]^{1.3}} \pm 0.5\%$$

Convert to dimensionless form: $\mathbf{C_d \approx 0.449 + \dfrac{2.8}{We^{0.65}}}$, where $\text{We} = \dfrac{\rho g H^2}{Y}$ *Ans.*

The data and this curve-fit are shown on the graph below.



10.120 The rectangular channel in Fig. P10.120 contains a V-notch weir as shown. The intent is to meter flow rates between 2.0 and 6.0 m^3/s with an upstream hook gage set to measure water depths between 2.0 and 2.75 m. What are the most appropriate values for the notch height *Y* and the notch half-angle α?

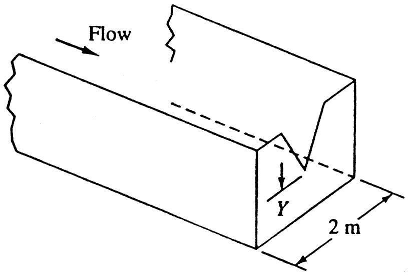


Fig.

P10.120

Solution: There is an *exact* solution to this problem which uses the full range of water depth to measure the full range of flow rates. Of course, there are also a wide variety of combinations of (α, Y) which do a good job but have somewhat less range and accuracy. Anyway, for the "solution" to this problem, match each flow to each depth, with H = y – Y and using Table 10.2(c) for the flow-rate correlation:

$$y = 2\text{ m:}\quad Q = 2\ \frac{\text{m}^3}{\text{s}} = 0.44\tan\alpha\sqrt{g}(2.0 - Y)^{5/2};$$

Divide these two to get **Y ≈ 0.64 m** *Ans.* Back substitute for $\alpha \approx$ **34°** *Ans.*

10.121 Water flow in a rectangular channel is to be metered by a thin-plate weir with side contractions, as in Table 10.2b, with L = 6 ft and Y = 1 ft. It is desired to measure flow rates between 1500 and 3000 gal/min with only a 6-in change in upstream water depth. What is the most appropriate length for the weir width b?

Solution: We are given Y = 1 ft, so water level y_1 and weir width b are the unknowns. Apply Table 10.2(b) to each flow rate, noting that $y_2 = y_1 + 0.5$ ft:

$$Q_1 = 1500\text{ gpm} = 3.342\ \frac{\text{ft}^3}{} = 0.581[b - 0.1(y_1 - 1)]\sqrt{32.2}(y_1 - 1)^{3/2}$$

$$Q_2 = 3000\text{ gpm} = 6.684\ \frac{ft^3}{s} = 0.581[b - 0.1(y_1 - 0.5)]\sqrt{32.2}(y_1 - 0.5)^{3/2}$$

Solve simultaneously by iteration for $y_1 \approx 1.80$ ft and **b ≈ 1.50 ft** *Ans.*

10.122 In 1952 E. S Crump developed the triangular weir shape shown in Fig. P10.122 [Ref. 19, chap. 4]. The front slope is 1:2 to avoid sediment deposition, and the rear slope is 1:5 to maintain a stable tailwater flow. The beauty of the design is that it has a unique discharge correlation up to near-drowning conditions, $H_2/H_1 \le 0.75$:

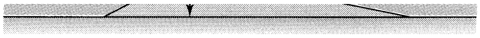


Fig. P10.122 **The Crump weir [19, chap. 4]**

$$Q = C_d b g^{1/2}\left(H_1 + \frac{V_1^2}{2g} - k_h\right)^{3/2} \quad \text{where } C_d \approx 0.63 \quad \text{and} \quad k_h \approx 0.3\text{ mm}$$

The term k_h is a low-head loss factor. Suppose that the weir is 3 m wide and has a crest height $Y = 50$ cm. If the water depth upstream is 65 cm, estimate the flow rate in gal/min.

Solution: We are given weir height Y = 50 cm and upstream depth y_1 = 65 cm, hence $H_1 = 65 - 50 = 15$ cm. Apply the formula, which has an unknown (but low) velocity:

$$Q = 0.63(3.0)\sqrt{9.81}\left(0.15 + \frac{V_1^2}{2(9.81)} - 0.0003\right)^{3/2}, \quad \text{where } V_1 = \frac{Q}{by_1} = \frac{Q}{3(0.65)}$$

Very slight iteration is needed to find $Q \approx 0.349\ m^3/s \approx$ **5500** $\frac{\textbf{gal}}{\textbf{min}}$ *Ans.*

10.123 The Crump weir in Prob. 10.122 is for *modular* flow, that is, when the flow rate is independent of downstream tailwater. When the weir becomes drowned, the flow rate decreases by the following factor:

$$Q = Q_{modular} f \qquad \text{where} \quad f \approx 1.035\left[0.817 - \left(\frac{H_2^*}{H_1^*}\right)^4\right]^{0.0647}$$

for $0.70 \le H_2^*/H_1^* \le 0.93$, where H^* denotes $H_1 + V_1^2/(2g) - k_h$ for brevity. The weir is then *double-gaged* to measure both H_1 and H_2. Suppose that the weir crest is 1 m high and 2 m wide. If the measured upstream and downstream water depths are 2.0 and 1.9 m, respectively, estimate the flow rate in gal/min. Comment on the possible uncertainty of your estimate.

Solution: Again, as in Prob. 10.123, we do not know the velocities (which are fairly low) so we have to iterate the formula slightly:

$$Q = 0.63(2.0)\sqrt{9.81}\left(H_2^*\right)^{3/2} f, \quad \text{where } H^* = H + \frac{V^2}{\quad} - k_h,\ V = \frac{Q}{\quad} \text{ for both 1 and 2}$$

$$\text{Given} \quad H_1 = 2 - 1 = 1 \text{ m and } H_2 = 1.9 - 1 = 0.9 \text{ m, iterate slightly to}$$

$$\mathbf{Q \approx 3.84\ m^3/s \approx 61000\ gal/min} \quad \textit{Ans.}$$

This estimate is uncertain to at least ±5%. The formula itself is a curve-fit and therefore probably uncertain. In addition, the formula is very sensitive to the measured values of y_1 and y_2. For example, a **1%** error in these measurements causes a **10%** change in Q. *Ans.*

10.124 Water flows at 600 ft^3/s in a rectangular channel 22 ft wide with $n \approx 0.024$ and a slope of 0.1°. A dam increases the depth to 15 ft, as in Fig. P10.124. Using gradually varied theory, estimate the distance L upstream at which the water depth will be 10 ft. What type of solution curve are we on? What should be the water depth asymptotically far upstream?

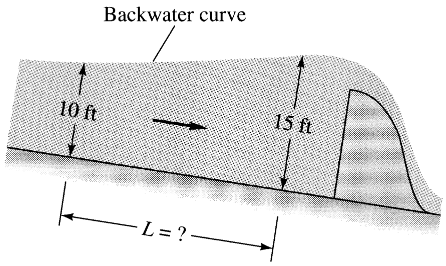


Fig. P10.124

Solution: With depth given just upstream of the dam, we do not have to make a "weir" calculation, but instead go directly to the gradually-varied-flow calculation. Note first that

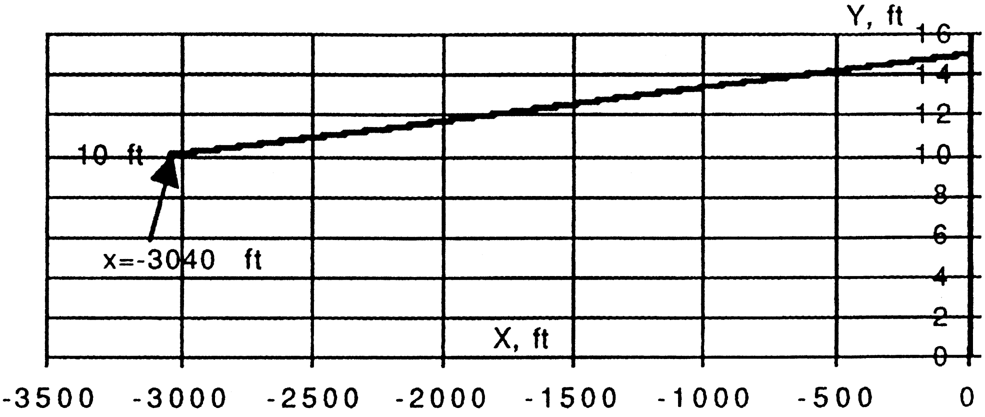
$$Q = 600 = \frac{1.486}{0.024}(22y_n)\left(\frac{22y_n}{\quad}\right)^{2/3}(\sin 0.1°)^{1/2}, \quad \text{solve for normal depth} \quad y_n \approx 4.74$$

$$\text{Also,}\quad y_c = (q^2/g)^{1/3} = \left[\frac{(600/22)^2}{32.2}\right]^{1/3} \quad \text{or} \quad \text{critical depth} \quad y_c \approx 2.85$$

Therefore, since $y_c < y_n < y$, we are on a mild-slope **M-1 curve**. *Ans*. (Fig. 10.4c) The basic differential equation is:

$$\frac{dy}{dx} = \frac{S_o - n^2q^2\Big/\left(2.208y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)},$$

The graph of y(x) is shown below. We reach y = 10 ft at **x = –3040 ft**. *Ans*.
If we keep integrating backward, we reach $v = v_n \approx$ **4.74 ft**.



10.125 The Tupperware dam on the Blackstone River is 12 ft high, 100 ft wide, and sharp-edged. It creates a backwater similar to Fig. P10.124. Assume that the river is a weedy-earth rectangular channel 100 ft wide with a flow rate of 800 ft^3/s. Estimate the water-depth 2 mi upstream of the dam if $S_o = 0.001$.

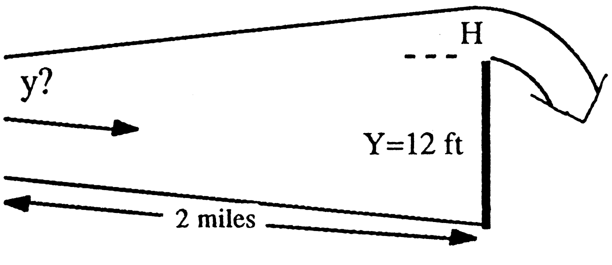


Fig. P10.125

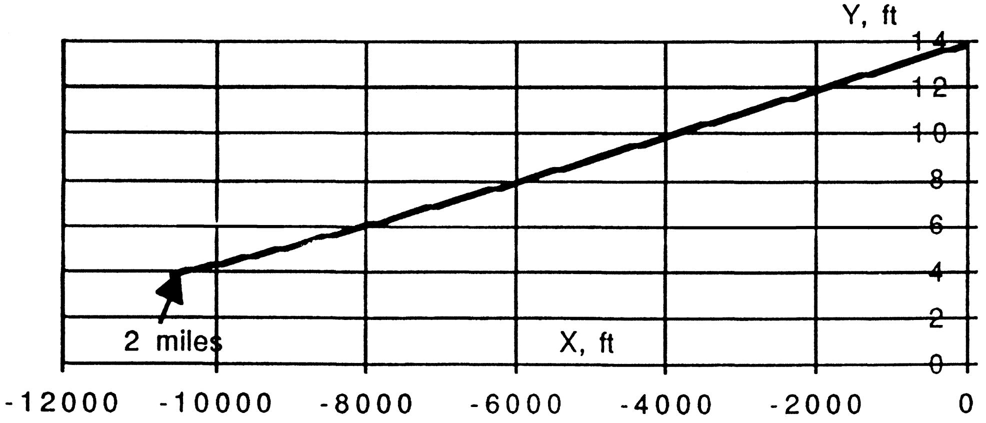
Solution: First use the weir formula to establish the water depth just upstream of the dam:

$$Q = 800\ \frac{ft^3}{s} = C_d b\sqrt{g}\, H^{3/2} = \left(0.564 + 0.0846\frac{H}{Y}\right)(100)\sqrt{32.2}\, H^{3/2},$$

Therefore the initial water depth is y(0) = Y + H = 12 + 1.81 = 13.81 ft. For weedy earth, take *n* ≈ 0.030. We are on an **M-1 curve,** with a basic differential equation

$$\frac{dy}{} = \frac{S_o - n^2q^2\Big/\left(2.208y^2R_h^{4/3}\right)}{},\quad \text{where } q = \frac{800}{},\ R_h = \frac{100y}{},\ n = 0.030,\ S_o = 0.001$$

to be integrated backwards (Δx < 0) for 2 miles (10560 ft). The result is shown in the graph below.



At x = −2 miles = −10560 ft, the water depth is **y ≈ 3.8 ft**. *Ans.*
[Another mile upstream and we would asymptotically reach the normal depth of 2.71 ft.]

10.126 Suppose that the rectangular channel of Fig. P10.120 is made of riveted steel with a flow of 8 m^3/s. If α = 30° and Y = 50 cm, estimate, from gradually-varied theory, the water depth 100 meters upstream.

Solution: For riveted steel, take *n* ≈ 0.015. First use the weir formula to get y(0):

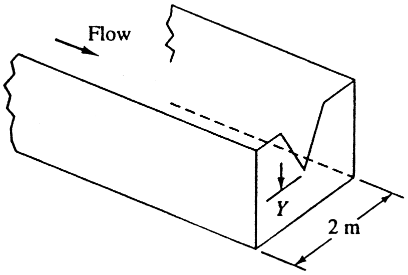


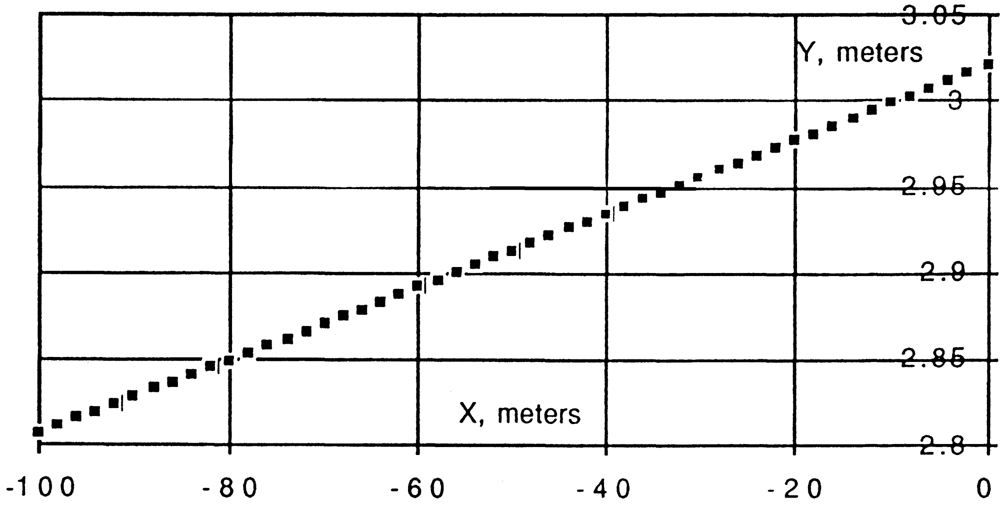
Fig. P10.120

$$Q = 8\ \frac{m}{\ } = 0.44\tan 30°\sqrt{9.81}H^{5/2},\quad \text{solve } H \approx 2.52 \text{ m},\ y(0) = 2.52 + 0.5 = \mathbf{3.02\ m}$$

The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/\left(y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)},\quad \text{where } q = \frac{8}{2},\ R_h = \frac{2y}{2+2y},\ n = 0.015,\ S_o = \sin 0.15°$$

Integrate backwards (Δx < 0) for 100 m. The result is shown in the graph below:



At x = –100 m, the water depth is **y ≈ 2.81 m**. *Ans.*
[Another 1000 m upstream and we asymptotically reach the normal depth of 1.62 m.]

P10.127 A clean earth river is 50 ft wide and averages 600 ft^3/s. It contains a dam that increases the water depth to 8 ft, to provide head for a hydropower plant. The bed slope is 0.0025. (*a*) What is the normal depth of this river? (*b*) Engineers propose putting flashboards on the dam to raise the water level to 10 ft. Residents a half mile upstream are worried about flooding above their present water depth of about 2.2 ft. Using Eq. (10.52) in one big half-mile step, estimate the new water depth upstream.

Solution: For clean earth, from Table 10.1, $n = 0.022$. (*a*) Evaluate the normal flow depth:

$$Q = 600\frac{ft^3}{s} = \frac{\alpha}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1.486}{0.022}(50\,y_n)(\frac{50\,y_n}{50+2y_n})^{2/3}\sqrt{0.0025}$$

Iterate slightly to obtain $R_h = 2.03\,ft$ and $y_n = \mathbf{2.21}\ ft$ *Ans.*(*a*)

(*b*) Let point 1 be the dam, and point 2 be a half mile upstream. Prepare for Eq. (10.52):

$$V_1 = \frac{Q}{b\,y_1} = \frac{600}{(50)(10)} = 1.2\frac{ft}{s}\ ;\quad E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{(1.2)^2}{2(32.2)} = 10.02\,ft$$

$$E_2 = y_2 + \frac{V_2^2}{2g}\ ,\quad V_2 = \frac{V_1\,y_1}{y_2} = \frac{(1.2)(10)}{y_2}\ ,\quad \text{but we don't know } y_2 \text{ yet.}$$

$$S_o = 0.0025\ ;\quad S_{avg} = \frac{n^2\,V_{avg}^2}{\alpha^2\,R_{h,avg}^2}\ ;\ V_{avg} = \frac{V_1+V_2}{2}\ ;\ R_{h,avg} = \frac{R_{h1}+R_{h2}}{2}$$

With some iteration needed to find y_2, and thence V_{avg}, $R_{h,avg}$, and S_{avg}, we have

$$\Delta x \approx \frac{E_2 - 10.02}{0.0025 - S_{avg}} = \frac{1}{2}\text{mile} = 2640\,ft$$

This half-mile jump is too big for real engineering accuracy. Students may get a spread of answers, depending upon their approximations. The writer used EES and obtained

$$y_2 \approx \mathbf{3.6}\,ft\ ,\quad \Delta y = y_2(10\,ft\text{ dam}) - y_2(8\,ft\text{ dam}) = 3.6\,ft - 2.2\,ft \approx \mathbf{1.4}\,ft\quad \textit{Ans.}$$

Upstream, the water will be 1.4 ft higher than without the flashboards. The residents may be right in their fears of flooding when the river discharge is high.

10.128 A rectangular channel 4 m wide is blocked by a broadcrested weir 2 m high, as in Fig. P10.128. The channel is horizontal for 200 m upstream and then slopes at 0.7° as shown. The flow rate is 12 m^3/s, and *n* = 0.03. Compute the water depth *y* at 300 m upstream from gradually varied theory.

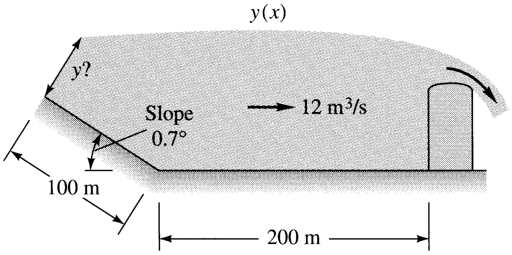


Fig. P10.128

Solution: First use (smooth) weir theory to establish the depth just upstream of the dam:

$$Q = 12\,\frac{m^3}{} \approx 0.544(4.0)\sqrt{9.81}H^{3/2}, \quad \text{solve for } H \approx 1.46\text{ m}, \quad \therefore\ y(0) = H + Y = 3.46\text{ m}$$

$$\frac{dy}{} = \frac{S_o - n^2q^2/\left(y^2R_h^{4/3}\right)}{}, \quad \text{where } q = \frac{12}{}, \ R_h = \frac{4y}{}, \ n = 0.03, \text{ two values of } S_o$$

Integrate backwards ($\Delta x < 0$) for 200 m with $S_o = 0$, then for 100 m with $S_o = \sin 0.7°$. After 200 m, the depth is y = 3.56 m. Then at x = –300 m, **y ≈ 2.37 m**. *Ans.*

Both water profiles are nearly linear:

x, m:	0	–50	–100	–150	–200	–225	–250	–275	–300
y, m:	3.46	3.49	3.51	3.53	3.56	3.26	2.96	2.66	**2.37 m**

[At x ≈ –400 m, the sloped flow will approach its normal depth of 1.05 m.]

FUNDAMENTALSOFENGINEERINGEXAMPROBLEMS:Answers

FE10.1 Consider a rectangular channel 3 m wide laid on a 1° slope. If the water depth is 2 m, the hydraulic radius is

(a) 0.43 m (b) 0.6 m (**c**) **0.86 m** (d) 1.0 m (e) 1.2 m

FE10.2 For the channel of Prob. FE10.1, the most efficient water depth (best flow for a given slope and resistance) is

(a) 1 m (**b**) **1.5 m** (c) 2 m (d) 2.5 m (e) 3 m

FE10.3 If the channel of Prob. FE10.1 is built of rubble cement (Manning's $n \approx 0.020$), what is the uniform flow rate when the water depth is 2 m?

(a) 6 m^3/s (b) 18 m^3/s (**c**) **36** $\mathbf{m^3/s}$ (d) 40 m^3/s (e) 53 m^3/s

FE10.4 For the channel of Prob. FE10.1, if the water depth is 2 m and the uniform flow rate is 24 m^3/s, what is the approximate value of Manning's roughness factor n?

(a) 0.015 (b) 0.020 (c) 0.025 (**d**) **0.030** (e) 0.035

FE10.5 For the channel of Prob. FE10.1, if Manning's roughness factor $n \approx 0.020$ and $Q \approx 24\ m^3/s$, what is the normal depth y_n?

(a) 1 m (**b**) **1.5 m** (c) 2 m (d) 2.5 m (e) 3 m

FE10.6 For the channel of Prob. FE10.1, if $Q \approx 24\ m^3/s$, what is the critical depth y_c?

(a) 1.0 m (b) 1.26 m (c) 1.5 m (**d**) **1.87 m** (e) 2.0 m

FE10.7 For the channel of Prob. FE10.1, if $Q \approx 24\ m^3/s$ and the depth is 2 m, what is the Froude number of the flow?

(a) 0.50 (b) 0.77 **(c) 0.90** (d) 1.00 (e) 1.11

COMPREHENSIVEAPPLIEDPROBLEMS

C10.1 February 1998 saw the failure of the earthen dam impounding California Jim's Pond in southern Rhode Island. The resulting flood raised temporary havoc in the nearby village of Peace Dale. The pond is 17 acres in area and 15 ft deep and was full from heavy rains. The breach in the dam was 22 ft wide and 15 ft deep. Estimate the time required to drain the pond to a depth of 2 ft.

22 ft

Solution: Unfortunately, Table 10.2, item *b*, does not really apply, because the breach is so *deep*:, H = y > 0.5Y = 0. Nevertheless, it's all we have, and ponds don't rupture every day, so let's use it! A control volume around the pond yields

$$\frac{d}{dt}\left(\int d\upsilon_{pond}\right) + Q_{out} = 0,$$

$$or: \quad A_{pond}\frac{dy}{dt} = -Q_{out} = -0.581(b - 0.1y)g^{1/2}y^{3/2},$$

$$b = 22 \text{ ft}, \quad A_{pond} = 17 \text{ acres} = 740{,}520 \text{ ft}^2$$

If we neglect the "edge contraction" term "–0.1y" compared to b = 22 ft, this first-order differential equation has the solvable form

$$\frac{dy}{dt} \approx -Cy^{3/2}, \quad \text{where } C = \frac{0.581(22 \text{ ft})(32.2)^{1/2}}{740520} \approx 9.8\text{E–5 ft}^{-1/2}\text{sec}^{-1}$$

$$\text{Separate and integrate:} \quad \int^{2\,ft} \frac{dy}{y^{3/2}} = -C\int^{t} dt, \quad or: \quad \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{15}} = Ct$$

$$\text{Finally, solve} \quad t_{drain\text{-}to\text{-}2\,ft} \approx \frac{1.414 - 0.516}{9.8\text{E–5}} = 9160 \text{ s} = \mathbf{2.55\ h} \quad \textit{Ans.}$$

If we used a spreadsheet and kept the term "–0.1y", we would predict a *time-to-drain-to-2 ft* or about **2.61** hours. The theory is too crude to distinguish between these estimates.

Fig. C10.2

C10.2 A circular, unfinished concrete drainpipe is laid on a slope of 0.0025 and is planned to carry from 50 to 300 ft^3/s of run-off water. Design constraints are that (1) the water depth should be no more than 3/4 of the diameter; and (2) the flow should always be subcritical. What is the appropriate pipe diameter to satisfy these requirements? If no commercial pipe is exactly this calculated size, should you buy the next smallest or the next largest pipe?

Solution: For unfinished concrete $n \approx 0.014$. From the geometry of Fig. 10.6, 3/4 - full corresponds to an angle $\theta = 120°$. This level should be able to carry the maximum 300 ft^3/s flow:

$$\theta = 120°: \quad A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 2.53R^2; \quad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2}\right) = 0.603R$$

$$\text{Try} \quad Q = 300\ \frac{ft^3}{} = \frac{1.49}{}AR_h^{2/3}S_o^{1/2} = \frac{1.49}{0.014}(2.53R^2)(0.603R)^{2/3}\sqrt{0.0025}$$

$$\text{Solve for} \quad R_{3/4\text{-}full} \approx \mathbf{3.64\ ft} \quad \textit{Ans.}$$

Now check to see if this flow is subcritical:

$$A_c = \left(\frac{b_0 Q^2}{}\right)^{1/3} = \left[\frac{(1.732)(3.64)(Q_c)^2}{32.2}\right]^{1/3} = 2.53(3.64)^2$$

$$\text{Solve for} \quad Q_c = 438\ \frac{ft^3}{} > 300, \quad \therefore \text{ flow is subcritical}$$

Even at max flow, three-quarters full, the Froude number is only 0.68. Scanning the other flow rates, down to 50 ft^3/s, yields *smaller* drainpipes. Therefore we conclude that a pipe of diameter **D ≈ 7.3 ft** will do the job. Pick the nearest ***larger*** available size. *Ans.*

Fig. P10.72

C10.3 Extend Prob. 10.72, whose solution was $V_2 = 4.33$ m/s. Use gradually-varied theory to estimate the water depth 10 m down at section 3 for (a) the 5° unfinished concrete slope shown in the figure; and (b) for an *upward* (–5°) adverse slope. (c) When you find out that (b) is *impossible*, explain why and repeat for an adverse slope of (–1°).

Solution: For unfinished concrete take $n \approx 0.014$. Note that y(0) = 0.0462 m. For the given flow rate $q = 0.2$ m^3/s·m, first calculate reference depths:

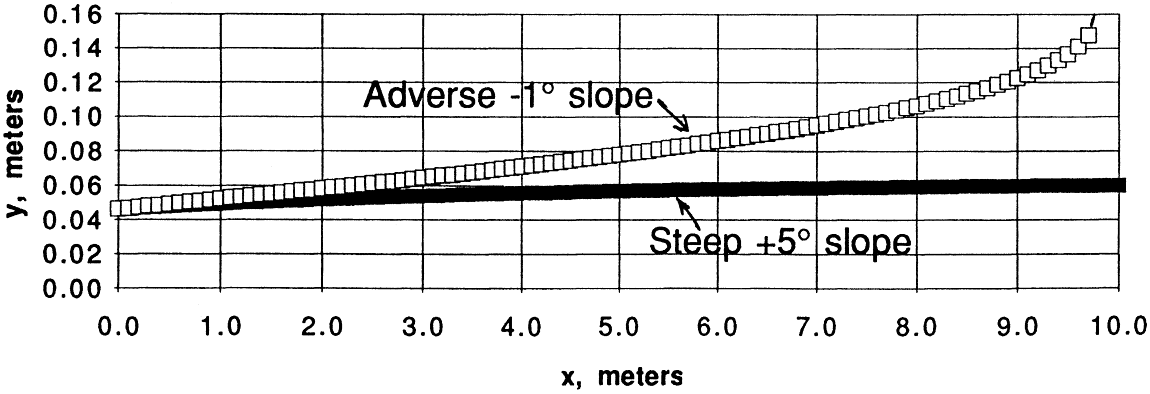
$$q = 0.2 = \frac{1}{0.014}(y)(y^{2/3})\sqrt{\sin 5°}, \quad \text{solve normal depth } y_n \approx 0.0611 \text{ m}$$

$$\text{Critical depth: } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.2^2}{9.81}\right)^{1/3} = 0.160 \text{ m} \quad \therefore \text{ Steep S-3 curve.}$$

The channel is "wide," so the formulation of Example 10.8 is appropriate:

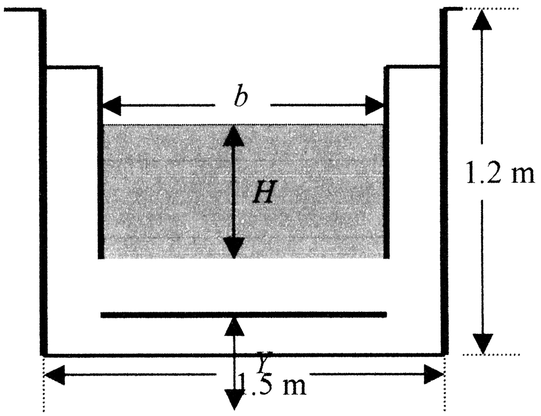
$$\frac{dy}{dx} = \frac{S_0 - n^2q^2/y^{10/3}}{1 - q^2/(gy^3)}, \quad S_o = \sin \pm 5°, \; n = 0.014, \; q = 0.2, \; y(0) = 0.0462 \text{ m}, \; y(10 \text{ m}) = ?$$

Numerical integration, by Excel or MATLAB or whatever, for the *steep* (**S-3**) slope yields ***y* = 0.060 m** at x = 10 m, nearly critical, as shown below. *Ans.* (a) Integration for the adverse (–5°) slope goes to *critical* at about x = 4.5 m—**the theory fails for x > 4.5 m**. *Ans.* (b) If we go back and take a *smaller* adverse slope of (–1°), we obtain **y ≈ 0.16 m** at x = 9.8 m—the flow goes critical near x = L, as shown in the graph below. *Ans.* (c)



C10.4 It is desired to meter an asphalt rectangular channel of width 1.5 m which is designed for uniform flow at a depth of 70 cm and a slope of 0.0036. The vertical sides of the channel are 1.2 m high. Consider using a thin-plate rectangular weir, either full or partial width (Table 10.2*a*, *b*) for this purpose. Sturm [7, p. 51] recommends, for accurate correlation, that such a weir have $Y \geq 9$ cm and $H/Y \leq 2.0$. Determine the feasibility of installing such a weir which will be accurate and yet not cause the water to overflow the sides of the channel.

Solution: For asphalt take $n = 0.016$. We have only one partial-width formula, and that is from Table 10.2b. We are slightly outside the limits of applicability, but we will use it anyway:



$$Q_{weir} = 0.581(b\ -\ 0.1H)g^{1/2}H^{3/2}$$

where b and H are shown in the figure.

Meanwhile, calculate the total flow rate in the expected normal flow:

$$Q = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{0.016}[1.5\ m(0.7\ m)]\left[\frac{1.5\ m(0.7\ m)}{1.5\ m + 2(0.7\ m)}\right]^{2/3}\sqrt{0.0036} = \mathbf{2.00}\ \frac{\mathbf{m^3}}{}$$

The weir discharge must equal this flow rate. Let us begin by making b equal to the full available width of 1.5 m and holding Y to the minimum height of 9 cm. The weir formula is:

$$Q = 2.0\ \frac{m^3}{} = 0.581(1.5\ m - 0.1H)(9.81\ m/s^2)^{1/2}H^{3/2},\quad \text{solve for}\ \ H = 0.845\ m$$

Note that H is independent of Y, but the ratio $H/Y = 0.845/0.09 = 9.4$, which far exceeds: Sturm's recommendation $H/Y \le 2.0$. If we raise Y to $H/2 = 0.423$ m, the total upstream water depth is $H + Y = 1.27$ m, which overflows the channel walls. If we back down to Y = 0.35 m, the upstream depth is only 1.195 m, so **a wide-weir design is possible with *H/Y* = 2.4, not bad**.

Similarly, we can try shorter values of b, but either (1) the upstream depth will exceed 1.2 m, or (2) the ratio H/Y will exceed 2.0. Here are some possible scenarios:

$\boldsymbol{b} = \mathbf{1.4\ m}$: $H = 0.89$ m; $H + Y \approx 1.2$ m if $\boldsymbol{Y} = \mathbf{0.31\ m}$ and $H/Y = 2.9$

$\boldsymbol{b} = \mathbf{1.25\ m}$: $H = 0.97$ m; $H + Y \approx 1.2$ m if $\boldsymbol{Y} = \mathbf{0.23\ m}$ and $H/Y = 4.2$

The first two of these are plausible, although H/Y is larger than 2.0. The third result is not recommended because Y is too far below 9 cm. **We conclude that reasonable designs are possible, but they slightly violate the constraints on the formulas we are using**.

C10.5 Figure C10.5 shows a hydraulic model of a *compound weir,* that is, one which combines two different shapes. (a) Other than measurement, for which it might be useful, what could be the engineering reason for such a weir? (b) For the prototype river, assume that both sections have sides at a 70° angle to the vertical, with the bottom section having a base width of 2 m and the upper section having a base width of 4.5 m, including the cut-out portion. The heights of lower and upper horizontal sections are 1 m and 2 m, respectively. Use engineering estimates and make a plot of upstream water depth versus Petaluma River flow rate in the range 0 to 4 m^3/s. (c) For what river flow rate will the water overflow the top of the dam?

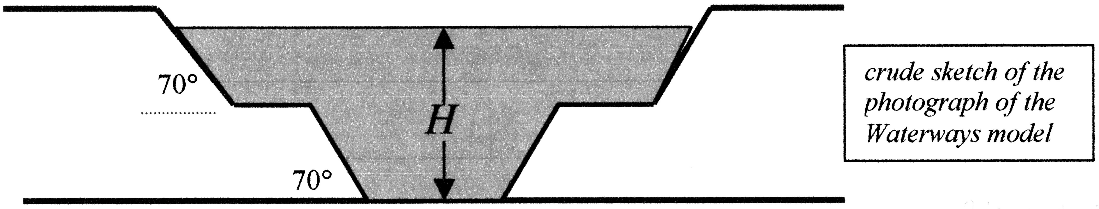


Fig. C10.5

Solution: We have no formulas in the text for a compound weir shape, but we can still use the concept of weir flow and estimate the discharge for various water depths.

(a) A good reason for using a narrow bottom portion of the weir is to maintain a reasonable upstream depth at low flow, then widen to maintain a lower depth at high flow. It also allows a more accurate flow measurement during low flow. *Ans.* (a)

(b) Rather than derive a whole new theory for compound weirs, we will assume that the bottom portion is more or less rectangular, based on average width *b*, with the top portion also assumed rectangular with its flow rate added onto the lower portion. For example, if $H = 1$ m (the top of the lower portion), the flow rate is estimated by Table 10.2*b*:

$$b_{avg} = \frac{2.728 + 2.0\ m}{2} = 2.364\ m,\ \ Q \approx 0.58(b_{avg} - 0.1H)g^{1/2}H^{3/2}$$

$$\text{or: } Q_{lower} = 0.58[2.364\ m - 0.1(1\ m)](9.81\ m/s^2)^{1/2}(1\ m)^{3/2} \approx 4.1\ m^3/s$$

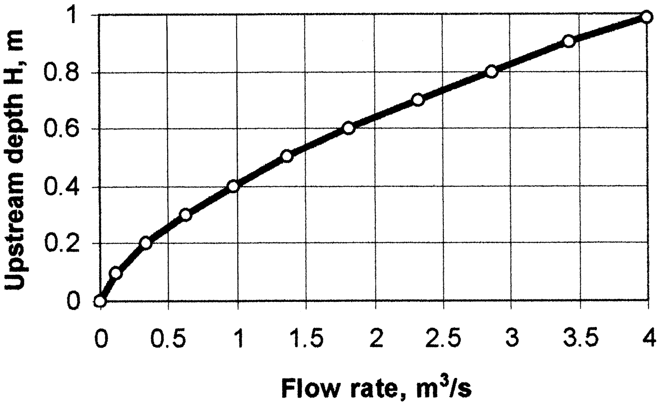
Then if $H = 2$ m > 1 m, we figure Q_{upper} the same way and add on the lower portion flow. Again take $H = 1$ m, that is, the height of the flow above the lower part of the weir:

$$b_{avg} = \frac{4.5 + 5.23\ m}{2} = 4.87\ m,\quad Q \approx 0.58(b_{avg} - 0.1H)g^{1/2}H^{3/2}$$

$$\textbf{\textit{or}}: \quad Q_{upper} = 0.58[4.87\ m - 0.1(1\ m)](9.81\ m/s^2)^{1/2}(1\ m)^{3/2} \approx 8.7\ m^3/s$$

$$\textbf{\textit{Thus}} \quad Q_{total} = Q_{lower} + Q_{upper} = 4.1 + 8.7 \approx \mathbf{12.8\ m^3/s}$$

Flow rates greater than this value of 12.8 m^3/s will **overflow the top of the weir**. *Ans.* (b)
A plot of Q versus H for the range $0 < Q < 4$ m^3/s is shown below.



ProblemC10.5

C10.6 Figure C10.6 shows a horizontal flow of water through a sluice gate, a hydraulic jump, and over a 6-ft-high sharp-crested weir. Channel, gate, jump, and weir are all 8 ft wide unfinished concrete. Determine (*a*) the flow rate, (*b*) the normal depth, (*c*) y_2, (*d*) y_3, and (*e*) y_4.

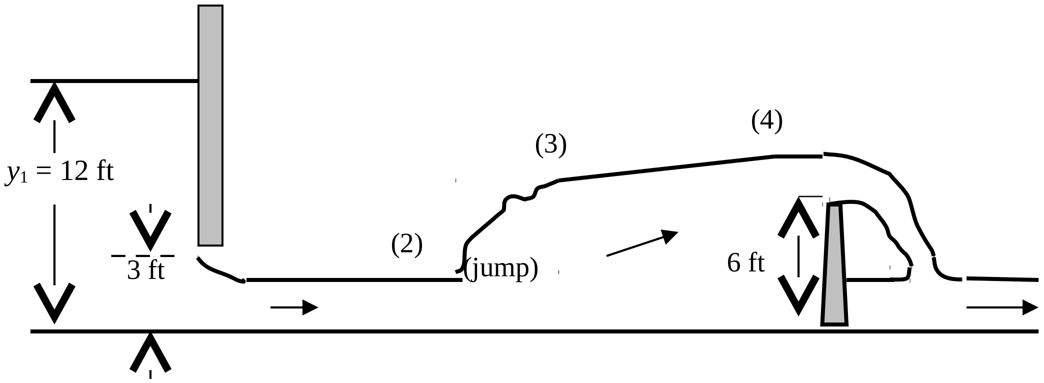


Fig. C10.6

[*Note to instructor: This problem combines the three parts of Problems P10.76, 10.90, and 10.114.*]

Solution: (*a*) The flow rate follows from the sluice gate correlation, Eq. (10.41):

$$Q = C_d Hb\sqrt{2gy_1}\ ,\quad C_d = (\frac{0.61}{\sqrt{1+0.61H_1/y_1}}) = (\frac{0.61}{\sqrt{1+0.61(3ft)/12ft}}) = 0.568$$

$$\text{or}:\qquad Q = (0.568)(3ft)(8ft)\sqrt{2(32.2ft/s^2)(12ft)} \approx \mathbf{379\ \frac{ft^3}{s}}\qquad Ans.(a)$$

(*b*) Find the normal depth. This is a trick question! Ordinarily, if you know the flow rate, you can find the normal depth. But this channel is *horizontal*! **It has no normal depth**. *Ans*.(*b*)

(*c*) With the flow rate known, we can compute V_1 and E_1 and equate them to section 2 values:

$$\frac{Q}{b} = \frac{379\,ft^3/s}{8\,ft} = 47.39\frac{ft^2}{s} = V_1 y_1 = V_1(12ft)\ ,\quad V_1 = 3.95\frac{ft}{s}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 12ft + \frac{(3.95ft/s)^2}{2(32.2ft/s^2)} = 12.24\,ft$$

$$Thus\quad \frac{Q}{b} = V_2 y_2 = 47.39\frac{ft^2}{s}\quad and\quad E_2 = y_2 + \frac{V_2^2}{2g} = 12.24\,ft$$

As we did in Chapter 3 Bernoulli problems, we can solve the last two equations for

$$y_2 = \mathbf{1.83\,ft}\quad Ans.(c)\quad ;\quad V_2 = 25.9\frac{ft}{s}\quad ;\quad Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{25.9}{\sqrt{32.2(1.83)}} = 3.37$$

As expected, the flow in section 2 is highly *supercritical*. Now use hydraulic jump theory:

$$\frac{2y_3}{y_2} = \frac{2y_3}{(1.83ft)} = -1+\sqrt{1+8Fr_2^2} = -1+\sqrt{1+8(3.37)^2} = 8.593;\ \ Solve\quad y_3 = \mathbf{7.86ft}\quad Ans.($$

(*e*) With the flow rate and weir width known, we can go directly to Eqs. (10.55, 56):

$$C_{d,sharp-crested} = 0.564 + 0.0846\frac{H_4}{Y_{weir}} \;=\; 0.564 + 0.0846\left(\frac{H_4}{6ft}\right) \quad (= 0.642 \text{ after iteration})$$

$$Q \;=\; 379\frac{ft^3}{s} \approx C_d b\sqrt{g}\,H_4^{3/2} \;=\; C_d(8ft)\sqrt{32.2ft/s^2}\,H_4^{3/2}$$

Iteration or EES: $C_D = 0.642$; $H_4 = 5.53ft$; $y_4 = 5.53 + 6ft$ = **11.53 ft** *Ans.(e)*

C10.7 Consider the vee-channel in Fig, C10.7, with an arbitrary angle θ. Make a continuity and momentum analysis of a small disturbance $\delta y << y$, as in Fig. 10.4. Show that the wave propagation speed in this channel is independent of θ and does *not* equal the wide-channel result $c_o = (gy)^{1/2}$.

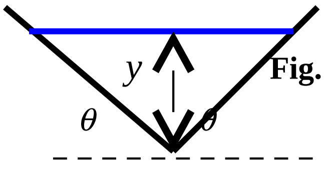
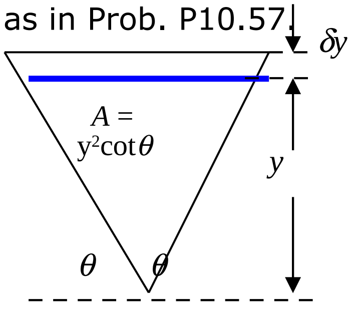


Fig. C10.7

Solution: NOTE: This derivation is heavy algebra, but the result is rewarding. A much easier derivation is to use critical-flow concepts, as in Prob. P10.57. Figure 10.4 is exactly applicable, except that the areas on either side of the wave are *triangles*, not rectangles.



$$\text{Continuity: } \rho C y^2 \cot\theta = \rho(c - \delta V)(y + \delta y)^2 \cot\theta,$$

$$\text{or: } \quad \delta V = c\,\frac{2y\delta y + \delta y^2}{y^2 + 2y\delta y + \delta y^2} \approx 2c\frac{\delta y}{y}$$

Momentum: Note: The centroid of the triangle is ***one-third*** of the way down from the surface:

$$\Sigma F_x = \frac{1}{3}\rho g y(y^2\cot\theta) - \frac{1}{3}\rho g(y + \delta y)[(y + \delta y)^2\cot\theta] = \rho g y^2 \cot\theta(c - c - \delta V)$$

$$\text{Cancel } \rho \text{ and } \cot\theta\text{: } \quad \frac{1}{3}g(3y^2\delta y + 3y\,\delta y^2 + \delta y^3) = c^2 y^2\,\delta V \approx \frac{1}{3}g(3y^2\delta y + \ldots)$$

$$\text{Substitute } \delta V \text{ from continuity: } \quad c = \sqrt{\frac{gy}{2}} \qquad \textit{Ans.}$$

Compare this with the rectangular-channel relation $c = (gy)^{1/2}$ from Eq. (10.10). The density and vee-angle do not appear, but the triangular shape changes the relationship.

Chapter 11 • Turbomachinery

11.1 Describe the geometry and operation of a human peristaltic PDP which is cherished by every romantic person on earth. How do the two ventricles differ?

Solution: Clearly we are speaking of the *human heart,* driven periodically by travelling compression of the heart walls. One ventricle serves the brain and the rest of one's extremities, while the other ventricle serves the lungs and promotes oxygenation of the blood. *Ans.*

11.2 What would be the technical classification of the following turbomachines:

(a) a household fan = **an axial flow fan**. *Ans.* (a)
(b) a windmill = **an axial flow turbine**. *Ans.* (b)
(c) an aircraft propeller = **an axial flow fan**. *Ans.* (c)
(d) a fuel pump in a car = **a positive displacement pump (PDP)**. *Ans.* (d)
(e) an eductor = **a liquid-jet-pump** (special purpose). *Ans.* (e)
(f) a fluid coupling transmission = **a double-impeller energy transmission device**. *Ans.* (f)
(g) a power plant steam turbine = **an axial flow turbine**. *Ans.* (g)

11.3 A PDP can deliver almost any fluid, but there is always a limiting very-high *viscosity* for which performance will deteriorate. Can you explain the probable reason?

Solution: High-viscosity fluids take a long time to enter and fill the inlet cavity of a PDP. Thus a PDP pumping high-viscosity liquid should be run slowly to ensure filling. *Ans.*

11.4 An interesting turbomachine is the *torque converter* [58], which combines both a pump and a turbine to change torque between two shafts. Do some research on this concept and describe it, with a report, sketches and performance data, to the class.

Solution: As described, for example, in Ref. 58, the torque converter transfers torque T from a pump runner to a turbine runner such that $\omega_{\text{pump}} T_{\text{pump}} \approx \omega_{\text{turbine}} T_{\text{turbine}}$. Maximum efficiency occurs when the turbine speed ω_{turbine} is approximately one-half of the pump speed ω_{pump}. *Ans.*

11.5 What type of pump is shown in Fig. P11.5? How does it operate?

Fig. P11.5

Solution: This is a *flexible-liner* pump. The rotating eccentric cylinder acts as a "squeegee." *Ans.*

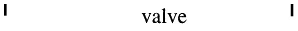


Fig. P11.6

11.6 Fig. P11.6 shows two points a half-period apart in the operation of a pump. What type of pump is this? How does it work? Sketch your best guess of flow rate versus time for a few cycles.

Solution: This is a **diaphragm** pump. As the center rod moves to the right, opening A and closing B, the check valves allow A to fill and B to discharge. Then, when the rod moves to the left, B fills and A discharges. Depending upon the exact oscillatory motion of the center rod, the flow rate is fairly steady, being higher when the rod is faster. *Ans.*

11.7 A piston PDP has a 5-in diameter and a 2-in stroke and operates at 750 rpm with 92% volumetric efficiency. (a) What is the delivery, in gal/min? (b) If the pump delivers SAE 10W oil at 20°C against a head of 50 ft, what horsepower is required when the overall efficiency is 84%?

Solution: For SAE 10W oil, take $\rho \approx 870\ \text{kg/m}^3 \approx 1.69\ \text{slug/ft}^3$. The volume displaced is

$$\upsilon = \frac{\pi}{4}(5)^2(2) = 39.3\ \text{in}^3,$$

$$\left(\quad \text{in}^3 \right)\left(1\ \text{gal} \right)\left(\quad \text{strokes} \right)$$

or: $Q \approx$ **117 gal/min** *Ans.* (a)

$$\text{Power} = \rho gQH = 1.65(32.2)\left(449\ \text{ft}^3/\text{s}\right)(30) = 846\ \frac{\text{ft}\cdot\text{lbf}}{} \div 550 \approx \mathbf{1.54\ hp} \quad \textit{Ans. (b)}$$

11.8 A centrifugal pump delivers 550 gal/min of water at 20°C when the brake horsepower is 22 and the efficiency is 71%. (a) Estimate the head rise in ft and the pressure rise in psi. (b) Also estimate the head rise and horsepower if instead the delivery is 550 gal/min of *gasoline* at 20°C.

Solution: (a) For water at 20°C, take $\rho \approx 998\ \text{kg/m}^3 \approx 1.94\ \text{slug/ft}^3$. The power relation is

$$\text{Power} = 22(550) = 12100\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} = \frac{\rho gQH}{\eta} = \frac{(62.4)\left(\dfrac{550}{449}\ \dfrac{\text{ft}^3}{\text{s}}\right)H}{0.71},$$

$$\text{Pressure rise}\ \ \Delta p = \rho gH = (62.4)(112) = 7011\ \text{psf} \div 144 \approx \mathbf{49\ psi} \quad \textit{Ans.}\ \text{(a)}$$

(b) For gasoline at 20°C, take $\rho \approx 680\ \text{kg/m}^3 \approx 1.32\ \text{slug/ft}^3$. If viscosity (Reynolds number) is not important, the operating conditions (flow rate, impeller size and speed) are exactly the same and hence the head is the same and the power scales with the density:

$$H \approx \mathbf{112\ ft}\ \text{(of } \textit{gasoline}\text{)}; \quad \text{Power} = P_{\text{water}}\frac{\rho_{\text{gasoline}}}{} = 22\left(\frac{680}{998}\right) \approx \mathbf{15\ hp} \quad \textit{Ans.}\ \text{(b)}$$

11.9 Figure P11.9 shows the measured performance of the Vickers Inc. Model PVQ40 piston pump when delivering SAE 10W oil at 180°F ($\rho \approx 910\ \text{kg/m}^3$). Make some general observations about these data vis-à-vis Fig. 11.2 and your intuition about PDP behavior.

Solution: The following are observed:

(a) The discharge Q is almost linearly proportional to speed Ω and slightly less for the higher heads (H or Δp).

(b) The efficiency (volumetric or overall) is nearly independent of speed Ω and again slightly less for high Δp.

(c) The power required is linearly proportional to the speed Ω and also to the head H (or Δp). *Ans*.

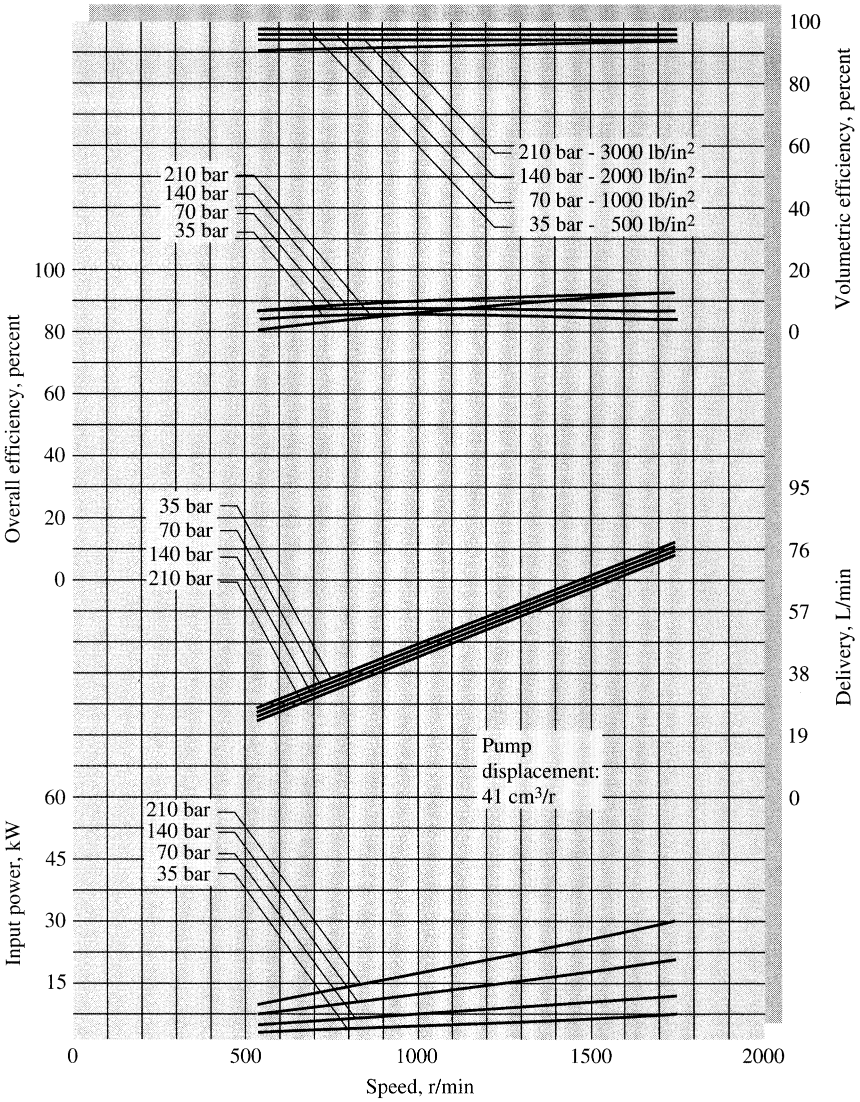


Fig. P11.9

11.10 Suppose that the pump of Fig. P11.9 is run at 1100 r/min against a pressure rise of 210 bar. (a) Using the measured displacement, estimate the theoretical delivery in gal/min. From the chart, estimate (b) the actual delivery; and (c) the overall efficiency.

Solution: (a) From Fig. P11.9, the pump displacement is 41 cm^3. The theoretical delivery is

$$Q = \left(1100\ \frac{\mathrm{r}}{\cdot}\right)\left(41\ \frac{\mathrm{cm}^3}{\cdot}\right) = 45100\ \frac{\mathrm{cm}^3}{\cdot} = 45\ \frac{\mathrm{L}}{\cdot} = \mathbf{11.9}\ \frac{\mathbf{gal}}{\cdot} \quad \textit{Ans.}\ \text{(a)}$$

(b) From Fig. P11.9, at 1100 r/min and $\Delta p = 210$ bar, read

$$Q \approx 47\text{ L/min} \approx \mathbf{12\ gal/min}. \quad \textit{Ans.}\text{ (b)}$$

(c) From Fig. P11.9, at 1100 r/min and $\Delta p = 210$ bar, read $\boldsymbol{\eta_{\text{overall}} \approx 87\%}$. *Ans*. (c)

11.11 A pump delivers 1500 L/min of water at 20°C against a pressure rise of 270 kPa. Kinetic and potential energy changes are negligible. If the driving motor supplies 9 kW, what is the overall efficiency?

Solution: With pressure rise given, we don't need density. Compute "water" power:

$$P_{water} = \rho g Q H = Q\Delta p = \left(\frac{1.5}{60}\ \frac{\text{m}^3}{\text{s}}\right)\left(270\ \frac{\text{kN}}{\text{m}^2}\right) = 6.75\text{ kW}, \quad \therefore\ \eta = \frac{6.75}{9.0} = \mathbf{75\%} \quad \textit{Ans.}$$

11.12 In a test of the pump in the figure, the data are: $p_1 = 100$ mmHg (vacuum), $p_2 = 500$ mmHg (gage), $D_1 = 12$ cm, and $D_2 = 5$ cm. The flow rate is 180 gal/min of light oil (SG = 0.91). Estimate (a) the head developed; and (b) the input power at 75% efficiency.

Fig. P11.12

Solution: Convert 100 mmHg = 13332 Pa, 500 mmHg = 66661 Pa, 180 gal/min = 0.01136 m^3/s. Compute $V_1 = Q/A_1 = 0.01136/[(\pi/4)(0.12)^2] = 1.00$ m/s. Also, $V_2 = Q/A_2 = 5.79$ m/s. Calculate $\gamma_{oil} = 0.91(9790) = 8909$ N/m^3. Then the head is

$$H = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - \frac{p_1}{\gamma} - \frac{V_1^2}{2g} - z_1$$

$$= \frac{66661}{8909} + \frac{(5.79)^2}{2(9.81)} + 0.65 - \frac{-13332}{8909} - \frac{(1.00)^2}{2(9.81)} - 0, \quad \text{or:} \quad \mathbf{H = 11.3\ m} \quad \textit{Ans.}\text{ (a)}$$

$$Power = \frac{\gamma Q H}{\eta} = \frac{8909(0.01136)(11.3)}{0.75} = \mathbf{1520\ W} \quad \textit{Ans.}\text{ (b)}$$

P11.13 A 3.5 hp pump delivers 1140 lbf of ethylene glycol at 20°C in 12 seconds, against a head of 17 ft. Calculate the efficiency of the pump.

Solution: From Table A.3 for ethylene glycol, ρ = 1117 kg/m^3 = 2.167 slug/ft^3. The specific weight is thus γ= (2.167)(32.2) = 69.8 lbf/ft^3. We then may calculate the volume flow rate:

$$Q = \frac{\dot{w}}{\gamma} = \frac{(1140\,lbf/12s)}{69.8\ lbf/ft^3} = 1.36\,\frac{ft^3}{s}$$

This gives us enough information to calculate the pump efficiency:

$$P = (3.5hp)(550\frac{ft-lbf}{hp}) = 1925\frac{ft-lbf}{s} = \frac{\gamma QH}{\eta} = \frac{(69.8lbf/ft^3)(1.36ft^3/s)(17\,ft)}{\eta}$$

$$\text{Solve for} \quad \eta = \frac{1614}{1925} = 0.84 \quad \text{or} \quad \mathbf{84\%} \quad \textit{Ans.}$$

11.14 A pump delivers gasoline at 20°C and 12 m^3/h. At the inlet, p_1 = 100 kPa, z_1 = 1 m, and V_1 = 2 m/s. At the exit p_2 = 500 kPa, z_2 = 4 m, and V_2 = 3 m/s. How much power is required if the motor efficiency is 75%?

Solution: For gasoline, take ρg ≈ 680(9.81) = 6671 N/m^3. Compute head and power:

$$H = \frac{p_2}{} + \frac{V_2^2}{2} + z_2 - \frac{p_1}{} - \frac{V_1^2}{2} - z_1 = \frac{500000}{6671} + \frac{(3)^2}{2(9.81)} + 4 - \frac{100000}{6671} - \frac{(2)^2}{2(9.81)} - 1,$$

$$\text{or:} \quad H \approx 63.2\text{ m} \quad \text{Power} = \frac{\rho gQH}{} = \frac{6671\left(\frac{12}{3600}\right)(63.2)}{} \approx \mathbf{1870\ W} \quad \textit{Ans.}$$

11.15 A lawn sprinkler can be used as a simple turbine. As shown in Fig. P11.15, flow enters normal to the paper in the center and splits evenly into $Q/2$ and V_{rel} leaving each nozzle. The arms rotate at angular velocity ω and do work on a shaft. Draw the velocity diagram for this turbine. Neglecting friction, find an expression for the power delivered to the shaft. Find the rotation rate for which the power is a maximum.

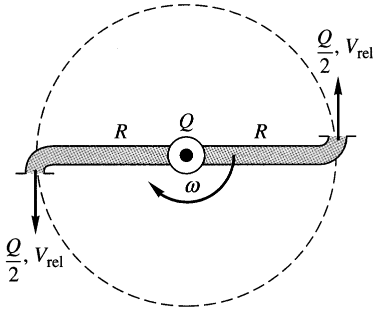
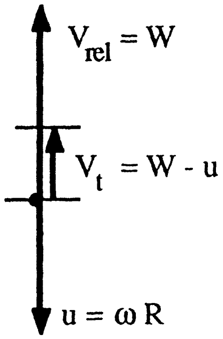


Fig. P11.15



Solution: Utilizing the velocity diagram at right, we apply the Euler turbine formula:

$$P = \rho Q(u_2 V_{t2} - u_1 V_{t1}) = \rho Q[u(W - u) - 0]$$

$$\text{or:} \quad \mathbf{P = \rho Q \omega R(V_{rel} - \omega R)} \quad \textit{Ans.}$$

$$\frac{dP}{d\omega} = \rho Q(V_{rel} - 2u) = 0 \quad \text{if} \quad \omega = \frac{\mathbf{V_{rel}}}{\mathbf{2R}} \quad \textit{Ans.}$$

where $P_{max} = \rho Qu(2u - u) = \rho Q(\omega R)^2$

P11.16 The centrifugal pump in Fig. P11.16 has $r_1 = 15$ cm, $r_2 = 25$ cm, $b_1 = b_2 = 6$ cm, and rotates *counterclockwise* at 600 r/min. A sample blade is shown. Assume $\alpha_1 = 90°$. Estimate the theoretical flow rate and head produced, for water at 20°C, and comment.

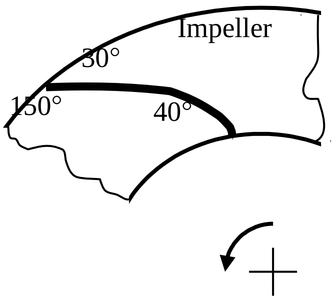


Fig. P11.16

Solution: For water, take $\rho = 998$ kg/m^3. For counterclockwise rotation, this is a *forward-facing* impeller, that is, looking at the figure, $\beta_2 = 150°$, not 30°. If $\alpha_1 = 90°$, then $V_{t1} = 0$. Convert 600 r/min × 2π/60 = 62.8 rad/s. Evaluate $u_1 = \omega r_1 = (62.8)(0.15) =$ 9.425 m/s. Then

$$V_{n1} = u_1 \tan(\beta_1) = (9.425 m/s)\tan(40^o) = 7.91\ m/s$$

And $Q = 2\pi r_1 b_1 |V_{n1}| = 2\pi(0.15m)(0.06m)(7.91m/s) = 0.447\ \frac{m^3}{s} \approx \mathbf{7100}\ \frac{\mathbf{gal}}{\mathbf{min}}$ *Ans.*

Evaluate $u_2 = \omega r_2 = (62.8)(0.25) = 15.71$ m/s. From Eq. (3.18), the head produced is

$$H = \frac{u_2^2}{g} - \frac{u_2 \cot\beta_2\, Q}{2\pi r_2 b_2 g} = \frac{(15.71)^2}{2(9.81)} - \frac{(15.71)\cot(150^o)(0.447)}{2\pi(0.25)(0.06)(9.81)} = 25.15 - (-13.15) = \mathbf{38.3\,m} \quad \textit{Ans.}$$

The water power is $P = \rho g Q H = (998)(9.81)(0.447)(38.3) = 168{,}000$ ft-lbf/s ≈ 305 hp.

11.17 A centrifugal pump has $d_1 = 7$ in, $d_2 = 13$ in, $b_1 = 4$ in, $b_2 = 3$ in, $\beta_1 = 25°$, and $\beta_2 =$ 40° and rotates at 1160 r/min. If the fluid is gasoline at 20°C and the flow enters the blades radially, estimate the theoretical (a) flow rate in gal/min, (b) horsepower, and (c) head in ft.

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3. Compute ω = 1160 rpm = 121.5 rad/s.

$V_{n1} = u_1 \tan\beta_1 = 35.4 \tan 25° \approx 16.5$ ft/s

(12)(12)

$\approx \mathbf{10}\ \frac{\mathbf{ft^3}}{\ }$ *Ans.* (a)

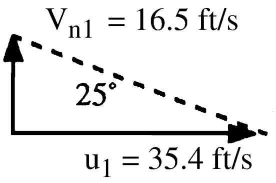
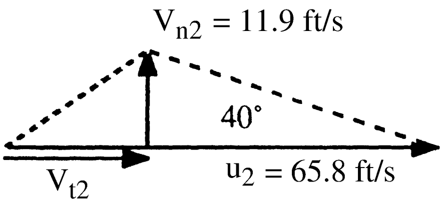


Fig. P11.17



Finally, $\mathbf{P_{ideal}} = \rho Q u_2 V_{t2} = 1.32(10.0)(65.8)(51.7) = 44900 \div 550 \approx$ **82 hp**. *Ans.* (b)
Theoretical head $\mathbf{H} = P/(\rho g Q) = 44900/[1.32(32.2)(10.0)] \approx$ **106 ft**. *Ans.* (c)

11.18 A jet of velocity V strikes a vane which moves to the right at speed V_c, as in Fig. P11.18. The vane has a turning angle θ. Derive an expression for the power delivered to the vane by the jet. For what vane speed is the power maximum?

Fig. P11.18

Solution: The jet approaches the vane at relative velocity $(V - V_c)$. Then the force is

$F = \rho A(V - V_c)^2(1 - \cos\theta)$, and Power $= FV_c = \boldsymbol{\rho A V_c (V - V_c)^2 (1 - \cos\theta)}$ *Ans.* (a)

Maximum power occurs when $\frac{dP}{dV_c} = 0,$

$$\text{or:}\quad \mathbf{V_c = \frac{1}{2} V_{jet}} \quad \textit{Ans. (b)} \quad \left(P = \frac{4}{27}\rho A V^3[1 - \cos\theta]\right)$$

11.19 A centrifugal water pump has r_2 = 9 in, b_2 = 2 in, and β_2 = 35° and rotates at 1060 r/min. If it generates a head of 180 ft, determine the theoretical (a) flow rate in gal/min and
(b) horsepower. Assume near-radial entry flow.

Solution: For water take ρ = 1.94 slug/ft³. Convert ω = 1060 rpm = 111 rad/s. Then

$$u_2 = \omega r_2 = 111\left(\frac{9}{12}\right) = 83.3\ \frac{ft}{s};$$

$$\text{or:}\quad P = 62.4QH = 1.94Q(83.3)\left[83.3 - \frac{Q}{2\pi(9/12)(2/12)}\cot 35^\circ\right]\quad \text{with}\ \ H = 180$$

$$\text{Solve for}\quad \mathbf{Q} = 7.5\ \text{ft}^3/\text{s} \approx \mathbf{3360\ gal/min}\quad \textit{Ans.}\ \text{(a)}$$

With Q and H known, $\mathbf{P} = \rho gQH = 62.4(7.5)(180) \div 550 \approx \mathbf{153\ hp}$. *Ans.* (b)

11.20 Suppose that Prob. 11.19 is reversed into a statement of the theoretical power P = 153 hp. Can you then compute the theoretical (a) flow rate; and (b) head? Explain and resolve the difficulty that arises.

Solution: With power known, the basic theory becomes quadratic in flow rate:

$$u_2 = 83.3\ \frac{\text{ft}}{\text{s}},\quad P = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)$$

$$\text{Clean up: } Q^2 - 45.8Q + 287 = 0,\quad \text{two roots: } Q_1 = \mathbf{7.5}\ \frac{\mathbf{ft^3}}{};\quad Q_2 = \mathbf{38.3}\ \frac{\mathbf{ft^3}}{}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{These correspond to}\ \ H_1 = \mathbf{180\ ft};\quad H_2 = \mathbf{35\ ft}\quad \textit{Ans.}\ \text{(b)}$$

So the ideal pump theory admits to two valid combinations of Q and H which, for the given geometry and speed, give the theoretical power of 153 hp. Prob. 11.19 was solution **1**.

11.21 The centrifugal pump of Fig. P11.21 develops a flow rate of 4200 gpm with gasoline at 20°C and near-radial absolute inflow. Estimate the theoretical (a) horsepower; (b) head rise; and (c) appropriate blade angle at the inner radius.

Solution: For gasoline take $\rho \approx 1.32$ slug/ft^3.

Convert

Q = 4200 gal/min = 9.36 ft^3/s and ω = 1750 rpm = 183 rad/s. Note r_2 = 6 in and β_2 = 30°. The ideal power is computed as

Fig. P11.21

$$P = \rho Q u_2\left[u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right], \quad \text{where } u_2 = \omega r_2 = 183\left(\frac{6}{12}\right) \approx 91.6 \text{ ft/s.} \quad \text{Plug in:}$$

$$P = 1.32(9.36)(91.6)\left[91.6 - \frac{9.36}{2\pi(6/12)(3/12)}\cot 30^\circ\right] = 80400 \div 550 \approx \mathbf{146\ hp} \quad \textit{Ans. (a)}$$

$$H = \frac{P}{\rho g Q} = \frac{80400}{1.32(32.2)(9.36)} \approx \mathbf{202\ ft} \quad \textit{Ans. (b)}$$

Compute $V_{n1} = Q/[2\pi r_1 b_1] = 9.36/[2\pi(4/12)(3/12)] \approx 17.9$ ft/s, $u_1 = \omega r_1 = 183(4/12) \approx 61.1$ ft/s. For purely radial inflow, $\boldsymbol{\beta_1} = \tan^{-1}(17.9/61.1) \approx \mathbf{16^\circ}$. *Ans.* (c)

11.22 A 37-cm-diameter centrifugal pump, running at 2140 rev/min with water at 20°C produces the following performance data:

Q, m^3/s:	0.0	0.05	0.10	0.15	0.20	0.25	0.30
H, m:	105	104	102	100	95	85	67
P, kW:	100	115	135	171	202	228	249
η:	**0%**	**44%**	**74%**	**86%**	**92%**	**91%**	**79%**

(a) Determine the best efficiency point. (b) Plot C_H versus C_Q. (c) If we desire to use this same pump family to deliver 7000 gal/min of kerosene at 20°C at an input power of 400 kW, what pump speed (in rev/min) and impeller size (in cm) are needed? What head will be developed?

Solution: Efficiencies, computed by $\eta = \rho gQH/\text{Power}$, are listed above. The best efficiency point (BEP) is approximately **92% at Q = 0.2 m^3/s**. *Ans.* (a)

The dimensionless coefficients are $C_Q = Q/(nD^3)$, where n = 2160/60 = 36 rev/s and D = 0.37 m, plus $C_H = gH/(n^2D^2)$ and $C_P = P/(\rho n^3D^5)$, where ρ_{water} = 998 kg/m^3. BEP values are $C_Q^* = 0.111$, $C_H^* = 5.35$, and $C_P^* = 0.643$. **A plot of C_H versus C_Q is below**. The values are similar to Fig. 11.8 of the text. *Ans.* (b)

(c) For kerosene, $\rho_k = 804$ kg/m^3. Convert 7000 gal/min = 0.442 m^3/s. At BEP, we require the above values of dimensionless parameters:

$$\frac{Q}{nD^3} = \frac{0.442}{nD^3} = 0.111; \quad \frac{P}{\rho n^3 D^5} = \frac{400000}{804 n^3 D^5} = 0.643;$$

$$\text{Solve} \quad n = 26.1\ \frac{\text{rev}}{\text{s}} = \mathbf{1560}\ \frac{\mathbf{rev}}{\mathbf{min}}; \quad D = \mathbf{0.534\ m} \quad \textit{Ans.}\ \text{(c)}$$

$$\text{Also,} \quad H^* = C_H^*(n^2D^2)/g = 5.35(26.1)^2(0.534)^2/9.81 = \mathbf{106\ m} \quad \textit{Ans.}\ \text{(c)}$$

P11.23 At what speed, when pumping water, should the 41.5-inch pump of Fig. 11.7*b* be run to realize, at highest efficiency, (*a*) a head of 100 ft? For this condition, what are the resulting (*b*) flow rate, and (*c*) brake horsepower?

Solution: This is the same pump, same fluid, hence $\rho_1 = \rho_2$ and $D_1 = D_2$. Thus the scaling from Eqs. (11.28) is relatively simple. First read the 41.5-inch BEP values as best the writer can:

$$\eta = 89\% \quad ; \quad n_1 = 710\ r/min \ ; \quad H_1 \approx 270\,ft \quad ; \quad Q_1 \approx 21500\ gal/min \quad ; P_1 \approx 1650\ bhp$$

Then get the new rotation rate from the new head, using Eq.(11.28):

$$\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{\cancel{D_2}}{\cancel{D_1}}\right)^2 = \frac{100\,ft}{270\,ft} = \left(\frac{n_2}{710}\right)^2, \quad n_2 = 710\left(\frac{100}{270}\right)^{1/2} = \mathbf{430}\ \frac{r}{min} \quad Ans.(a)$$

The flow rate and power follow in like manner:

$$\frac{Q_2}{Q_1} = \frac{Q_2}{21500\,gpm} = \frac{n_2}{n_1}\left(\frac{\cancel{D_2}}{\cancel{D_1}}\right)^3 = \frac{430}{710}, \quad Q_2 = (21500)\left(\frac{430}{710}\right) \approx \mathbf{13{,}000}\ \frac{gal}{min} \quad Ans.(b)$$

$$\frac{P_2}{P_1} = \frac{P_2}{1650\,bhp} = \left(\frac{\cancel{\rho_2}}{\cancel{\rho_1}}\right)\left(\frac{n_2}{n_1}\right)^3 \left(\frac{\cancel{D_2}}{\cancel{D_1}}\right)^5 = \left(\frac{430}{710}\right)^3, \quad P_2 = (1650)(0.222) = \mathbf{370}\ bhp \quad Ans.(c)$$

These numbers are slightly uncertain because of the difficulty of reading Fig. 11.7*b*.

11.24 Figure P11.24 shows performance data for the Taco, Inc., model 4013 pump. Compute the ratios of measured shutoff head to the ideal value U^2/g for all seven impeller sizes. Determine the average and standard deviation of this ratio and compare it to the average for the six impellers in Fig. 11.7.

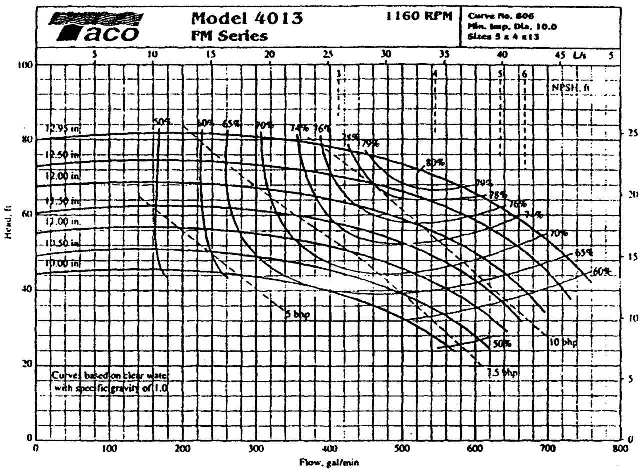


Fig. P11.24 Performance data for a centrifugal pump. (*Courtesy of Taco. Inc., Cranston, Rhode Island.*)

Solution: All seven pumps are run at 1160 rpm = 121 rad/s. The 7 diameters are given. Thus we can easily compute U = ωr = ωD/2 and construct the following table:

D, inches:	10.0	10.5	11.0	11.5	12.0	12.5	12.95
U, ft/s:	50.6	53.1	55.7	58.2	60.7	63.3	65.5
H_o, ft:	44	49	53	61	67	73	80
$H_o/(U^2/g)$:	**0.553**	**0.559**	**0.551**	**0.580**	**0.585**	**0.587**	**0.599**

The average ratio is **0.573** *Ans.* (a), and the standard deviation is **0.019**. *Ans.* (b)

For the six pumps of Fig. 11.7, the average is *0.611*, the standard deviation is *0.025*. These results are true of most centrifugal pumps: we can take an average of *0.60* ± *0.04*.

11.25 At what speed in rpm should the 35-in-diameter pump of Fig. 11.7(b) be run to produce a head of 400 ft at a discharge of 20000 gal/min? What brake horsepower will be required? *Hint*: Fit H(Q) to a formula.

Solution: A curve-fit formula for H(Q) for this pump is H(ft) ≈ 235 – 0.125Q^2, with Q in kgal/min. Then, for constant diameter, the similarity rules predict

U.S. gallons per minute × 1000
(*b*)

Fig. 11.7b

$$H_1 = H_2\left(\frac{n_1}{}\right) \quad \text{and} \quad Q_1 = Q_2\left(\frac{n_1}{}\right), \quad \text{or:} \quad H_1 = 400\left(\frac{710}{}\right) \approx 235 - 0.125\left(20\left(\frac{710}{}\right)\right)$$

$$\text{Solve for} \quad n_2 = \left(\frac{2.27E8}{}\right)^{1/2} \approx \mathbf{980\ rpm} \quad \textit{Ans.}$$

Now backtrack to compute $H_1 \approx$ *210* ft and $Q_1 \approx$ *14500* gal/min. From Fig. 11.7(b) we may then read the power $P_1 \approx$ *900* bhp (or compute this from $\eta_1 \approx 0.85$). Then, by similarity, $P_2 = (n_2/n_1)^3 = 900(980/710)^3 \approx$ **2400 bhp**. *Ans.*

P11.26 Would the smallest, or the largest, of the seven Taco Inc. pumps in Fig. P11.24 be better (*a*) for producing, near best efficiency, a water flow rate of 600 gal/min and a head of 95 ft? (*b*) At what speed, in r/min, should this pump run? (*c*) What input power is required?

Solution: First try the smallest pump, D = 10.00 in. Read Fig. P11.24: Q^* = 390 gal/min, H^* = 41 ft, η_{max} = 65%. To scale up to 600 gpm, use the similarity rules for constant D:

$$\frac{Q_2}{Q_1} = \frac{600}{390\,gpm} = \frac{n_2}{n_1} = \frac{n_2}{1160\,rpm}, \quad \text{solve} \quad n_2 = 1785\,r/min$$

$$\text{Then} \quad \frac{H_2}{H_1} = \frac{H_2}{41\,ft} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{1785}{1160}\right)^2, \quad \text{solve} \quad H_2 = 97\,ft$$

$$Power_2 = \frac{\gamma Q_2 H_2}{\eta} = \frac{(62.4)(600/448.8)(97)}{0.65} = 12450\,\frac{ft-lbf}{s} = 22.6\,hp$$

So the small pump will do the job, using 22.6 hp. Next try the largest pump, D = 12.95 in. Read Fig. P11.24: Q^* = 525 gal/min, H^* = 72 ft, η_{max} = 80%. Again scale up to 600 gpm, using the similarity rules at constant diameter:

$$\frac{Q_2}{Q_1} = \frac{600}{525\,gpm} = \frac{n_2}{n_1} = \frac{n_2}{1160\,rpm}, \quad \text{solve} \quad n_2 = 1325\,r/min$$

$$\text{Then} \quad \frac{H_2}{H} = \frac{H_2}{72\,ft} = \left(\frac{n_2}{n}\right)^2 = \left(\frac{1325}{1160}\right)^2, \quad \text{solve} \quad H_2 = 94\,ft$$

So the largest pump *is **better to do the job***, with slower speed and less power. *Ans*. (*a*) The proper large-pump speed is **1325 r/min**. *Ans*. (*b*). The power is **17.8 hp**. *Ans*. (*c*) [NOTE that if we specify a different head, like 200 ft, neither pump can do the job.]

11.27 The 12-in pump of Fig. P11.24 is to be scaled up in size to provide a head of 90 ft and a flow rate of 1000 gal/min at BEP. Determine the correct (a) impeller diameter; (b) speed in rpm; and (c) horsepower required.

Solution: From the chart, at BEP, Q ≈ 480 gpm, H ≈ 60 ft, and $\eta \approx 76.5\%$, from which

$$P = \frac{\rho g Q H}{\eta} = \frac{(62.4)(480/449)(60)}{0.765} \approx 5234\ \frac{ft\cdot lbf}{s},$$

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{480/449}{} \approx 0.0553; \quad C_H^* = \frac{gH^*}{n^2D^2} = \frac{(32.2)(60)}{} \approx 5.17$$

$$\text{Larger pump, } H = 90,\ Q = 1000:\quad \frac{32.2(90)}{} \approx 5.17, \quad \frac{1000/449}{} \approx 0.0553$$

Solve for $D \approx 1.305$ ft ≈ **15.7 in** *Ans.* (a) and $n \approx 18.15$ rps ≈ **1090 rpm** *Ans.* (b)

$$P^* = C_P^* \rho n^3 D^5 = 0.373(1.94)(18.15)^3(1.305)^5 \approx 16300\ \frac{ft\cdot lb}{} \div 550 \approx \mathbf{30\ bhp} \quad \textit{Ans. (c)}$$

11.28 Tests by the Byron Jackson Co. of a 14.62-in centrifugal water pump at 2134 rpm yield the data below. What is the BEP? What is the specific speed? Estimate the maximum discharge possible.

Q, ft^3/s:	0	2	4	6	8	10
H, ft:	340	340	340	330	300	220
bhp:	135	160	205	255	330	330

Solution: The efficiencies are computed from $\eta = \rho g Q H/(550\text{ bhp})$ and are as follows:

Q:	0	2	4	**6**	8	10
η:	0	0.482	0.753	**0.881**	0.825	0.756

Thus the BEP is, even without a plot, close to **Q ≈ 6 ft^3/s**. *Ans.* The specific speed is

$$N_s \approx \frac{nQ^{*1/2}}{} = \frac{2134[(6)(449)]^{1/2}}{} \approx \mathbf{1430} \quad \textit{Ans.}$$

For estimating Q_{max}, the last three points fit a Power-law to within ±0.5%:

$$H \approx 340 - 0.00168Q^{4.85} = 0 \quad \text{if} \quad Q \approx \mathbf{12.4}\ \frac{\mathbf{ft^3}}{} = Q_{max} \quad \textit{Ans.}$$

11.29 If the scaling laws are applied to the Byron Jackson pump of Prob. 11.28 for the same impeller diameter, determine (a) the speed for which the shut-off head will be 280 ft; (b) the speed for which the BEP flow rate will be 8.0 ft^3/s; and (c) the speed for which the BEP conditions will require 80 horsepower.

Solution: From the table in Prob. 11.28, the shut-off head at 2134 rpm is 340 ft. Thus

$$\text{If } D_1 = D_2, \quad n_2 = n_1(H_2/H_1)^{1/2} = 2134(280/340)^{1/2} \approx \mathbf{1940\ rpm} \quad \textit{Ans.}\text{ (a)}$$

$$\text{If } Q_2^* = 8\ \frac{ft^2}{}, \quad n_2 = n_1(Q_2^*/Q_1^*) = 2134(8.0/6.0) \approx \mathbf{2850\ rpm} \quad \textit{Ans.}\text{ (b)}$$

Finally, if $\rho_1 = \rho_2$ (water) and the diameters are the same, then Power ∝ n^3, or, at BEP,

$$n_2 = n_1(P_2^*/P_1^*)^{1/3} = 2134(80/255)^{1/3} \approx \mathbf{1450\ rpm} \quad \textit{Ans.}\text{ (c)}$$

P11.30 A pump, geometrically similar to the 38-in-diameter model in Fig. 11.7*b*, has a diameter of 24 in and is to develop 300 bhp at BEP when pumping *gasoline* (not water). Determine (*a*) the appropriate speed, in r/min; (*b*) the BEP head; and (*c*) the BEP flow rate.

Solution: For water, ρ_1 = 1.94 slug.ft^3. For gasoline, from Table A.3, ρ_2 = 680 kg/m^3 = 1.32 slug.ft^3. From Fig. 11.7*a*, read, as best you can, the BEP properties:

$$P_1 \approx 1250\ bhp\ ;\quad Q_1 \approx 20{,}000\ gal/min\ ;\quad H_1 \approx 225\ ft\ ;\quad n_1 = 710\ rpm$$

(*a*) With the BEP power and the diameter known, the power ratio yields the speed:

$$\frac{P_2}{P_1} = \frac{300\,bhp}{1250\,bhp} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{1.32}{1.94}\right)\left(\frac{n_2}{710}\right)^3\left(\frac{24\,in}{38\,in}\right)^5 \quad , \text{ or :}$$

$$(1.91E-10)\ n_2^3 = 0.24\ , \quad solve \quad n_2 \approx \mathbf{1080\ \frac{r}{min}} \quad \textit{Ans.}(a)$$

(*b*, *c*) With n_2 known, the head and flow rate at BEP follow from the scaling laws:

$$\frac{H_2}{H_1} = \frac{H_2}{225\,ft} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1080}{710}\right)^2\left(\frac{24}{38}\right)^2, \quad H_2 = (225)(0.923) \approx \mathbf{208}\,ft \qquad Ans.(b)$$

$$\frac{Q_2}{Q_1} = \frac{Q_2}{20{,}000\,gpm} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 = \left(\frac{1080}{710}\right)\left(\frac{24}{38}\right)^3, \quad Q_2 = (20000)(0.383) \approx \mathbf{7{,}700}\ \frac{gal}{min} \qquad Ans.(c)$$

These answers have a slight uncertainty due to the difficulty of reading Fig. 11.7*b*.

11.31 A centrifugal pump with backward-curved blades has the following measured performance when tested with water at 20°C:

Q, gal/min:	0	400	800	1200	1600	2000	2400
H, ft:	123	115	108	101	93	81	62
P, hp:	30	36	40	44	47	48	46

(a) Estimate the best efficiency point and the maximum efficiency. (b) Estimate the most efficient flow rate, and the resulting head and brake horsepower, if the diameter is doubled and the rotation speed increased by 50%.

Solution: (a) Convert the data above into efficiency. For example, at $Q = 400$ gal/min,

$$\eta = \frac{\gamma QH}{P} = \frac{(62.4\text{ lbf/ft}^3)(400/448.8\text{ ft}^3\text{/s})(115\text{ ft})}{(36\times 550\text{ ft·lbf/s})} = 0.32 = 32\%$$

When converted, the efficiency table looks like this:

Q, gal/min:	0	400	800	1200	1600	2000	2400
η, %:	0	32%	55%	70%	80%	**85%**	82%

So maximum efficiency of **85%** occurs at ***Q* = 2000 gal/min.** *Ans.* (a)

(b) We don't know the values of C_Q^* or C_H^* or C_P^*, but we can set them equal for conditions 1 (the data above) and 2 (the performance when *n* and *D* are changed):

$$C_Q^* = \frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3} = \frac{Q_2}{(1.5n_1)(2D_1)^3},$$

$$C_H^* = \frac{gH_1}{n_1^2 D_1^2} = \frac{gH_2}{n_2^2 D_2^2} = \frac{gH_2}{(1.5n_1)^2 (2D_1)^2},$$

$$C_P^* = \frac{P_1}{\rho n_1^3 D_1^5} = \frac{P_2}{\rho n_2^3 D_2^5} = \frac{P_2}{\rho (1.5n_1)^3 (2D_1)^5},$$

11.32 The data of Prob. 11.31 correspond to a pump speed of 1200 r/min. (Were you able to solve Prob. 11.31 without this knowledge?) (a) Estimate the diameter of the impeller [*HINT*: See Prob. 11.24 for a clue.]. (b) Using your estimate from part (a), calculate the BEP parameters C_Q^*, C_H^*, and C_P^* and compare with Eq. (11.27). (c) For what speed of this pump would the BEP head be 280 ft?

Solution: **Yes**, we were able to solve Prob. 11.31 by simply using *ratios*.
(a) Prob. 11.24 showed that, for a wide range of centrifugal pumps, the shut-off head $H_o \approx 0.6U^2/g \pm 6\%$, where U is the impeller blade tip velocity, $U = \omega D/2$. Use this estimate with the shut-off head and speed of the pump in Prob. 11.31:

$$\omega = 2\pi(1200 \text{ rpm})/60 = 126 \text{ rad/s}, \quad H_o \approx 123 \text{ ft} = 0.6[(126D/2)^2/32.2 \text{ ft/s}^2]$$

$$\text{Solve for} \quad D \approx 1.29 \text{ ft} \approx \mathbf{15.5\ in} \quad \textit{Ans.}\text{ (a)}$$

(b) With diameter $D \approx 1.29$ ft estimated and speed $n = 1200/60 = 20$ r/s given, we can calculate:

$$C_Q^* = \frac{(2000/448.8)\text{ ft}^3/\text{s}}{(20 \text{ r/s})(1.29 \text{ ft})^3} \approx \mathbf{0.103}; \quad C_H^* = \frac{(32.2 \text{ ft/s}^2)(81 \text{ ft})}{} \approx \mathbf{3.9}$$

$$C_P^* = \frac{(48)(550 \text{ ft}\times\text{lbf/s})}{(1.94 \text{ slug/ft}^3)(20 \text{ r/s})^3(1.29 \text{ ft})^5} \approx \mathbf{0.47} \quad \textit{Ans.}\text{ (b)}$$

(c) Use the estimate of C_H^* to estimate the speed needed to produce 280 ft of head:

$$C_H^* \approx 3.9 = \frac{(32.2\text{ ft/s}^2)(280\text{ ft})}{}, \quad \text{solve for } n = 37\text{ r/s} \approx \mathbf{2230\ r/min} \quad \textit{Ans.}\text{ (c)}$$

11.33 In Prob. P11.31, the pump BEP flow rate is 2000 gal/min, the impeller diameter is 16 in, and the speed is 1200 r/min. Scale this pump with the similarity rules to find (*a*) the diameter and (*b*) the speed that will deliver a BEP water flow rate of 4000 gal/min and a head of 180 ft. (*c*) What brake horsepower will be required for this new condition?

Solution: Take the specific weight of water to be γ= 62.4 lbf/ft^3. Find the efficiency:

$$\eta_1 \approx \eta_2 = \frac{\gamma Q_1 H_1}{} = \frac{(62.4 lbf\,/\,ft^3)(2000/448.8\,ft^3/s)(81\,ft)}{} = 0.853 \quad or \quad 85.3\%$$

Meanwhile use the scaling laws for BEP flow rate and head:

$$\frac{Q_1}{n_1 D_1^3} = \frac{2000}{(1200)(16)^3} = \frac{Q_2}{n_2 D_2^3} = \frac{4000}{n_2 D_2^3}$$

$$\frac{gH_1}{n_1^2 D_1^2} = \frac{(32.2)(81)}{(1200)^2(16)^2} = \frac{gH_2}{n_2^2 D_2^2} = \frac{(32.2)(180)}{n_2^2 D_2^2}$$

Solve simultaneously for D_2 = **18.5 in** *Ans.*(*a*) and n_2 = **1540 r/min** *Ans.*(*b*).

(*c*) Finally, determine the required horsepower (which we could have done above, using η_1):

$$P_2 = \frac{\gamma Q_2 H_2}{\eta_2} = \frac{(62.4)(4000/448.8)(180)}{0.853} = 118{,}000\,\frac{ft-lbf}{s} \div 550 = \ \mathbf{214}\ hp \quad Ans.(c)$$

11.34 Consider a pump geometrically similar to the 9-in-diameter pump of Fig. P11.34 to deliver 1200 gal/min of *kerosene* at 1500 rpm. Determine the appropriate (a) impeller diameter; (b) BEP horsepower; (c) shut-off head; and (d) maximum efficiency.

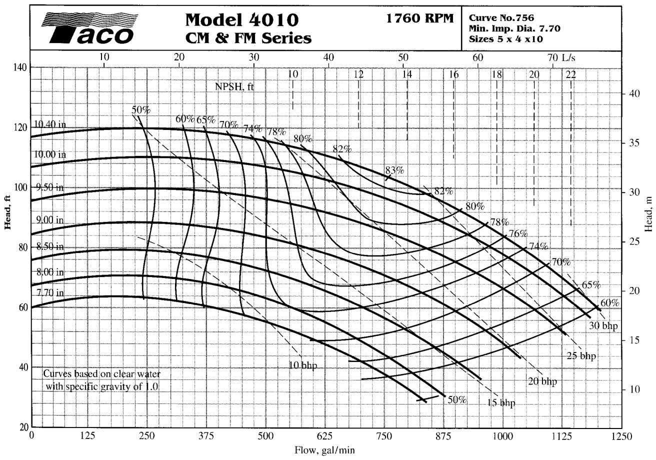


Fig. P11.34 Performance data for a family of centrifugal pump impellers. (*Courtesy of Taco, Inc., Cranston, Rhode Island.*)

Solution: For kerosene, take $\rho \approx 1.56$ slug/ft^3, whereas for water $\rho \approx 1.94$ slug/ft^3. From Fig. P11.34, at BEP, read Q* ≈ 675 gpm, H* ≈ 76 ft, and $\eta_{max} \approx 0.77$. Then

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{675/449}{(1760/60)(9/12)^3} \approx 0.122 = \frac{1200/449}{(1500/60)D^3},$$

$$\text{Shut-off:}\quad \frac{gH_o}{n^2D^2} = \frac{32.2(84\text{ ft})}{(1760/60)^2(9/12)^2} = \frac{32.2H_o}{(1500/60)^2(0.96)^2},$$

Moody: $\frac{1-\eta_2}{1-0.77} \approx \left(\frac{9.0}{11.5}\right)^{1/4}$, solve for $\eta_2 \approx 0.784$ **=78%** *Ans.* (d) (crude estimate)

Fig. P11.34: Read H* ≈ 76 ft, whence $\frac{32.2(76)}{(29.3)^2(0.75)^2} = \frac{32.2\ H^*_{new}}{(25)^2(0.96)^2}$,

Then $P^*_{new} = \frac{1.56(32.2)(1200/449)(90.1)}{0.784} \approx 15440\ \frac{ft\cdot lbf}{s} \div 550 \approx$ **28 bhp** *Ans.* (b)

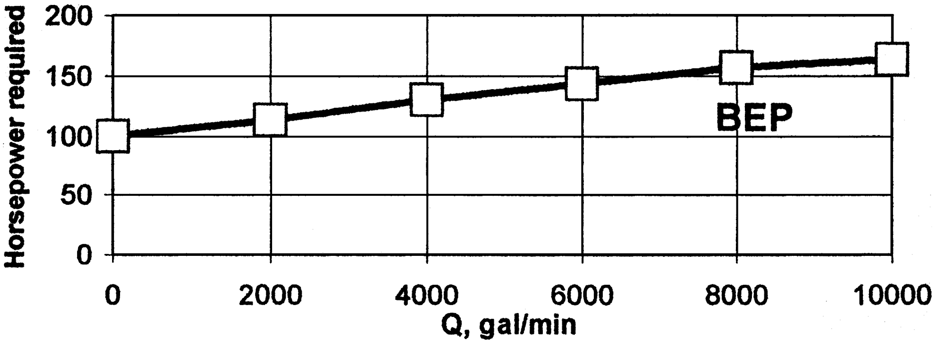
11.35 An 18-in-diameter centrifugal pump, running at 880 rev/min with water at 20°C, generates the following performance data:

Q, gal/min:	0.0	2000	4000	6000	8000	10000
H, ft:	92	89	84	78	68	50
P, hp:	100	112	130	143	156	163
η:	0%	40%	65%	83%	**88%**	78%

Determine (a) the BEP; (b) the maximum efficiency; and (c) the specific speed. (d) Plot the required input power versus the flow rate.

Solution: We have computed the efficiencies and listed them. The BEP is the next-to-last point: **Q = 8000 gal/min, η_{max} = 88%**. *Ans.* (a, b) The specific speed is $N'_s = nQ^{*1/2}/(gH^*)^{3/4} = (880/60)(8000/448.83)^{1/2}/[32.2(68)]^{3/4} \approx$ **0.193**, or **N_s = 3320** (probably a centrifugal pump). *Ans.* (c)

The plot of input horsepower versus flow rate is shown below—there are no surprises in this plot. *Ans.* (d)



P11.36 The pump of Prob. P11.35 has a maximum efficiency of 88% at 8000 gal/min. (*a*) Can we use this pump, at the same diameter but a different speed, to generate a BEP head of 150 ft and a BEP flow rate of 10,000 gal/min? (*b*) If not, what diameter is appropriate?

Solution: We are still pumping *water*, ρ = 1.94 slug/ft^3. Try scaling laws for head and then for flow:

$$H_1 = 68\,ft \;;\; Q_1 = 8000\,\frac{gal}{min} = 17.8\,\frac{ft^3}{s} \;;\; P_1 = 156\,bhp \;;\; D_1 = 1.5\,ft \;;\; n_1 = 880\,rpm = 14.7\,\frac{r}{s}$$

$$D_1 = D_2:\; \frac{H_2}{H_1} = \frac{150}{68} = \left(\frac{n_2}{14.7}\right)^2 \;,\; n_2 = 21.8\,\frac{r}{s} = 1307\,\frac{r}{min} \quad \text{(from the head)}$$

$$D_1 = D_2:\; \frac{Q_2}{Q_1} = \frac{10000}{8000} = \left(\frac{n_2}{14.7}\right) \;,\; n_2 = 18.3\frac{r}{s} = 1100\,\frac{r}{min} \quad \text{(from the flow rate)}$$

These rotation rates are not the same. **Therefore we must change the diamete**r. Ans.(*a*)

(*b*) Allow for a different diameter in both head and flow rate scaling:

$$\frac{H_2}{H_1} = \frac{150\,ft}{68\,ft} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{n_2}{14.67\,r/s}\right)^2\left(\frac{D_2}{1.5\,ft}\right)^2 \;,\quad or:\; n_2^2 D_2^2 = 1068$$

$$\frac{Q_2}{Q_1} = \frac{10,000\,gpm}{8,000\,gpm} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 = \left(\frac{n_2}{14.67}\right)\left(\frac{D_2}{1.5\,ft}\right)^3 \;,\quad or:\; n_2 D_2^3 = 61.88$$

Solve simultaneously to obtain

$$n_2 = 16.0\; r/s = \textbf{960 rpm} \;;\; D_2 = 2.04\,ft = \textbf{24.5 inches} \qquad Ans.(b)$$

The power required increases to 380 bhp.

11.37 Consider the two pumps of Problems P11.28 and P11.35, respectively. If the diameters are not changed, which is more efficient for delivering water at 3000 gal/min and a head of 400 ft? What is the appropriate rotation speed for the better pump?

Solution: Unless we are brilliant, what can we do but try them both? The scaling law for constant diameter states that Q is proportional to n, and H is proportional to n^2. The pump of Prob. P11.28 has BEP at Q^* = 6 ft^3/s and H^* = 330 ft. Scale from there:

$$\frac{Q_2}{Q_1} = \frac{3000}{6\text{x}448.83} = \frac{n_2}{n_1} = \frac{n_2}{2134\,r/\text{min}}, \quad Solve \quad n_2 \approx 2380\,r/min$$

$$\frac{H_2}{H_1} = \frac{H_2}{330\,ft} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{2380}{2134}\right)^2, \quad Solve \quad H_2 \approx 410\,ft$$

It seems that the **Prob. P11.28 pump can do the job quite well**. *Answer*.

Try the Prob. P11.35 pump, whose BEP is at Q* = 8000 gal/min and *H** = 68 ft. Then

$$\frac{Q_2}{Q_1} = \frac{3000}{8000} = \frac{n_2}{n_1} = \frac{n_2}{880\, r/\min}, \quad \textit{Solve} \quad n_2 \approx 330\, r/min$$

This is not just *worse*, it is ridiculous – too slow, almost no head, totally inefficient.

11.38 A 6.85-in pump, running at 3500 rpm, has the measured performance at right for water at 20°C. (a) Estimate the horsepower at BEP. If this pump is rescaled in water to provide 20 bhp at 3000 rpm, determine the appropriate (b) impeller diameter; (c) flow rate; and (d) efficiency for this new condition.

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	**81**	79	74

Solution: The BEP of 81% is at about Q = 350 gpm and H = 169 ft. Hence the power is

$$P^* = \frac{\rho g Q^* H^*}{\eta} = \frac{62.4(350/449)(169)}{0.81} = 10150\ \frac{ft\cdot lbf}{s} \div 550 \approx \mathbf{18.5\ bhp} \quad \textit{Ans.}\ (a)$$

If the new conditions are 20 hp at *n* = 3000 rpm = 50 rps, we equate power coefficients:

$$C_P^* = \frac{10150}{1.94(3500/60)^3(6.85/12)^5} = 0.435 \overset{?}{=} \frac{20\times 550}{1.94(50)^3 D^5},$$

With diameter known, the flow rate is computed from BEP flow coefficient:

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{350/449}{(3500/60)(6.85/12)^3} = 0.0719 \overset{?}{=} \frac{Q^*}{50(0.636)^3},$$

$$\text{Solve} \quad Q^* = 0.926\ ft^3/s \approx \mathbf{415\ gal/min} \quad \textit{Ans.}\ (c)$$

Finally, since $D_1 \approx D_2$, we can assume the same maximum efficiency: **81%**. *Ans.* (d)

11.39 The Allis-Chalmers D30LR centrifugal compressor delivers 33,000 ft^3/min of SO_2 with a pressure change from 14.0 to 18.0 lbf/in^2 absolute using an 800-hp motor at 3550 r/min. What is the overall efficiency? What will the flow rate and Δp be at 3000 r/min? Estimate the diameter of the impeller.

Solution: For SO_2, take $M = 64.06$, hence $R = 49720/64.06 \approx 776$ ft·lbf/(slug·°R). Then

$$\Delta p = (18-14)(144) = 576 \text{ psf}, \quad \text{Power} = Q\Delta p = 576\left(\frac{33000}{60}\right) \div 550 \approx 576 \text{ hp delivered}$$

$$\text{Then} \quad \eta = P_{delivered}/P_{motor} = 576/800 \approx \mathbf{72\%} \quad \textit{Ans. (a)}$$

$$\text{If } n_2 = 3000 \text{ rpm}, \quad Q_2 = Q_1\left(\frac{n_2}{n_1}\right) = 33000\left(\frac{3000}{3550}\right) \approx \mathbf{27900\ \frac{ft^3}{min}} \quad \textit{Ans. (b)}$$

$$\Delta p_2 = \Delta p_1(n_2/n_1)^2 = (4\text{ psi})\left(\frac{3000}{3550}\right)^2 \approx \mathbf{2.86\ psi} \quad \textit{Ans. (c)}$$

To estimate impeller diameter, we have little to go on except the specific speed:

$$\rho_{avg} \approx \frac{16(144)}{776(520)} \approx 0.0057\ \frac{slug}{ft^3}, \quad H = \frac{\Delta p}{\rho g} = \frac{4(144)}{0.0057(32.2)} \approx 3133 \text{ ft},$$

$$N_s = \frac{\text{rpm(gpm)}^{1/2}}{(\text{H-ft})^{3/4}} = \frac{3550[33000(449)/60]^{1/2}}{(3133)^{3/4}} \approx 4212: \quad \text{Fig. P11.49: } C_Q^* \approx 0.45$$

$$\text{Crudely,} \quad C_Q^* \approx 0.45 = \frac{33000/60}{(3550/60)D^3}, \quad \text{solve for } D_{impeller} \approx \mathbf{2.7\ ft} \quad \textit{Ans. (d)}$$

Clearly this last part depends upon the ingenuity and resourcefulness of the student.

11.40 The specific speed N_s, as defined by Eq. (11.30), does not contain the impeller diameter. How then should we size the pump for a given N_S? Logan [7] suggests a parameter called the *specific diameter* D_S, which is a dimensionless combination of Q, (gH), and D. (a) If D_S is proportional to D, determine its form. (b) What is the relationship, if any, of D_S to C_{Q^*}, C_{H^*}, and C_{P^*}? (c) Estimate D_S for the two pumps of Figs. 11.8 and 11.13.

Solution: If we combine C_Q and C_H in such a way as to eliminate speed *n*, and also to make the result linearly proportional to *D*, we obtain Logan's result:

$$\text{Specific diameter}\quad \mathbf{D_s = \frac{D(gH^*)^{1/4}}{Q^{*1/2}}}\quad \textit{Ans.}\text{ (a)}\quad \mathbf{D_s = \frac{C_H^{*\,1/4}}{C_Q^{*\,1/2}}}\quad \textit{Ans.}\text{ (b)}$$

(c) For the pumps of Figs. 11.8 and 11.13, we obtain

$$D_{s\textbf{-Fig.11.8}} = \frac{(5.0)^{1/4}}{(0.115)^{1/2}} = \mathbf{4.41};\quad D_{s\textbf{-Fig.11.13}} = \frac{(1.07)^{1/4}}{(0.55)^{1/2}} = \mathbf{1.37}\quad \textit{Ans.}\text{ (c)}$$

11.41 It is desired to build a centrifugal pump geometrically similar to Prob. 11.28 (data below) to deliver 6500 gal/min of gasoline at 1060 rpm. Estimate the resulting (a) impel-ler diameter; (b) head; (c) brake horsepower; and (d) maximum efficiency.

Q, ft^3/s:	0	2	4	6	8	10
H, ft:	340	340	340	330	300	220
bhp:	135	160	205	255	330	330

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3. From Prob. 11.28, BEP occurs at $Q^* \approx 6$ ft^3/s, $\eta_{max} \approx 0.88$. The data above are for $n = 2134$ rpm $= 35.6$ rps and $D = 14.62$ in.

$$\text{Then}\quad C_Q^* = \frac{6.0}{35.6(14.62/12)^3} = 0.0933 \overset{?}{=} \frac{6500/449}{(1060/60)D^3},$$

$$C_H^* = \frac{32.2(330)}{(35.6)^2(14.62/12)^2} = 5.66 \overset{?}{=} \frac{32.2H}{(1060/60)^2(2.06)^2},\quad \text{solve for } H \approx \mathbf{233\ ft}\quad \textit{Ans.}\text{ (b)}$$

Step-up the efficiency with Moody's correlation, Eq. (11.29*a*), for $D_1 = 14.62/12 \approx 1.22$ ft:

$$\frac{1-\eta_2}{1-0.88} \approx \left(\frac{D_1}{D_2}\right)^{1/4} = \left(\frac{1.22}{2.06}\right)^{1/4} = 0.877,\quad \text{solve for } \eta_2 \approx 0.895$$

$$\text{Then}\quad P_2 = \frac{\rho g Q_2 H_2}{\eta} = \frac{1.32(32.2)(6500/449)(233)}{0.895} = 160200 \div 550 \approx \mathbf{290\ bhp}\quad \textit{Ans.}\text{ (c)}$$

11.42 An 8-inch model pump delivering water at 180°F at 800 gal/min and 2400 rpm begins to cavitate when the inlet pressure and velocity are 12 psia and 20 ft/s, respectively. Find the required NPSH of a prototype which is 4 times larger and runs at 1000 rpm.

Solution: For water at 180°F, take $\rho g \approx 60.6$ lbf/ft^3 and $p_v \approx 1600$ psfa. From Eq. 11.19,

$$\text{NPSH}_{\text{model}} = \frac{p_i - p_v}{\rho g} + \frac{V_i^2}{2g} = \frac{12(144) - 1600}{60.6} + \frac{(20)^2}{2(32.2)} = 8.32 \text{ ft}$$

$$\text{Similarity:} \quad \text{NPSH}_{\text{proto}} = \text{NPSH}_m \left(\frac{n_p}{n_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2 = 8.32 \left(\frac{1000}{2400}\right)^2 \left(\frac{4}{1}\right)^2 \approx \mathbf{23\ ft} \quad \textit{Ans.}$$

11.43 The 2-in-diameter pump in Fig. 11.7*a* at 1170 r/min is used to pump water at 20°C through a piping system at 14,000 gal/min. (a) Determine the required brake horsepower. The average friction factor is 0.018. (b) If there is 65 ft of 12-in-diameter pipe upstream of the pump, how far below the surface should the pump inlet be placed to avoid cavitation?

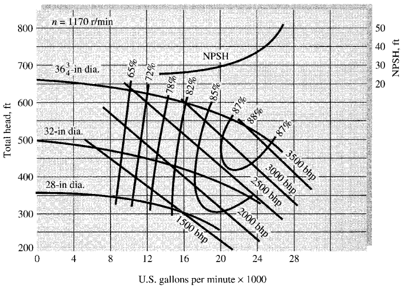


Fig. 11.7a

Solution: For water at 20°F, take $\rho g \approx 62.4$ lbf/ft^3 and $p_v \approx 49$ psfa. From Fig. 11.7a (above), at 28″ and 14000 gpm, read H ≈ 320 ft, $\eta \approx 0.81$, and P ≈ **1400** bhp. *Ans.*

$$\text{Or:} \quad \text{Required bhp} = \frac{\rho g Q H}{\eta} = \frac{(62.4)(14000/449)(320)}{0.81} = 769000 \div 550 \approx \mathbf{1400\ bhp} \quad \textit{Ans.}$$

From the figure, at 14000 gal/min, read NPSH ≈ 25 ft. Assuming $p_a = 1$ atm $= 2116$ psf,

$$\text{Eq. 11.20:} \quad \text{NPSH} = \frac{p_a - p_v}{\rho g} - Z_i - h_{fi} = \frac{2116 - 49}{62.4} - Z_i - h_{fi} \approx 25 \text{ ft}, \quad h_{fi} = f\frac{L}{D}\frac{V^2}{2g},$$

11.44 The pump of Prob. 11.28 is scaled up to an 18-in-diameter, operating in water at BEP at 1760 rpm. The measured NPSH is 16 ft, and the friction loss between the inlet and the pump is 22 ft. Will it be sufficient to avoid cavitation if the pump inlet is placed 9 ft below the surface of a sea-level reservoir?

Solution: For water at 20°C, take $\rho g = 62.4\ \text{lbf/ft}^3$ and $p_v = 49$ psfa. Since the NPSH is *given*, there is no need to use the similarity laws. Merely apply Eq. 11.20:

$$\text{NPSH} \le \frac{p_a - p_v}{\rho g} - Z_i - h_{fi},\quad \text{or:}\quad Z_i \le \frac{2116 - 49}{62.4} - 22 - 16 = -4.9\ \text{ft},\ \textbf{OK,}$$

$$\mathbf{Z_{actual} = -9\ ft}\quad \textit{Ans.}$$

This works. Putting the inlet 9 ft below the surface gives 4 ft of margin against cavitation.

11.45 Determine the specific speeds of the seven Taco, Inc. pump impellers in Fig. P11.24. Are they appropriate for centrifugal designs? Are they approximately equal within experimental uncertainty? If not, why not?

Solution: Read the BEP values for each impeller and make a little table for *1160* rpm:

D, inches:	10.0	10.5	11.0	11.5	12.0	12.5	12.95
Q*, gal/min:	390	420	440	460	480	510	530
H*, ft:	41	44	49	56	60	66	72
Specific speed N_S:	**1414**	**1392**	**1314**	**1215**	**1179**	**1131**	**1080**

These are well within the centrifugal-pump range ($N_S < 4000$) but they are not equal because they are not geometrically similar (7 different impellers within a single housing). *Ans.*

11.46 The answer to Prob. 11.40 is that the dimensionless "specific diameter" takes the form $D_S = D(gH^*)^{1/4}/Q^{*1/2}$, evaluated at the BEP. Data collected by the writer for 30 different pumps indicates, in Fig. P11.46, that D_S correlates well with specific speed N_S. Use this figure to estimate the appropriate impeller diameter for a pump which delivers 20,000 gal/min of water and a head of 400 ft running at 1200 rev/min. Suggest a curve-fit formula to the data (*Hint*: a hyperbola).

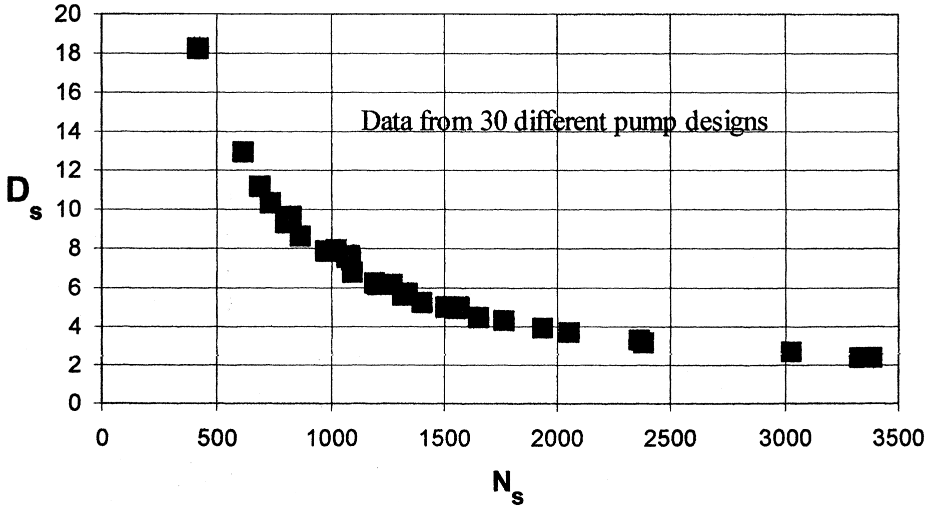


Fig. P11.46

Solution: We see that the data are very well correlated by a single curve. (NOTE: These are all *centrifugal* pumps—a slightly different correlation holds for mixed- and axial-flow pumps.) The data are well fit by a hyperbola:

$$\text{Figure P11.46:}\quad \mathbf{D_s} \gg \frac{\mathbf{Const}}{\mathbf{N}},\quad \text{where}\ \ \text{Const} \approx 7800 \pm 500 \quad \textit{Ans.}$$

For the given pump-data example, we compute

$$N_s = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{(H_{head},\ \text{ft})^{3/4}} = \frac{1200(20000)^{1/2}}{(400)^{3/4}} = 1897,$$

$$\text{Hence}\quad D_s \approx \frac{7800}{1897} \approx 4.11 = \frac{D[32.2(400)]^{1/4}}{(20000/448.83)^{1/2}},\quad \text{solve}\ \ \mathbf{D \approx 2.6 \pm 0.1\ ft} \quad \textit{Ans.}$$

P11.47 A pump must be designed to deliver 6 m^3/s of water against a head of 28 m. The specified shaft speed is 20 r/s. What type of pump do you recommend?

Solution: Change units to suit the practicing-engineer version of specific speed: 6 m^3/s = 95,100 gal/min, 20 r/s = 1200 r/min, and 28 m = 91.9 ft. Then the specific speed is

$$N_s = \frac{(rpm)(gpm)^{1/2}}{(H - ft)^{3/4}} = \frac{(1200\,rpm)(95100\,gpm)^{1/2}}{(91.9\,ft)^{3/4}} = \mathbf{12,500}$$

From Fig. 11.11*a*, an axial-flow pump is appropriate for this job. *Ans.*

11.48 A commercial pump runs at 1750 r/min and delivers, near BEP, a flow of 2300 gal/min and a head of 40 m. (*a*) What type of pump is this? (*b*) Estimate the impeller diameter using the data of Prob. P11.46. (*c*) Estimate C_Q* and add another data point to Fig. P11.49.

Solution: (a) To find the type of pump from Fig. 11.11, we need the head in feet: H = 40 m = 131 ft. Then we can calculate the traditional (dimensional) specific speed:

$$N_S = \frac{(rpm)(gpm)^{1/2}}{[H(ft)]^{3/4}} = \frac{(1750\,rpm)(2300\,gpm)^{1/2}}{(131\,ft)^{3/4}} \approx 2170$$

From Fig. 11.11, **this is a centrifugal pump**. *Ans*. (*a*)

(*b*) Use Fig. P11.46 to estimate the specific diameter and hence the impeller diameter. The curve-fit recommended in the solution to Prob. P11.46 is

$$N_S \quad 2170 \qquad Q^{1/2} \qquad (2300/448.8\ ft^3/s)^{1/2}$$

The accuracy is unusually good, not typical. The actual impeller diameter was 12 inches! (*c*) Calculate the flow coefficient, which we are told is near BEP:

$$C_Q^* = \frac{Q^*}{\ } = \frac{(2300/448.8\ ft^3/s)}{\ } = 0.17$$

Add this point to Fig. P11.49 at $N_S = 2170$ and we have an excellent fit to the other data.

11.49 Data collected by the writer for flow coefficient at BEP for 30 different pumps are plotted at right in Fig. P11.49. Determine if the values of C_Q^* fit this correlation for the pumps of Problems P11.24, P11.28, P11.35, and P11.38. If so, suggest a curve fit formula.

N_S

Fig. P11.49 Flow coefficient at BEP for 30 commercial pumps.

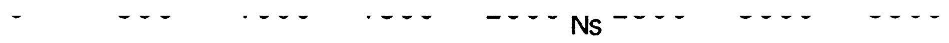
Solution: Make a table of these values:

	Q*, gpm	**D, inches**	***n*, rpm**	**N_S**	**$C_Q^* = Q^*/(nD^3)$**
Prob. 11.28:	2692	14.62	2134	**1430**	*0.0933*
Prob. 11.35:	8000	18.0	880	**3320**	*0.360*
Prob. 11.38:	350	6.85	3500	**1400**	*0.0719*
Fig. P11.24:	460	11.5	1160	**1215**	*0.0602* *Ans.*

When added to the plot shown below, all four data points seem to fit quite well. The data are useful for predicting general centrifugal-pump behavior and are well fit to either a 2nd-order polynomial or a single-term Power-law slightly less than parabolic:

Polynomial: $C_Q^* \approx 1.97E{-}5N_S + 2.58E{-}8N_S^2$

(Correlation $R^2 \approx 0.99$) *Ans.*



11.50 Data collected by the writer for power coefficient at BEP for 30 different pumps are plotted at right in Fig. P11.50. Determine if the values of C_Q^* for the FOUR pumps of Prob. 11.49 above fit this correlation.

Ns

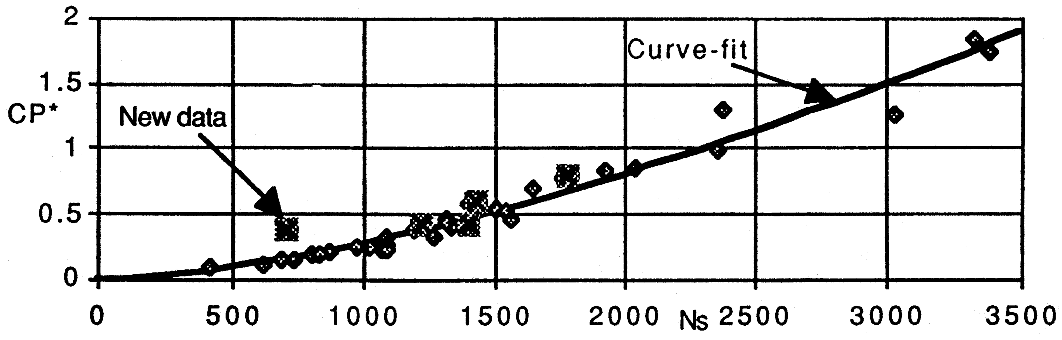
Fig. P11.50 Power coefficient at BEP for 30 commercial pumps.

Solution: Make a table of these values, similar to Prob. 11.50:

	P*, bhp	**D, inches**	***n*, rpm**	**Ns**	$\mathbf{C_P^* = P^*/(\rho n^3 D^5)}$
Prob. 11.28:	255	14.62	2134	**1430**	*0.600*
Prob. 11.35:	156	18.0	880	**3320**	*1.85*
Prob. 11.38:	18.5	6.85	3500	**1400**	*0.435*
Fig. P11.24:	8.7	11.5	1160	**1215**	*0.421* *Ans.*

When added to the plot shown below, all four of them seem to fit reasonably well. The data are moderately useful for predicting general centrifugal-pump behavior and can be fit to either a 2nd-order polynomial or a single-term Power-law:

Polynomial: $C_P^* \approx 2.12E\text{–}4N_s + 9.5E\text{–}8N_s^2$ (poorer correlation, $R^2 \approx 0.96$) *Ans.*



11.51 An axial-flow pump delivers 40 ft^3/s of air which enters at 20°C and 1 atm. The flow passage has a 10-in outer radius and an 8-in inner radius. Blade angles are $\alpha_1 = 60°$

and $\beta_2 = 70°$, and the rotor runs at 1800 rpm. For the first stage, compute (a) the head rise; and (b) the power required.

Solution: Assume an average radius of (8 + 10)/2 = 9 inches and compute the blade speed:

$$u_{avg} = \omega r_{avg} = \left(1800\frac{2\pi}{60}\right)\left(\frac{9}{12}\right) \approx 141\ \frac{ft}{};\quad V_n = \frac{Q}{A} = \frac{40\ ft^3/s}{[(10/12)^2 - (8/12)^2]} \approx 50.9\ \frac{ft}{}$$

$$\text{Theory: } gH = u^2 - uV_n(\cot\alpha_1 + \cot\beta_2) = (141)^2 - 141(50.9)(\cot 60° + \cot 70°),$$

$$\mathbf{H \approx 410\ ft} \quad \textit{Ans. (a)}$$

$$P_{theory} = \rho gQH = \left[\frac{2116}{1717(528)}\right](32.2)(40)(410) = 1232 \div 550 \approx \mathbf{2.24\ hp} \quad \textit{Ans. (b)}$$

11.52 An axial-flow fan operates in sea-level air at 1200 r/min and has a blade-tip diameter of 1 m and a root diameter of 80 cm. The inlet angles are $\alpha_1 = 55°$ and $\beta_1 = 30°$, while at the outlet $\beta_2 = 60°$. Estimate the theoretical values of the (a) flow rate, (b) horse-power, and (c) outlet angle α_2.

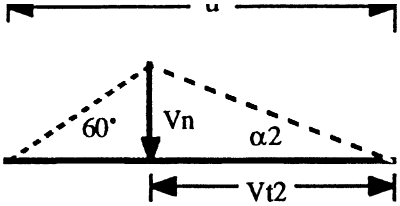


Fig. P11.52

Solution: For air, take $\rho \approx 1.205$ kg/m^3. The average radius is 0.45 m. Thus

$$u = \omega R = \left(1200\frac{2\pi}{60}\right)(0.45) \approx 56.6\ \frac{m}{s} = V_n(\cot\alpha_1 + \cot\beta_1) = V_{n2}(\cot\alpha_2 + \cot\beta_2)$$

$$\text{Solve}\quad V_{n1} = V_{n2} = \frac{56.6}{\cot 55° + \cot 30°} \approx 23.2\ \frac{m}{s} \quad\text{and}\quad \alpha_2 \approx \mathbf{28.3°} \quad \textit{Ans. (c)}$$

$$\text{Then}\quad Q = V_nA = (23.2)[\pi\{(0.5)^2 - (0.4)^2\}] \approx \mathbf{6.56\ \frac{m^3}{}} \quad \textit{Ans. (a)}$$

$$gH = u^2 - uV_n(\cot\alpha_1 + \cot\beta_2) = (56.6)^2 - 56.6(23.2)(\cot 55° + \cot 60°) = 1520\ m^2/s^2$$

$$\text{Finally,}\quad \mathbf{P} = \rho QgH = 1.205(6.56)(1520) = \mathbf{12{,}000\ W} \quad \textit{Ans. (b)}$$

11.53 Figure P11.46 is an example of a centrifugal pump correlation, where D_S is defined in the problem. Logan [3] suggests the following correlation for axial-flow pumps and fans:

$$D_S \approx \frac{130}{N_S^{0.485}} \qquad \text{for}\quad N_S > 8000$$

where N_S is the *dimensional* specific speed, Eq. (11.30*b*). Use this correlation to find the appropriate size for a fan that delivers 24,000 ft³/min of air at sea-level conditions when running at 1620 r/min with a pressure rise of 2 inches of water. [HINT: Express the fan head in feet of *air*, not feet of water.]

Solution: At sea level, take ρ_{air} = 1.2255 kg/m³ = 0.00238 slug/ft³. Convert Q = 24,000 ft³/min = 400 ft³/s = 175,900 gal/min and n = 1620 r/min = 27 r/s. Convert the head H into feet of air:

$$H_{air} = \frac{\Delta p}{\rho_{air} g} = \frac{\rho_{water} g \Delta h_{water}}{\rho_{air} g} = \frac{(1.94\,slug/ft^3)(32.2\,ft/s^2)(2/12\,ft)}{(0.00238\,slug/ft^3)(32.2\,ft/s^2)} = 136\,ft\ \ of\ air$$

Now compute the dimensional specific speed of the fan:

$$N_S = \frac{(rpm)(gal/\min)^{1/2}}{(ft\ of\ head)^{3/4}} = \frac{(1620)(175,900)^{1/2}}{(136)^{3/4}} = 17060 \quad (yes,\ axial\ flow)$$

Finally, use Logan's correlation to find the specific diameter and hence the fan size:

$$D_S \approx \frac{130}{(17060)^{0.485}} = 1.15 = \frac{D(gH)^{1/4}}{Q^{1/2}} = \frac{D[(32.2\,ft/s^2)(136\,ft)]^{1/4}}{(400\,ft^3/s)^{1/2}}$$

P11.54 It is desired to pump 50 ft³/s of water at a speed of 22 r/s, against a head of 80 ft. (*a*) What type of pump would you recommend? Estimate (*b*) the required impeller diameter and (*c*) the brake horsepower.

Solution: Convert to English units and evaluate the specific speed:

$$Q = 50\frac{ft^3}{s} = 22,440\ \frac{gal}{min}\ ;\ n = 22\frac{r}{s} = 1320\ rpm\ ;\ H = 80\ ft\ \ (OK)$$

$$N_s = \frac{(rpm)(gal/min)^{1/2}}{(H-ft)^{3/4}} = \frac{(1320 rpm)(22440 gpm)^{1/2}}{(80\,ft)^{3/4}} = \mathbf{7400}$$

From Fig. 11.11*a*, **this pump should have a mixed-flow impeller**. *Ans*.(*a*)

(*b*) The writer estimated impeller diameter from the "specific diameter" correlation in Prob. 11.53:

$$\text{BG units}: D_s = \frac{D(gH)^{1/4}}{Q^{1/2}} = \frac{D[32.2(80\,ft)]^{1/4}}{(50\,ft^3/s)^{1/2}} \approx \frac{130}{N_s^{\,0.485}} = \frac{130}{(7400)^{0.485}} = 1.73$$

$$\text{Solve} \quad D_{impeller} = (1.73)\left(\frac{7.07}{7.12}\right) = \mathbf{1.72}\ ft = 20.6 \text{ inches} \quad Ans.(b)$$

(*c*) According to Fig. 11.11*a*, a really good mixed-flow impeller, N_s = 7400, would have an efficiency of about 92 per cent. Thus we can estimate the power required:

$$P_{input} = \frac{\gamma Q H}{\eta} = \frac{(62.4\,lbf/ft^3)(50\,ft^3/s)(80\,ft)}{0.92} = 271{,}000\,\frac{ft-lbf}{s} \approx \mathbf{493\ bhp} \quad Ans.(c)$$

P11.55 Suppose that the axial-flow pump of Fig. 11.13, with *D* = 18 in, runs at 1800 r/min. (*a*) Could it efficiently pump 25,000 gal/min of water? (*b*) If so, what head would result? (*c*) If a head of 120 ft is desired, what values of *D* and *n* would be better?

Solution: Convert *n* = 1800 r/min = 30 r/s. The flow coefficient at BEP is approximately 0.55. Check this against the data:

$$C_{Q*} = \frac{Q}{nD^3} = \frac{(25000/449\ \ ft^3/s)}{(30\,r/s)(1.5\,ft)} \approx 0.55 \quad (\text{Yes, this is at } \mathbf{BEP}) \quad Ans.(a)$$

(*b*) Since we are at BEP, we can use the BEP head coefficient to get *H*:

$$C_{H*} \approx 1.07 = \frac{gH}{n^2D^2} = \frac{(32.2)H}{(30)^2(1.5)^2}; \quad \text{Solve} \quad H \approx \mathbf{67\ ft} \quad Ans.(b)$$

(*c*) For *D* = 18 in, *H* = 120 ft is far off BEP, $C_H \approx 1.9$, efficiency only 40 per cent. So let diameter be unknown and set both head and flow coefficients equal to BEP values:

$$C_{Q*} = 0.55 = \frac{(25000/449\, ft^3/s)}{n\,D^3}, \quad C_{H*} = 1.07 = \frac{(32.2)(120\,ft)}{n^2\,D^2}$$

Solve simultaneously for $n = 46.3\,\frac{r}{s} = \mathbf{2780\,\frac{r}{min}}$, $D = 1.30\,ft = \mathbf{15.6\ in}$ *Ans.*(*c*)

11.56 A pump is needed to deliver 40,000 gpm of gasoline at 20°C against a head of 90 ft. Find the impeller size, speed, and brake horsepower needed to use the pump families of (a) Fig. 11.8; and (b) Fig. 11.13. Which is the better design?

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3.

(a) For the *centrifugal* design, $C_Q^* \approx 0.115 = \frac{40000/449}{\;}$ and $C_H^* \approx 5.0 = \frac{32.2(90)}{\;}$,

Solve for $n \approx 4.24$ rps $\approx$ **255 rpm** and $D_{impeller} \approx$ **5.67 ft** *Ans.* (a)

$$P^* = C_P^*\rho n^3 D^5 = 0.65(1.32)(4.24)^3(5.67)^5 \div 550 \approx \mathbf{700\ bhp} \quad \textit{Ans.}\,(a)$$

(b) For the axial-flow design, Fig. 11.13,

$$C_Q^* = 0.55 = \frac{40000/449}{nD^3}, \quad C_H^* = 1.07 = \frac{32.2(90)}{n^2D^2}$$

$$P^* = C_P^*\rho n^3 D^3 = 0.70(1.32)(29.5)^3(1.76)^5 \div 550 \approx \mathbf{740\ bhp} \quad \textit{Ans.}\,(b)$$

The **axial-flow design** (**b**) is far better for this system: smaller and faster. *Ans.*

11.57 Performance data for a 21-in-diameter air blower running at 3550 rpm are shown below. What is the specific speed? How does the performance compare with Fig. 11.13? What are C_Q^*, C_H^*, C_P^*?

Δ*p*, in H_2O:	29	30	28	21	10
Q, ft^3/min:	500	1000	2000	3000	4000
bhp:	6	8	12	18	25

Solution: Assume 1-atm air, $\rho \approx 0.00233$ slug/ft^3. Convert the data to dimensionless form and put the results into a table:

Δp, psf:	151	156	146	109	52
Q, ft^3/min:	500	1000	2000	3000	4000
Q, gal/min:	3740	7480	14960	22440	29920
H, ft (of air):	2010	2080	1940	1455	693
C_Q:	0.0263	0.0526	**0.105**	0.158	0.210
C_H:	6.04	6.25	**5.83**	4.37	2.08
C_P:	0.417	0.555	**0.833**	1.25	1.74
η:	0.381	0.592	**0.735**	0.552	0.251

Close enough without plotting: $C_Q^* \approx \mathbf{0.105}$, $C_H^* \approx \mathbf{5.83}$, $C_P^* \approx \mathbf{0.833}$ *Ans.*

$$\text{Specific speed} \quad N_s = \frac{(3550\text{ rpm})(14960\text{ gpm})^{1/2}}{(1940\text{ ft})^{3/4}} \approx \mathbf{1485} \quad \textit{Ans.}$$

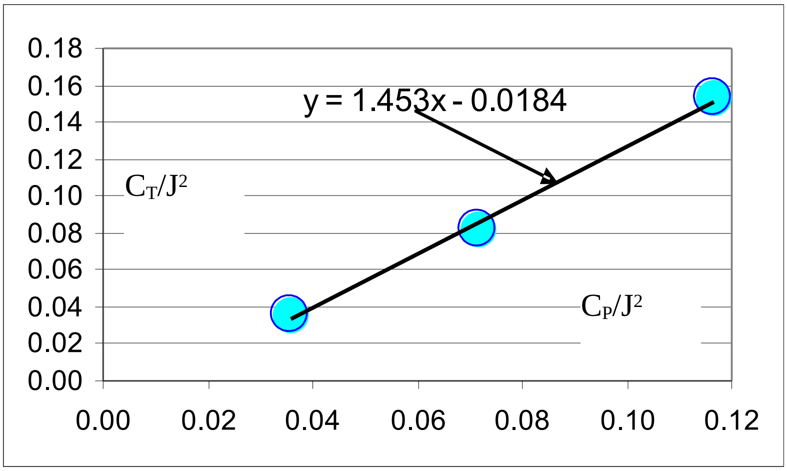
This centrifugal pump is **very similar** to the dimensionless data of **Fig. 11.8**. *Ans.*

P11.58 Aircraft propeller specialists claim that dimensionless propeller data, when plotted as (C_T/J^2) versus (C_P/J^2), form a nearly straight line, $y = mx + b$. (*a*) Test this hypothesis for the data of Fig. 11.16, in the high-efficiency range $J = V/(nD)$ equal to 0.6, 0.7, and 0.8. (*b*) If successful, use this straight line to predict the rotation rate *n*, in r/min, for a propeller with $D = 5$ ft, $P = 30$ hp, $T = 95$ lbf, and $V = 95$ mi/h, for sea-level standard conditions. Comment.

Solution: For sea-level air, take $\rho = 0.00237$ slug/ft^3. The writer read Fig. 11.16 as best he could and came up with the following data, rewritten in the desired form:

J	C_T	C_P	C_P/J^2	C_T/J^2
0.6	0.055	0.042	0.1167	0.1528
0.7	0.040	0.035	0.0714	0.0816
0.8	0.023	0.023	0.0359	0.0359

The plot is, sure enough, reasonably linear:



The specialists were correct; this propeller data fits the simple formula

$$\frac{C_T}{J^2} = \frac{T}{\rho D^2 V^2} \approx 1.453\left(\frac{C_P}{J^2}\right) - 0.0184 = 1.453\left(\frac{P}{\rho n D^3 V^2}\right) - 0.0184$$

We have all the data except n. Convert to BG units: 95 mi/h = 139 ft/s and 30 hp = 16,500 ft-lbf/s. Put these values into our straight-line formula:

$$\frac{95}{(0.00237)(5.0)^2(139)^2} \approx 1.453\left[\frac{16,500}{(0.00237)\,n\,(5.0)^3(139)^2}\right] - 0.0184 \quad ,$$

$$\text{or}: \quad 0.0826 \approx \frac{4.17}{n} - 0.0184\,, \quad n \approx \frac{4.17}{0.101} \approx 41\ \frac{r}{s} \approx \mathbf{2500}\ \frac{r}{min} \qquad \textit{Ans.}$$

This is reasonably accurate and close to maximum efficiency, *V*/*nD* ≈ 0.68. However, **beware**! This clever correlation has reduced the propeller data from three variables (C_T, C_P, and *V*/*nD*) to only two (C_T/J^2 and C_P/J^2), without any theoretical justification. Thus, information is lost. *Do not use this straight line as a general problem solver.* It will fail miserably if fed with data that does not match the conditions in Fig. 11.16.

11.59 Suppose it is desired to deliver 700 ft^3/min of propane gas (molecular weight = 44.06) at 1 atm and 20°C with a single-stage pressure rise of 8.0 in H_2O. Determine the appropriate size and speed for using the pump families of (a) Prob. 11.57 and (b) Fig. 11.13. Which is the better design?

Solution: For propane, with *M* = 44.06, the gas constant *R* = 49720/44.06 ≈ 1128 ft·lbf/(slug·°R). Convert $\Delta p = 8$ inH_2O = (62.4)(8/12) = 41.6 psf. The propane density and head rise are

$$\rho_{gas} = \frac{p}{RT} = \frac{2116 \text{ psf}}{1128(528)} \approx 0.00355 \ \frac{\text{slug}}{\text{ft}^3},$$

$$\text{Hence} \quad H_{pump} = \frac{41.6}{0.00355(32.2)} \approx 364 \text{ ft propane}$$

$$\text{(a) Prob. 11.57:} \quad C_Q^* \approx 0.105 = \frac{700/60}{D^3} \quad \text{and} \quad C_H^* \approx 5.83 = \frac{32.2(364)}{{}^2D^2}$$

$$\text{Solve for} \quad n = 28.5 \text{ rps} \approx \mathbf{1710\ rpm} \quad \text{and} \quad D \approx \mathbf{1.57\ ft} \quad \textit{Ans.}\text{ (a)} \quad \text{(centrifugal pump)}$$

$$\text{(b) Fig. 11.13:} \quad C_Q^* \approx 0.55 \quad \text{and} \quad C_H^* \approx 1.07 \text{ yield}$$

$$n \approx \mathbf{14000\ rpm}, \quad D \approx \mathbf{0.45\ ft} \quad \textit{Ans.}\text{ (b)} \quad \text{(axial flow)}$$

The **centrifugal pump** (**a**) is the better design—nice size, nice speed. The axial flow pump is much smaller but runs too fast. *Ans.*

P11.60 Performance curves for a certain free propeller, comparable to Fig. 11.16, can be plotted as shown in Fig. P11.60, for thrust *T* versus speed *V* for constant power *P*. (*a*) What is striking, at least to the writer, about these curves? (*b*) Can you deduce this behavior by rearranging, or replotting, the data of Fig. 11.16?

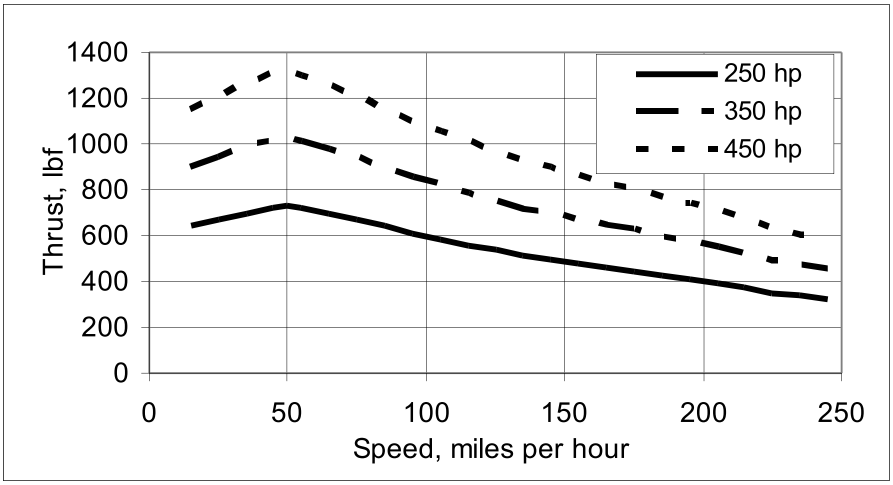
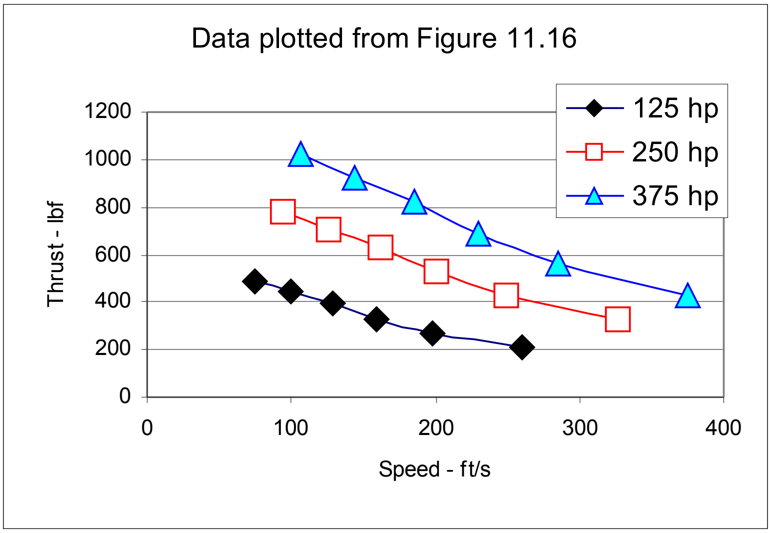


Fig. P11.60

Solution: (*a*) The three curves look exactly alike! In fact, we deduce, for this propeller, that the thrust, at constant speed, is linearly proportional to the horsepower! This implies that the dimensionless thrust, C_T, is proportional to the dimensionless power, C_P, for this propeller. We can go back and see *that this is not the case for Fig. 11.16.*
(*b*) Here is a plot of thrust versus speed for the actual data of Fig. 11.16. We see that the curves are similar in shape, but **the thrust is *not* proportional to the horsepower**. *Ans.* (*b*)



11.61 A mine ventilation fan delivers 500 m^3/s of sea-level air at 295 rpm and Δp = 1100 Pa. Is this fan axial, centrifugal, or mixed? Estimate its diameter in feet. If the flow rate is increased 50% for the same diameter, by what percent will Δp change?

Solution: For sea-level air, take $\rho g \approx 11.8\ N/m^3$, hence $H = \Delta p/\rho g = 1100/11.8 \approx 93\ m \approx 305\ ft$. Convert 500 m^3/s to 7.93E6 gal/min and calculate the specific speed:

$$N_s = \frac{rpm(gal/min)^{1/2}}{(head)^{3/4}} = \frac{295(7.93E6)^{1/2}}{(305)^{3/4}} \approx \mathbf{11400\ \ (axial\text{-}flow\ pump)} \quad \mathit{Ans.}\,(a)$$

$$\text{Estimate}\ \ C_Q^* \approx 0.55 = \frac{500\ m^3/s}{(295/60)D^3},\quad \text{solve}\ \ D_{impeller} \approx\ 5.7\ m \approx \mathbf{19\ ft} \quad \mathit{Ans.}\,(b)$$

At constant D, $Q \propto n$ and Δp (or H) $\propto n^2$. Therefore, if Q increases 50%, so does n, and therefore Δp increases as $(1.5)^2 = 2.25$, or **a 125% increase**. *Ans.* (c)

11.62 The actual mine-ventilation fan in Prob. 11.61 had a diameter of 20 ft [Ref. 20, p. 339]. What would be the proper diameter for the pump family of Fig. 11.14 to provide 500 m^3/s at 295 rpm and BEP? What would be the resulting pressure rise in Pa?

Solution: For sea-level air, take $\rho g \approx 11.8$ N/m^3. As in Prob. 11.61 above, the specific speed of this fan is **11400**, hence an *axial-flow* fan. Figure 11.14 indicates an efficiency of about 90%, and the only values we know for performance are from Fig. 11.13:

$$N_s \approx 12000\text{:}\ \ C_Q^* \approx 0.55 = \frac{500}{(295/60)D^3},\quad \text{solve}\ \ D_{\mathbf{impeller}} \approx 5.7\text{ m} \approx \mathbf{18.7\ ft}\quad \textit{Ans.}\ \text{(a)}$$

$$C_H^* \approx 1.07,\quad H = 1.07\frac{(295/60)^2(20\times 0.3048)^2}{9.81} = 98\text{ m},$$

P11.63 A good curve-fit to the head vs. flow for the 32-inch pump in Fig. 11.7*a* is

$$H\,(\text{in ft}) \ \approx\ 500 \ -\ (2.9E-7)Q^2 \quad,\quad Q \ \text{ in gal/min}$$

Assume the same rotation rate, 1170 r/min, and estimate the flow rate this pump will provide to deliver water from a reservoir, through 900 ft of 12-inch pipe, to a point 150 ft above the reservoir surface. Assume a friction factor $f = 0.019$.

Solution: The system head would be the elevation change, 150 ft, plus the friction head loss:

$$H_{pump} \approx\ 500 - (2.9\text{E}-7)Q^2 = H_{system} = \Delta z + f\,\frac{L}{D}\frac{V^2}{2g} = 150 + (0.019)(\frac{900}{1})\{\frac{[4Q/\pi/(1)^2/449]^2]}{2(32.2)}\}$$

$$\text{or}:\ \ 500 - (2.9\text{E}-7)Q^2 \ =\ 150 + (2.14\text{E}-6)Q^2$$

$$\textit{Solve}\quad Q^2 = \frac{350}{2.43\text{E}-6} = 1.44\text{E}8\ ,\quad Q \approx\ \mathbf{12{,}000}\ \frac{\text{gal}}{\text{min}}\qquad \textit{Ans.}$$

This is not a BEP match, but the efficiency is about 75%, not too bad.

11.64 A leaf blower is essentially a centrifugal impeller exiting to a tube. Suppose that the tube is smooth PVC pipe, 4 ft long, with a diameter of 2.5 in. The desired exit velocity is 73 mi/h in sea-level standard air. If we use the pump family of Eq. (11.27) to drive the blower, what approximate (a) diameter and (b) rotation speed are appropriate? (c) Is this a good design?

Solution: Recall that Eq. (11.27) gave BEP coefficients for the pumps of Fig. 11.7:

$$C_Q^* \approx 0.115; \quad C_H^* \approx 5.0; \quad C_P^* \approx 0.65$$

Apply these coefficients to the leaf-blower data. Neglect minor losses, that is, let the pump head match the pipe friction loss. For air, take $\rho = 1.2\ \text{kg/m}^3$ and $\mu = 1.8\text{E}{-}5$ kg/m·s. Convert 73 mi/h = 32.6 m/s, 4 ft = 1.22 m and 2.5 in = 0.0635 m:

$$h_f = H_{pump} = 5.0\frac{n^2D^2}{9.81\ \text{m/s}^2} = f\frac{L}{d_{pipe}}\frac{V_{pipe}^2}{2g} = f\left(\frac{1.22\ \text{m}}{0.0635\ \text{m}}\right)\left[\frac{(32.6\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}\right], \quad f = f(\text{Re}_d)$$

$$Q = \frac{\pi}{4}d_{pipe}^2V = \frac{\pi}{4}(0.0635\ \text{m})^2(32.6\ \text{m/s}) = 0.103\ \frac{\text{m}^3}{\text{s}} = \mathbf{0.115}nD^3$$

We know the Reynolds number, $\text{Re}_d = \rho Vd/\mu = (1.2)(32.6)(0.0635)/(1.8\text{E}{-}5) = 138{,}000$, and for a smooth pipe, from the Moody chart, calculate $f_{smooth} = 0.0168$. Then $H = h_f = 17.5$ m, and the two previous equations can then be solved for

$$D_{pump} \approx \mathbf{0.39\ m}\ (15.4\ \text{in}); \quad n \approx 15\ \text{r/s} = \mathbf{900\ r/min} \quad \textit{Ans.}\ \text{(a, b)}$$

(c) This blower is **too slow and too large**, a better (mixed or axial flow) pump can be designed. *Ans.* (c)

11.65 An 11.5-in diameter centrifugal pump, running at 1750 rev/min, delivers 850 gal/min and a head of 105 ft at best efficiency (82%). (*a*) Can this pump operate efficiently when delivering water at 20°C through 200 m of 10-cm-diameter smooth pipe? Neglect minor losses. (*b*) If your answer to (*a*) is negative, can the speed *n* be changed to operate efficiently? (*c*) If your answer to (*b*) is also negative, can the impeller diameter be changed to operate efficiently and still run at 1750 rev/min?

Solution: For water at 20°C, take ρ = 998 kg/m3 and μ = 0.001 kg/m-s. Convert to SI units: Q = 850 gal/min = 0.0536 m³/s, D = 11.5 in = 0.292 m, H = 105 ft = 32.0 m. Compute Re_D and f:

$$V = \frac{Q}{A} = \frac{0.0536}{(\pi/4)(0.1)^2} = 6.83\frac{m}{s};\ \text{Re}_D = \frac{\rho VD}{\mu} = \frac{998(6.83)(0.1)}{(0.1)^2} = 681{,}000;\ f_{smooth} = 0.0125$$

(*a*) Now compute the friction head loss at 850 gal/min and see if it matches the pump head:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.0125)\left(\frac{200m}{0.1m}\right)\left[\frac{(6.83m/s)^2}{2(9.81m/s^2)}\right] = 59.2m = 194\,ft \neq h_{pump} = 105\,ft$$

The answer to part (a) is **No**, the pump will operate far off-design to lower the flow rate and h_f.

(*b*) Can we just change the pump speed? This answer is also **No**. At BEP, (gH/n^2D^2) is constant, therefore pump head is proportional to n^2. But the pipe friction loss is also approximately proportional to n^2. Thus, for any speed n, the pipe head will always be about twice as large as the pump head and **not operate efficiently**. *Ans.*(*b*)

(*c*) Can we change the impeller diameter, still at 1750 rev/min, and operate efficiently? This time the answer is **Yes**. First note the BEP dimensionless parameters, n = 1750/60 = 29.2 rps:

$$C_{H^*} = \frac{gH^*}{n^2D^2} = \frac{(9.81m/s^2)(32.0m)}{(29.2/s)^2(0.292m)^2} = 4.33;\ \ C_{Q^*} = \frac{Q^*}{nD} = \frac{(0.0536m^3/s)}{(29.2/s)(0.292m)^3} = 0.0738$$

Now combine these with the fact that h_f is approximately proportional to n^2. In SI units,

$$h_f(m) \approx 20600Q^2(in\ m^3/s) = H_{pump} = 4.33\frac{n^2D_p^2}{g} \approx 20600[0.0738n\ D_p^3]^2$$

$$n \text{ cancels, solve for } D_p^4 \approx 0.00356\ m^4,\quad D_p \approx 0.244m \approx \mathbf{9.6\ inches}\quad \text{Ans. }(c)$$

This is approximately correct. Check: the pump head is 22.4 m (74 ft) and flow rate is 497 gal/min. For this flow rate, the pipe head loss is 22.3 m (73 ft). Good match! *Ans.* (*c*)

NOTE: Part (*c*) is *almost* independent of *n*. For example, if n = 20 rev/s, the best efficiency converges to an impeller diameter of 0.24 m (9.45 in), with $Q \approx$ 323 gal/min and $H = h_f \approx$ 10.2 m.

11.66 It is proposed to run the pump of Prob. 11.35 at 880 rpm to pump water at 20°C through the system of Fig. P11.66. The pipe is 20-cm diameter commercial steel. What flow rate in ft^3/min results? Is this an efficient operation?

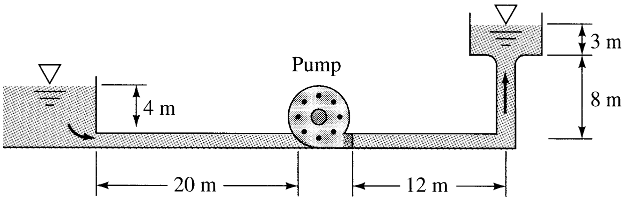


Fig. P11.66

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.0010$ kg/m·s. For commercial steel, take $\varepsilon = 0.046$ mm. Write the energy equation for the system:

$$H_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 11 - 4 + f\left(\frac{40}{0.2}\right)\frac{[Q/(\pi(0.2)^2/4)]^2}{2(9.81)} = 7 + 10328fQ^2, \quad \left(Q\ in\ \frac{m^3}{s}\right)$$

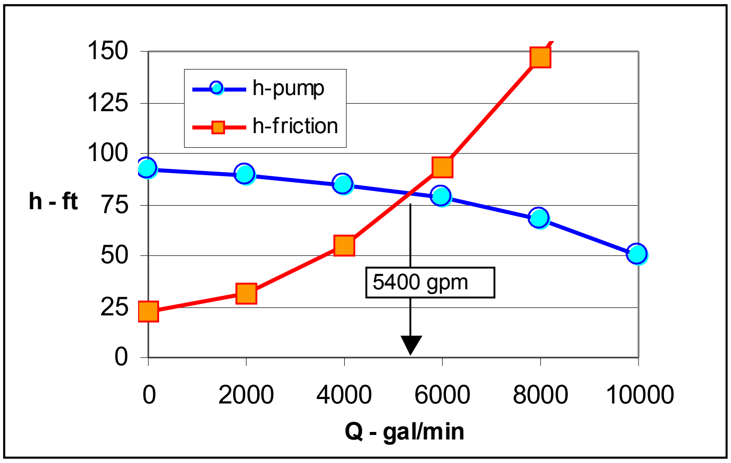
$$\text{Meanwhile,}\ H_p = fcn(Q)_{Prob.\ 11.35} \quad \text{and} \quad f = fcn\left(Re_d, \frac{\varepsilon}{d}\right)_{Moodychart}$$

where ε/d = 0.046/200 = 0.00023. If we guess f as the fully rough value of 0.0141, we find that H_p is about 25 m and Q is about 5500 gal/min (0.35 m^3/s). To do better would require some very careful plotting and interpolating, or: **EES** is made for this job! Iteration with EES leads to the more accurate solution:

$$f = 0.01456; \quad V = 10.8\ \frac{m}{s}; \quad P_{pump} = 139\ \text{hp}; \quad H_p = 80.1\,ft = 24.4\ m$$

The efficiency is **78%**, considerably off the maximum of 88% - not a very good system fit. *Ans.*

Here is a plot of the pump and system curves:



11.67 The pump of Prob. 11.35, running at 880 r/min, is to pump water at 20°C through 75 m of horizontal galvanized-iron pipe ($\varepsilon = 0.15$ mm). All other system losses are neglected. Determine the flow rate and input power for (a) pipe diameter = 20 cm; and (b) the pipe diameter yielding maximum pump efficiency.

Solution: (a) There is no elevation change, so the pump head matches the friction:

$$H_p = f\frac{L}{d}\frac{V^2}{2} = f\frac{75}{0.2}\frac{[Q/(\pi(0.2)^2/4)]^2}{2(9.81)} = 19366 fQ^2, \quad Re_d = \frac{4\rho Q}{d}, \quad \frac{\varepsilon}{d} = \frac{0.15}{200} = 0.00075$$

But also H$_p$ = fcn(Q) from the data in Prob. 11.35. Guessing *f* equal to the fully rough value of 0.0183 yields Q of about 7000 gal/min. Use **EES** to get closer:

$$f = 0.0185; \quad V = 14.4\ \frac{\text{m}}{\text{s}}; \quad Re_d = 2.87E6; \quad H_p = 73\ \text{m}$$

The efficiency is 87%, not bad! (b) If we vary the diameter but hold the pump at maximum efficiency (Q* = 8000 gal/min), we obtain a best **d = 0.211 m**. *Ans.* (b)

11.68 Suppose that we use the axial-flow pump of Fig. 11.13 to drive the leaf blower of Prob. 11.64. What approximate (a) diameter and (b) rotation speed are appropriate? (c) Is this a good design?

Solution: Recall that Fig. 11.13 gave BEP coefficients for an axial-flow pump:

$$C_Q^* \approx 0.55; \quad C_H^* \approx 1.07; \quad C_P^* \approx 0.70$$

Apply these coefficients to the leaf-blower data. Neglect minor losses, that is, let the pump head match the pipe friction loss. For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Convert 73 mi/h = 32.6 m/s, 4 ft = 1.22 m and 2.5 in = 0.0635 m:

$$h_f = H_{pump} = \mathbf{1.07}\frac{n^2D^2}{\ } = f\frac{L}{d}\frac{V_{pipe}^2}{2} = f\left(\frac{1.22\ \text{m}}{0.0635}\right)\left[\frac{(32.6\ \text{m/s})^2}{\ }\right], \quad f = f(\text{Re}_d)$$

$$Q = \frac{\pi}{\ }d_{pipe}^2 V = \frac{\pi}{\ }(0.0635\ \text{m})^2(32.6\ \text{m/s}) = 0.103\ \frac{\text{m}^3}{\ } = \mathbf{0.55}nD^3$$

We know the Reynolds number, $\text{Re}_d = \rho Vd/\mu = (1.2)(32.6)(0.0635)/(1.8\text{E}{-}5) = 138{,}000$, and for a smooth pipe, from the Moody chart, calculate $f_{\text{smooth}} = 0.0168$. Then $H = h_f = 17.5$ m, and the two equations above can then be solved for:

$$D_{\text{pump}} \approx \mathbf{0.122\ m}\ \ (4.8\text{ in});\quad n \approx 104\text{ r/s} = \mathbf{6250\ r/min}\quad \textit{Ans.}\ (\text{a, b})$$

This blower is **too fast and too small**, a better (mixed flow) pump can be designed. *Ans.* (c)

11.69 The pump of Prob. 11.38, running at 3500 rpm, is used to deliver water at 20°C through 600 ft of cast-iron pipe to an elevation 100 ft higher. Find (a) the proper pipe diameter for BEP operation; and (b) the flow rate that results if the pipe diameter is 3 inches.

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	**81**	79	74

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. (a) The data above *are* for 3500 rpm, with BEP at 350 gal/min:

$$H^* = 169\text{ ft} = \Delta z + f\frac{L}{}\frac{V^2}{} = 100 + f\frac{600}{}\frac{[Q/(\pi d^2/4)]^2}{} = 100 + \frac{9.18f}{},\quad Q = Q^* = \frac{350}{}\frac{\text{ft}^3}{}$$

$$\text{Iterate, converges to } Re_d = 2.87\text{E}5,\quad \frac{\varepsilon}{d} = 0.00265,\quad f = 0.0258,$$

(b) If $d = 3$ inches, the above solution changes to a new flow rate:

$$\text{Curve-fit:}\quad H \approx 201 - 61Q^{2.7} = 100 + \frac{600}{}\frac{[Q/(\pi(0.25)^2/4)]^2}{} = 100 + 15466fQ^2$$

$$\text{Iterate or EES: } f = 0.0277,\quad Re = 2.21\text{E}5,\quad H = 193\text{ ft},$$

ft^3 **gal**

11.70 The pump of Prob. 11.28, operating at 2134 rpm, is used with water at 20°C in the system of Fig. P11.70. The diameter is 8 inches. (a) If it is operating at BEP, what is the proper elevation z_2? (b) If $z_2 = 225$ ft, what is the flow rate?

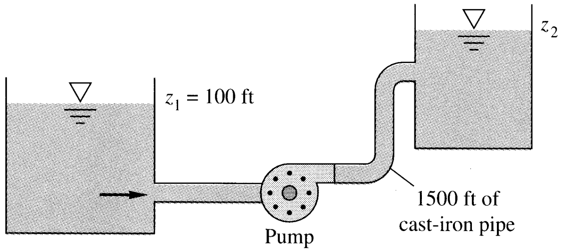


Fig. P11.70

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For cast iron, take $\varepsilon \approx$ 0.00085 ft, hence ε/d ≈ 0.00128. (a) At BEP from Prob. 11.28, Q* ≈ 6 ft^3/s and H* ≈ 330 ft. Then the pipe head loss can be determined:

$$V = \frac{Q}{A} = \frac{6}{(\pi/4)(8/12)^2} = 17.2\ \frac{ft}{s},\quad Re_d = \frac{17.2(8/12)}{1.08E{-}5} = 1.06E6,\quad f_{Moody} \approx\ 0.0211$$

$$H_{syst} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = z_2 - 100 + 0.0211\left(\frac{1500}{8/12}\right)\frac{(17.2)^2}{2(32.2)} = z_2 - 100 + 218 \overset{?}{=} H^* = 330\ ft$$

$$\text{Solve for}\quad z_2 \approx \mathbf{212\ ft}\quad \textit{Ans.}\ (a)$$

(b) If z_2 = 225 ft, the flow rate will be slightly lower and we will be barely off-design:

$$H = H(Q)_{Table} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 225 - 100 + f\frac{1500}{8/12}\frac{V^2}{2(32.2)},\quad V = \frac{Q}{(\pi/4)(8/12)^2}$$

$$\text{Converges to}\ \ f \approx 0.0211,\ \ V \approx 16.6\ \frac{ft}{s},\ \ H \approx 331\ ft,\ \ Q \approx \mathbf{5.8\ \ ft^3/s}\quad \textit{Ans.}\ (b)$$

11.71 The pump of Prob. 11.38, running at 3500 r/min, delivers water at 20°C through 7200 ft of horizontal 5-in-diameter commercial-steel pipe. There are a sharp entrance, sharp exit, four 90° elbows, and a gate valve. Estimate (a) the flow rate if the valve is wide open and (b) the valve closing percentage which causes the pump to operate at BEP. (c) If the latter condition holds continuously for 1 year, estimate the energy cost at 10 ¢/kWh.

Data at 3500 rpm:

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	**81**	79	74

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d \approx 0.00036$. The data above show BEP at 350 gal/min. The minor losses are a sharp entrance (K = 0.5), sharp exit (K = 1.0), 4 elbows (4 × 0.28), and an open gate valve (K = 0.1), or $\sum K \approx$ **2.72**. Pump and systems heads are equal:

$$H_p = H(Q)_{Table} = H_{syst} = \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right) \quad \text{where } V = Q/[(\pi/4)(5/12)^2],\ \frac{L}{d} \approx 17280$$

The friction factor $f \geq 0.0155$ depends slightly upon Q through the Reynolds number.

(a) Iterate on Q until both heads are equal. The result is

$$f \approx 0.0178,\ Re_d \approx 227000,\ H_p = H_{syst} \approx 167 \text{ ft},\ \mathbf{Q \approx 359\ gal/min} \quad \textit{Ans. (a)}$$

(b) Bring Q down to BEP, ≈ 350 gpm, by increasing the gate-valve loss. The result is $f \approx 0.0179$, $Re_d \approx 221000$, $H \approx 169$ ft, $Q \approx 350$ gpm, $\mathbf{K_{valve} \approx 21}$ **(25% open)** *Ans.* (b)

(c) Continue case (b) for 1 year. What does it cost at 10¢ per kWh? Well, we know the power level is exactly BEP, so just figure the energy:

$$P = \frac{\rho g Q H}{\eta} = \frac{62.4(350/449)(169)}{0.81} \approx 10152\ \frac{ft \cdot lbf}{s} = 13.8 \text{ kW}$$

$$\text{Annual cost} = 13.8(365 \text{ days}*24 \text{ hours})(\$0.1/\text{kWh}) \approx \mathbf{\$12{,}100} \quad \textit{Ans. (c)}$$

11.72 Performance data for a small commercial pump are shown below. The pump supplies 20°C water to a horizontal 5/8-in-diameter garden hose ($\varepsilon \approx 0.01$ in), which is 50 ft long. Estimate (a) the flow rate; and (b) the hose diameter which would cause the pump to operate at BEP.

Q, gal/min:	0	10	20	30	40	50	60	70
H, ft:	75	75	74	72	68	62	47	24

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. Given $\varepsilon/d = 0.01/(5/8) \approx 0.016$, so f(fully rough) ≈ 0.045. Pump and hose heads must equate, including the velocity head in the outlet jet:

$$H_p = H(Q)_{Table} = H_{syst} = \frac{V^2}{2g}\left(f\frac{L}{d} + 1\right) \quad \frac{L}{d} = \frac{50}{5/8/12} \approx 960, \quad V = \frac{Q}{(\pi/4)(5/8/12)^2}$$

(a) Iterate on Q until both heads are equal. The result is:

$$f \approx 0.0456, \quad Re_d \approx 50440, \quad V \approx 10.4\ \frac{ft}{s}, \quad H \approx 75 \text{ ft}, \quad \mathbf{Q \approx 10\ gal/min} \quad \textit{Ans. (a)}$$

The hose is too small.

(b) We don't know *exactly* where the BEP is, but it typically lies at about 60% of maximum flow rate (0.6 × 77 ≈ **45** gpm). Iterate on hose diameter D to make the flow rate lie between 40 and 50 gal/min. The results are:

Q = 40 gal/min,	H = 68 ft,	f ≈ 0.0362,	Re ≈ 104000,	D_{hose} ≈ 1.22 inches	
45	65	0.0354	109000	1.30 inches	*Ans.* (b)
50	62	0.0348	114000	1.38 inches	

11.73 The Cessna 172 aircraft has a wing area of 174 ft², an aspect ratio of 7.38, and a basic drag coefficient $C_{D\infty}$ = 0.037. Its propeller, whose data is shown in Fig. 11.16, has a diameter of 6.25 ft. If it weighs 2300 lbf and flies at 180 ft/s at 1500 m standard altitude, estimate (*a*) the appropriate propeller speed, in rev/min, and (*b*) the power required. Is the propeller efficient? [HINT: The efficiency is good, but not best.]

Solution: At 1500 m altitude, take ρ = 1.0583 kg/m³ = 0.00205 slug/ft³. The lift and drag are

$$C_L = \frac{W}{(1/2)\rho V^2 A} = \frac{2300\,lbf}{(1/2)(0.00205 slug/ft^3)(180\,ft/s)^2(174\,ft^2)} = 0.397$$

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi \ \ AR} = 0.037 + \frac{(0.397)^2}{\pi(7.38)} = \ 0.037 + 0.0068 \ = 0.0438$$

$$Drag = Thrust \ = C_D\frac{\rho}{2}V^2 A = (0.0438)(\frac{0.00205}{2})(180)^2(174) \ = 253\,lbf$$

Since *T* and *n* are related (strongly), we have to look for the right system match on the performance chart, Fig. 11.16. The HINT says it is pretty efficient. Try *best efficiency*:

$$J(\,for\ \eta_{\max}) = 0.7 = \frac{V}{nD} = \frac{180\,ft/s}{n(6.25\,ft)}, \quad for\ which \quad n = 41.1 rev/s$$

So the thrust is too small, we need larger C_T, hence a smaller *J*. Iterate to this solution:

$$J \approx 0.67\ yields \quad n \approx 43 rev/s \quad C_T = 0.044 \quad and \quad T \approx 254\,lbf \quad (OK, a\ match)$$

The appropriate propeller speed is *n* = 43 rev/s ≈ **2580 rev/min**. *Ans.*(*a*)

(*b*) At *J* = 0.67, read, from Fig. 11.16, C_P ≈ 0.029. Then the required power is

$$P = C_P \rho n^3 D^5 = (0.029)(0.00205)(43)^3(6.25)^5 = 45{,}100\frac{ft-lbf}{} \approx \mathbf{82hp} \quad Ans.(b)$$

11.74 The 32-in-diameter pump in Fig. 11.7*a* is used at 1170 rpm in a system whose head curve is $H_s(\text{ft}) \approx 100 + 1.5Q^2$, with Q in kgal/min. Find the discharge and brake horsepower required for (a) one pump; (b) two pumps in parallel; and (c) two pumps in series. Which configuration is best?

Solution: Assume plain old water, $\rho g \approx 62.4\ \text{lbf/ft}^3$. A reasonable curve-fit to the pump head is taken from Fig. 11.7*a*: $H_p(\text{ft}) \approx 500 - 0.3Q^2$, with Q in kgal/min. Try each case:

(a) **One** pump: $H_p = 500 - 0.3Q^2 = H_s = 100 + 1.5Q^2$: $Q \approx$ **14.9 kgal/min** *Ans.* (a)

(b) Two pumps in **parallel**: $500 - 0.3(Q/2)^2 = 100 + 1.5Q^2$, $Q \approx$ **15.9 kgal/min** *Ans.* (b)

(c) Two pumps in **series**: $2(500 - 0.3Q^2) = 100 + 1.5Q^2$: $Q \approx$ **20.7 kgal/min** *Ans.* (c)

Clearly **case(c) is best,** because it is very near the BEP of the pump. *Ans.*

11.75 Two 35-inch pumps from Fig. 11.7*b* are installed in parallel for the system of Fig. P11.75. Neglect minor losses. For water at 20°C, estimate the flow rate and power required if (a) both pumps are running; and (b) one pump is shut off and isolated.

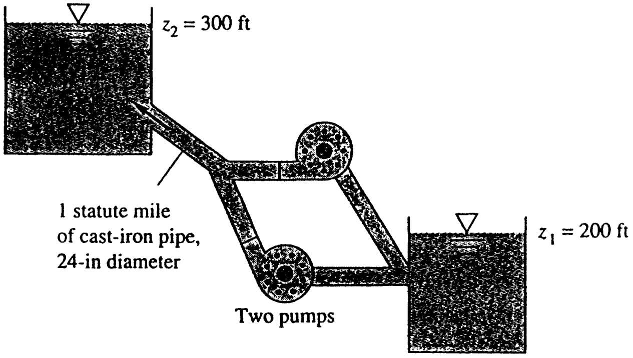


Fig. P11.75

Solution: For water at 20°C, take $\rho = 1.94\ \text{slug/ft}^3$ and $\mu = 2.09\text{E}{-5}\ \text{slug/ft·s}$. For cast iron, $\varepsilon \approx 0.00085$ ft, or $\varepsilon/d \approx 0.000425$. The 35-inch pump has the curve-fit head relation $H_p(\text{ft}) \approx 235 - 0.006Q^3$, with Q in kgal/min. In parallel, each pump takes Q/2:

$$H_p = \text{fcn}\left(\frac{1}{2}Q\right)_{\text{curve-fit}} = H_{\text{syst}} = \Delta z + f\frac{L}{d}\frac{V^2}{2g}, \quad \frac{L}{d} = 2640$$

(a) Two pumps: Iterate on Q (for Q/2 each) until both heads are equal. The results are:

gal

$$P = 2P_{each} = 2\left[\frac{62.4(9800/449)(229)}{0.73}\right] = 855000 \div 550 \approx \mathbf{1550\ bhp} \quad \textit{Ans.}\ \text{(a)}$$

(b) One pump: Iterate on Q alone until both heads are equal. The results are:

$$f \approx 0.0164,\ \ Re \approx 2.3E6,\ \ H \approx 203\ \text{ft},\ \ Q \approx \mathbf{17500\ gal/min} \quad \textit{Ans.}\ \text{(b)}$$

$$P = 62.4(17500/449)(203)/0.87 \div 550 \approx \mathbf{1030\ bhp} \quad \textit{Ans.}\ \text{(b)}$$

The pumps in parallel give 12% more flow at the expense of **50**% more power.

11.76 Two 32-inch pumps are combined in parallel to deliver water at 20°C through 1500 ft of horizontal pipe. If *f* = 0.025, what pipe diameter will ensure a flow rate of 35,000 gal/min at 1170 rpm?

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. As in Prob. 11.74, a reasonable curve-fit to the pump head is taken from Fig. 11.7*a*: H_p(ft) ≈ 500 – 0.3Q^2, with Q in kgal/min. Each pump takes half the flow, 17,500 gal/min, for which

$$H_p \approx 500 - 0.3(17.5)^2 \approx 408\ \text{ft.} \quad \text{Then}\ \ Q_{pipe} = \frac{35000}{} = 78\ \frac{ft^3}{}\ \text{and the pipe loss is}$$

$$H_{syst} = 0.025\left(\frac{1500}{}\right)\frac{[78/(\pi d^2/4)]^2}{} = \frac{5740}{} = 408\ \text{ft,} \quad \text{solve for}\ \mathbf{d \approx 1.70\ ft} \quad \textit{Ans.}$$

11.77 Two pumps of the type tested in Prob. 11.22 are to be used at 2140 r/min to pump water at 20°C vertically upward through 100 m of commercial-steel pipe. Should they be in series or in parallel? What is the proper pipe diameter for most efficient operation?

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For commercial steel take ε = 0.046 mm. **Parallel operation is not feasible**, as the pump can barely generate 100 m of head and the friction loss must be added to this. For **series** operation, assume BEP operation, Q* = 0.2 m^3/s, H* = 95 m:

$$H_{2\,pumps} = 2(95) = \Delta z + f\frac{L}{}\frac{V^2}{} = 100 + f\frac{100}{}\frac{[0.20/(\pi d^2/4)]^2}{} = 100 + \frac{0.3305f}{}$$

Given $\text{Re}_d = 4\rho Q/(\pi\mu d) = 4(998)(0.2)/[\pi(0.001)d] = 254000/d$ and $\varepsilon/d = 0.000046/d$, we can iterate on f until d is obtained. The **EES** result is:

$$f = 0.0156;\ \ \text{Re}_d = 1.8\text{E}6;\ \ V = 12.7\ \text{m/s},\ \ \boldsymbol{d_{\textbf{best}} = 0.142\ \textbf{m}} \quad \textit{Ans.}$$

P11.78 Consider the axial-flow pump of Fig. 11.13, running at 4200 r/min, with an impeller diameter of 36 inches. The fluid is propane gas (molecular weight 44.06). (a) How many pumps in series are needed to increase the gas pressure from 1 atm to 2 atm? (b) Estimate the mass flow of gas.

Solution: Find the gas constant and initial density of the propane. Assume $T = 68°\text{F} = 528°\text{R}$.

$$R_{propane} = \frac{49{,}700}{44.06} = 1128\ \frac{ft-lbf}{slug\ {}^oR}\ ;\ \rho = \frac{p}{RT} = \frac{2116\,lbf/ft^2}{(1128)(528^o R)} = 0.00355\ \frac{slug}{ft^3}$$

From Fig. 11.13, at BEP, $C_{Q*} = 0.55$ and $C_{H*} = 1.07$. Calculate the head rise for one stage:

$$\frac{g\,H}{n^2 D^2} = 1.07 = \frac{(32.2)\,H}{(70 r/s)^2 (3.0\,ft)^2}\ ,\quad \text{or}:\ \ H = 1465\,ft$$

$$\Delta p_{1st\,stage} = \rho\,g\,H = (0.00355)(32.2)(1465) = 168\ \frac{lbf}{ft^2}$$

(*a*) Crudely, then, to increase pressure by 1 atm = 2116 psf, we need 2116/168 = 12.6, or round off to13 stages. However, the pressure rises about 8 per cent per stage, so a better estimate would be ln(2.0)/ln(1.08) = **10 stages.** *Ans.*(*a*)

(*b*) The mass flow follows from the flow coefficient:

$$C_{Q*} = 0.55 = \frac{Q}{n\,D^3} = \frac{Q}{(70)(3.0)^3}\ ,\quad or:\ \ Q = 1040\ \frac{ft^3}{s}$$

$$\dot{m}_{propane} = \rho Q = (0.00355\frac{slug}{ft^3})(1040\frac{ft^3}{s}) = \mathbf{3.7\ \frac{slug}{s}} \quad \textit{Ans.}(b)$$

11.79 Two 32-inch pumps from Fig. 11.7*a* are to be used in *series* at 1170 rpm to lift water through 500 ft of vertical cast-iron pipe. What should the pipe diameter be for most efficient operation? Neglect minor losses.

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. From Fig. 11.7a, read H* ≈ 385 ft at Q* ≈ 20,000 gal/min. Equate

$$\mathrm{H_p = 2H^* = 2(385) = H_{syst} = \Delta z + f\frac{L}{d}\frac{V^2}{2g}}$$

$$\left\lceil 20000 \right\rceil^2$$

Iterate, guessing f ≈ 0.02 to get *d*, then get $\mathrm{Re_d}$ and ε/d and repeat. The final result is

$$\mathrm{f \approx 0.0185,\quad V \approx 45.8\ ft/s,\quad Re_d \approx 4.72E6,\quad \mathbf{d \approx 1.11\ ft}}\quad \textit{Ans.}$$

11.80 Determine if either (*a*) the smallest, or (*b*) the largest of the seven Taco pumps in Fig.P11.24, running in series at 1160 r/min, can efficiently pump water at 20°C through 1 km of horizontal 12-cm-diameter commercial steel pipe.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. For commercial steel pipe, from Table 6.1, ε = 0.046 mm. Then ε/D = 0.046 mm/120 mm = 0.000383.

(*a*) For the smallest pump in Fig. P11.24, the BEP is at about 400 gal/min, with a head of about 41 ft. See what friction head loss results from this flow rate:

$$V = \frac{Q}{A} = \frac{(400\times 6.309E-5)m^3/s}{(\pi/4)(0.12m)^2} = 2.23\frac{m}{s};\ \mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(998)(2.23)(0.12)}{0.001} = 267{,}000$$

$$\frac{\varepsilon}{D} = 0.000383;\quad Eq.(6.48):\ f_{Moody}\ \approx\ 0.0177\ ,$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.0177)(\frac{1000}{0.12})\frac{(2.23)^2}{2(9.81)} = 37.4m = 123\,ft$$

So **three small pumps in series**, each with 41 ft of head, would be an efficient system. *Ans*. (*a*)

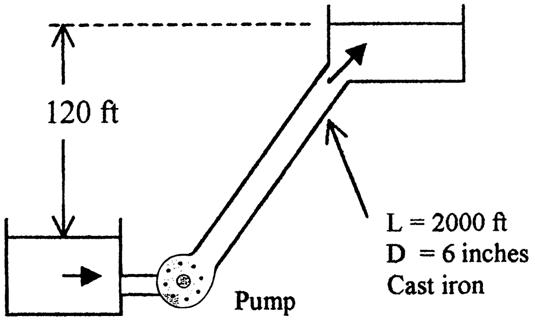
(*b*) For the largest pump in Fig. P11.24, the BEP is at about 525 gal/min, with a head of about 72 ft. See what friction head loss results from this flow rate:

$$\frac{\varepsilon}{D} = 0.000383; \quad Eq.\,(6.48): \quad f_{Moody} \approx 0.0173,$$

So **three large pumps in series**, each with 71 ft of head, would be an efficient system. *Ans*. (*b*) The largest pump is a better solution because of its higher efficiency (80% compared to 65%).

11.81 Reconsider the system of Fig. P6.62. Use the Byron Jackson pump of Prob. 11.28 running at 2134 r/min, no scaling, to drive the flow. Determine the resulting flow rate between the reservoirs. What is the pump efficiency?

Solution: For water take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For cast iron take $\varepsilon = 0.00085$ ft, or $\varepsilon/\text{d} = 0.00085/0.5 = 0.0017$. The energy equation, written between reservoirs, is the same as in Prob. 6.62:



$$H_p = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\text{ ft} + f\frac{2000}{0.5}\frac{[Q/(\pi(0.5)^2/4)]^2}{2(32.2)} = 120 + 1611 fQ^2$$

$$\text{where} \quad f = f_{Moody} = fcn\left(Re_d, \frac{\varepsilon}{d}\right) \quad \text{with} \quad Re_d = \frac{4\rho Q}{\pi\mu d}$$

From the data in Prob. 11.28 for the pump, Hp = fcn(Q) and is of the order of 300 ft. Guessing $f \approx 0.02$, we can estimate a flow rate of about **Q ≈ 2.4 ft^3/s**, down in the low range of the Byron-Jackson pump. Get a closer result with EES:

$$H_p = 340\text{ ft}; \quad f = 0.0228; \quad Re_d = 579{,}000;$$

$$\text{ft} \qquad \mathbf{ft^3}$$

Interpolating, the pump efficiency is $\boldsymbol{\eta \approx 56\%}$. Ans. The flow rate is too low for this particular pump.

11.82 The S-shaped head-versus-flow curve in Fig. P11.82 occurs in some axial-flow pumps. Explain how a fairly *flat* system-loss curve might cause instabilities in the operation of the pump. How might we avoid instability?

Solution: The stability of pump operation is nicely covered in the review article by Greitzer (Ref. 41 of Chap. 11). Generally speaking, there is little danger of instability if the slope of the pump-head curve, dH/dQ, is *negative*, unless there are two such points. In Fig. P11.82 above, a flat system curve may cross the pump curve at *three* points (a, b, c). Of these 3, **point *b* is statically unstable** and cannot be maintained. Consider a small disturbance near point *b*: Suppose the flow rate drops slightly—then the system head decreases, but the pump head decreases *even more*. Then the flow rate will drop still *more*, etc., and we move away from the operating point, which therefore is *unstable*. The general rule is:

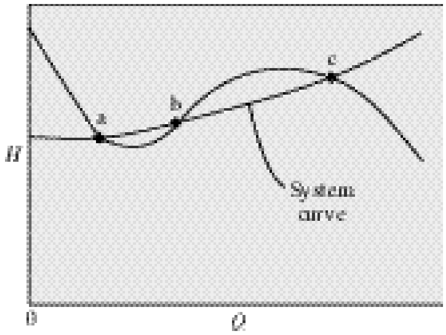


Fig. P11.82

> A pump operating point is statically unstable if the (positive) slope of the pump-head curve is greater than the (positive) slope of the system curve.

By this criterion, both points *a* and *c* above are statically stable. However, if the points are close together or there are large disturbances, a pump can “hunt” or oscillate between points *a* and *c*, so this could also be considered **unstable to large disturbances**.

Finally, even a steep system curve (not shown above) which crosses at only a single point *b* on the positive-slope part of the pump-head curve can be dynamically unstable, that is, it can trigger an energy-feeding oscillation which diverges from point *b*. See Greitzer’s article for further details of this and other turbomachine instabilities.

11.83 The low-shutoff head-versus-flow curve in Fig. P11.83 occurs in some centrifugal pumps. Explain how a fairly flat system-loss curve might cause instabilities in the operation of the pump. What additional vexation occurs when two of these pumps are in parallel? How might we avoid instability?

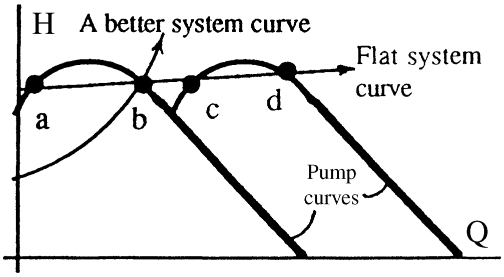


Fig. P11.83

Solution: As discussed, for one pump with a flat system curve, point *a* is statically unstable, point *b* is stable. A 'better' system curve only passes through *b*.

For two pumps in parallel, both points *a* and *c* are unstable (see above). Points *b* and *d* are stable but for large disturbances the system can 'hunt' between the two points.

11.84 Turbines are to be installed where the net head is 400 ft and the flow rate is 250,000 gal/min. Discuss the type, number, and size of turbine which might be selected if the generator selected is (a) 48-pole, 60-cycle (*n* =150 rpm); or (b) 8-pole (*n* = 900 rpm). Why are at least two turbines desirable from a planning point of view?

Solution: We select **two** turbines, of about half-flow each, so that one is still available for power generation if the other is shut down for maintenance or repairs. *Ans.*

Assume $\eta \approx 90\%$:

$$P_{total} = \eta\rho gQH \approx 0.9(62.4)\left(\frac{250000}{449}\right)(400) \div 550 \approx 22750 \text{ hp (each turbine} \approx 11375 \text{ hp)}$$

(a) n = 150 rpm:

$$N_{sp} = \frac{nP^{1/2}}{H^{5/4}} = \frac{150(11375)^{1/2}}{(400)^{5/4}} \approx \mathbf{8.9} \text{ (select two } \textbf{impulse turbines}\text{)} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Estimate} \quad \phi \approx 0.47 = \frac{\pi nD}{\sqrt{2gH}} = \frac{\pi(150/60)D}{\sqrt{2(32.2)(400)}}, \quad \text{or:} \quad \mathbf{D \approx 9.6\ ft} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } n = 900 \text{ rpm:} \quad N_{sp} = \frac{900(11375)^{1/2}}{(400)^{5/4}} \approx \mathbf{54} \text{ (select } \textbf{two Francis turbines}\text{)} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Fig. 11.21:} \quad C_p^* \approx 2.6 = \frac{P^*}{\rho n^3 D^5} = \frac{11375 \times 550}{1.94\left(900\right)^3 D^5}, \quad \text{solve } \mathbf{D \approx 3.26\ ft} \quad \textit{Ans.}\text{ (b)}$$

11.85 For a high-flow site with a head of 45 ft, it is desired to design a single 7-ft-diameter turbine that develops 4000 bhp at a speed of 360 r/min and 88% efficiency. It is decided first to test a geometrically similar model of diameter 1 ft, running at 1180 r/min. (*a*) What likely type of turbine is the prototype? What are the appropriate (*b*) head, and (*c*) flow rate for the model test? (*d*) Estimate the power expected to be delivered by the model turbine.

Solution: For water, take ρ = 1.94 slug/ft^3. (*a*) We have enough information to determine power specific speed:

$$N_{sp} = \frac{(rpm)(bhp)^{1/2}}{[H(ft)]^{5/4}} = \frac{(360 rpm)(4000 bhp)^{1/2}}{(45\ ft)^{5/4}} = 195$$

For a water-flow application, this number is characteristic of a **propeller turbine**. *Ans.* (*a*)

(*b*) The similarity rules give us the required model head by equating the head coefficients:

$$C_H = \frac{gH_1}{n_1^2\ D_1^2} = \frac{(32.2)(45)}{(360)^2(7.0)^2} = \frac{gH_2}{n_2^2\ D_{21}^2} = \frac{(32.2)\ \ H_2}{(1180)^2(1.0)^2},\quad Solve\ H_2 \approx 9.9\ ft \quad Ans.(b)$$

[We didn't bother changing *n* to r/s because the factors of "60" would have cancelled out.]

(c) To get the model flow rate, first calculate the *prototype* flow rate from the known power:

$$P = \eta\rho gHQ = (4000x550\ \ ft-lbf/s) = (0.88)(1.94 slug/ft^3)(32.2\, ft/s^2)(45\, ft)Q_1$$

$$Solve\ \ for\ Q_1 = 890\ \ ft^3/s$$

$$Then\quad C_Q = \frac{Q_1}{n_1D_1^3} = \frac{(890)}{(360)(7.0)^3} = \frac{Q_2}{n_2D_2^3} = \frac{Q_2}{(1180)(1.0)^3}\qquad Solve\, Q_2 = \ 8.5\frac{ft^3}{s}\quad Ans.(c)$$

If we assume the efficiency is unchanged, the model power output will be

$$P_2 = \eta\rho gH_2Q_2 = (0.88)(1.94)(32.2)(9.9\, ft)(8.5\, ft^3/s) = \ 4630\frac{ft-lbf}{} = 8.4\, bhp\quad Ans.(d)$$

[The Moody size-effect formula, Eq. (11.29a), suggests $\eta_2 \approx 0.805$, or $P_2 \approx$ 7.7 bhp.]

11.86 The Tupperware hydroelectric plant on the Blackstone River has four 36-inch-diameter turbines, each providing 447 kW at 200 rpm and 205 ft^3/s for a head of 30 ft. What type of turbines are these? How does their performance compare with Fig. 11.21?

Solution: Convert P* = 447 kW = 599 hp. Then, for D = 36″ = 3.0 ft,

$$N_{sp} = \frac{nP^{1/2}}{H^{5/4}} = \frac{200(599)^{1/2}}{(30)^{5/4}} \approx \mathbf{70} \text{ (These are } \textbf{Francis turbines}\text{)} \quad \textit{Ans.}$$

Use the given data to compute the dimensionless BEP coefficients:

$$C_Q^* = \frac{205}{(200/60)(3.0)^3} \approx \mathbf{2.3}; \quad C_P^* = \frac{599(550)}{1.94(200/60)^3(3)^5} \approx \mathbf{19} \text{ (both are } \textbf{quite different!}\text{)}$$

$$C_H^* = \frac{32.2(30)}{(200/60)^2(3)^2} \approx \mathbf{9.7}\ \ \mathbf{(OK)}; \quad \eta_{max} = \frac{599(550)}{62.4(205)(30)} \approx \mathbf{86\%}\ \ \mathbf{(OK)} \quad \textit{Ans.}$$

Fig. P11.87

11.87 An idealized radial turbine is shown in Fig. P11.87. The absolute flow enters at 30° and leaves radially inward. The flow rate is 3.5 m^3/s of water at 20°C. The blade thickness is constant at 10 cm. Compute the theoretical power developed at 100% efficiency.

Solution: For water, take $\rho \approx 998$ kg/m^3. With reference to Fig. 11.22 and Eq. 11.35,

$$u_2 = \omega r_2 = 135\left(\frac{2\pi}{60}\right)(0.7) = 9.90\ \frac{m}{s},\ \alpha_2 = 30°,\ \alpha_1 = 90°,\ V_{n2} = \frac{3.5}{2\pi(0.7)(0.1)} \approx 7.96\ \frac{m}{s}$$

$$V_{t2} = \frac{V_{n2}}{\tan\alpha_2} = \frac{7.96}{\tan 30°} = 13.8\ \frac{m}{s} \quad \text{and} \quad V_{t1} = \frac{V_{n1}}{\tan 90°} = 0$$

$$\text{Thus } P_{theory} = \rho Q u_2 V_{t2} = 998(3.5)(9.90)(13.8) = 477000\ W \approx \mathbf{477\ kW} \quad \textit{Ans.}$$

11.88 Performance data for a very small (*D* = 8.25 cm) model water turbine, operating with an available head of 49 ft, are as follows:

Q, m^3/h:	18.7	18.7	18.5	18.3	17.6	**16.7**	15.1	11.5
rpm:	0	500	1000	1500	2000	**2500**	3000	3500
η:	0	14%	27%	38%	50%	**65%**	61%	11%

(a) What type of turbine is this likely to be? (b) What is so different about this data compared to the dimensionless performance plot in Fig. 11.21*d*? Suppose it is desired to use a geometrically similar turbine to serve where the available head and flow are 150 ft and 6.7 ft^3/s, respectively. Estimate the most efficient (c) turbine diameter; (d) rotation speed; and (e) horsepower.

Solution: (a) Convert *Q* = 16.7 m^3/h = 0.164 ft^3/s. Use BEP data to calculate power specific speed:

$$bhp = \rho g Q H \eta = (62.4\ \text{lbf/ft}^3)(0.164\ \text{ft}^3/\text{s})(49\ \text{ft})(0.65) = 326\ \text{ft·lbf/s} = 0.593\ \text{hp}$$

$$N_{sp} = \frac{\text{rpm}(\text{bhp})^{1/2}}{(H, \text{ft})^{5/4}} = \frac{(2500\ \text{rpm})(0.593\ \text{bhp})^{1/2}}{(49\ \text{ft})^{5/4}} \approx 15\ (\textbf{Francis turbine}) \quad \textit{Ans.}\ (a)$$

(b) This data is different because it has ***variable speed***. Our other data is at constant speed. *Ans*. (b)

(c, d, e) First establish the BEP coefficients from the small-turbine data:

$$nD \quad (2500/60\ \text{r/s})(0.0825\ \text{m})$$

$$H \quad (9.81)(40*0.3048\ \text{m})$$

$$C_P^* = \frac{P}{\rho n^3 D^5} = \frac{(326*1.3558\ \text{W})}{(998\ \text{kg/m}^3)(2500/60\ \text{r/s})^3(0.0825\ \text{m})^5} = \mathbf{1.60}$$

Now enter the new data, in *English* units, to the flow and head coefficients:

$$C_Q^* = \frac{6.7\ \text{ft}^3/\text{s}}{D^3} = 0.198; \quad C_H^* = \frac{(32.2\ \text{ft/s}^2)(150\ \text{ft})}{{}^2D^2} = 12.4$$

$$\text{Solve for}\quad n_2 = 15.1\ \text{r/s} = \mathbf{904\ r/min}; \quad D_2 = 1.31\ \text{ft} = \mathbf{15.7\,in} \quad \textit{Ans.}\ \text{(c, d)}$$

$$\text{Then}\quad P_2 = C_P^*\rho n_2^3 D_2^5 = 1.60(1.94\ \text{slug/ft}^3)(15.1\ \text{r/s})^3(1.31\ \text{ft})^5$$

$$\text{ft}\times\text{lbf}$$

Actually, since the new turbine is 4.84 times larger, we could use the Moody step-up formula, Eq. (11.29a), to predict $(1 - \eta_2) \approx (1 - 0.65)/(4.84)^{1/4} = 0.236$, or $\eta_2 = 0.764 = 76.4\%$. We thus expect more power from the larger turbine:

$$P_2 = (74\ \text{hp})(76\%/65\%) \approx \mathbf{87\ hp}\ \textit{Better} \quad \textit{Ans.}\ \text{(e)}$$

11.89 A Pelton wheel of 12-ft pitch diameter operates under a new head of 2000 ft. Estimate the speed, power output, and flow rate for best efficiency if the nozzle exit diameter is 4 inches.

Solution: First get the jet velocity and then assume BEP at $\phi \approx 0.47$:

$$C_v \approx 0.94, \quad \text{so:}\quad V_{jet} = C_v\sqrt{2gH} = 0.94\sqrt{2(32.2)(2000)} \approx 337\ \text{ft/s}$$

$$\text{BEP at}\quad \phi \approx 0.47 = \frac{\pi nD}{\sqrt{2gH}} = \frac{\pi n(12.0)}{\sqrt{2(32.2)(2000)}},$$

$$Q = V_{jet}A_{nozzle} = 337\left(\frac{\pi}{4}\right)\left(\frac{4}{12}\right)^2 = 29.4\ \frac{\text{ft}^3}{\text{s}} \approx \mathbf{13200}\ \frac{\mathbf{gal}}{\mathbf{min}} \quad \textit{Ans.}$$

$$u_{best} = V_{jet}/2 \approx 169\ \text{ft/s}$$

$$P_{theory} = \rho Qu(V_j - u)(1 - \cos\beta)$$

$$1.94(29.4)(169)(337 - 169)(1 \quad 165°) \quad 550 \quad 5800\ \text{h}$$

$$P_{turb} \approx 4500 \text{ bhp}, \quad N_{sp} \approx \frac{208(4500)}{(3000)^{5/4}} \approx 1.34: \quad \text{Fig. 11.27, read } \eta_{max} < 80\%, \text{ say, } 75\%$$

$$\text{Actual power output} \approx \eta P_{100\%} = 0.75(5800) \approx \mathbf{4350\ bhp} \quad \textit{Ans.}$$

11.90 An idealized radial turbine is shown in Fig. P11.90. The absolute flow enters at 25° with the blade angles as shown. The flow rate is 8 m^3/s of water at 20°C. The blade thickness is constant at 20 cm. Compute the theoretical power devel-oped at 100% efficiency.

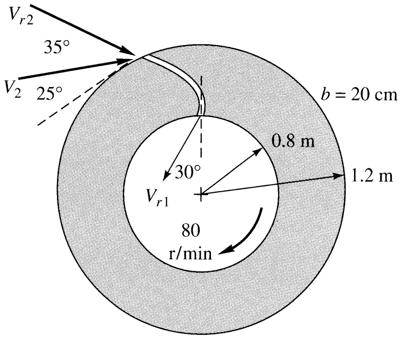
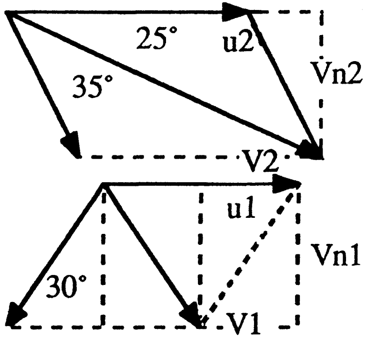


Fig. P11.90

Solution: The inlet (2) and outlet (1) velocity vector diagrams are shown at right. The normal velocities are



$$V_{n2} = Q/A_2 = \frac{8.0}{2\pi(1.2)(0.2)} = 5.31 \text{ m/s}$$

$$V_{n1} = Q/A_1 = \frac{8.0}{2\pi(0.8)(0.2)} = 7.96 \text{ m/s}$$

From these we can compute the tangential velocities at each section:

$$u_2 = \omega r_2 = 80\left(\frac{2\pi}{60}\right)(1.2) = 10.1 \text{ m/s};$$

$$u_1 = \omega r_1 = 80\left(\frac{2\pi}{60}\right)(0.8) = 6.70 \text{ m/s}$$

$$V_{t2} = V_{n2}\cot 25° = 11.4 \text{ m/s}; \qquad V_{t1} = u_1 - V_{n1}\tan 30° = 2.11 \text{ m/s}$$

$$P_{theory} = \rho Q(u_2V_{t2} - u_1V_{t1}) = 998(8)[10.1(11.4) - 6.7(2.11)] \approx \mathbf{800{,}000\ W} \quad \textit{Ans.}$$

11.91 The flow through an axial-flow *turbine* can be idealized by modifying the stator-rotor diagrams of Fig. 11.12 for energy absorption. Sketch a suitable blade and flow arrangement and the associated velocity vector diagrams. For further details, see Chap. 8 of Ref. 25.

Solution: Some typical velocity diagrams are shown below, where u = ωr = blade speed. The power delivered to the turbine, at 100% ideal shock-free flow, is

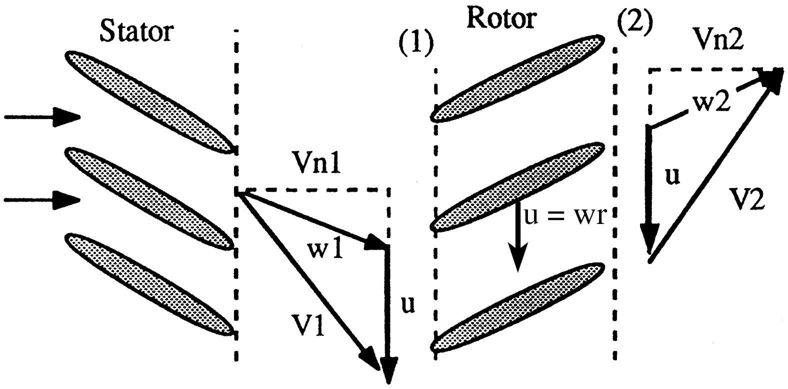


Fig. P11.91

11.92 A dam on a river is being sited for a hydraulic turbine. The flow rate is 1500 m^3/h, the available head is 24 m, and the turbine speed is to be 480 r/min. Discuss the estimated turbine size and feasibility for (a) a Francis turbine; and (b) a Pelton wheel.

Solution: Assume $\eta \approx 89\%$, as in Fig. 11.21*d*. The power generated by the turbine would be $P = \eta\gamma QH = (0.89)(62.4\ \text{lbf/ft}^3)(14.7\ \text{ft}^3/\text{s})(78.7\ \text{ft}) = 64{,}300\ \text{ft-lbf/s} = 117\ \text{hp}$. Now compute $\mathbf{N_{sp}} = (480\ \text{rpm})(117\ \text{hp})^{1/2}/(78.7\ \text{ft})^{5/4} \approx \mathbf{22}$, appropriate for a Francis turbine. (a) A Francis turbine, similar to Fig. 11.21*d*, would have $C_Q^* \approx 0.34 = (14.7\ \text{ft}^3/\text{s})/[(480/60\ \text{r/s})D^3]$. Solve for a turbine diameter of about **1.8 ft**, which would be excellent for the task. *Ans*. (a)

(b) A Pelton wheel at best efficiency (half the jet velocity) would only be **18 inches** in diameter, with a huge nozzle, $d \approx 6$ inches, which is too large for the wheel. We conclude that a Pelton wheel would be a poor design. *Ans*. (b)

11.93 Figure P11.93 shown on the following page, shows a *crossflow* or “Banki” turbine [Ref. 55], which resembles a squirrel cage with slotted curved blades. The flow enters at about 2 o’clock, passes through the center and then again through the blades, leaving at about 8 o’clock.

Report to the class on the operation and advantages of this design, including idealized velocity vector diagrams.

Brief Discussion (not a "Solution"):
The crossflow turbine is ideal for small dam owners, because of its simple, inexpensive design. It can easily be constructed by a novice (such as the writer) from wood and plastic.

It is not especially efficient (≈ 60%) but makes good, inexpensive use of a small stream to produce electric power. For details, see Ref. 55 or the paper "Design and Testing of an Inexpensive Crossflow Turbine," by W. Johnson et al., ASME Symposium on Small Hydropower Fluid Machinery, Phoenix, AZ, Nov. 1982, ASME vol. H00233, pp. 129–133.

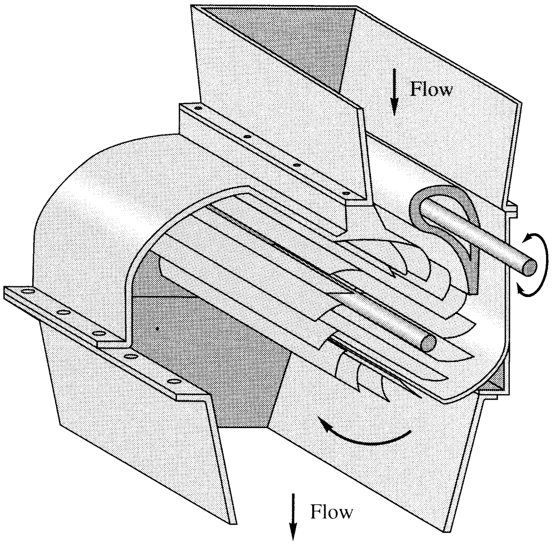


Fig. P11.93

11.94 A simple crossflow turbine, Fig. P11.93 above, was constructed and tested at the University of Rhode Island. The blades were made of PVC pipe cut lengthwise into three 120°-arc pieces. When tested in water at a heads of 5.3 ft and a flow rate of 630 gal/min, the measured power output was 0.6 hp. Estimate (a) the efficiency; and (b) the power specific speed if *n* ≈ 200 rpm.

Solution: We have sufficient information to compute the available water power:

$$P_{\text{avail}} = \rho g Q H = (62.4)\left(\frac{630}{449}\right)(5.3) = 464 \div 550 = 0.844\ \text{hp}, \quad \therefore\ \eta = \frac{0.6}{0.844} \approx \mathbf{71\%} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{At 200 rpm,}\quad N_{sp} = \frac{\text{rpm}(\text{hp})^{1/2}}{(\text{ft})^{5/4}} = \frac{200(0.6)^{1/2}}{(5.3)^{5/4}} \approx \mathbf{19} \quad \textit{Ans.}\ \text{(b)}$$

11.95 One can make a theoretical estimate of the proper diameter for a penstock in an impulse turbine installation, as in Fig. P11.95. Let *L* and *H* be known, and let the turbine performance be idealized by Eqs. (11.38) and (11.39). Account for friction loss h_f in the penstock, but neglect minor losses. Show that (a) the maximum power is generated when

$h_f = H/3$, (b) the optimum jet velocity is $(4gH/3)^{1/2}$, and (c) the best nozzle diameter is $D_j = [D^5/(2fL)]^{1/4}$, where f is the pipe-friction factor.

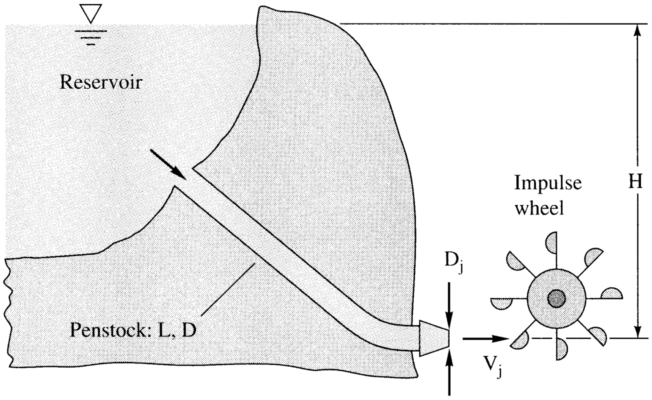


Fig. P11.95

Solution: From Eqs. 11.38 and 39, maximum power is obtained when $u = V_j/2$, or:

$$P_{max} = \rho Q \frac{V_j}{2}\left(V_j - \frac{V_j}{2}\right)(1-\cos\beta) = \rho A_j\left(\frac{1-\cos\beta}{4}\right)V_j^3 = CV_j^2V_j, \quad C = \text{constant}$$

Now apply the steady-flow energy equation between the reservoir and the outlet jet:

$$\Delta z = H = f\frac{L}{D}\frac{V_{pipe}^2}{2g} + \frac{V_j^2}{2g}, \quad \text{or:} \quad V_j^2 = 2gH - f\frac{L}{D}\left(\frac{D_j}{D}\right)^4 V_j^2 \quad \text{since } V_j\frac{\pi}{4}D_j^2 = V_p\frac{\pi}{4}D^2$$

$$\text{Thus} \quad P_{max} = C\left[2gHV_j - f\frac{L}{D}\left(\frac{D_j}{D}\right)^4 V_j^3\right]; \quad \text{Differentiate: } \frac{dP_{max}}{dV_j} = 0 \quad \text{if } 2gH = 3f\frac{L}{D}V_p^2$$

$$\text{or if:} \quad H = 3h_{f,pipe}! \quad \text{The pipe head loss} = \mathbf{H/3} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Continuing,}\ V_j^2\Big|_{optimum} = 2g(H - h_f) = 2g(H - H/3), \quad \text{or:} \quad V_{j,optimum} = \sqrt{\frac{\mathbf{4}}{\mathbf{3}}\mathbf{gH}} \quad \textit{Ans.}\ \text{(b)}$$

Then the correct pipe flow speed is obtained by back-substitution:

$$f\frac{L}{D}\frac{V_{pipe}^2}{2g} = \frac{H}{3}, \quad \text{or:} \quad V_{pipe} = \sqrt{\frac{\mathbf{2gH}}{\mathbf{3fL/D}}}$$

$$\text{Continuing,}\ \ V_p^2 = V_j^2\left(\frac{D_j}{D}\right)^4 = \frac{2gH}{3fL/D},\quad \text{solve for}\ \ D_{jet} = \left(\frac{\mathbf{D_p^5}}{\mathbf{2fL}}\right)^{\mathbf{1/4}}\quad \textit{Ans.}\ \text{(c)}$$

11.96 Apply the results of Prob. 11.95 to determining the optimum (a) penstock diameter, and (b) nozzle diameter for the data of Prob. 11.92, with a commercial-steel penstock of length 1500 ft. [H = 800 ft, Q = 40,000 gal/min.]

Solution: For water, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For commercial steel, $\varepsilon \approx 0.00015$ ft. We can't find f_{Moody} until we know D, so iteration is required. We can immediately compute $V_{jet} = (4gH/3)^{1/2} \approx 185$ ft/s, but this wasn't asked! Anyway,

$$h_f = H/3 = 267\ \text{ft} = f\frac{L}{D}\frac{V_p^2}{2g} = f\frac{1500}{D}\frac{V_p^2}{2(32.2)},\quad \text{where}\ \ V_{pipe} = \frac{40000/449}{(\pi/4)D^2}\ \text{(ft/s)}$$

$$\text{Iterate to find}\quad f \approx 0.0120,\ \ Re_D \approx 6.24\text{E}6,\ \text{and}\quad D = D_{penstock} \approx \mathbf{1.68\ ft}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{Then}\quad D_{jet} = \left(\frac{D^5}{2fL}\right)^{1/4} = \left[\frac{(1.68)^5}{2(0.0120)(1500)}\right]^{1/4} \approx \mathbf{0.78\ ft = 9.4\ inches}\quad \textit{Ans.}\ \text{(b)}$$

11.97 Consider the following non-optimum version of Prob. 11.95: $H = 450$ m, $L = 5$ km, $D = 1.2$ m, $D_j = 20$ cm. The penstock is concrete, $\varepsilon = 1$ mm. The impulse wheel diameter is 3.2 m. Estimate (a) the power generated by the wheel at 80% efficiency; and (b) the best speed of the wheel in r/min. Neglect minor losses.

Solution: For water take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. This is a non-optimum condition, so we simply make a standard energy and continuity analysis. Refer to the figure on the next page for the notation:

$$\Delta z = H = h_f + \frac{V_j^2}{2g},\quad V_jD_j^2 = V_{pipe}D^2,\quad \text{combine and solve for jet velocity:}$$

$$V_j^2\left[1 + f\frac{L}{D}\left(\frac{D_j}{D}\right)^4\right] = 2gH,\quad \text{where}\quad f = fcn\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right),\quad \frac{\varepsilon}{D} = \frac{0.001}{1.2} = 8.33\text{E}{-4}$$

For example, guessing $f \approx 0.02$, we estimate $V_j \approx 91.2$ m/s. Using **EES** yields

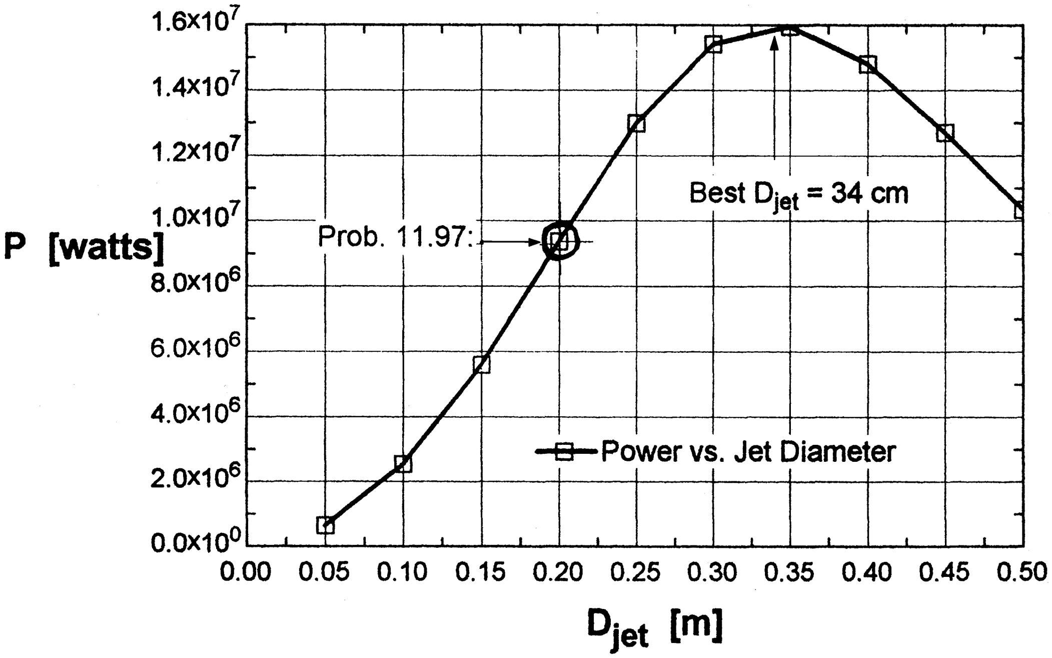
$$f = 0.0189,\quad Re_d = 3.03\text{E}6,\quad V_{jet} = 91.23\ \frac{\text{m}}{},\quad \boldsymbol{Q = 2.87}\ \frac{\mathbf{m^3}}{}$$

The power generated (at 80% efficiency) and best wheel speed are

$$Power = \rho Q u_{\text{wheel}}(V_j - u_{\text{wheel}})(1-\cos\beta)(0.8), \quad u_{\text{best}} = \frac{1}{2}V_{\text{jet}} = \Omega_{\text{wheel}}\frac{D_{\text{wheel}}}{2}, \quad \beta \approx 165°$$

$$If \;\; D_{\text{wheel}} = 3.2 \text{ m}, \quad \text{solve for } \Omega_{\text{wheel}} = 28.5 \frac{\text{rad}}{\text{s}} = \mathbf{272} \frac{\mathbf{rev}}{\mathbf{min}},$$

As shown on the figure below, which varies D_j, the optimum jet diameter is *34 cm*, not 20 cm, and the optimum power would be *16 MW*, or 70% more!



11.98 Francis and Kaplan (enclosed) turbines are often provided with *draft tubes*, which lead the exit flow into the tailwater region, as in Fig. P11.98. Explain at least two advantages to using a draft tube.

Solution: Draft tubes have two big ad-vantages:
(1) They reduce the *exit loss*, since a draft tube is essentially a diffuser, as in Fig. 6.23 of the text, so more of the water head is converted to power.

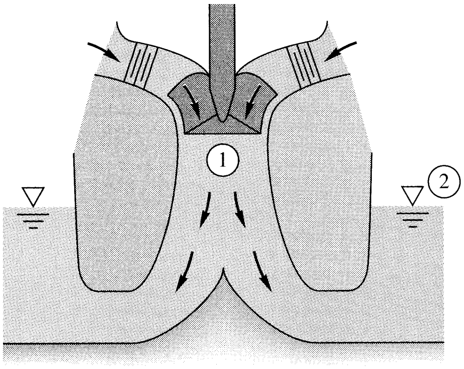


Fig. P11.98

(2) They reduce total losses *downstream of the turbine*, so that the turbine runner can be placed higher up without the danger of cavitation.

11.99 Like pumps, turbines can also cavitate when the pressure at point 1 in Fig. P11.98 drops too low. With NPSH defined by Eq. 11.20, the empirical criterion given by Wislicenus [Ref. 4] for cavitation is

$$N_{ss} = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{[\text{NPSH(ft)}]^{3/4}} \geq 11{,}000$$

Use this criterion to compute how high $(z_1 - z_2)$ the impeller eye in Fig. P11.98 can be placed for a Francis turbine, with a head of 300 ft, $N_{sp} = 40$, and $p_a = 14$ psia, before cavitation occurs in 60°F water.

Solution: For water at 60°F, take $\rho g = 62.4\ \text{lbf/ft}^3$ and $p_v \approx 37$ psfa $= 0.25$ psia. Then

$$\text{Eq. 11.20: NPSH} = \frac{p_a - p_v}{\rho g} - \Delta z - h_{fi} = \frac{(14.0 - 0.25)(144)}{62.4} - \Delta z - 0 = 31.7\ \text{ft} - \Delta z$$

Now we need the NPSH, which we find between the two specific-speed criteria:

$$N_{sp} = 40 = \frac{n(P\ \text{in hp})^{1/2}}{(300\ \text{ft})^{5/4}} \quad \text{with } P = \eta\rho g(Q/449)(300\ \text{ft}) \div 550$$

$$N_{ss} = 11000 = \frac{n(Q\ \text{in gpm})^{1/2}}{(\text{NPSH})^{3/4}}. \quad \text{Iterate to solve by assuming } \eta \approx 90\%$$

(We can't find n or Q, only $nQ^{1/2}$). Result: NPSH ≈ 45.0 ft, $\Delta z = 31.7 - 45 \approx$ **−13.3 ft**. *Ans.*

11.100 Consider the large wind turbine in the chapter-opener photo. Pretend that you do not know that 5M means five megawatts. Use the meager photo-caption data, plus your knowledge of wind turbine theory and experiment, to estimate the power delivered by the turbine for a rated wind speed of 12 m/s and a rotation rate of 10 rev/min.

Solution: Meager data: It is a HAWT with D = 126 m. For sea level air, take ρ = 1.22 kg/m^3. Convert ω = 10 rev/min = (10)(2π)/60 = 1.05 rad/s. Then calculate the speed ratio:

$$\frac{\omega R}{V} = \frac{(1.05\,rad/s)(63\,m)}{12\,m/s} = 5.5$$

Now find the experimental power coefficient on Fig. 11.31 of the text. This giant turbine is probably not “high speed”, but use the “high speed HAWT” curve: $C_p \approx 0.43$. Compute

$$\text{Power} = C_p \frac{\rho}{2} A V^3 = (0.43)(\frac{1.22}{2})[\frac{\pi}{4}(126)^2](12)^3 = 5,650,000\,W \approx 5.7\,\text{MW} \quad \textit{Ans.}$$

This is a good estimate, considering that we have no data for the 5MW turbine.

11.101 A Darrieus VAWT in operation in Lumsden, Saskatchewan, that is 32 ft high and 20 ft in diameter sweeps out an area of 432 ft^2. Estimate (a) the maximum power and (b) the rotor speed if it is operating in 16 mi/h winds.

Solution: For air in Saskatchewan (?), take $\rho \approx$ 0.0023 slug/ft^3. Convert 16 mi/h = 23.5 ft/s. From Fig. 11.34 for the Darrieus VAWT, read optimum C_P and speed ratio:

$$C_{P,\max} \approx 0.42 = \frac{P}{(1/2)\rho A V^3} = \frac{P_{\max}}{(1/2)(0.0023)(432)(23.5)^3}$$

$$\text{Solve for} \quad P_{\max} \approx 2696\ \text{ft·lbf/s} \approx \mathbf{3.7\ kW} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{At}\ P_{\max},\ \frac{\omega r}{V} \approx 4.1 = \frac{\omega(20/2)}{23.5},\quad \text{or:}\quad \omega \approx 9.62\ \frac{\text{rad}}{s} \times \frac{60}{2\pi} \approx \mathbf{92\ rpm} \quad \textit{Ans.}\ \text{(b)}$$

11.102 An American 6-ft diameter multiblade HAWT is used to pump water to a height of 10 ft through 3-in-diameter cast-iron pipe. If the winds are 12 mi/h, estimate the rate of water flow in gal/min.

Solution: For air in “America” (?), take $\rho \approx$ 0.0023 slug/ft^3. Convert 12 mi/h = 17.6 ft/s. For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/(ft·s). From Fig. 11.34 for the American multiblade HAWT, read optimum C_P and speed ratio:

$$C_{P,\max} \approx 0.29 \text{ at } \frac{\omega r}{V} \approx 0.9:\quad P_{\max} \approx 0.29\left(\frac{1}{2}\right)(0.0023)\frac{\pi}{4}(6)^2(17.6)^3 \approx 51.4\ \frac{\text{ft·lbf}}{s}$$

$$\text{If } \eta_{pump} \approx 80\%, \quad P_{pump} \approx 0.8(51.4) = \rho_{water} gQH_{syst} = 62.4Q\left| \Delta z + f\frac{L}{D}\frac{V_{pipe}^2}{2g} \right|$$

$$\text{where} \quad V_{pipe} = \frac{Q}{(\pi/4)(3/12)^2}, \quad \frac{L}{D} = \frac{10}{3/12} = 40, \quad \frac{\varepsilon}{D} = \frac{0.00085}{3/12} \approx 0.0034$$

Clean up to: $0.659 = Q(10 + 258 f_{Moody} Q^2)$, with Q in ft^3/s. Iterate to obtain

$$f \approx 0.0305, \ V_{pipe} \approx 1.34 \ \frac{ft}{s}, \ Re \approx 31000, \ Q \approx 0.0657 \ \frac{ft^3}{s} \approx \mathbf{29\ gal/min} \quad \textit{Ans.}$$

P11.103 The new 3-blade HAWT municipal wind turbine in Portsmouth, RI has a tower 213 ft high, with a turbine diameter of 252 ft. The newspaper rates the turbine at 1.5 MW. (*a*) What wind velocity, and turbine rotation speed, would generate this power? (*b*) Estimate the annual power production for a steady wind speed of 12 mi/h.

Solution: From Fig. 11.32, best efficiency for a HAWT occurs at $\omega r/V \approx 5.5$, where the power coefficient is about 0.45. Assume sea-level density of 0.00237 slug/ft³ – all of Rhode Island is pretty much sea-level. Convert 1.5 MW ÷ 1.3558 = 1.106E6 ft-lbf/s. Then find the velocity:

$$P = 1.108\text{E}6 \ \frac{ft-lbf}{s} = C_P \frac{\rho}{2} A V^3 = (0.45)(\frac{0.00237}{2})[\frac{\pi}{4}(252 ft)^2]V^3$$

$$\textit{Solve for} \quad V = (41598)^{1/3} = 34.6 \ \frac{ft}{s} \approx \mathbf{24} \ \frac{\mathbf{mi}}{\mathbf{h}} \quad \textit{Ans.}(a)$$

$$\text{Also, } \frac{\omega r}{V} = 5.5 = \frac{\omega(126 ft)}{34.8 \ ft/s}, \quad \text{Solve } \omega = 1.52 \ \frac{rad}{s} = \mathbf{14.5} \ \frac{\mathbf{r}}{\mathbf{min}} \quad \textit{Ans.}(a)$$

(*b*) Carry out the same calculations for a steady V = 12 mi/h = 17.6 ft/s:

$$P = C_P \frac{\rho}{2} A V^3 = (0.45)(\frac{0.00237}{2})[\frac{\pi}{4}(252 ft)^2](17.6 ft/s)^3 = 145{,}000 \frac{ft-lbf}{s} = 197 \ kW$$

$$\text{Annual production} = (197 kW)(24 h/day)(365 day/year) = \mathbf{1{,}730{,}000 \ \ kWh/year} \quad \textit{Ans.}(b)$$

That’s a lot of electric power. Of course, Rhode Island isn’t that windy all the time.

11.104 The controversial Cape Wind project proposes 130 large wind turbines in Nantucket Sound, intended to provide 75% of the electric power needs of Cape Cod and the Islands. The turbine diameter is 328 ft. For an average wind velocity of 14 mi/h, what are the best rotation rate (r/min) and total power output (MW) estimates for (*a*) a HAWT; and (*b*) a VAWT?

Solution: Convert 14 mi/h = 20.53 ft/s. Use Fig. 11.32 of the text to make your estimates. (*a*) HAWT geometry. Best efficiency is at ($\omega r/V$) ≈ 5.8, where $C_P \approx 0.45$:

$$\frac{\omega r}{V} = 5.8 = \frac{\omega(328/2\ ft)}{20.53\,ft/s},\ \text{Solve}\ \ \omega = 0.726\frac{rad}{s},\ RPM = \frac{60(0.726)}{2\pi} = 6.9\frac{r}{\min}\quad Ans.(a)$$

$$Power = C_P\frac{\rho}{2}AV^3 = 0.45(\frac{0.00238}{2})\frac{\pi}{4}(328)^2(20.53)^3 = 392,000\frac{ft-lbf}{s} = 531\,kW$$

$$\text{Total}\quad \text{power} = (0.531\ MW)(130\ \text{units}) = 69\ MW\quad Ans.(a)$$

(*b*) Darrieus VAWT geometry. Best efficiency is at ($\omega r/V$) ≈ 4.0, where $C_P \approx 0.40$:

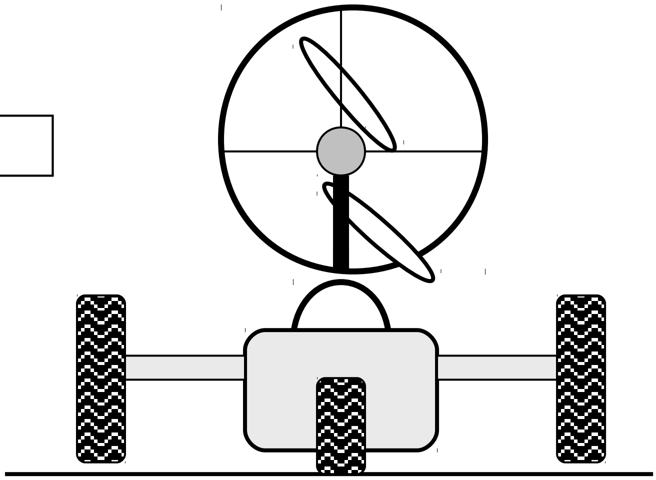
$$\frac{\omega r}{V} = 4.0 = \frac{\omega(328/2\ ft)}{20.53\ ft/s},\ \text{Solve}\ \ \omega = 0.501\frac{rad}{s},\ \ RPM = \frac{60(0.501)}{2\pi} = 4.8\frac{r}{\min}\ \ Ans.(b)$$

$$Power = C_P\frac{\rho}{\ }AV^3 = 0.40(\frac{0.00238}{\ })\frac{\pi}{\ }(328)^2(20.53)^3 = 348,000\frac{ft-lbf}{\ } = 472\,kW$$

Primarily because of the variability of Cape winds, both of these are *over*estimates. The official power estimate for the Cape Wind (HAWT) project is 42 MW.

P11.105 In 2007, a wind-powered-vehicle contest, held in North Holland [64], was won with a design by students at the University of Stuttgart. A schematic of the winning 3-wheeler is shown in Fig. P11.105. It is powered by a shrouded wind turbine, not a propeller, and, unlike a sailboat, can move directly into the wind. (*a*) How does it work? (*b*) What if the wind is off to the side? (*c*) Cite some design questions you might have.

Fig. P11.105



Solution: (*a*) The wind turns the turbine. The turbine turns a shaft that is geared to the wheels and drives the vehicle.

(*b*) If the wind is from the side, the entire turbine swivels to face into the wind.

(*c*) Design questions might include:

1. How large to make the turbine? Answer: D = 6 ft.
2. How to design the drive train? Answer: Two sets of bicycle gears and a chain drive.
3. How to balance the weight to resist the overturning moment of the turbine thrust? Answer: More weight in the front – total weight was about 290 lbf.
4. How to manage variable winds? Answer: The turbine blades could change pitch if the wind speed changed.
5. How to make light but strong blades? Answer: The students molded the blades from composite materials.

P11.106 Analyze the wind-powered-vehicle of Fig. P11.105 with the following data: turbine diameter D = 6 ft, power coefficient (Fig. 11.32) = 0.3, vehicle C_DA = 4.5 ft^2, and turbine rotation 240 r/min. The vehicle moves directly into a head wind, W = 25 mi/h. The wind backward thrust on the turbine is approximately $T \approx C_T(\rho/2)V_{\text{rel}}^2 A_{\text{turbine}}$, where

V_{rel} is the air velocity relative to the turbine and $C_T \approx 0.7$. Eighty per cent of the turbine power is delivered by gears to the wheels, to propel the vehicle. Estimate the sea-level vehicle velocity V, in mi/h.

Solution: At sea level take $\rho = 0.00237$ slug/ft^3. Recall the definition of power coefficient from Eq. (11.46). The air velocity relative to the moving turbine is $(W+V)$. Hence

$$P_{turbine} = C_P \frac{\rho}{2} A_{turbine} V_{rel}^3 = C_P \frac{\rho}{2}(\frac{\pi}{4})D^2 (W+V)^3 \;, \quad C_P \approx 0.4 \;\; (Fig.11.32)$$

$$\text{Also,} \quad \eta P_{turbine} = P_{vehicle} = F_{vehicle} V \;, \quad \text{where} \;\; \eta \approx 0.8$$

The retarding force on the vehicle is the sum of drag on the vehicle and thrust on the turbine:

$$F_{vehicle} = F_{drag} + F_{thrust} = \frac{\rho}{2}(C_D A)(W+V)^2 + C_T \frac{\rho}{2}\frac{\pi}{4}D^2 (W+V)^2$$

where $C_T \approx 0.7$. Combine these two equations and we find that both $(W+V)^2$ and ρ cancel, and the turbine area can be divided out. The result is

$$\eta C_P (W+V) = V[\frac{C_D A}{(\pi/4)D^2} + C_T] \;, \; or: \; V = \frac{W}{(\frac{1}{\eta C_P})[\frac{C_D A}{(\pi/4)D^2} + C_T] - 1} \quad \text{(analytic answer)}$$

$$\text{or:} \quad V = \frac{W}{[\frac{1}{0.8(0.4)}][\frac{4.5}{(\pi/4)(6)^2} + 0.7] - 1} = \frac{W}{2.68 - 1} = \frac{25\, mi/h}{1.68} = \mathbf{14.8\ mi/h} \quad \textit{Ans.}$$

This is a simplified analysis, independent of density and RPM, in reasonable agreement with the actual performance of the Stuttgart student design, which averaged 15 mi/h during the contest.

P11.107 Figure 11.32 showed the typical *power* performance of a wind turbine. The wind also causes a *thrust* force that must be resisted by the structure. The thrust coefficient C_T of a wind turbine may be defined as follows:

$$C_T = \frac{Thrust\ \ force}{(\rho/2)\,A\,V^2} = \frac{T}{(\rho/2)[(\pi/4)D^2]\,V^2}$$

Values of C_T for a typical horizontal-axis wind turbine are shown in Fig. P11.107. The abscissa is the same as in Fig. 11.32. Consider the turbine of Prob. P11.103. If the wind is 35 mi/h and the rotation rate 24 r/min, estimate the bending moment about the tower base.

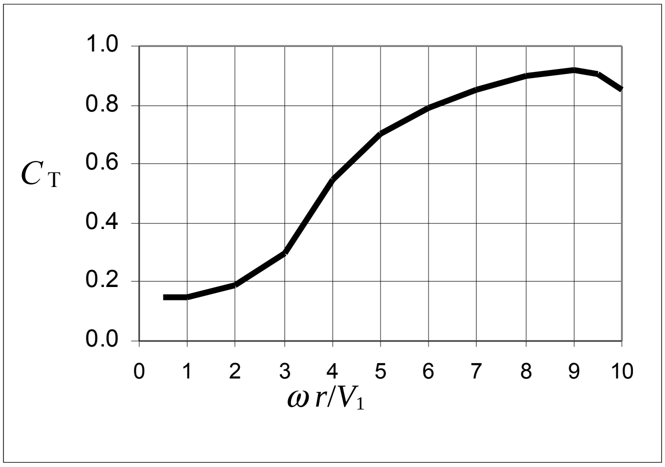


Fig. P11.107

Solution: Assume sea-level density of 0.00237 slug/ft^3 – all of Rhode Island is pretty much sea-level. Convert 24 r/min x 2π/60 = ω = 2.51 rad/s. Convert wind speed = 35 mi/h = 51.3 ft/s. Now find where we are on the graph:

$$\frac{\omega r}{V} = \frac{(2.51/s)(126ft)}{51.3\ ft/s} = 6.2$$

From Fig. P11.107, read $C_T \approx 0.8$. Then the thrust force is, approximately,

$$Thrust = C\,\frac{\rho}{2}A\,V^2 = (0.8)(\frac{0.00237}{2})[\pi(126ft)^2](51.3)^2 \approx 126,000\ lbf$$

$$Moment\ about\ the\ base = T\,h = (126,000lbf)(213ft) = \mathbf{2.65E7}\ \ ft-lbf \qquad Ans.$$

COMPREHENSIVE PROBLEMS

C11.1 The net head of a little aquarium pump is given by the manufacturer as a function of volume flow rate as listed:

Q, m^3/s:	0	1E–6	2E–6	3E–6	4E–6	5E–6
H, mmH$_2$O:	1.10	1.00	0.80	0.60	0.35	0.0

What is the maximum achievable flow rate if you use this pump to pump water from the lower reservoir to the upper reservoir as shown in the figure?
NOTE: The tubing is smooth, with an inner diameter of 5 mm and a total length of 29.8 m. The water is at room temperature and pressure, and minor losses are neglected.

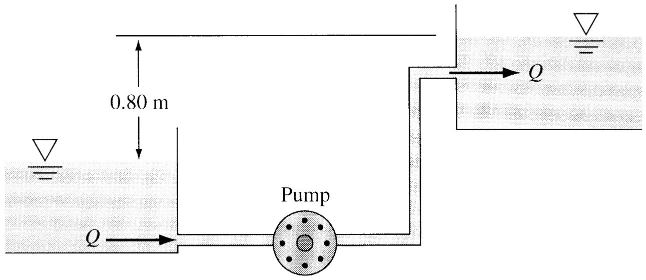


Fig. C11.1

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. NOTE: The tubing is so small that the flow is *laminar*, even at the highest pump flow rate:

$$Re_{\max} = \frac{4\rho Q_{\max}}{\pi\mu d} = \frac{4(998)(5E{-}6)}{\pi(0.001)(0.005)} = 1270 < 2000 \quad \therefore \ \textbf{Laminar}$$

The energy equation shows that the pump must fight both friction and elevation:

$$H_{\text{pump}} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = \Delta z + h_{f,\text{lam}} = \Delta z + \frac{128\mu LQ}{\pi\rho g d^4} = 0.8 + \frac{128(.001)(29.8)Q}{\pi(0.005)^4(998)(9.81)},$$

$$\text{or:} \quad H_p = 0.8 + 198400Q = H_p(Q) \quad \text{from the pump data above}$$

One can plot the two relations, as at right, or use EES with a look-up table to get the final result for flow rate and head:

$$H_{\text{p}} = 1.00\ \text{m}$$

$\boldsymbol{Q = 1.0E{-}6\ \text{m}^3/\text{s}}$ *Ans.*

The EES print-out gives the results Re$_d$ = 255, $H = 0.999$ m, $Q = 1.004E{-}6$ m^3/s.

Fig. C11.2

C11.2 Reconsider Prob. 6.62 as an exercise in pump selection. Select an impeller size and rotational speed from the Byron Jackson pump family of Prob. 11.28 which will deliver a flow rate of 3 ft^3/s to the system of Fig. P6.62 at minimum input power. Calculate the horsepower required.

Solution: For water take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron take $\varepsilon = 0.00085$ ft, or $\varepsilon/d = 0.00085/0.5 = 0.0017$. The energy equation, written between reservoirs, is the same as in Prob. 6.62:

$$H_p = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\ \text{ft} + f\frac{2000}{0.5}\frac{[Q/(\pi/4)/(0.5)^2]^2}{2(32.2)} = 120 + 1611fQ^2$$

$$\text{where}\quad f = f_{\text{Moody}} = fcn\left(Re_d, \frac{\varepsilon}{d}\right)\ \text{with}\ Re_d = \frac{4\rho Q}{\pi\mu d}$$

If, as given, Q = **3 ft^3/s**, then, from above, $f = 0.0227$ and H_p = **449.6 ft**.

Now we have to *optimize*: From Prob. 11.28, Q* = 6 ft^3/s, H* = 330 ft, and P* = 255 bhp when n = 2134 rpm (35.57 rps) and D = 14.62 inches (1.218 ft). Thus, at BEP:

$$C_Q^* = \frac{Q^*}{nD^3} = 0.0933;\quad C_H^* = \frac{gH^*}{n^2D^2} = 5.66;\quad C_P^* = \frac{P^*}{\rho n^3D^5} = 0.599$$

For the system above, (3.0)/[nD^3] = 0.0932, or $nD^3 = 32.15$, and (32.2)(449.6)/n^{2}D^2 = 5.66, or $n^2D^2 = 2558$. Solve simultaneously:

$$n = 63.4\ \text{rps} = 3800\ \frac{\text{rev}}{\text{min}};\quad D_p = 0.798\ \text{ft};\quad Power = 0.599\rho n^3 D_p^5 = \mathbf{174\ hp}\quad \textit{Ans.}$$

C11.3 Reconsider Prob. 6.77 as an exercise in turbine selection. Select an impeller size and rotational speed from the Francis turbine family of Fig. 11.21*d* which will deliver maximum power generated by the turbine. Calculate the water turbine power output and remark on the practicality of your design.

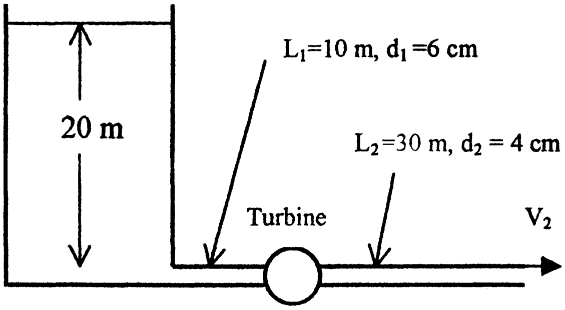


Fig. C11.3

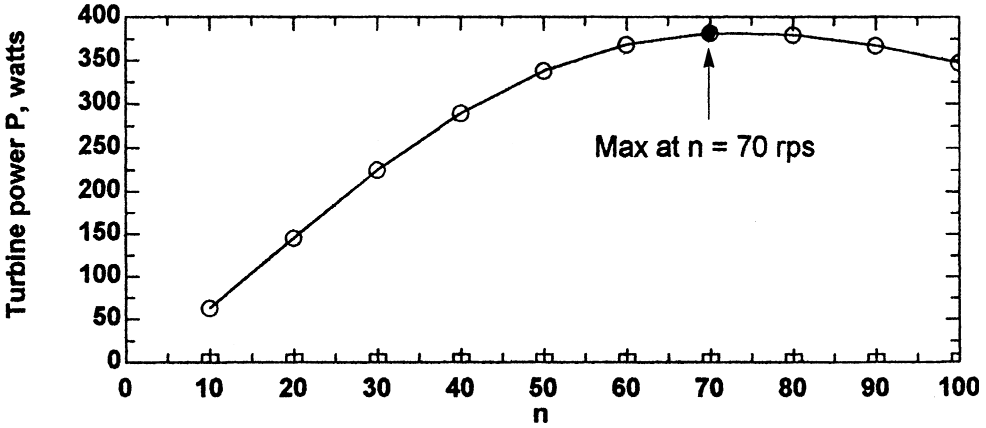
Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron take $\varepsilon = 0.046$ m, or $\varepsilon/d_1 = 0.046/60 = 0.000767$ and $\varepsilon/d_2 = 0.046/40 = 0.00115$. The energy and continuity equations yields

$$\Delta z = 20\text{ m} = \frac{V_2^2}{2g} + f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} + f_2\frac{L_2}{d_2}\frac{V_2^2}{2g} + h_{turbine};\quad V_2 = V_1\left(\frac{d_1}{d_2}\right)^2 = 2.25V_1$$

with the friction factors to be determined by the respective Reynolds numbers and roughness ratios. We use the BEP coefficients from Eq. (11.36) for this turbine:

$$C_Q^* = \frac{Q^*}{nD^3} = 0.34;\quad C_H^* = \frac{gH^*}{n^2D^2} = 9.03;\quad C_P^* = \frac{P^*}{\rho n^3D^5} = 2.70$$

We know from Prob. 6.76 that the friction factors are approximately 0.022. With only *one* energy equation (above) and *two* unknowns (n and D_t), we vary, say, n from 0 to 100 rev/s and find the resulting turbine diameter and power extracted. The power is shown in the graph below, with a maximum of **381 watts** at n = **70 rev/s**, with a resulting turbine diameter $\boldsymbol{D_t}$ **= 5.3 cm**. This is a fast, small turbine! *Ans.*



C11.4 The system of Fig. C11.4 is designed to deliver water at 20°C from a sea-level reservoir to another through new cast iron pipe of diameter 38 cm. Minor losses are $\Sigma K_1 = 0.5$ before the pump entrance and $\Sigma K_2 = 7.2$ after the pump exit. (a) Select a pump from either Figs. 11.7*a* or 11.7*b*, running at the given speeds, which can perform this task at maximum efficiency. Determine (b) the resulting flow rate; (c) the brake horsepower; and (d) whether the pump as presently situated is safe from cavitation.

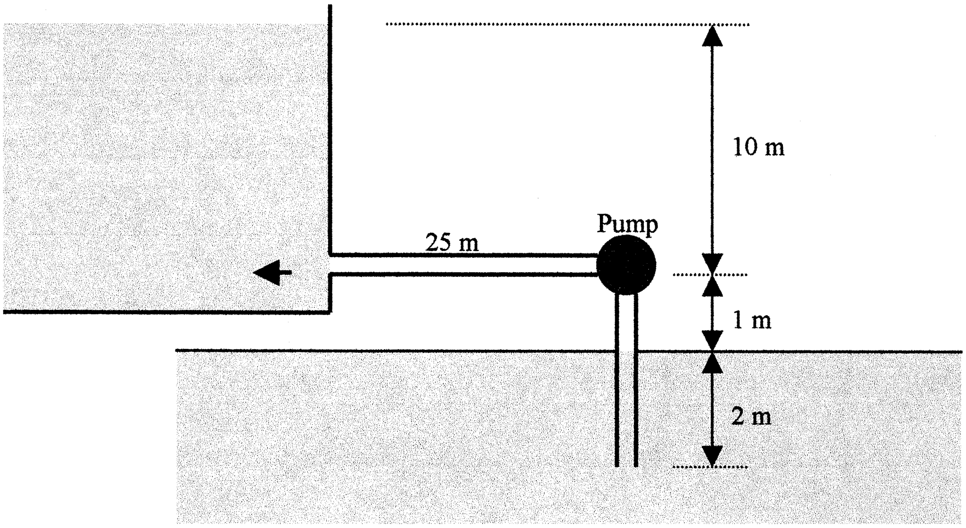


Fig. C11.4

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. First establish the *system curve* of head loss versus flow rate. For cast iron take ε = 0.26 mm, hence ε/d = 0.26/380 = 0.000684. The pumps of Figs. 11.7*a,b* deliver flows of 4000 to 28000 gal/min, no doubt turbulent flow. The energy equation, written from the lower free surface to the upper surface, gives

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=0+0+0=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f-H_{pump}=0+0+11\text{ m}+\frac{V_{pipe}^2}{2g}\left(f\frac{L}{d}+\sum K\right)$$

$$\text{Where}\quad f^{-1/2}=-2.0\log_{10}\left(\frac{\varepsilon/d}{3.7}+\frac{2.51}{\mathrm{Re}_d\sqrt{f}}\right),\quad \mathrm{Re}_d=\frac{\rho V_{pipe}d}{\mu}\quad\text{and}\quad V_{pipe}=\frac{Q}{(\pi/4)d^2}$$

Take, for example, Q = 22,000 gal/min = 1.39 m^3/s. Then V_{pipe} = 12.24 m/s, Re_d = 4.64E6, f = 0.0180, hence the head loss becomes

$$h_f=\frac{(12.24)^2}{2(9.81)}\left[(0.0180)\left(\frac{28\text{ m}}{0.38}\right)+0.5+7.2\right]=68.9\text{ m}=226\text{ ft},$$

$$\text{Thus a match requires}\quad H=h_f+z_2=68.9\text{ m}+11\text{ m}=79.9\text{ m}=262\text{ ft}$$

No pump in Fig. 11.7 exactly matches this, but the 28-inch pump in Fig. 11.7*a* and the 41.5-inch pump in Fig. 11.7*b* are pretty close, especially the latter. We can continue the calculations:

Q, gal/min:	4000	8000	12000	16000	**20000**	22000	24000	28000
h_f, ft:	44	66	103	156	**223**	262	305	402

(a) The best match seems to be **the 38-inch pump of Fig. 11.7*b* at a flow rate of 20,000 gal/min**, near maximum efficiency of 88%. *Ans*. (a)
(b, c) The appropriate flow rate is **20,000 gal/min**. *Ans*. (b)
The horsepower is **1250 bhp**. *Ans*. (c)
(d) Use Eq. (11.20) to check the NPSH for cavitation. For water at 20°C, $p_v = 2337$ Pa. The velocity in the pipe is $V = Q/A = 11.13$ m/s. The theoretical minimum net positive suction head is:

$$NPSH = \frac{p_a}{\gamma} - z_i - h_{fi} - \frac{p_v}{\gamma} = \frac{101350\text{ Pa}}{9790\text{ N/m}^3} - 1\text{ m} - \frac{(11.13\text{ m/s})^2}{2(9.81\text{ m/s}^2)}(1+0.5) - \frac{2337\text{ Pa}}{9790\text{ N/m}^3}$$

In Fig. 11.7b, at Q = 20,000 gal/min, read NPSH ≈ 16 m >> −0.36 m! So this pump, when placed in its present position, **will surely cavitate**! *Ans*. (d) A new pump placement is needed.

C11.5 In Prob. 11.23, estimate the efficiency of the pump in two ways: (a) read it directly from Fig. 11.7*b* (for the dynamically similar pump); (b) calculate it from Eq. (11.5) for the actual kerosene flow. Compare your results and discuss any discrepancies.

Solution: Problem 11.23 used the 38-inch-pump in Fig. 11.7b to deliver 22000 gal/min of kerosene at 850 rpm. (a) The problem showed that the *dynamically similar water* pump, at 710 rpm, had a flow rate of 18,400 gal/min.

(a) Figure 11.7*b*: Read $\boldsymbol{\eta} \approx \mathbf{88.5\%}$ *Ans.* (a)

(b) For kerosene, take $\rho = 804\text{ kg/m}^3 = 1.56\text{ slug/ft}^3$. The solution to Prob. 11.23 gave a kerosene head of 340 ft and a brake horsepower of 1600 hp. Together with the known flow rate, we can calculate the kerosene efficiency by definition:

$$\eta_{kerosene} = \frac{\rho_{kerosene} gQH}{\text{Power}} = \frac{(1.56)(32.2)(22000/448.83)(340)}{(1600)(550)\text{ ft·lbf/s}} = 0.95 \quad \text{or} \quad \mathbf{95\%} \quad \textit{Ans.}\text{ (b)}$$

This is significantly different from 88.5% in part (a) above. The main reason (the author thinks) is the **difficulty of reading bhp from Fig. 11.7b**. The actual kerosene bhp for Prob. 11.23 is probably about *1700*, not 1600. *Ans.*

C11.6 An interesting turbomachine [58] is the *fluid coupling* of Fig. C11.6, which delivers fluid from a primary pump rotor into a secondary turbine on a separate shaft. Both rotors have radial blades. Couplings are common in all types of vehicle and machine transmissions and drives. The *slip* of the coupling is defined as the dimensionless difference between shaft rotation rates, $s = 1 - \omega_s/\omega_p$. For a given percentage of fluid filling, the torque T transmitted is a function of s, ρ, ω_p, and impeller diameter D. (a) Non-dimensionalize this function into two pi groups, with one pi proportional to T. Tests on a 1-ft-diameter coupling at 2500 r/min, filled with hydraulic fluid of density 56 lbm/ft^3, yield the following torque versus slip data:

Slip, *s*:	0%	5%	10%	15%	20%	25%
Torque *T*, ft·lbf:	0	90	275	440	580	680

(b) If this coupling is run at 3600 r/min, at what slip value will it transmit a torque of 900 ft·lbf? (c) What is the proper diameter for a geometrically similar coupling to run at 3000 r/min and 5% slip and transmit 600 ft·lbf of torque?

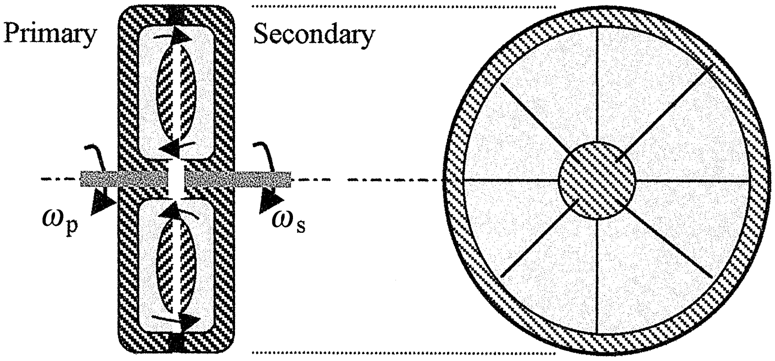


Fig. C11.6

Solution: (a) List the dimensions of the five variables, from Table 5.1:

Variable:	T	s	ρ	ω_p	D
Dimension:	{ML^2/T^2}	{1}	{M/L^3}	{1/T}	{L}

Since s is already dimensionless, the other four must form a single pi group. The result is:

$$\boldsymbol{\frac{T}{\rho\omega_p^2 D^5} = \mathrm{fcn}(s)} \quad \textit{Ans.}\ \text{(a)}$$

Now, to work parts (b) and (c), put the data above into this dimensionless form. Convert ρ_{oil} = 56 lbm/ft^3 = 1.74 slug/ft^3. Convert ω_p = 2500 r/min = 41.7 r/s. The results are:

Slip, *s*:	0%	5%	10%	15%	20%	25%
$T/\left(\rho\omega_p^2 D^5\right)$:	0	0.0298	0.0911	0.146	0.192	0.225

(b) With $D = 1$ ft and $\omega_p = 3500$ r/min $= 58.3$ r/s and $T = 900$ ft·lbf,

$$\frac{T}{\rho\omega^2 D^5} = \frac{900\ \text{ft}\cdot\text{lbf}}{(1.74\ \text{slug/ft}^3)(58.3\ \text{r/s})^2(1\ \text{ft})^5} = 0.152, \quad \text{Estimate}\ \ \mathbf{s \approx 15\%} \quad \textit{Ans.}\ \text{(b)}$$

(c) With D unknown and $s = 5\%$ and $\omega_p = 3000$ r/min $= 50$ r/s and $T = 600$ ft·lbf,

$$\frac{T}{\rho\omega^2 D^5} = \frac{600\ \text{ft}\cdot\text{lbf}}{(1.74\ \text{slug/ft}^3)(50\ \text{r/s})^2(D)^5} = 0.0298, \quad \text{Solve}\ \ \mathbf{D \approx 1.36\ ft} \quad \textit{Ans.}\ \text{(c)}$$

C11.7 Report to the class on the *Cordier method* [63] for optimal design of turbomachinery. The method is related to, and greatly expanded from, Prob. P11.46 and uses both software and charts to develop an efficient design for any given pump or compressor application.

Solution: The Cordier method, developed in the textbook G. T. Csanady, *Theory of Turbomachines*, McGraw-Hill, New York, 1964, is an extensive correlation of the best features of various pumps (or turbines) to design an optimal machine for a given set of conditions. Its modern use, with software, is discussed in Ref. 63.

C11.8 A *pump-turbine* is a reversible device that uses a reservoir to generate power in the daytime and then pumps water back up to the reservoir at night. Let us reset Prob. P6.62 as a pump-turbine. Recall that $\Delta z = 120$ ft, and the water flows through 2000 ft of 6-in-diameter cast iron pipe. For simplicity, assume that the pump operates at BEP (92%) with $H^*_p = 200$ ft and the turbine operates at BEP (89%) with $H^*_t = 100$ ft. Neglect minor losses. Estimate (*a*) the input power, in watts, required by the pump; and (*b*) the power, in watts, generated by the turbine. For further technical reading, consult the URL www.usbr.gov/pmts/hydraulics_lab/pubs/EM/EM39.pdf.

Solution: For water, take $\rho = 1.94$ slug/ft³ and $\mu = 2.09\text{E-}5$ slug/ft-s. (a) Write the steady flow energy equation for pump operation, neglecting minor losses:

$$h_p = 200\,ft = \Delta z + f\,\frac{L}{D}\frac{V^2}{2} = 120\,ft + f\left(\frac{2000\,ft}{0.5\,ft}\right)\left[\frac{V^2}{2(32.2)}\right],\ V = \frac{Q_p}{(\pi/4)(0.5\,ft)^2}$$

For cast iron, $\varepsilon = 0.00085$ ft, hence $\varepsilon/D = 0.00085\text{ft}/0.5\text{ft} = 0.0017$. Iterate to find the friction factor and complete the pump head equation. EES quickly finds the result:

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(1.94)V(0.5)}{2.09E-5} = 347,000;\ f = 0.0230; V = 7.48\frac{ft}{s}; Q = 5.87\frac{ft^3}{s}; H = 200\,ft$$

Hence $Power_{pump} = \frac{\gamma Q H}{\eta} = \frac{(62.4)(5.87)(200)}{0.92} = 80,000\frac{ft-lbf}{s} \approx \mathbf{108,000\ W}$ $\quad Ans.(a)$

(*b*) Write the steady flow energy equation for turbine operation, neglecting minor losses:

$$h_t = 100\,ft\ = \Delta z - f\frac{L}{D}\frac{V^2}{2} = 120\,ft - f\left(\frac{2000\,ft}{0.5\,ft}\right)\left[\frac{V^2}{2(32.2)}\right],\quad V = \frac{Q_p}{(\pi/4)(0.5\,ft)^2}$$

Iterate again and obtain a considerably lower flow rate:

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(1.94)V(0.5)}{2.09E-5} = 171000;\ f = 0.0236; V = 3.69\frac{ft}{s}; Q = 2.90\frac{ft^3}{s}; H = 100\,ft$$

Hence $Power_{pump} = \eta\gamma Q H = (0.88)(62.4)(5.87)(200) = 16,100\frac{ft-lbf}{s} \approx \mathbf{21,000\ W}$ $Ans.(b)$

The pumping power is much greater than the power generated by the turbine, but the pump’s required electricity is very cheap because usage is so small at night.
